sasoptpy Documentation

Release 0.2.0

SAS Institute Inc.

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sasoptpy is a Python package providing a modeling interface for SAS Viya and SAS/OR Optimization solvers. It provides a quick way for users to deploy optimization models and solve them using SAS Viya Optimization Action Set.

sasoptpy can handle linear optimization, mixed integer linear optimization, and nonlinear optimization problems. Users can benefit from native Python structures like dictionaries, tuples, and list to define an optimization problem. sasoptpy supports Pandas objects extensively.

Under the hood, *sasoptpy* uses swat package to communicate SAS Viya, and uses saspy package to communicate SAS 9.4 installations.

sasoptpy is an interface to SAS Optimization solvers. Check SAS/OR and PROC OPTMODEL for more details about optimization tools provided by SAS and an interface to model optimization problems inside SAS.

See our SAS Global Forum paper: Optimization Modeling with Python and SAS Viya

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CHAPTER

ONE

OVERVIEW

1.1 What's New

This page outlines changes from each release.

1.1.1 v0.2.0 (July 30, 2018)

New Features

- Support for the new runOptmodel CAS action is added
- Nonlinear optimization model building support is added for both SAS 9.4 and SAS Viya solvers.
- Abstract model building support is added when using SAS Viya solvers
- New object types, Set, SetIterator, Parameter, ParameterValue, ImplicitVar, ExpressionDict, and Statement are added for abstract model building
- Model.to_optmodel() method is added for exporting model objects into PROC OPTMODEL codes as a string
- Wrapper functions read_table() and read_data() are added to read CASTable and DataFrame objects into the models
- Math function wrappers are added
- _expr and _defn methods are added to all object types for producing OPTMODEL expression and definitions
- Multiple solutions are now being returned when using *solveMilp* action and can be grabbed using *Model*. get_solution() method
- Model.get_variable_value() is added to get solution values of abstract variables

Changes

- Variable and constraint naming schemes are replaced with OPTMODEL equivalent versions
- Variables and constraints now preserve the order they are inserted to the problem
- Model.to_frame() method is updated to reflect changes to VG and CG orderings
- Two solve methods, <code>Model.solve_on_cas()</code> and <code>Model.solve_on_viya()</code> are merged into <code>Model.solve()</code>
- Model.solve() method checks the available CAS actions and uses runOptmodel whenever possible

- As part of the merging process, lp and milp arguments are replaced with options argument in Model. solve() and Model.to_optmodel()
- An optional argument frame is added to Model.solve() for forcing to use MPS mode and solveLp-solveMilp actions
- Minor changes are applied to __str__ and __repr__ methods
- Creation indices for objects are being kept using the return of the register_name() function
- Objective constant values are now being passed using new CAS action arguments when posssible
- · A linearity check is added for models
- · Test folder is added to the repository

Bug Fixes

• Nondeterministic behavior when generating MPS files is fixed.

Notes

- Abstract and nonlinear models can be solved on Viya if only runOptmodel action is available on the CAS server.
- Three new examples are added which demonstrate abstract model building.
- Some minor changes are applied to the existing examples.

1.1.2 v0.1.2 (April 24, 2018)

New Features

- As an experimental feature, sasoptpy supports saspy connections now
- Model.solve_local() method is added for solving optimization problems using SAS 9.4 installations
- Model.get_constraint() and Model.get_constraints() methods are added to grab Constraint objects in a model
- Model.get_variables() method is added
- _dual attribute is added to the Expression objects
- Variable.get_dual() and Constraint.get_dual() methods are added
- Expression.set_name() method is added

Changes

- Session argument accepts saspy. SASsession objects
- VariableGroup.mult () method now supports pandas.DataFrame
- Type check for the Model.set_session() is removed to support new session types
- Problem and solution summaries are not being printed by default anymore, see Model. get_problem_summary() and Model.get_solution_summary()

• The default behavior of dropping the table after each solve is changed, but can be controlled with the drop argument of the Model.solve() method

Bug Fixes

- Fixed: Variables do not appear in MPS files if they are not used in the model
- Fixed: Model.solve() primalin argument does not pass into options

Notes

- A .gitignore file is added to the repository.
- A new example is added: Decentralization.
- Both CAS/Viya and SAS versions of the new example are available.
- There is a known issue with the nondeterministic behavior when creating MPS tables. This will be fixed with a hotfix after the release.
- A new option (no-ex) is added to makedocs script for skipping examples when building docs.

1.1.3 v0.1.1 (February 26, 2018)

New Features

- Initial value argument 'init' is added for Variable objects
- Variable.set_init() method is added for variables
- Initial value option 'primalin' is added to Model.solve() method
- Table name argument 'name', table drop option 'drop' and replace option 'replace' are added to <code>Model.solve()</code> method
- Decomposition block implementation is rewritten, block numbers does not need to be consecutive and ordered Model.upload_user_blocks()
- VariableGroup.get_name() and ConstraintGroup.get_name() methods are added
- Model.test_session() method is added for checking if session is defined for models
- quick_sum() function is added for faster summation of Expression objects

Changes

• methods.py is renamed to utils.py

Bug Fixes

- Fixed: Crash in VG and CG when a key not in the list is called
- Fixed: get_value of pandas is depreceated
- Fixed: Variables can be set as temporary expressions
- Fixed: Ordering in get_solution_table() is incorrect for multiple entries

1.1. What's New 5

1.1.4 v0.1.0 (December 22, 2017)

· Initial release

1.2 Installation

1.2.1 Python version support and dependencies

sasoptpy is developed and tested for Python version 3.5+.

It depends on the following packages:

- numpy
- saspy (Optional)
- swat
- pandas

1.2.2 Getting swat

swat package is a requirement to use SAS Viya solvers.

swat releases are listed at https://github.com/sassoftware/python-swat/releases. After downloading the platform-specific release file, it can be installed using pip:

```
pip install python-swat-X.X.X-platform.tar.gz
```

1.2.3 Getting saspy

saspy package is a requirement to use SAS 9.4 solvers.

saspy releases are listed at https://github.com/sassoftware/saspy/releases. The easiest way to download the latest stable version of saspy is to use:

```
pip install saspy
```

1.2.4 Getting sasoptpy

The latest available version of *sasoptpy* can be obtained from the online repository. Call:

```
git clone https://github.com/sassoftware/sasoptpy.git
```

Then inside the sasoptpy folder, call:

```
pip install .
```

Alternatively, you can use:

```
python setup.py install
```

1.2.5 Step-by-step installation

1. Installing pandas and numpy

First, download and install numpy and pandas using pip:

```
pip install numpy
pip install pandas
```

2. Installing the swat package

First, check the swat release page to find the latest release of the SAS-SWAT package for your environment.

Then install it using

```
pip install python-swat-X.X.X.platform.tar.gz
```

As an example, run

```
wget https://github.com/sassoftware/python-swat/releases/download/v1.2.1/python-swat-1.2.1-linux64.tar.gz
pip install python-swat-1.2.1-linux64.tar.gz
```

to install the version 1.2.1 of the swat package for 64-bit Linux environments.

3. Installing sasoptpy

Finally you can install *sasoptpy* by downloading the latest archive file and install via pip.

```
wget *url-to-sasoptpy.tar.gz*
pip install sasoptpy.tar.gz
```

Latest release file is available at Github releases page.

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CHAPTER

TWO

GETTING STARTED

Solving an optimization problem via *sasoptpy* starts with having a running CAS Server or having a SAS 9.4 installation. It is possible to model a problem without a server but solving a problem requires access to SAS/OR solvers.

