Fuzzy Project Report

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1. **Inputs:**

The inputs to the fuzzy controller are the robot's starting position and target position in the form of x, y coordinates.

2. **Outputs:**

The fuzzy controller outputs the weights for the robot's distance, velocity, and angle membership functions as well as the new force, new angular velocity, current distance to target, angle offset, and current time every 100ms. The angle offset if the difference between the angle the robot is facing and the angle the target is at.

3. **Constraints:**

The controller is constrained by the fact that it can only control the robot's movement by changing its forward force and angular velocity every 100ms. The controller must also use a fuzzifier, fuzzy inference engine, and defuzzifier to determine the robot's forward force and angular velocity from its current distance from the target, velocity, and angle.

4. **What I defined:**

One element of this project that I defined was the way in which the robot moves. The robot moves straight with constant acceleration between each 100 ms interval and its angular velocity defines how fast its head spins. The robot's head spins continuously but the direction in which the robot moves is defined by the direction the robot's head is facing at the beginning of every 100ms interval.

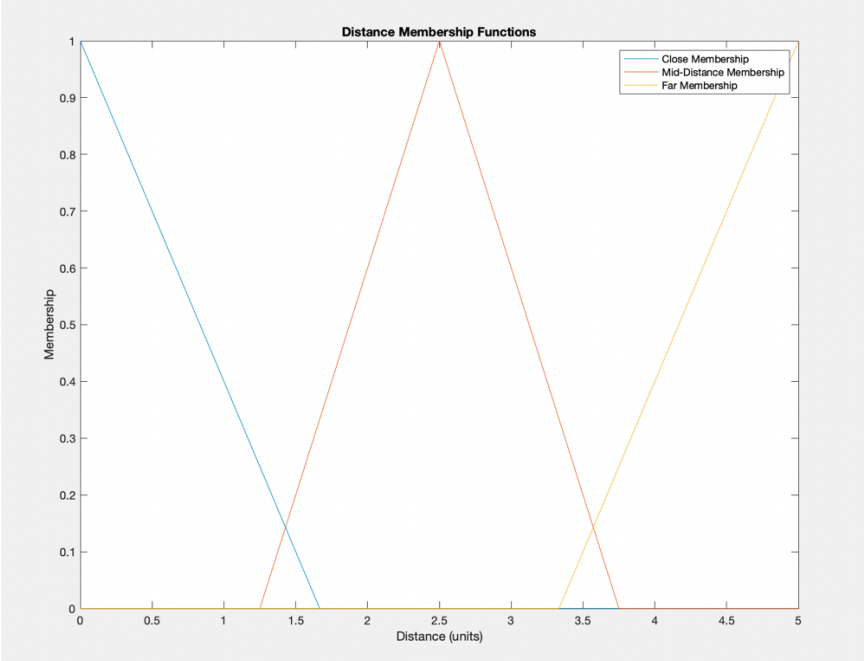
I had to additionally define the way that the fuzzifier, fuzzy inference engine, and defuzzifier, use the robot's current distance to the target, velocity, and angle to determine the force the push the robot with and the angular velocity to the spin the head in.

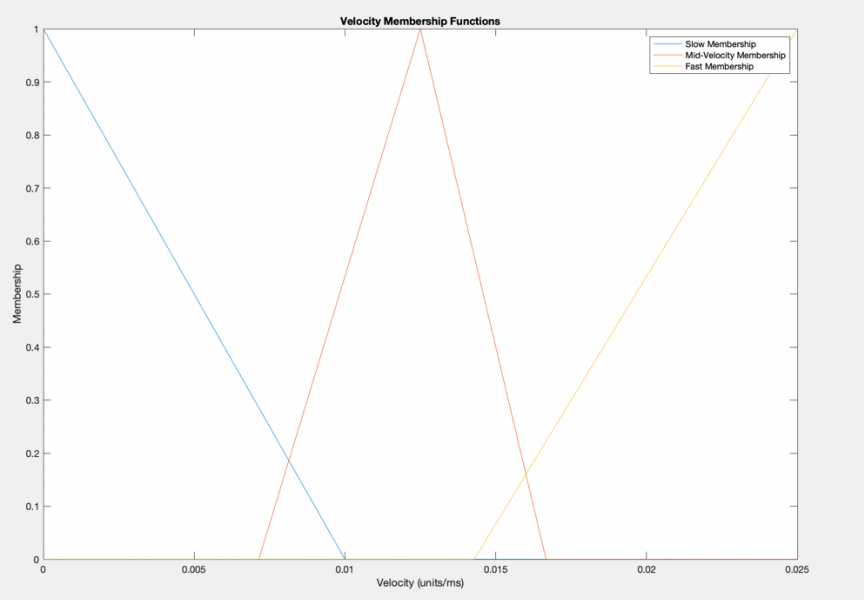
5. **Physics equations for the robot's movement:**

