Jared Blanco . HW2 - Written

lab section 03

1)

```
int total = 0
for( int i=0; i<n; i += 2)
    total += i;
```

n	i	total	# of additions
7	~~0~~	~~0~~	IIII
	~~2~~	~~0~~	
	~~4~~	~~2~~	
	~~6~~	~~6~~	
	8	12	

n	i	total	# of additions
8	~~0~~	~~0~~	IIII
	~~2~~	~~0~~	
	~~4~~	~~2~~	
	~~6~~	~~6~~	
	8	12	

$\Rightarrow$ $\frac{n}{2}$ additions

$$f(k) = \begin{cases} \frac{n}{2} & n = 2a \\ \frac{n+1}{2} & n \neq 2a \end{cases}$$

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2)

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int total = 0;
for (int i=0; i<n ; i++)
   for( int j = i; j>0 ; j--)
      total += j;
```

n	total	i	j	# of additions
8	0	~~0~~	0	1
		~~1~~	~~1~~ 0 -1	2
		2	~~2~~ 2 0 -1	3
		3	~~3~~ ~~2~~ 1 0 1	4
		4	~~4~~ ~~3~~ ~~2~~ ~~1~~ 0 1	5

i = 0, 1, 2, 3, ... n-1
j = 1, 2, 3, 4, ... i-1

$\frac{n(n+1)}{2}$

$f(n) = 1 + 2 + 3 + 4 \ldots n$
$f(n) \quad n \quad n-1 \quad n-2 \quad n-3 \quad 1$

$\frac{\cancel{2}f(n)}{\cancel{2}} = \frac{(n+1)n}{2}$

3) $d(n)$ is $O(f(n))$ $\quad F(n) \le k\, g(n)$ for $n \ge n_0$

$f(n)$ is $O(g(n))$ $\Rightarrow$ $d(n) = O(g(n))$

$d(n) \le k_1 f(n)$

$f(n) \le k_2 g(n)$, $k_1 f(n) \le k_1 k_2 g(n)$

$d(n) \le k_1 f(n) \le k_1 k_2 g(n)$

$d(n) \le k_1 k_2 g(n)$ $\quad$ now $k_3 = k_1 k_2$

$d(n) \le k_3 g(n)$

$d(n)$ is $O(g(n))$

4) $f(n) = 5n^2 + 4n - 2$, show $f(n)$ is $O(n^2)$, $\Omega(n^2)$, and $\Theta(n^2)$

$O(n)$: $f(n) \le k\, g(n)$

$\frac{5n^2 + 4n - 2}{n^2} \le \frac{k_1 \cancel{n^2}}{\cancel{n^2}}$ $\Rightarrow$ $\frac{5\cancel{n^2}}{\cancel{n^2}} + \frac{4\cancel{n}}{n^{\cancel{2}}} - \frac{2}{n^2} \le k$

$5 + \frac{4}{n} - \frac{2}{n^2} \le k$ $\quad$ let $n = 1$

$5 + 4 - 2 = 7 \le k$

now plug k into the function

$5n^2 + 4n - 2 \le 7n^2$

$\frac{0}{2} \le \frac{2n^2 - 4n + 2}{2}$

$0 \le n^2 - 2n + 1$

$\frac{0}{n-1} \le \frac{(n-1)^{\cancel{2}}}{\cancel{n-1}}$ $\Rightarrow$ $0 \le n-1$ $\Rightarrow$ $1 \le n$

$k = 7$, $n_0 = 1$

Ω : $f(n) \geq k_1 g(n)$

$\frac{5n^2+4n-2}{n^2} \geq \frac{k\,\cancel{n^2}}{\cancel{n^2}} \Rightarrow 5 + \frac{4}{n} - \frac{2}{n^2} \geq k$ let n be 1

$5+4-2 \geq k \Rightarrow 7 \geq k \quad k = 5$

I choose $k=5$ because k just has to be less than 7

$5n^2 + 4n - 2 \geq 5n^2$ Plug Back In to find n

$4n - 2 \geq 0$

$\frac{\cancel{4}n}{\cancel{4}} \geq \frac{2}{4}$

$n \geq \frac{1}{2} \quad n_0 = 1$