2.1 Creating a session

2.1.1 Creating a SAS Viya session

sasoptpy uses the CAS connection provided by the swat package. After installation simply use

```
In [1]: from swat import CAS
In [2]: s = CAS(hostname, port, userid, password)
```

The last two parameters are optional for some use cases. See swat Documentation for more details.

2.1.2 Creating a SAS 9.4 session

To create a SAS 9.4 session, see saspy Documentation. After customizing the configurations for your setup, a session can be created as follows:

```
import saspy
s = saspy.SASsession(cfgname='winlocal')
```

2.2 Initializing a model

After having an active CAS/SAS session, an empty model can be defined as follows:

```
In [3]: import sasoptpy as so
In [4]: m = so.Model(name='my_first_model', session=s)
NOTE: Initialized model my_first_model.
```

This command creates an empty model.

2.3 Processing input data

The easisest way to work with *sasoptpy* is to define problem inputs as Pandas DataFrames. Objective and cost coefficients, and lower and upper bounds can be defined using the DataFrame and Series objects. See Pandas Documentation to learn more.

Set PERIODS and other fields demand, min_production can be extracted as follows

```
In [9]: PERIODS = prob_data.index.tolist()
In [10]: demand = prob_data['demand']
In [11]: min_production = prob_data['min_prod']
```

2.4 Adding variables

You can add a single variables or a set of variables to Model objects.

• Model.add_variable() method is used to add a single variable.

When working with multiple models, you can create a variable independent of the model, such as

```
>>> production_cap = so.Variable(name='production_cap', vartype=so.INT, lb=0)
```

and add it to an existing model using

```
>>> m.include(production_cap)
```

• Model.add variables () method is used to add a set of variables.

When passed as a set of variables, individual variables can be obtained by using individual keys, such as production['Period1']. To create multi-dimensional variables, simply list all the keys as

```
>>> multivar = m.add_variables(KEYS1, KEYS2, KEYS3, name='multivar')
```

2.5 Creating expressions

Expression objects keep mathematical expressions. Although these objects are mostly used under the hood when defining a model, it is possible to define a custom Expression to use later.

```
In [14]: totalRevenue = production.sum('*')*price_per_product
In [15]: totalCost = production_cap * capacity_cost
```

The first thing to notice is the use of the *VariableGroup.sum()* method over a variable group. This method returns the sum of variables inside the group as an *Expression* object. Its multiplication with a scalar profit_per_product gives the final expression.

Similarly, totalCost is simply multiplication of a Variable object with a scalar.

2.6 Setting an objective function

Objective functions can be written in terms of expressions. In this problem, the objective is to maximize the profit, so <code>Model.set_objective()</code> method is used as follows:

Notice that you can define the same objective using

```
>>> m.set_objective(production.sum('*')*price_per_product - production_cap*capacity_

cost, sense=so.MAX, name='totalProfit')
```

The mandatory argument sense should be assigned the value of either so.MIN or so.MAX for minimization or maximization problems, respectively.

2.7 Adding constraints

In *sasoptpy*, constraints are simply expressions with a direction. It is possible to define an expression and add it to a model by defining which direction the linear relation should have.

There are two methods to add constraints. The first one is <code>Model.add_constraint()</code> where a single constraint can be inserted into a model.

The second one is Model.add_constraints() where multiple constraints can be added to a model.

Here, the first term provides a Python generator, which then gets translated into constraints in the problem. The symbols <=, >=, and == are used for less than or equal to, greater than or equal to, and equal to constraints, respectively. Range constraints can be inserted using == and a list of 2 values representing lower and upper bounds.

2.8 Solving a problem

Defined problems can be simply sent to CAS server or SAS sesion by calling the <code>Model.solve()</code> method. See the solution output to the problem.

```
In [19]: m.solve()
NOTE: Added action set 'optimization'.
NOTE: Converting model my_first_model to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 4 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 4 integer variables.
NOTE: The problem has 6 linear constraints (6 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 9 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed all variables and constraints.
NOTE: Optimal.
NOTE: Objective = 400.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 4 rows and 6
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
Out [19]:
                   var lb
                                        ub value rc
        production_cap -0.0 1.797693e+308 25.0 NaN
1 production[Period1] 5.0 1.797693e+308 25.0 NaN
  production[Period2] 5.0 1.797693e+308 15.0 NaN
  production[Period3] -0.0 1.797693e+308
```

At the end of the solve operation, the solver returns both Problem Summary and Solution Summary tables. These tables can be later accessed using m.get_problem_summary() and m.get_solution_summary.

(continues on next page)

```
Relative Gap
                                     0
Absolute Gap
                                     0
Primal Infeasibility
                                     0
Bound Infeasibility
                                     0
Integer Infeasibility
                                     0
Best Bound
                                   400
Nodes
                                     0
Solutions Found
                                     1
Iterations
                                    0
Presolve Time
                                  0.02
Solution Time
                                  0.02
```

The Model.solve() method returns the primal solution when available, and None otherwise.

2.9 Printing solutions

Solutions provided by the solver can be obtained using <code>sasoptpy.get_solution_table()</code> method. It is strongly suggested to group variables and expressions that share the same keys in a call.

As seen, a Pandas Series and a Variable object that has the same index keys are printed in this example.

2.10 Next steps

You can browse *Examples* to see various uses of aformentioned functionality.

If you have a good understanding of the flow, then check API Reference to access API details.

CHAPTER

THREE

HANDLING DATA

sasoptpy can work with native Python types and pandas objects for all data operations. Among pandas object types, sasoptpy works with pandas. DataFrame and pandas. Series objects to construct and manipulate model components.

3.1 Indices

Methods like Model.add_variables() can utilize native Python object types like list and range as variable and constraint indices. pandas.Index can be used as index as well.

3.1.1 List

```
In [1]: m = so.Model(name='demo')
NOTE: Initialized model demo.

In [2]: SEASONS = ['Fall', 'Winter', 'Spring', 'Summer']

In [3]: prod_lb = {'Fall': 100, 'Winter': 200, 'Spring': 100, 'Summer': 400}

In [4]: production = m.add_variables(SEASONS, lb=prod_lb, name='production')

In [5]: print(production)
Variable Group (production) [
  [Fall: production[Fall]]
  [Spring: production[Spring]]
  [Summer: production[Summer]]
  [Winter: production[Winter]]
]
```

```
In [6]: print(repr(production['Summer']))
sasoptpy.Variable(name='production[Summer]', 1b=400, vartype='CONT')
```

Note that if a list is being used as the index set, associated fields like *lb*, *ub* should be accesible using the index keys. Accepted types are dict and pandas. Series.

3.1.2 Range

```
In [7]: link = m.add_variables(range(3), range(2), vartype=so.BIN, name='link')

In [8]: print(link)
Variable Group (link) [
   [(0, 0): link[0, 0]]
   [(0, 1): link[0, 1]]
   [(1, 0): link[1, 0]]
   [(1, 1): link[1, 1]]
   [(2, 0): link[2, 0]]
   [(2, 1): link[2, 1]]
]
```

```
In [9]: print(repr(link[2, 1]))
sasoptpy.Variable(name='link[2,1]', ub=1, vartype='BIN')
```

3.1.3 pandas.Index

```
In [14]: print(x)
Variable Group (x) [
  [0: x[0]]
  [1: x[1]]
  [2: x[2]]
]
```

```
In [15]: df2 = df.set_index([['r1', 'r2', 'r3']])
In [16]: y = m.add_variables(df2.index, lb=df2['col_lb'], ub=df2['col_ub'], name='y')
```

```
In [17]: print(y)
Variable Group (y) [
  [r1: y[r1]]
  [r2: y[r2]]
  [r3: y[r3]]
]
```

```
In [18]: print(repr(y['r1']))
sasoptpy.Variable(name='y[r1]', lb=5, ub=9, vartype='CONT')
```

3.1.4 Set

sasoptpy can work with data on the server and generate abstract expressions. For this purpose, you can use Set objects to represent PROC OPTMODEL sets.

```
In [19]: m2 = so.Model(name='m2')
NOTE: Initialized model m2.

In [20]: I = m2.add_set(name='I')

In [21]: u = m2.add_variables(I, name='u')

In [22]: print(I, u)
I Variable Group (u) [
   [I: u[I]]
]
```

See *Workflows* for more information on working with server-side models.