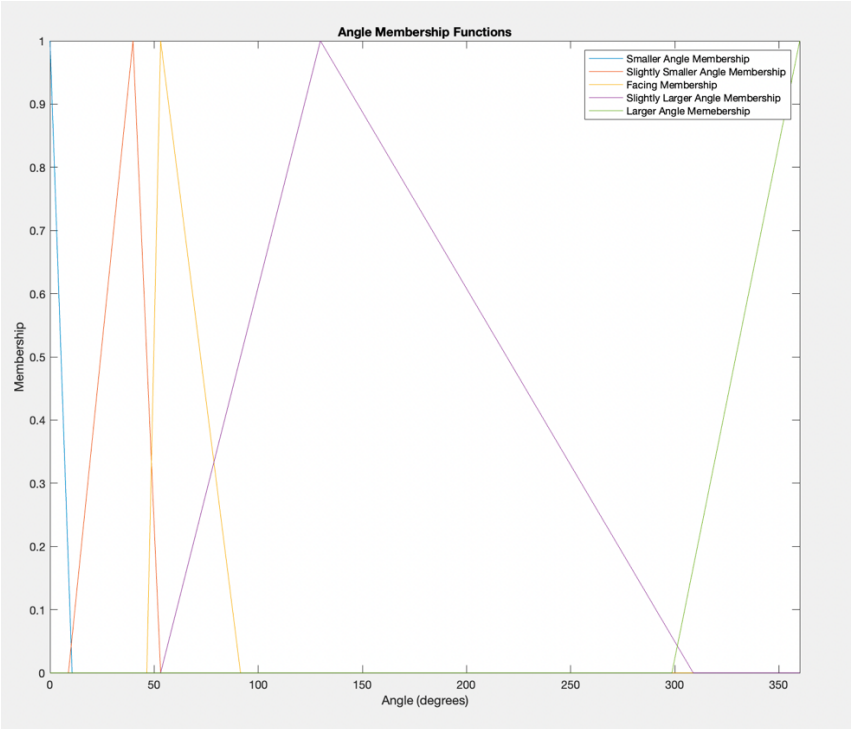
$$(1)\ F = ma$$
$$(2)\ d = v_0 t + \frac{1}{2} a t^2$$
$$(3)\ v = v_0 + at$$
$$(4)\ \theta = \theta_0 + \omega t$$
$$F = Force$$
$$a = acceleration$$
$$d = distance$$
$$t = time$$
$$v = velocity \quad v_0 = initial\ velocity$$
$$m = mass$$
$$\theta = angle\ facing \quad \theta_0 = initial\ angle\ facing$$
$$\omega = angular\ velocity$$

Equation 1 was used to find the acceleration of the robot, since I assumed that the robot weighs 1 kg, the force value was the same as the acceleration. Equation 2 was used to find the distance the robot traveled over a 100ms interval with the assumption that the robot travels straight and it's acceleration is constant. Equation 3 was used to find the speed of the robot after that interval. Finally, equation 4 was used to determine the angle the robot was facing at the end of every 100ms.

6. Fuzzifier Design







The plots above show the membership functions for the distance from the target, velocity and angle of the robot when the starting distance from the target was 5 units. The distance and velocity plot are scaled based on the robot's starting distance from the target. The max distance in the distance membership plot is the starting distance from the target and the mid-distance membership function reaches a value of 1 when the distance from the target is half the starting distance. The max velocity in the velocity membership plot has the value of (start distance)/200ms so the robot is considered to be moving fast when it can travel its starting distance in 200ms. The mid velocity membership function reaches 1 when the robot is traveling (start distance)/400ms.