Θ : $\underbrace{k' g(n) \leq f(n)}_{\Omega(n^2)} \leq k'' g(n)$, with $\underbrace{f(n) \leq k'' g(n)}_{O(n^2)}$

Plug In 1 for n because n_0 is max of $\{\frac{1}{2}, 1\}$

$5n^2 \leq 5n^2 + 4n - 2 \leq 7n^2$

$5 \leq 5 + 4 - 2 \leq 7$

$5 \leq 7 \leq 7$ True!

5) $T(n) = 2 + T(n/3)$, $T(2) = 2$, $T(7) = 2$

$T(n) = 2 + \left(2 + T\left(\frac{n}{9}\right)\right)$

$T(n) = 2 + 2 + \left(2 + T\left(\frac{n}{27}\right)\right)$

$2k + T\left(\frac{n}{3^k}\right)$

$T(n) = 2k + T\left(\frac{n}{3^k}\right)$

$3^k \leq n$

$\log_3(3^k) \leq \log_3 n$

$k \leq \log_3 n$

$T(n) = 2\log_3 n + T\left(\frac{n}{\cancel{3}^{\log_3(n)}}\right)$

$T(n) = 2\log_3 n + T\left(\frac{n}{n}\right)$

$T(n) = 2\log_3 n + 1$

$T(n) = 2 + T(n/3)$

- the two is from the two index's in our array
- and the $\frac{1}{3}$ comes from the $\frac{1}{3}$ of the array being searched.

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6) "Brute Force"

a) explain all possible strings with n length

$\underbrace{\sqcup}_{26\text{ letters}} + \underbrace{\sqcup}_{26} \cdot \underbrace{\sqcup}_{26} + \underbrace{\sqcup}_{26}\ \underbrace{\sqcup}_{26}\ \underbrace{\sqcup}_{26}$

$\sum_{n=0}^{m} r^n = \frac{1-r^m}{1-r}$

$\Rightarrow \sum_{i=1}^{n} 26^i = \frac{1-26^n}{1-26} \cdot 26$

B) 1,000,000,000 $\frac{\text{strings}}{\text{sec}}$, $n=100$, years to process = ? $1 \times 10^9 \frac{\text{string}}{\text{sec}}$

$\frac{1-26^{100}}{1-26} \cdot 26 = 3.27 \times 10^{141}$ strings

$1.0 \times 10^9 \frac{\text{string}}{\cancel{\text{sec}}} \cdot \frac{3.15 \times 10^7 \cancel{\text{sec}}}{\text{year}} = 3.15 \times 10^{16} \frac{\text{string}}{\text{year}}$

$\frac{3.27 \times 10^{141} \text{ strings}}{3.15 \times 10^{16} \frac{\text{string}}{\text{year}}} = 1.04 \times 10^{125}$ year

c) max time = 1 minute, n_{max} = ?

$$\frac{1-26^n}{1-26} \cdot 26 \text{ string} = 60 \cancel{\text{seconds}} \cdot \frac{10^9 \text{ strings}}{\cancel{\text{seconds}}}$$

$$\frac{(1-26^n)\cdot 26}{-25} = 6 \cdot 10^{10} \text{ strings}$$

$$(1-26^n)(-1.04) = 6 \cdot 10^{10} \text{ strings}$$

$$1-26^n = \frac{6 * 10^{10} \text{ strings}}{(-1.04)}$$

$$1-26^n = -5.77 * 10^{10}$$

$$(5.77 \times 10^{10}) = 26^n$$

$$\log_{26}(5.77 * 10^{10}) = \log_{26}(26)^n$$

$$\log_{26}(5.77 * 10^{10}) = n \cancel{\log_{26}(26)}$$

$$7.61 \text{ strings} = n$$

$$7 \text{ string} = n$$