3.2 Data

sasoptpy can work with both client-side and server-side data. Here are some options to load data into the optimization models.

3.2.1 pandas DataFrame

pandas. DataFrame is the preferred object types when passing data into sasoptpy models.

```
In [23]: data = [
  ....: ['clock', 8, 4, 3],
           ['mug', 10, 6, 5],
   . . . . :
           ['headphone', 15, 7, 2],
           ['book', 20, 12, 10],
           ['pen', 1, 1, 15]
            ]
In [24]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit']).set_
→index(['item'])
In [25]: get = so.VariableGroup(df.index, ub=df['limit'], name='get')
In [26]: print(get)
Variable Group (get) [
  [book: get[book]]
  [clock: get[clock]]
  [headphone: get[headphone]]
  [mug: get[mug]]
  [pen: get[pen]]
```

3.2.2 Dictionaries

Lists and dictionaries can be used in expressions and when creating variables.

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```
In [28]: limits = {'clock': 3, 'mug': 5, 'headphone': 2, 'book': 10, 'pen': 15}
In [29]: get2 = so.VariableGroup(items, ub=limits, name='get2')
In [30]: print(get2)
Variable Group (get2) [
  [book: get2[book]]
  [clock: get2[clock]]
  [headphone: get2[headphone]]
  [mug: get2[mug]]
  [pen: get2[pen]]
]
```

3.2.3 CASTable

When a data is available on the server-side, a reference to the object can be passed. Note that, using CASTable and Abstract Data requires SAS Viya version 3.4.

```
In [31]: m2 = so.Model(name='m2', session=session)
NOTE: Initialized model m2.
```

```
In [32]: table = session.upload_frame(df)
NOTE: Cloud Analytic Services made the uploaded file available as table TMPD8T7Q6DG_
→in caslib CASUSERHDFS(casuser).
NOTE: The table TMPD8T7Q6DG has been created in caslib CASUSERHDFS(casuser) from_
→binary data uploaded to Cloud Analytic Services.
```

3.2.4 Abstract Data

If you would like to model your problem first and then load data, you can pass a string for the data sets that will be available later. See following:

Notice that the key set is created as a reference. We can later solve the problem after having the data available with the same name, e.g. using the *upload_frame* function.

3.3 Operations

Lists, pandas.Series, and pandas.DataFrame objects can be used for mathematical operations like VariableGroup.mult().

```
In [42]: sd = [3, 5, 6]
In [43]: z = m.add_variables(3, name='z')
```

```
In [44]: print(z)
Variable Group (z) [
  [0: z[0]]
  [1: z[1]]
  [2: z[2]]
]
```

```
In [45]: print(repr(z))
sasoptpy.VariableGroup([0, 1, 2], name='z')
```

```
In [46]: e1 = z.mult(sd)
In [47]: print(e1)
3 * z[0] + 5 * z[1] + 6 * z[2]
```

```
In [48]: ps = pd.Series(sd)
In [49]: e2 = z.mult(ps)
In [50]: print(e2)
3 * z[0] + 5 * z[1] + 6 * z[2]
```

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CHAPTER

FOUR

SESSIONS AND MODELS

4.1 Sessions

4.1.1 CAS Sessions

A swat.cas.connection.CAS session is needed to solve optimization problems with *sasoptpy* using SAS Viya OR solvers. See SAS documentation to learn more about CAS sessions and SAS Viya.

A sample CAS Session can be created using the following commands.

4.1.2 SAS Sessions

A saspy.SASsession session is needed to solve optimization problems with *sasoptpy* using SAS/OR solvers on SAS 9.4 clients.

A sample SAS session can be created using the following commands.

```
>>> import sasoptpy as so
>>> import saspy
>>> sas_session = saspy.SASsession(cfgname='winlocal')
>>> m = so.Model(name='demo', session=sas_session)
>>> print(repr(m))
sasoptpy.Model(name='demo', session=saspy.SASsession(cfgname='winlocal'))
```

4.2 Models

4.2.1 Creating a model

An empty model can be created using the Model constructor:

```
In [1]: import sasoptpy as so
In [2]: m = so.Model(name='model1')
NOTE: Initialized model model1.
```

4.2.2 Adding new components to a model

Adding a variable:

```
In [3]: x = m.add_variable(name='x', vartype=so.BIN)
In [4]: print(m)
Model: [
 Name: model1
 Objective: MIN [0]
 Variables (1): [
 Constraints (0): [
In [5]: y = m.add_variable(name='y', lb=1, ub=10)
In [6]: print(m)
Model: [
 Name: model1
 Objective: MIN [0]
 Variables (2): [
   У
 Constraints (0): [
 ]
```

Adding a constraint:

```
In [7]: c1 = m.add_constraint(x + 2 * y <= 10, name='c1')
In [8]: print(m)
Model: [
  Name: model1
  Objective: MIN [0]
  Variables (2): [
        x
        y
   ]
  Constraints (1): [
        2 * y + x <= 10
  ]
]</pre>
```

4.2.3 Adding existing components to a model

A new model can use existing variables. The typical way to include a variable is to use the <code>Model.include()</code> method:

```
In [9]: new_model = so.Model(name='new_model')
NOTE: Initialized model new_model.
In [10]: new_model.include(x, y)
In [11]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (2): [
  1
  Constraints (0): [
In [12]: new_model.include(c1)
In [13]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (2): [
    Х
    У
  1
  Constraints (1): [
    2 * y + x <= 10
In [14]: z = so.Variable(name='z', vartype=so.INT, lb=3)
In [15]: new_model.include(z)
In [16]: print(new_model)
Model: [
  Name: new_model
  Objective: MIN [0]
  Variables (3): [
    У
  Constraints (1): [
    2 * y + x <= 10
```

Note that variables are added to <code>Model</code> objects by reference. Therefore, after <code>Model.solve()</code> is called, values of variables will be replaced with optimal values.

4.2. Models 25

4.2.4 Accessing components

You can get a list of model variables using Model.get_variables() method.

Similarly, you can access a list of constraints using Model.get_constraints() method.

```
In [18]: c2 = m.add_constraint(2 * x - y >= 1, name='c2')
In [19]: print(m.get_constraints())
[sasoptpy.Constraint(2 * y + x <= 10, name='c1'), sasoptpy.Constraint(- y + 2 * x >= 4, name='c2')]
```

To access a certain constraint using its name, you can use Model.get_constraint() method:

```
In [20]: print(m.get_constraint('c2'))
- y + 2 * x >= 1
```

4.2.5 Dropping components

A variable inside a model can simply be dropped using <code>Model.drop_variable()</code>. Similarly, a set of variables can be dropped using <code>Model.drop_variables()</code>.