The angle membership functions are scaled based on the angle between the robot's current position and the target. This angle is measured relative to the positive x-axis where the robot's current position can be considered the origin. The large angle membership function always reaches the value of 1 at 360 degrees. The facing membership function always reaches a 1 at the current angle between the robot and the target. This plot therefore changes after every 100ms when the robot's position is updated.

The fuzzifier works by outputting membership values for each piecewise function in the corresponding plot when given the robot's current distance to the target, velocity, and angle it is facing.

7. Fuzzy Inference Engine Design

Once the values from the fuzzifier are obtained the below matrix is used to determine how each membership affects the weight of corresponding force components.

	Far distance membership	Mid-distance membership	Close-distance membership
Fast velocity membership	Weight = wfastfar Force = noPush	Weight = wfastmid Force = negSoft	Weight = wfastclose Force = negHard
Mid-velocity membership	Weight = wmidfar Force = posSoft	Weight = wmidmid Force = noPush	Weight = wmidclose Force = negSoft
Slow velocity membership	Weight = wslowfar Force = posHard	Weight = wslowmid Force = posSoft	Weight = wslowclose Force = noPush

The weights in each square are found by taking the max of the corresponding distance and velocity membership functions, for example:

$$wfastfar = \max(\mu_{fast}, \mu_{far})$$

The max of the two weights is used to prevent the case that all weights are 0.

The fuzzy inference engine for the angle membership functions is structured such that the value of each membership function is paired with an angular velocity component a shown in the matrix below:

Weight = Smaller Angle membership	Weight = Slightly Smaller Angle membership	Weight = Facing Target membership	Weight = Slightly Larger Angle membership	Weight = Larger Angle membership
Angular Velocity = CounterclkFast	Angular Velocity = CounterclkSlow	Angular Velocity = noturn	Angular Velocity = clkSlow	Angular Velocity = clkFast

8. Defuzzifier

The defuzzifier outputs a force and an angular velocity based on the results of the fuzzy inference engine. It does this using the following equations:

$$outForce = \frac{\sum(weight * force)}{\sum weight}$$

$$outAngVelocity = \frac{\sum(weight * angularVelocity)}{\sum weight}$$

Where the weight-force or weight-angular velocity pairs are determined by the fuzzy inference engine matrices. The force weights are given below where the units for each weight are kg * (distance unit)/ms^2. Negative forces will cause the robot to accelerate in the opposite direction it is facing:

noPush = 0
posHard = startDistance/(80000)
posSoft = startDistance/(140000)
negHard = -startDistance/(100000)
negSoft = -startDistance/(170000)

The angular velocity weights are given below where the units for each weight are degrees/ms. Positive velocities cause a rotation in the counterclockwise direction and negative velocities cause a rotation in the clockwise direction:

CounterclkFast = 30/100;
CounterclkSlow = 20/100;
noTurn = 0;
clkFast = -30/100;
clkSlow = -20/100;

9. Controller Outputs

Below shows the output of three different inputs to the controller. A different starting and ending point in the form of x and y coordinates were input each time. Every 100ms, the controller outputs a matrix of distance membership values, a velocity membership matrix, an angular velocity membership matrix, the output force, output angular velocity, distance to target, time, and angle offset which is the difference between the target angle and the angle the robot is facing. The controller continues to update the position of the robot every 100ms until its distance to the target is less that 1/8 its starting distance. The values of the membership functions within each membership matrices take the following order.

