I. Model

The locomotive crankshaft apparatus demonstrates kinetic relationships for linked mechanisms. Using data obtained from sensors of the rotational motion, a model for the translation motion of the collar can be created and compared with experimental data.

Point A rotates about the center of the disk, Point O. Point A is connected to the collar with a straight rod of length l at an angle β with respect to the collar. Point A is a fixed point on the disk, as Point A rotates around the center of the disk, Point B travels up or down the collar dependent on the position of Point A relative to Point O. A figure of the locomotive crank is shown below.

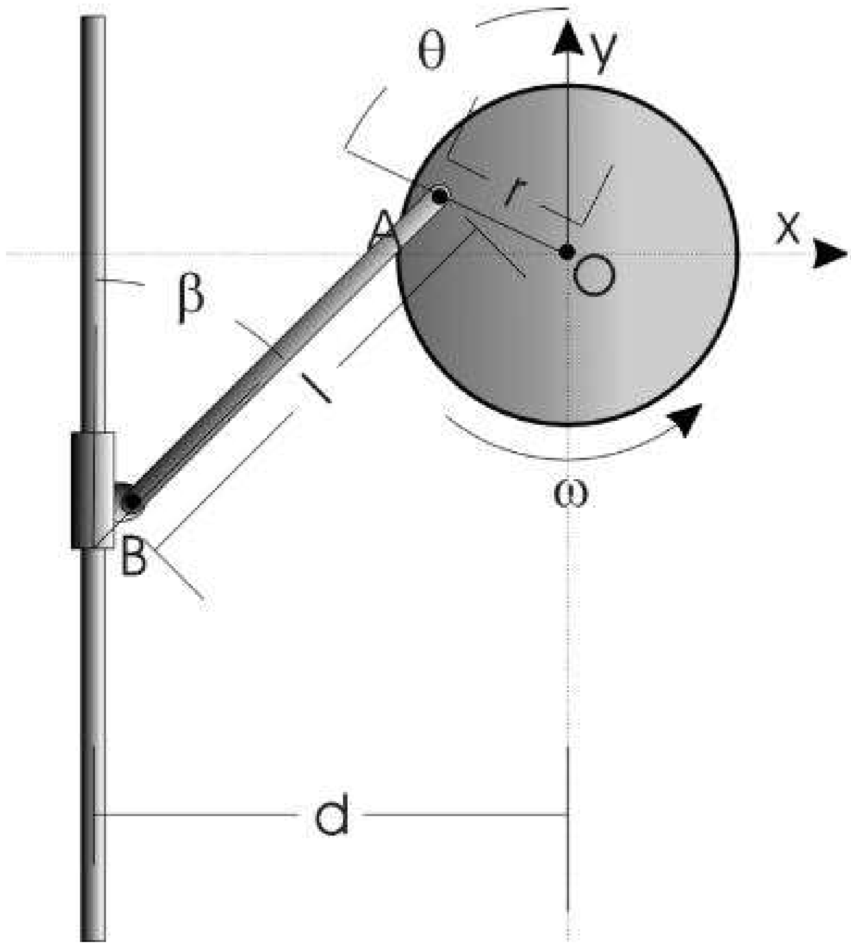


Fig. 1 Locomotive Crank Diagram

The first step to model the velocity of the collar is to derive an expression for the angle β in terms of θ. Although θ changes as Point A rotates, we assume that it stays constant. Using sine, the length of the bar, and the horizontal length of the bar, β can be found using trigonometric rules for sine

$$sin(\beta) = \frac{d - x}{l} \tag{1}$$

The factor $d - x$ represents the horizontal length component of the bar whereas l represents the length of the bar. The constant d is the distance from the collar to Point O and the value x is a placeholder value to be solved for in terms of θ.

The value x represents the horizontal distance from Point O to Point A. This distance can also be found using trigonometric rules for sine

$$sin(\theta) = \frac{x}{r} \tag{2}$$

Solving for x

$$x = rsin(\theta) \tag{3}$$

Plugging this equation into equation (1) results in

$$sin(\beta) = \frac{d - rsin(\theta)}{l} \tag{4}$$

Solving for β

$$\beta = sin^{-1}(\frac{d - rsin(\theta)}{l}) \tag{5}$$

The angle β has been derived and can be found for any θ value since d, r, and l are constants.