```
In [21]: m.drop_variable(y)

In [22]: print(m)

Model: [
   Name: model1
   Objective: MIN [0]
   Variables (1): [
        x
   ]
   Constraints (2): [
        2 * y + x <= 10
        - y + 2 * x >= 1
   ]
]
```

```
In [23]: m.include(y)

In [24]: print(m)

Model: [
   Name: model1
   Objective: MIN [0]
   Variables (2): [
        x
        y
   ]
   Constraints (2): [
        2 * y + x <= 10
        - y + 2 * x >= 1
   ]
]
```

A constraint can be dropped using <code>Model.drop_constraint()</code> method. Similarly, a set of constraints can be dropped using <code>Model.drop_constraints()</code>.

```
In [28]: m.include(c1)

In [29]: print(m)
Model: [
   Name: model1
   Objective: MIN [0]
   Variables (2): [
        x
        y
   ]
   Constraints (1): [
        2 * y + x <= 10
   ]
]</pre>
```

4.2.6 Copying a model

An exact copy of the existing model can be obtained by including the Model object itself.

Note that all variables and constraints are included by reference.

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4.2.7 Solving a model

A model is solved using the *Model.solve()* method. This method converts Python definitions into an MPS file and uploads to a CAS server for the optimization action. The type of the optimization problem (Linear Optimization or Mixed Integer Linear Optimization) is determined based on variable types.

```
>>> m.solve()
NOTE: Initialized model model_1
NOTE: Converting model model_1 to DataFrame
NOTE: Added action set 'optimization'.
...
NOTE: Optimal.
NOTE: Objective = 124.343.
NOTE: The Dual Simplex solve time is 0.01 seconds.
```

4.2.8 Solve options

Solver Options

Both PROC OPTMODEL solve options and solveLp, solveMilp action options can be passed using options argument of the <code>Model.solve()</code> method.

```
>>> m.solve(options={'with': 'milp', 'maxtime': 600})
>>> m.solve(options={'with': 'lp', 'algorithm': 'ipm'})
```

The only special option for the <code>Model.solve()</code> method is with. If not passed, PROC OPTMODEL chooses a solver that depends on the problem type. Possible with options are listed in SAS/OR documentation: http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_optmodel_syntax11.htm% docsetVersion=14.3&locale=en#ormpug.optmodel.npxsolvestmt

See specific solver options at following links:

- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_lpsolver_syntax02.htm% docsetVersion=14.3&locale=en for a list of LP solver options.
- See MILP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_syntax02.htm&docsetVersion=14.3&locale=en for a list of NLP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_qpsolver_syntax02.htm% docsetVersion=14.3&locale=en for a list of QP solver options.
- See http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_clpsolver_syntax01.htm% docsetVersion=14.3&locale=en for a list of CLP solver options.

The options argument can also pass solveLp and solveMilp action options when frame=True is used when calling the <code>Model.solve()</code> method.

- See http://go.documentation.sas.com/?cdcId=vdmmlcdc&cdcVersion=8.11&docsetId=casactmopt&docsetTarget=casactmopt_solvelp_syntax.htm&locale=en for a list of LP options.
- See http://go.documentation.sas.com/?cdcId=vdmmlcdc&cdcVersion=8.11&docsetId=casactmopt&docsetTarget=casactmopt_solvemilp_syntax.htm&locale=en for a list of MILP options.

Package Options

Besides the options argument, there are 7 arguments that can be passed into Model.solve() method:

- name: Name of the uploaded problem information
- drop: Option for dropping the data from server after solve
- replace: Option for replacing an existing data with the same name
- primalin: Option for using the current values of the variables as an initial solution
- submit: Option for calling the CAS / SAS action
- frame: Option for using frame (MPS) method (if False, it uses OPTMODEL)
- verbose: Option for printing the generated OPTMODEL code before solve

When primalin argument is True, it grabs <code>Variable</code> objects <code>_init</code> field. This field can be modified with <code>Variable.set_init()</code> method.

4.2.9 Getting solutions

After the solve is completed, all variable and constraint values are parsed automatically. A summary of the problem can be accessed using the <code>Model.get_problem_summary()</code> method, and a summary of the solution can be accessed using the <code>Model.get_solution_summary()</code> method.

To print values of any object, get_solution_table() can be used:

```
>>> print(so.get_solution_table(x, y))
```

All variables and constraints passed into this method are returned based on their indices. See *Examples* for more details.

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CHAPTER

FIVE

MODEL COMPONENTS

In this part, several model components are discussed with examples. See *Examples* to learn more about how these components can be used to define optimization models.

5.1 Expressions

Expression objects represent linear and nonlinear mathematical expressions in sasoptpy.

5.1.1 Creating expressions

An Expression can be created as follows:

```
In [1]: profit = so.Expression(5 * sales - 3 * material, name='profit')
In [2]: print(repr(profit))
sasoptpy.Expression(exp = - 3 * material + 5 * sales, name='profit')
```

5.1.2 Nonlinear expressions

Expression objects are linear by default. It is possible to create nonlinear expressions, but there are some limitations.

```
In [3]: nonexp = sales ** 2 + (1 / material) ** 3
In [4]: print(nonexp)
(sales) ** (2) + ((1) / (material)) ** (3)
```

Currently, it is not possible to get or print values of nonlinear expressions. Moreover, if your model includes a nonlinear expression, you need to be using SAS Viya \geq 3.4 or any SAS version for solving your problem.

For using mathematical operations, you need to import sasoptpy.math functions.

5.1.3 Mathematical expressions

sasoptpy provides mathematical functions for generating mathematical expressions to be used in optimization models.

You need to import *sasoptpy.math* to your code to start using these functions. A list of available mathematical functions are listed at *Math Functions*.

```
In [5]: import sasoptpy.math as sm
In [6]: newexp = sm.max(sales, 10) ** 2
In [7]: print(newexp._expr())
(max(sales , 10)) ^ (2)
```

```
In [8]: import sasoptpy.math as sm
In [9]: angle = so.Variable(name='angle')
In [10]: newexp = sm.sin(angle) ** 2 + sm.cos(angle) ** 2
In [11]: print(newexp._expr())
(sin(angle)) ^ (2) + (cos(angle)) ^ (2)
```

5.1.4 Operations

Getting the current value

After the solve is completed, the current value of an expression can be obtained using the *Expression*. $get_value()$ method:

```
>>> print(profit.get_value())
42.0
```

Getting the dual value

Dual values of *Expression* objects can be obtained using *Variable.get_dual()* and Constraint.get_dual() methods.

```
>>> m.solve()
>>> ...
>>> print(x.get_dual())
1.0
```

Addition

There are two ways to add elements to an expression. The first and simpler way creates a new expression at the end:

```
In [12]: tax = 0.5
In [13]: profit_after_tax = profit - tax
```

```
In [14]: print(repr(profit_after_tax))
sasoptpy.Expression(exp = - 3 * material + 5 * sales - 0.5, name=None)
```

The second way, Expression.add() method, takes two arguments: the element to be added and the sign (1 or -1):

```
In [15]: profit_after_tax = profit.add(tax, sign=-1)
```

```
In [16]: print(profit_after_tax)
- 3 * material + 5 * sales - 0.5
```

```
In [17]: print(repr(profit_after_tax))
sasoptpy.Expression(exp = - 3 * material + 5 * sales - 0.5, name=None)
```

If the expression is a temporary one, then the addition is performed in place.

Multiplication

You can multiply expressions with scalar values:

```
In [18]: investment = profit.mult(0.2)
In [19]: print(investment)
- 0.6 * material + sales
```

Summation

For faster summations compared to Python's native sum function, sasoptpy provides sasoptpy.quick_sum().

