

A set is finite if

it is either empty or  
has  $n$  elements for  
some  $n$  in  $\mathbb{N}$

A set is infinite

if it is not finite

A set  $S$  has  $n$  elements  
if

there exists from the  
set  $N_n := \{1, \dots, n\}$  onto  
 $S$

$$A \setminus (B \text{ inter } C)$$

$$= (A \setminus B) \cup (A \setminus C)$$



$$A \setminus (B \cup C)$$

$$= (A \setminus B) \text{ inter } (A \setminus C)$$

# Cantors theorem

If  $A$  is any set , then  
there is no surj. of  $A$   
onto the set  $P(A)$  of all  
subsets of  $A$ .

# Countable

A set  $S$  is said to be countable if it either finite or denumerable.

# Denumerable

A set  $S$  is said to be  
denumerable (or  
countable) if  $\exists$  a  
bijection of  $\mathbb{N}$  onto  $S$ .



If  $A_m$  is a countable  
set for each  $m$  in  $\mathbb{N}$   
then...

the union of  $A :=$   
 $\bigcup_{m=1}^{\infty} A_m$   
is countable.

injection and how to  
prove

whenever  $x_1 \neq x_2$ ,  $f(x_1) \neq f(x_2)$

show for all  $x_1$  and  $x_2$  in  $A$ ,  
if  $f(x_1) = f(x_2)$  then  $x_1 = x_2$

# Principle of Mathematical Induction

Let  $S$  be a subset of  $N$  that possesses the 2 properties

- 1) the number 1 belongs to  $S$
- 2) For every  $k$  in  $N$ , if  $k$  belongs to  $S$ , then  $k+1$  belongs to  $S$

Then  $S = N$

# Principle of Strong Induction

Let  $S$  be a subset of  $N$  :

1) 1 belongs to  $S$

2) For every  $k$  in  $N$ , if  $\{1, \dots, k\}$   
belong to  $S$  then  $k+1$  belongs to  
 $S$

Then  $S=N$



Suppose that  $S$  and  $T$   
are sets and that  $T \subset S$

- 1) If  $S$  is a countable set,  
then  $T$  is a countable set
- 2) If  $T$  is an uncountable  
set, then  $S$  is an  
uncountable set

surjection

$$R(f) = B$$

show that each point  
in  $B$  maps to at least  
one point in  $A$

The following are  
equivalent

- 1)  $S$  is a countable set
- 2)  $E$  a surj. of  $N$  onto  $S$
- 3)  $E$  an inj. of  $S$  onto  $N$

The set  $N \dots$

of natural numbers is  
an infinite set.



The set  $\mathbb{N} \times \mathbb{N} \dots$

is denumerable.

The set  $Q$  of all  
rational numbers...

is denumerable.

# Triangle Inequality

If  $a, b$  is in  $R$ , then

$$\text{abs}(a+b) \leq \text{abs}(a) + \text{abs}(b)$$

# Uncountable

A set  $S$  is said to be  
uncountable if it is not  
countable.



# Uniqueness Theorem

If  $S$  is a finite set, then  
the number of  
elements in  $S$  is a  
unique number in  $\mathbb{N}$ .

# Well Ordering property of $\mathbb{N}$

Every non-empty  
subset of  $\mathbb{N}$  has a least  
element