The last step is to derive an expression for the velocity vector of the collar in terms of β, θ, and ω. The equation for relative velocities can be used since there is only one frame of reference, the equation for the relative velocity of a moving reference frame need not be used. The first problem arises with the point of origin, however, Point O can be assigned this role as it does not change. The equation for relative velocity is as follows

$$v_A = v_B + \omega \times r_{A/B} \tag{6}$$

The factor v_A can be found using the rotational velocity rule

$$v_A = r\omega_A \tag{7}$$

Next, $r_{a/B}$ is the distance of Point A with respect to Point B in vector format; this is found using trigonometric rules

$$r_{A/B} = lsin(\beta)\hat{\imath} + lcos(\beta)\hat{\jmath} \tag{8}$$

The last sub-step is to get r in vector terms using the geometric constants and trigonometric rules

$$r = -rsin(\theta)\hat{\imath} + rcos(\theta)\hat{\jmath} \tag{9}$$

Plugging equations 8 and 7 back into equation 6 and solving for v_B results in

$$v_b = \omega_A \times (-rsin(\theta)\hat{\imath} + rcos(\theta)\hat{\jmath}) - \omega \times (lsin(\beta)\hat{\imath} + lcos(\beta)\hat{\jmath}) \tag{10}$$

Solving the cross-products results in

$$v_B = (\omega lcos(\beta) - \omega_A rcos(\theta))\hat{\imath} - (\omega lsin(\beta) + \omega_A rsin(\theta))\hat{\jmath} \tag{11}$$

Because the collar can only move up or down, the velocity in the $\hat{\imath}$ must equate to zero. Thus, equating the $\hat{\imath}$ to zero and solving for ω, the following equation results

$$\omega = \frac{\omega_A rcos(\theta)}{lcos(\beta)} \tag{12}$$

With equations 5, 11, and 12, the velocity for the collar can be found using the geometric constants.

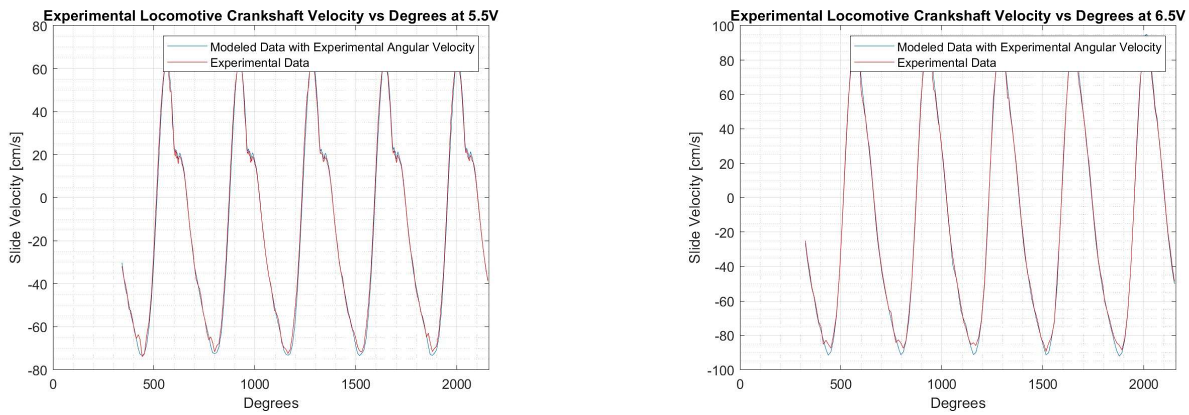
These three equations are put into MATLab where r, d, θ, l, and ω are inputs for the function. This code allows us to input these values and receive a velocity for the collar. We cab check if this function works by inputting reasonable values. We checked if the velocity was reasonable by making sure it was not very high, over 100 m/s, or too low, 0.005 mm/s.

II. Procedure

The experiment had already been completed for us so all the data was retrieved from Canvas. We also watched the videos posted to ensure we understood the concept of the lab. We used the dimensions and data of each voltage posted on Canvas for our function in MATLab. The specific variables used for the dimensions were r, d, and l. The variables used from the data were time, velocity, and degrees.