```
In [20]: import time
In [21]: x = m.add_variables(1000, name='x')

In [22]: t0 = time.time()

In [23]: e = so.quick_sum(2 * x[i] for i in range(1000))

In [24]: print(time.time()-t0)
0.009645700454711914

In [25]: t0 = time.time()

In [26]: f = sum(2 * x[i] for i in range(1000))

In [27]: print(time.time()-t0)
0.31011295318603516
```

5.1.5 Renaming an expression

Expressions can be renamed using Expression.set_name() method:

```
In [28]: e = so.Expression(x[5] + 2 * x[6], name='e1')
In [29]: print(repr(e))
sasoptpy.Expression(exp = x[5] + 2 * x[6], name='e1')
```

```
In [30]: e.set_name('e2');
In [31]: print(repr(e))
sasoptpy.Expression(exp = x[5] + 2 * x[6], name='e2')
```

5.1.6 Copying an expression

An Expression can be copied using Expression.copy().

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```
In [32]: copy_profit = profit.copy(name='copy_profit')
In [33]: print(repr(copy_profit))
sasoptpy.Expression(exp = - 3 * material + 5 * sales, name='copy_profit')
```

5.1.7 Temporary expressions

An Expression object can be defined as temporary, which enables faster Expression.sum() and Expression.mult() operations.

```
In [34]: new_profit = so.Expression(10 * sales - 2 * material, temp=True)
In [35]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales, name=None)
```

The expression can be modified inside a function:

```
In [36]: new_profit + 5
Out[36]: sasoptpy.Expression(exp = - 2 * material + 10 * sales + 5, name=None)
```

```
In [37]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales + 5, name=None)
```

As you can see, the value of new_profit is changed due to an in-place addition. To prevent the change, such expressions can be converted to permanent expressions using the <code>Expression.set_permanent()</code> method or constructor:

```
In [38]: new_profit = so.Expression(10 * sales - 2 * material, temp=True)
In [39]: new_profit.set_permanent()
Out[39]: 'expr_11'
In [40]: tmp = new_profit + 5
In [41]: print(repr(new_profit))
sasoptpy.Expression(exp = - 2 * material + 10 * sales, name='expr_11')
```

5.2 Objective Functions

5.2.1 Setting and getting an objective function

Any valid *Expression* can be used as the objective function of a model. An existing expression can be used as an objective function using the *Model.set_objective()* method. The objective function of a model can be obtained using the *Model.get_objective()* method.

```
>>> profit = so.Expression(5 * sales - 2 * material, name='profit')
>>> m.set_objective(profit, so.MAX)
>>> print(m.get_objective())
- 2.0 * material + 5.0 * sales
```

5.2.2 Getting the value

After a solve, the objective value can be checked using the Model.get_objective_value() method.

```
>>> m.solve()
>>> print(m.get_objective_value())
42.0
```

5.3 Variables

5.3.1 Creating variables

Variables can be created either separately or inside a model.

Creating a variable outside a model

The first way to create a variable uses the default constructor.

```
>>> x = so.Variable(vartype=so.INT, ub=5, name='x')
```

When created separately, a variable needs to be included (or added) inside the model:

```
>>> y = so.Variable(name='y', lb=5)
>>> m.add_variable(y)
```

and

```
>>> y = m.add_variable(name='y', lb=5)
```

are equivalent.

Creating a variable inside a model

The second way is to use <code>Model.add_variable()</code>. This method creates a <code>Variable</code> object and returns a pointer.

```
>>> x = m.add_variable(vartype=so.INT, ub=5, name='x')
```

5.3.2 Arguments

There are three types of variables: continuous variables, integer variables, and binary variables. Continuous variables are the default type and can be created using the vartype=so.CONT argument. Integer variables and binary variables can be created using the vartype=so.BIN arguments, respectively.

The default lower bound for variables is 0, and the upper bound is infinity. Name is a required argument. If the given name already exists in the namespace, then a different generic name can be used for the variable. The reset_globals() function can be used to reset sasoptpy namespace when needed.

5.3.3 Changing bounds

The Variable.set_bounds() method changes the bounds of a variable.

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```
>>> x = so.Variable(name='x', lb=0, ub=20)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')
>>> x.set_bounds(lb=5, ub=15)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=5, ub=15, vartype='CONT')
```

5.3.4 Setting initial values

Initial values of variables can be passed to the solvers for certain problems. The *Variable.set_init()* method changes the initial value for variables. This value can be set at the creation of the variable as well.

```
>>> x.set_init(5)
>>> print(repr(x))
sasoptpy.Variable(name='x', ub=20, init=5, vartype='CONT')
```

5.3.5 Working with a set of variables

A set of variables can be added using single or multiple indices. Valid index sets include list, dict, and pandas. Index objects. See *Handling Data* for more about allowed index types.

Creating a set of variables outside a model

Creating a set of variables inside a model

5.4 Constraints

5.4.1 Creating constraints

Similar to Variable objects, Constraint objects can be created inside or outside optimization models.

Creating a constraint outside a model

```
>>> c1 = so.Constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')
```

Creating a constraint inside a model

```
>>> c1 = m.add_constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')
```

5.4.2 Modifying variable coefficients

The coefficient of a variable inside a constraint can be updated using the Constraint.update_var_coef() method:

```
>>> c1 = so.Constraint(exp=3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')
>>> c1.update_var_coef(x, -1)
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y - x <= 10, name='c1')</pre>
```

5.4.3 Working with a set of constraints

A set of constraints can be added using single or multiple indices. Valid index sets include list, dict, and pandas. Index objects. See *Handling Data* for more about allowed index types.

Creating a set of constraints outside a model

Creating a set of constraints inside a model

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5.4.4 Range constraints

A range for an expression can be given using a list of two value (lower and upper bound) with an == sign:

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> c1 = m.add_constraint(x + 2*y == [2,9], name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( x + 2.0 * y == [2, 9], name='c1')
```

WORKFLOWS

sasoptpy can work both with client-side data and server-side data. Some limitations to the functionalities may apply in terms of which workflow is being used. In this part, overall flow of the package is explained.

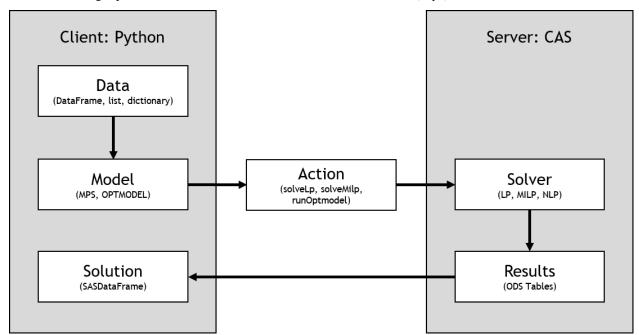
6.1 Client-side models

If the data is on the client-side (Python), then a concrete model is generated on the client-side and uploaded using one of the available CAS actions.

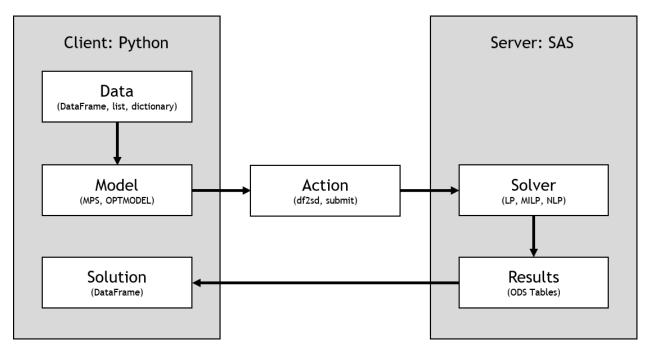
Using client-side models brings several advantages, such as accessing variables, expressions, and constraints directly. You may do more intensive operations like filter data, sort values, changing variable values, and print expressions more easily.