distanceMemberships = [close, mid, far]
velocityMemberships =[slow, mid, fast]
angleWeights = [smallerAngle, slightlySmallerAngle, facing, slightlyLargerAngle, largerAngle]

```
Enter robot starting x position  0
Enter robot starting y position  0
Enter robot target x position   3
Enter robot target y position   4

startDistance = 5
targetAngle = 53.1301

time = 0
distanceMemberships =  0    0    1
velocityMemberships = 1    0    0
AngleWeights =    1    0    0    0    0
outForce = 2.6786e-05
outAngVelocity =    0.3000
DistanceToTarget = 5
angleOffset = 53.1301

time = 100
distanceMemberships =  0       0    0.9525
velocityMemberships = 0.7321       0       0
AngleWeights = 0    0.6086       0       0       0
outForce = 2.7696e-05
outAngVelocity = 0.2000
DistanceToTarget = 4.9208
angleOffset = 24.3777

time = 200
distanceMemberships =    0    0    1
velocityMemberships =   0.4552       0       0
AngleWeights =       0    0.9234       0       0       0
outForce =   2.9274e-05
outAngVelocity =   0.2000
DistanceToTarget =   5.2184
angleOffset =   7.5061

time = 300
distanceMemberships =    0    0    1
velocityMemberships =   0.1624    0.2301       0
AngleWeights =       0       0    0.9550       0    0.0225
outForce =  2.7480e-05
outAngVelocity =   -0.0046
DistanceToTarget =    5.0563
angleOffset =   -4.9940

time = 400
distanceMemberships =       0       0    0.4712
velocityMemberships =       0    0.7431       0
AngleWeights =       0    0.6340       0       0       0
outForce =  1.6259e-05
outAngVelocity =    0.2000
DistanceToTarget =    4.1186
angleOffset =   -1.1810

time = 500
distanceMemberships =       0    0.3082    0.0188
velocityMemberships =       0    0.9401       0
AngleWeights =       0       0    0.6676       0    0.1662
outForce =  8.3477e-06
outAngVelocity =   -0.0399
DistanceToTarget =   3.3647
angleOffset =  -6.9010

time = 600
distanceMemberships =       0    0.6535       0
velocityMemberships =       0    0.7398       0
AngleWeights =       0       0    0.4032       0    0.2984
outForce = 7.1586e-06
outAngVelocity = -0.0851
DistanceToTarget =  2.0668
angleOffset =  -7.7382
```

```
time = 700
distanceMemberships =       0    0.3614       0
velocityMemberships =       0    0.5680    0.0013
AngleWeights =       0       0    0.1434       0    0.4283
outForce =  7.4950e-06
outAngVelocity = -0.1498
DistanceToTarget =  3.2982
angleOffset =  -13.6187

time = 800
distanceMemberships =       0    0.7401       0
velocityMemberships =       0    0.3881    0.0713
AngleWeights =       0    0.8586       0       0       0
outForce = 3.7220e-06
outAngVelocity =  0.2000
DistanceToTarget =  2.1751
angleOffset = -18.9472

time = 900
distanceMemberships =       0    0.9243       0
velocityMemberships =       0    0.2988    0.1060
AngleWeights =       0    0.9303       0       0       0
outForce =   2.1389e-06
outAngVelocity =    0.2000
DistanceToTarget =   2.5946
angleOffset =   -3.0373

time = 10000
distanceMemberships =       0    0.7493       0
velocityMemberships =       0    0.2474    0.1260
AngleWeights =       0    0.8197       0       0       0
outForce =   1.1402e-06
outAngVelocity =   0.2000
DistanceToTarget =    2.1866
angleOffset =   13.6264

time = 1100
distanceMemberships =   0.3475       0       0
velocityMemberships =       0    0.2201    0.1366
AngleWeights =       0    0.3642       0       0       0
outForce =  -1.2914e-05
outAngVelocity =    0.2000
DistanceToTarget =    1.0876
angleOffset =  36.2195

time = 1200
distanceMemberships =    0.7387       0       0
velocityMemberships =       0    0.5300    0.0161
AngleWeights =       0       0       0       0    0.8248
outForce =  -1.0520e-05
outAngVelocity =  -0.2000
DistanceToTarget =   0.4354
angleOffset = 179.8890
```

```
Enter robot starting x position -1
Enter robot starting y position 2
Enter robot target x position 0
Enter robot target y position -1
startDistance = 3.1623
targetAngle = 288.4349

time =  0
distanceMemberships =    0    0    1
velocityMemberships =    1    0    0
AngleWeights =    1    0    0    0    0