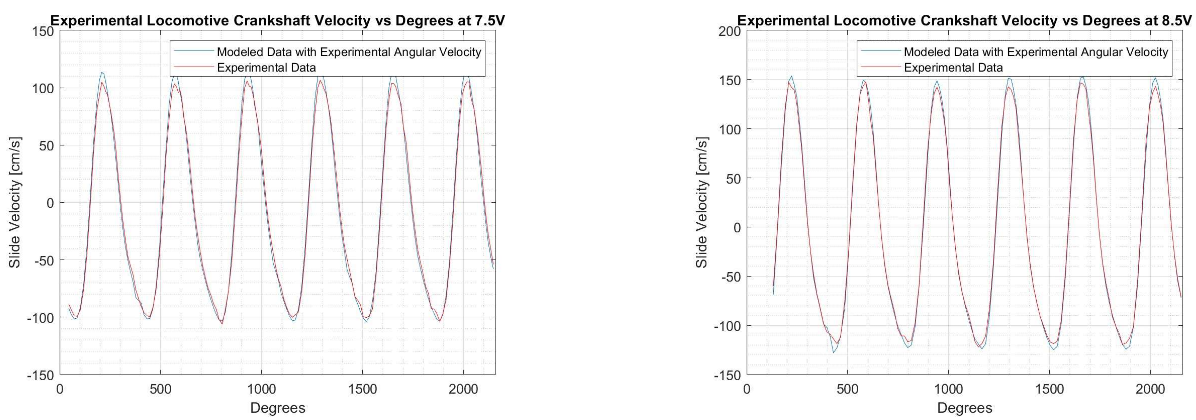
III. Results & Analysis

In order to compare experimental to observed collar velocities for each experiment, the data for angular position and angular velocity from the experiments was ran through our $lcsdata.m$ MATLAB function. This function takes a data file from the locomotive setup, converts to necessary units, and adjusts the angular position to start at 0^o and go up to 2160^o. It then outputs the experimental angular positions, angular velocities, collar velocities, and time. The experimental values for angular position and angular velocity are then ran through our $lcsmodel.m$ MATLAB function, which outputs expected collar velocities.



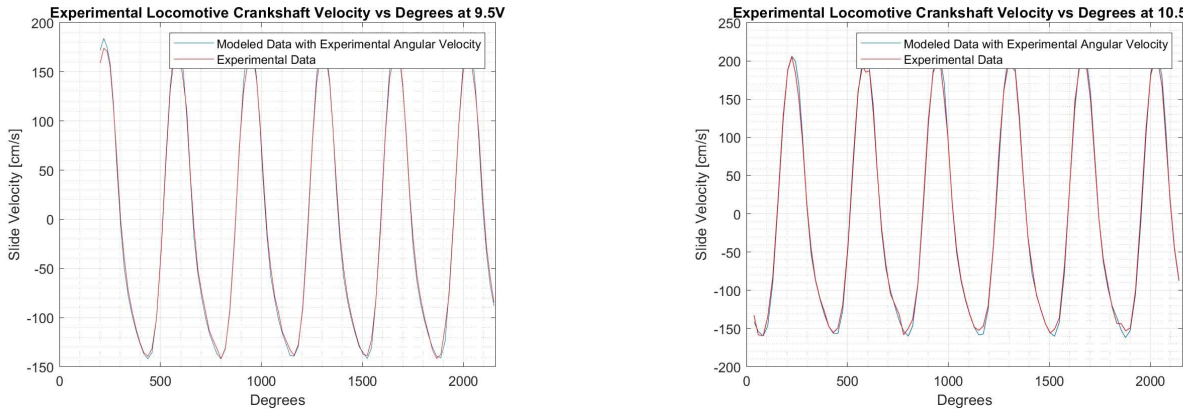
(a) $5.5V$ **(b)** $6.5V$

Fig. 2 5.5V and 6.5V Modeled and Experimental Data



(a) $7.5V$ **(b)** $8.5V$

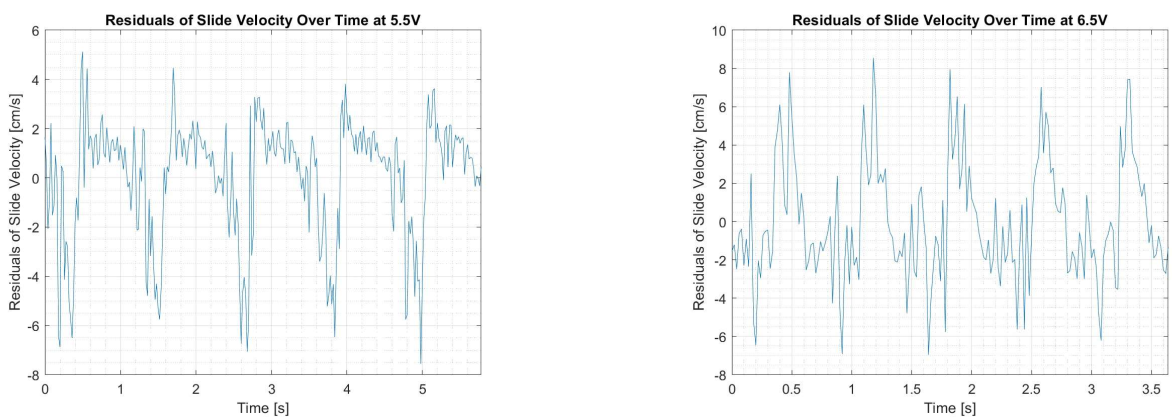
Fig. 3 7.5V and 8.5V Modeled and Experimental Data