There are two main disadvantages of working with client-side models. First, if your model is relatively big size, then the generated MPS DataFrame or OPTMODEL codes may allocate a large memory on your machine. Second, the information that needs to be passed from client to server might be bigger than using a server-side model.

See the following representation of the client-side model workflow for CAS (Viya) servers:



See the following representation of the client-side model workflow for SAS clients:



Steps of modeling a simple Knapsack problem is shown in the following subsections.

6.1.1 Reading data

```
In [1]: import sasoptpy as so
In [2]: import pandas as pd
In [3]: from swat import CAS
In [4]: session = CAS(hostname, port)
In [5]: m = so.Model(name='client_CAS', session=session)
NOTE: Initialized model client_CAS.
In [6]: data = [
   ...: ['clock', 8, 4, 3],
           ['mug', 10, 6, 5],
          ['headphone', 15, 7, 2], ['book', 20, 12, 10],
           ['pen', 1, 1, 15]
In [7]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit'])
In [8]: ITEMS, (value, weight, limit) = m.read_table(
           df, key=['item'], columns=['value', 'weight', 'limit'])
   . . . :
   . . . :
In [9]: total_weight = 55
```

```
In [10]: print(type(ITEMS), ITEMS)
<class 'list'> ['clock', 'mug', 'headphone', 'book', 'pen']
```

```
In [11]: print(type(total_weight), total_weight)
<class 'int'> 55
```

Here,

Instead of using Model.read_table() method, column values can be obtained one by one:

```
>>> df = df.set_index('item')
>>> ITEMS = df.index.tolist()
>>> value = df['value']
>>> weight = df['weight']
>>> limit = df['limit']
```

6.1.2 Model

```
# Variables
In [12]: get = m.add_variables(ITEMS, name='get', vartype=so.INT)
# Constraints
In [13]: m.add_constraints((get[i] <= limit[i] for i in ITEMS), name='limit_con');</pre>
In [14]: m.add_constraint(
   ....: so.quick_sum(weight[i] * get[i] for i in ITEMS) <= total_weight,</pre>
            name='weight con');
   . . . . :
# Objective
In [15]: total_value = so.quick_sum(value[i] * get[i] for i in ITEMS)
In [16]: m.set_objective(total_value, name='total_value', sense=so.MAX);
# Solve
In [17]: m.solve(verbose=True)
NOTE: Added action set 'optimization'.
NOTE: Converting model client_CAS to OPTMODEL.
var get {{'clock', 'mug', 'headphone', 'book', 'pen'}} integer >= 0;
con limit_con_clock : get['clock'] <= 3;</pre>
con limit_con_mug : get['mug'] <= 5;</pre>
con limit_con_headphone : get['headphone'] <= 2;</pre>
con limit_con_book : get['book'] <= 10;</pre>
con limit_con_pen : get['pen'] <= 15;</pre>
con weight_con : 7 * get['headphone'] + 4 * get['clock'] + get['pen'] + 12 * get['book
→'] + 6 * get['mug'] <= 55;</pre>
max total_value = 15 * get['headphone'] + 8 * get['clock'] + get['pen'] + 20 * get[
→'book'] + 10 * get['mug'];
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 5 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 5 integer variables.
NOTE: The problem has 6 linear constraints (6 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 10 linear constraint coefficients.
```

```
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 5 constraints.
NOTE: The MILP presolver removed 5 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 5 variables, 1 constraints, and 5 constraint.
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger BestBound Gap Time
              0
                   1 3 99.0000000 199.0000000 50.25%
               Ω
                      1
                             3
                                   99.0000000 102.3333333 3.26%
                                                                          0
                      1 3 99.000000 102.333333 3.26%
               0
                                                                          0
NOTE: The MILP presolver is applied again.
                      1 3 99.0000000
1 3 99.000000
               0 1
                                                 102.3333333
                                                                3.26%
                                    99.0000000 102.3333333 3.26%
               0
                                                                           1
NOTE: The MILP solver added 3 cuts with 7 cut coefficients at the root.
NOTE: Optimal.
NOTE: Objective = 99.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 6,
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
Out[17]:
             var lb
                                 ub value rc
      get[clock] -0.0 1.797693e+308
                                      3.0 NaN
        get[mug] -0.0 1.797693e+308
                                      4.0 NaN
  get[headphone] -0.0 1.797693e+308
                                      2.0 NaN
3
      get[book] -0.0 1.797693e+308 -0.0 NaN
        get[pen] -0.0 1.797693e+308 5.0 NaN
```

Using verbose option shows the generated OPTMODEL code. Here, we can see the coefficient values of the parameters inside the model.

6.1.3 Parsing results

After the solve, primal and dual solution tables are obtained. We can print the solution tables using the <code>Model.get_solution()</code> method.

It is also possible to print the optimal solution using the get_solution_table() function.

```
In [19]: print(so.get_solution_table(get, key=ITEMS));
In [20]: print('Total value:', total_value.get_value());
```

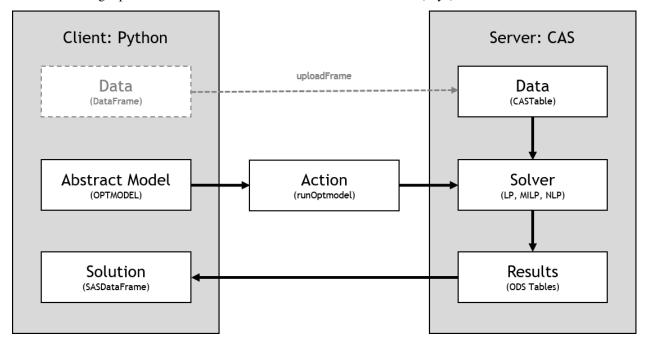
6.2 Server-side models

If the data is on the server-side (CAS or SAS), then an abstract model is generated on the client-side. This abstract model is later converted to PROC OPTMODEL code, which combines the data on the server.

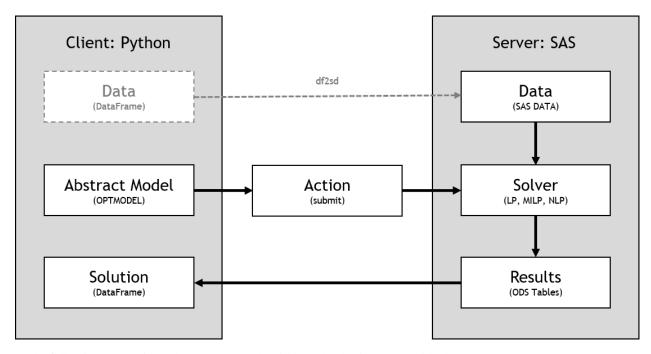
The main advantage of the server-side models is faster upload times compared to client-side. This is especially very noticable when using large chunks of variable and constraint groups.

The only disadvantage of using server-side models is that variables are often needs to be accessed directly from the resulting SASDataFrame objects. Since components of the models are abstract, accessing objects directly is often not possible.

See the following representation of the server-side model workflow for CAS (Viya) servers:



See the following representation of the server-side model workflow for SAS clients:



In the following subsections, the same example will be solved using server-side data.