outForce =  1.6941e-05
outAngVelocity =   0.3000
DistanceToTarget =   3.1623
angleOffset = 288.4349

time =  100
distanceMemberships =       0       0   0.9756
velocityMemberships =   0.7321       0       0
AngleWeights =   0.4786       0       0       0       0
outForce =  1.7567e-05
outAngVelocity =   0.3000
DistanceToTarget =   3.1365
angleOffset = 256.9669

time =  200
distanceMemberships =      0       0   0.7342
velocityMemberships =  0.4544       0       0
AngleWeights =       0   0.0615       0       0       0
outForce = 1.7956e-05
outAngVelocity =    0.2000
DistanceToTarget =   2.8821
angleOffset =  227.6869

time =  300
distanceMemberships =       0       0   0.7647
velocityMemberships =   0.1705   0.2151       0
AngleWeights =       0   0.1654       0       0       0
outForce =  1.6752e-05
outAngVelocity =   0.2000
DistanceToTarget =   2.9142
angleOffset = 216.2685

time =  400
distanceMemberships =       0       0   0.2996
velocityMemberships =       0   0.7096       0
AngleWeights =       0   0.2978       0       0       0
outForce =  8.8820e-06
outAngVelocity =    0.2000
DistanceToTarget =    2.4240
angleOffset =  204.0416

time =  500
distanceMemberships =       0   0.7386       0
velocityMemberships =       0   0.9717       0
AngleWeights =       0   0.4711       0       0       0
outForce =   4.6407e-06
outAngVelocity =    0.2000
DistanceToTarget =    1.7878
angleOffset = 173.8112

time =  600
distanceMemberships =       0   0.3391       0
velocityMemberships =       0   0.8603       0
AngleWeights =       0   0.5883       0       0       0
outForce =   5.0453e-06
outAngVelocity =    0.2000
DistanceToTarget =    2.1036
angleOffset = 131.8259
```

```
Time =  700
distanceMemberships =       0       0   0.7965
velocityMemberships =       0   0.6688       0
AngleWeights =       0   0.6595       0       0       0
outForce =  1.3274e-05
outAngVelocity =    0.2000
DistanceToTarget =    2.9478
angleOffset =  114.5859

time =  800
distanceMemberships =    0    0    1
velocityMemberships =       0   0.1651   0.1580
AngleWeights =       0   0.7150       0       0       0
outForce =   1.4053e-05
outAngVelocity =   0.2000
DistanceToTarget =   3.3515
angleOffset =  110.1931

time =  900
distanceMemberships =       0       0   0.7577
velocityMemberships =       0       0   0.3654
AngleWeights =       0   0.8041       0       0       0
outForce =  7.2959e-06
outAngVelocity =   0.2000
DistanceToTarget =   2.9069
angleOffset = 108.3532

time =       1000
distanceMemberships =       0   0.7929       0
velocityMemberships =       0       0   0.4731
AngleWeights =       0   0.7621       0       0       0
outForce =  -7.2237e-06
outAngVelocity =   0.2000
DistanceToTarget =   1.7449
angleOffset =  94.5984

time =       1100
distanceMemberships =       0   0.7052       0
velocityMemberships =       0       0   0.3665
AngleWeights =       0   0.3408   0.3184       0       0
outForce = -6.2826e-06
outAngVelocity =    0.1034
DistanceToTarget =    1.3481
angleOffset =  31.7788

time =       1200
distanceMemberships =       0       0   0.3037
velocityMemberships =       0       0   0.2738
AngleWeights =       0       0   0.2120       0   0.3940
outForce =  1.7949e-08
outAngVelocity =  -0.1300
DistanceToTarget =   2.4284
angleOffset =   12.0126

time =       1300
distanceMemberships =       0   0.7748       0
velocityMemberships =       0       0   0.2740
AngleWeights =       0       0       0       0   0.8902
outForce =  -4.1080e-06
outAngVelocity =   -0.2000
DistanceToTarget =    1.7592
angleOffset =    1.0218

time =       1400
distanceMemberships =       0   0.4727       0
velocityMemberships =       0   0.0227   0.2134
AngleWeights =       0       0       0       0   0.8963
outForce = -4.8955e-06
outAngVelocity =  -0.2000
DistanceToTarget =    1.1643
angleOffset = -15.1157

time =       1500
distanceMemberships =   0.7726       0       0
velocityMemberships =       0   0.2085   0.1412
AngleWeights =       0   0.7575       0       0       0
outForce = -1.1905e-05