(a) 9.5V **(b)** 10.5V

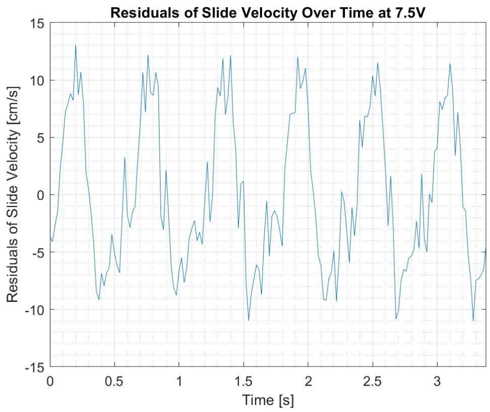
Fig. 4 9.5V and 10.5V Modeled and Experimental Data

Then, the experimental collared velocities are plotted against the modeled ones. As seen in Figs. 2-4, the data matches almost perfectly, with the exception of at the peaks and troughs. Figs. 5-7 then show the residuals between the experimental and modeled collar velocities. To do this, the modeled values were subtracted from experimental values.

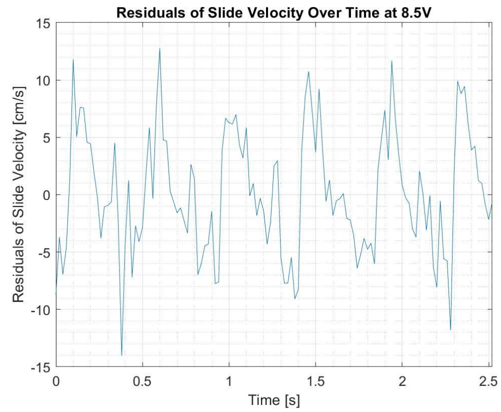


(a) 5.5V **(b)** 6.5V

Fig. 5 5.5V and 6.5V Residuals

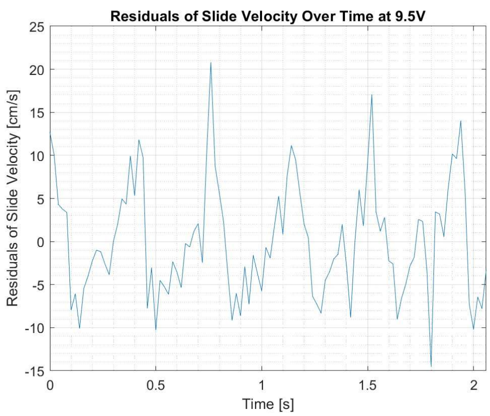


(a) 7.5V

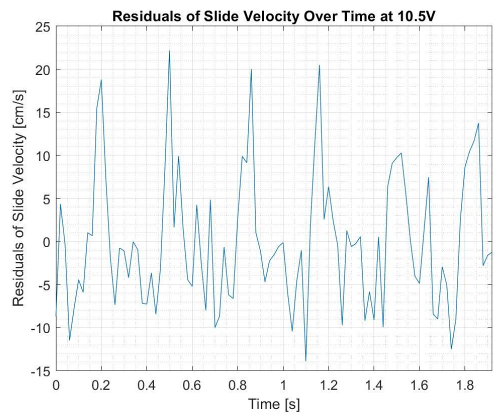


(b) 8.5V

Fig. 6 7.5V and 8.5V Residuals



(a) 9.5V



(b) 10.5V

Fig. 7 9.5V and 10.5V Residuals

The mean and standard deviations of the residuals were then taken to show the error in the experimental and modeled results. At first, the mean and standard deviation of the residuals contained outliers. The outliers were subsequently removed to give show a more accurate accounting of error. The difference in the data can be seen in the tables below.

Experiment	Mean	Std. Deviation
5.5V	0.00	2.4
6.5V	-0.04	3.1
7.5V	0.05	6.5
8.5V	0.12	5.3
9.5V	0.03	6.6
10.5V	-0.02	7.7

(a) Mean and Std. Deviation With Outliers

Experiment	Mean	Std. Deviation
5.5V	0.69	1.6
6.5V	-0.17	2.9
7.5V	0.05	6.5
8.5V	0.12	5.3
9.5V	-0.17	6.3
10.5V	-0.25	7.4

(b) Mean and Std. Deviation Without Outliers

The mean and standard deviations of each experiment show that generally, at lower voltages, the model is more accurate. Overall, however, the experimental data is very close to the modeled data. The most likely source of error is the fact that the locomotive setup vibrates, is loud, and has friction at the collar, so energy is being lost into the surroundings. The model does not account for this, so it is expected for the modeled results to be slightly higher than the experimental results, especially at higher collar velocities, where more energy is being lost into the surroundings. Another possible source of error is the uncertainty in the measurements of the actual locomotive setup. The dimensions were measured with a ruler, so there is a $\pm$ 0.05 cm error associated with the measurements of r, d, and l.

IV. Conclusion

V. Member Contributions

VI. Appendix A