6.2.1 Uploading data (Optional)

It is possible to upload client-side data to server-side when working with relatively big models.

sasoptpy supports using swat.cas.table.CASTable objects. The swat.cas.connection.CAS.upload_frame() method can be used to upload pandas.DataFrame objects to the CAS Server. Another way is to use read_table() function with upload=True option.

```
In [21]: session = CAS(hostname, port)
In [22]: m = so.Model(name='server_CAS', session=session)
NOTE: Initialized model server_CAS.
In [23]: data = [
  ....: ['clock', 8, 4, 3],
           ['mug', 10, 6, 5],
            ['headphone', 15, 7, 2],
           ['book', 20, 12, 10],
           ['pen', 1, 1, 15]
In [24]: df = pd.DataFrame(data, columns=['item', 'value', 'weight', 'limit'])
In [25]: ITEMS, (value, weight, limit) = m.read_table(
  ....: df, key=['item'], key_type='str', columns=['value', 'weight', 'limit'],
            upload=True, casout='df')
NOTE: Cloud Analytic Services made the uploaded file available as table DF in caslib.
→CASUSERHDFS (casuser) .
NOTE: The table DF has been created in caslib CASUSERHDFS(casuser) from binary data_
→uploaded to Cloud Analytic Services.
```

```
In [26]: total_weight = m.add_parameter(init = 55, name='total_weight')
In [27]: print(type(ITEMS), ITEMS)
<class 'sasoptpy.data.Set'> set_item
```

```
In [28]: print(type(total_weight), total_weight)
<class 'sasoptpy.data.ParameterValue'> total_weight
```

Since we use upload=True option, the data is uploaded to the server and we get a CASTable object. Similarly, total_weight is a parameter object here.

6.2.2 Model

```
# Variables
In [29]: get = m.add_variables(ITEMS, name='get', vartype=so.INT)
# Constraints
In [30]: m.add_constraints((get[i] <= limit[i] for i in ITEMS), name='limit_con');</pre>
In [31]: m.add_constraint(
            so.quick_sum(weight[i] * get[i] for i in ITEMS) <= total_weight,</pre>
             name='weight_con');
   . . . . :
# Objective
In [32]: total_value = so.quick_sum(value[i] * get[i] for i in ITEMS)
In [33]: m.set_objective(total_value, name='total_value', sense=so.MAX);
# Solve
In [34]: m.solve(verbose=True)
NOTE: Added action set 'optimization'.
NOTE: Converting model server_CAS to OPTMODEL.
set <str> set_item;
num value {set_item};
num weight {set_item};
num limit {set_item};
read data DF into set_item=[item] value weight limit;
num total_weight = 55;
var get {set_item} integer >= 0;
con limit_con {i_1 in set_item} : get[i_1] - limit[i_1] <= 0;</pre>
con weight_con : - sum {i_2 in set_item}(weight[i_2] * get[i_2]) + total_weight >= 0;
max total_value = sum {i_3 in set_item}(value[i_3] * get[i_3]);
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 5 rows read from table 'DF' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 5 variables (0 free, 0 fixed).
NOTE: The problem has 0 binary and 5 integer variables.
```

```
NOTE: The problem has 6 linear constraints (5 LE, 0 EQ, 1 GE, 0 range).
NOTE: The problem has 10 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 5 constraints.
NOTE: The MILP presolver removed 5 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 5 variables, 1 constraints, and 5 constraint,
→coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger BestBound Gap Time
               0 1 3 99.0000000 199.0000000 50.25%
                                   99.0000000 102.3333333 3.26%
               Ω
                       1
                                                                          Ω
                      1 3 99.0000000 102.3333333
               0
                                                              3.26%
                                                                          0
NOTE: The MILP presolver is applied again.
                   1 3 99.0000000
                                                 102.3333333
                                                                3.26%
NOTE: Optimal.
NOTE: Objective = 99.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 6 rows and 4 columns.
Out [34]:
             var lb
                                 ub value rc
                                     2.0 NaN
       get[book] -0.0 1.797693e+308
      get[clock] -0.0 1.797693e+308
                                      3.0 NaN
  get[headphone] -0.0 1.797693e+308
                                       2.0 NaN
3
        get[mug] -0.0 1.797693e+308 -0.0 NaN
        get[pen] -0.0 1.797693e+308 5.0 NaN
```

There is no differences in terms of how client-side and server-side models are written. However, the generated OPT-MODEL code is more compact for server-side models.

6.2.3 Parsing results

```
In [36]: print('Total value:', m.get_objective_value())
Total value: 99.0
```

Since there is no direct access to expressions and variables, the optimal solution is printed using the server response.

6.3 Limitations

- Nonlinear models can only be solved using runOptmodel action, hence requires SAS Viya version to be greater than or equal to 3.4.
- User defined decomposition blocks are only available in MPS mode, hence only works with client-side data.
- Mixed usage (client-side and server-side data) may not work in some cases. A quick fix would be transferring the data, in either direction.

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CHAPTER

SEVEN

EXAMPLES

Examples are provided from SAS/OR documentation.

7.1 Viya Examples / Concrete

7.1.1 Food Manufacture 1

Reference

 $http://go.documentation.sas.com/?docsetId=ormpex\&docsetTarget=ormpex_ex1_toc.htm\&docsetVersion=14.3\&locale=en$

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex01.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
    # Problem data
   OILS = ['veg1', 'veg2', 'oil1', 'oil2', 'oil3']
   PERIODS = range(1, 7)
    cost_data = [
       [110, 120, 130, 110, 115],
        [130, 130, 110, 90, 115],
        [110, 140, 130, 100, 95],
        [120, 110, 120, 120, 125],
        [100, 120, 150, 110, 105],
        [90, 100, 140, 80, 135]]
    cost = pd.DataFrame(cost_data, columns=OILS, index=PERIODS).transpose()
    hardness_data = [8.8, 6.1, 2.0, 4.2, 5.0]
   hardness = {OILS[i]: hardness_data[i] for i in range(len(OILS))}
   revenue_per_ton = 150
   veg\_ub = 200
   nonveg\_ub = 250
    store\_ub = 1000
    storage\_cost\_per\_ton = 5
```

```
hardness_1b = 3
hardness ub = 6
init_storage = 500
# Problem initialization
m = so.Model(name='food_manufacture_1', session=cas_conn)
# Problem definition
buy = m.add_variables(OILS, PERIODS, lb=0, name='buy')
use = m.add_variables(OILS, PERIODS, 1b=0, name='use')
manufacture = m.add_implicit_variable((use.sum('*', p) for p in PERIODS),
                                      name='manufacture')
last_period = len(PERIODS)
store = m.add_variables(OILS, [0] + list(PERIODS), lb=0, ub=store_ub,
                        name='store')
for oil in OILS:
    store[oil, 0].set_bounds(lb=init_storage, ub=init_storage)
    store[oil, last_period].set_bounds(lb=init_storage, ub=init_storage)
VEG = [i for i in OILS if 'veg' in i]
NONVEG = [i for i in OILS if i not in VEG]
revenue = so.quick_sum(revenue_per_ton * manufacture[p] for p in PERIODS)
rawcost = so.quick_sum(cost.at[o, p] * buy[o, p]
                       for o in OILS for p in PERIODS)
storagecost = so.quick_sum(storage_cost_per_ton * store[o, p]
                           for o in OILS for p in PERIODS)
m.set_objective(revenue - rawcost - storagecost, sense=so.MAX,
                name='profit')
# Constraints
m.add_constraints((use.sum(VEG, p) <= veg_ub for p in PERIODS),</pre>
                  name='veg_ub')
m.add_constraints((use.sum(NONVEG, p) <= nonveg_ub for p in PERIODS),</pre>
                  name='nonveg_ub')
m.add\_constraints((store[o, p-1] + buy[o, p] == use[o, p] + store[o, p])
                  for o in OILS for p in PERIODS),
                  name='flow_balance')
m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) >=
                  hardness_lb * manufacture[p] for p in PERIODS),
                  name='hardness_ub')
m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) <=</pre>
                  hardness_ub * manufacture[p] for p in PERIODS),
                  name='hardness_lb')
# Solver call
res = m.solve()
# With other solve options
m.solve(options={'with': 'lp', 'algorithm': 'PS'})
m.solve(options={'with': 'lp', 'algorithm': 'PS'})
m.solve(options={'with': 'lp', 'algorithm': 'PS'})
if res is not None:
    print(so.get_solution_table(buy, use, store))
return m.get_objective_value()
```

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Output