outAngVelocity =   0.2000
DistanceToTarget = 0.2397
angleOffset =  -44.2496
```

```
Enter robot starting x position0
Enter robot starting y position2
Enter robot target x position-1
Enter robot target y position1

startDistance =  1.4142
targetAngle =  225

time =  0
distanceMemberships =  0  0  1
velocityMemberships =  1  0  0
AngleWeights =  1  0  0  0  0
outForce =  7.5761e-06
outAngVelocity =  0.3000
DistanceToTarget =  1.4142
angleOffset =  225

time =  100
distanceMemberships =  0  0  1
velocityMemberships =  0.7321  0  0
AngleWeights =  0.3182  0  0  0  0
outForce =  7.8792e-06
outAngVelocity =  0.3000
DistanceToTarget =  1.4412
angleOffset = 193.9351

time =  200
distanceMemberships =  0  0  0.9239
velocityMemberships =  0.4536  0  0
AngleWeights =  0  0.1744  0  0  0
outForce =  8.2218e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.3783
angleOffset =  160.0137

time =  300
distanceMemberships =  0  0  0.5446
velocityMemberships =  0.1629  0.2293  0
AngleWeights =  0  0.3565  0  0  0
outForce =  6.9368e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.1995
angleOffset =  143.5573

time =  400
distanceMemberships =  0  0.1512  0.1366
velocityMemberships =  0  0.6871  0
AngleWeights =  0  0.5738  0  0  0
outForce =  3.0199e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.0072
angleOffset = 113.5591

time =  500
distanceMemberships =  0  0  0.5114
velocityMemberships =  0  0.8864  0
AngleWeights =  0  0.7269  0  0  0
outForce =  4.4353e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.1839
angleOffset =  79.4454

time =  600
distanceMemberships =  0  0  1
velocityMemberships =  0  0.7697  0
AngleWeights =  0  0.8195  0  0  0
outForce =  6.1195e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.5316
angleOffset =  63.1556

time =  700
distanceMemberships =  0  0  1
velocityMemberships =  0  0.2505  0.1248
AngleWeights =  0  0.8962  0  0  0
outForce =  6.3823e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.6648
angleOffset =  57.1530

time =  800
distanceMemberships =  0  0  0.9837
velocityMemberships =  0  0  0.3354
AngleWeights =  0  0.8880  0  0  0
outForce =  4.6943e-06
outAngVelocity =  0.2000
DistanceToTarget =  1.4065
angleOffset =  52.0686

time =  900
distanceMemberships =  0  0.5180  0
velocityMemberships =  0  0  0.4903
AngleWeights =  0  0  0.7845  0  0.1077
outForce =  -4.9359e-06
outAngVelocity =  -0.0241
DistanceToTarget =  0.8775
angleOffset =  31.3620

time =  1000
distanceMemberships =  0  0.6359  0
velocityMemberships =  0  0  0.3274
AngleWeights =  0  0.5385  0  0  0
outForce =  -2.7833e-06
outAngVelocity =  0.2000
DistanceToTarget =  0.8358
angleOffset =  -2.0137

time =  1100
distanceMemberships =  0  0.4591  0
velocityMemberships =  0  0  0.2356
AngleWeights =  0  0.3994  0.2013  0  0
outForce =  -2.7738e-06
outAngVelocity =  0.1330
DistanceToTarget =  0.5159
angleOffset =  10.7399

time =  1200
distanceMemberships =  0.8813  0  0
velocityMemberships =  0  0.2009  0.1441
AngleWeights =  0  0  0  0  0.5420
outForce =  -5.5926e-06
outAngVelocity =  -0.2000
DistanceToTarget =  0.0560
angleOffset =  10.8354
```

10. Ethical Discussion

The ethical concern that was addressed in this project was whether the robot would be a danger to humans while navigating to its target. This concern was addressed through how the velocity membership functions were defined. The robot is considered to be going fast at and beyond (startDistance)/200ms. At (startDistance)/200ms the robot would need two 100ms intervals to travel the entire starting distance. The robot will therefore attempt to slow down by applying the max force backwards at a conservative speed that likely would not be a danger to humans. In addition, the project assumes that the robot only weighs 1kg which is relatively light.

In future versions of this system this concern could be addressed further by designing the robot to be padded which would allow for a softer impact if it were to run into a human.

GitHub Project Page: https://github.com/Jared-01/fuzzyProject.git