

10 pt max:

Name:

Solutions

Lab Station:

8 / 10 AM

2 pts.

1. A sinusoidal signal with an amplitude of  $V_{in} = 10$  V is input to a circuit, and gets attenuation of -20 dB. What is the amplitude of the resulting output signal  $V_{out}$ ?

$$-20 \text{ dB} = 20 * \log\left(\frac{V_{out}}{V_{in}}\right) \Rightarrow -1 = \log\left(\frac{V_{out}}{V_{in}}\right)$$

$$\Rightarrow V_{out} = 10^{-1} \cdot V_{in} = 10^{-1} * 10 = 1 \text{ V}$$

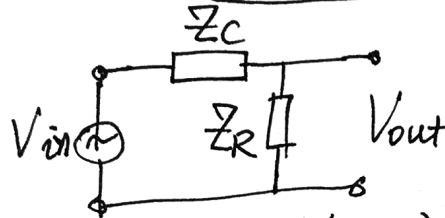
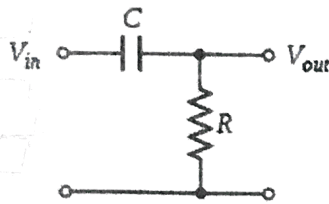
4 pts.

2. For a RC filter circuit below, the input is  $V_{in}$  and the output is  $V_{out}$ . Write out the complex impedance of the capacitor and resistor in the frequency domain, and then derive the transfer function of this circuit.

(Transfer function is defined as  $H(\omega) = \frac{V_{out}}{V_{in}}$ , and its magnitude is  $|H(\omega)| = \left|\frac{V_{out}}{V_{in}}\right|$ .) Identify

what kind of filter this circuit is, basically low-pass or high-pass filter?

+1  
This is a  
high-pass  
filter.



$$Z_C = \frac{1}{j\omega C}$$

$$Z_R = R$$

$$V_{out} = \frac{Z_R}{Z_C + Z_R} V_{in} \Rightarrow H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{Z_R}{Z_C + Z_R}$$

$$\Rightarrow H(\omega) = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC} \quad |H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

Not Required

4 pts.

3. A sinusoidal wave with frequency of 1 kHz and peak amplitude of 1 V is input to a RC low-pass filter as  $V_i$ , and the RC circuit has the following transfer function:  $H(\omega) = \frac{V_o}{V_i} = \frac{\omega_0}{\omega_0 + j\omega}$ , where

$\omega_0 = 2\pi f_0$  and  $f_0 = 500$  Hz is the -3dB frequency and  $\omega$  represents angular frequency.

- a) How much are the frequency and amplitude of the output signal  $V_o$ ?  
(Hint: consider the transfer function magnitude)
- b) If the resistor is  $R = 3183 \Omega$ , then what capacitance is needed to make the -3 dB frequency to be 500 Hz?

+1 a) The frequency of output signal  $V_o$  is the same as the input: 1 kHz

$$b) |V_o| = |H(\omega) V_i| = |H(\omega)| \cdot |V_i|$$

$$= \left| \frac{\omega_0}{\omega_0 + j\omega} \right| \cdot |V_i| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}} \cdot |V_i|$$

$$= \frac{1}{\sqrt{1 + (f/f_0)^2}} \cdot |V_i| = \frac{1}{\sqrt{1 + (1000/500)^2}} \cdot 1 \text{ V} = 0.447 \text{ V}$$

$$c) \omega_0 = 2\pi f_0 = \frac{1}{RC} \Rightarrow C = \frac{1}{2\pi f_0 R} = \frac{1}{2\pi \times 500 \times 3183} = 1 \times 10^{-7} \text{ F} = 0.1 \mu\text{F}$$