```
In [1]: from examples.food_manufacture_1 import test
In [2]: test(cas_conn)
NOTE: Initialized model food_manufacture_1.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint
→coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                             Objective
         Phase Iteration
                               Value
                                              Time
         D 2
                     1
                           1.019986E+06
                                               0
          D 2
                     54
                           1.233856E+05
                                                0
          P 2
                     70
                           1.078426E+05
                                                 0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Dual Simplex solve time is 0.02 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint.
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
                             Objective
         Phase Iteration
                               Value
                                              Time
          P 1
                           2.310290E+03
                     1
          P 2
                     47
                           4.266801E+04
                                                 0
                     57 8.634298E+04
                                                 0
```

```
D 2
                      71
                            1.078426E+05
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.02 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
 \rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS (casuser)' has 13 rows,
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint
→coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
                              Objective
         Phase Iteration
                                Value
                                               Time
          P 1
                            2.310290E+03
                      1
                      47
          P 2
                            4.266801E+04
                                                  0
                      57
          P 2
                            8.634298E+04
                                                  0
          D 2
                      71
                            1.078426E+05
                                                  0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6.
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
NOTE: Added action set 'optimization'.
NOTE: Converting model food_manufacture_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 95 variables (0 free, 10 fixed).
NOTE: The problem has 54 linear constraints (18 LE, 30 EQ, 6 GE, 0 range).
NOTE: The problem has 210 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 10 variables and 0 constraints.
NOTE: The LP presolver removed 10 constraint coefficients.
NOTE: The presolved problem has 85 variables, 54 constraints, and 200 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Primal Simplex algorithm is used.
                                                                          (continues on next page)
```

```
Objective
        Phase Iteration
                             Value
                                          Time
         P 1 1
                         2.310290E+03
                                           0
         P 2
                   47
                         4.266801E+04
                                             \cap
         P 2
                   57
                          8.634298E+04
                                             0
         D 2
                    71
                         1.078426E+05
                                             0
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Primal Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 95 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 54 rows and 4 columns.
          buy use
                           store
1
oil1 0
                       500.000000
oill 1
           0
                     0
                       500.000000
oil1 2
            0
                     0
                        500.000000
           0
                    0
oill 3
                       500.000000
                   0
oill 4
           0
                       500.000000
oil1 5
           0
                   0 500.000000
oil1 6
           0
                   0 500.000000
oil2 0
            _
                    - 500.000000
                   0 500.000000
oil2 1
           0
                   0 500.000000
0 500.000000
oil2 2
           0
oil2 3
           0
           0
oil2 4
                 250 250.000000
      0
750
                 250
oil2 5
                         0.000000
                 250 500.000000
oil2 6
oil3 0
                        500.000000
           0
                 250 250.000000
oil3 1
oil3 2
                 250
           0
                        0.000000
oil3 3 250 oil3 4 0
                       0.000000
                 250
                   0
                        0.000000
oil3 5 500
                   0 500.000000
oil3 6
          0
                    0 500.000000
veg1 0
                    - 500.000000
           0 85.1852 414.814815
vegl 1
veg1 2
           0 85.1852 329.629630
veg1 3
           0 85.1852 244.44444
           0 159.259
                       85.185185
veg1 4
       0 85.1852
veg1 5
                        0.000000
veg1 6 659.259 159.259
                       500.000000
                        500.000000
veg2 0
            0 114.815 385.185185
veg2 1
           0 114.815 270.370370
veg2 2
            0 114.815 155.55556
veg2 3
           0 40.7407 114.814815
veg2 4
veg2 5
           0 114.815
                        0.000000
veg2 6 540.741 40.7407 500.000000
Out[2]: 107842.592593
```

7.1.2 Food Manufacture 2

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex2_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex02.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
    # Problem data
   OILS = ['veg1', 'veg2', 'oil1', 'oil2', 'oil3']
   PERIODS = range(1, 7)
    cost_data = [
        [110, 120, 130, 110, 115],
        [130, 130, 110, 90, 115],
        [110, 140, 130, 100, 95],
        [120, 110, 120, 120, 125],
        [100, 120, 150, 110, 105],
        [90, 100, 140, 80, 135]]
   cost = pd.DataFrame(cost_data, columns=OILS, index=PERIODS).transpose()
   hardness_data = [8.8, 6.1, 2.0, 4.2, 5.0]
   hardness = {OILS[i]: hardness_data[i] for i in range(len(OILS))}
   revenue_per_ton = 150
   veg\_ub = 200
   nonveg\_ub = 250
   store\_ub = 1000
   storage_cost_per_ton = 5
   hardness_lb = 3
   hardness\_ub = 6
   init_storage = 500
   max_num_oils_used = 3
   min_oil_used_threshold = 20
    # Problem initialization
   m = so.Model(name='food_manufacture_2', session=cas_conn)
    # Problem definition
   buy = m.add_variables(OILS, PERIODS, lb=0, name='buy')
   use = m.add_variables(OILS, PERIODS, lb=0, name='use')
   manufacture = m.add_implicit_variable((use.sum('*', p) for p in PERIODS),
                                          name='manufacture')
   last_period = len(PERIODS)
   store = m.add_variables(OILS, [0] + list(PERIODS), lb=0, ub=store_ub,
                            name='store')
    for oil in OILS:
       store[oil, 0].set_bounds(lb=init_storage, ub=init_storage)
        store[oil, last_period].set_bounds(lb=init_storage, ub=init_storage)
```

```
VEG = [i for i in OILS if 'veg' in i]
NONVEG = [i for i in OILS if i not in VEG]
revenue = so.quick_sum(revenue_per_ton * manufacture[p] for p in PERIODS)
rawcost = so.quick_sum(cost.at[o, p] * buy[o, p]
                       for o in OILS for p in PERIODS)
storagecost = so.quick_sum(storage_cost_per_ton * store[o, p]
                           for o in OILS for p in PERIODS)
m.set_objective(revenue - rawcost - storagecost, sense=so.MAX,
                name='profit')
# Constraints
m.add_constraints((use.sum(VEG, p) <= veg_ub for p in PERIODS),</pre>
                  name='veg_ub')
m.add_constraints((use.sum(NONVEG, p) <= nonveg_ub for p in PERIODS),</pre>
                  name='nonveg_ub')
m.add\_constraints((store[o, p-1] + buy[o, p] == use[o, p] + store[o, p]
                  for o in OILS for p in PERIODS),
                  name='flow_balance')
m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) >=
                  hardness_lb * manufacture[p] for p in PERIODS),
                  name='hardness_ub')
m.add_constraints((so.quick_sum(hardness[o]*use[o, p] for o in OILS) <=</pre>
                  hardness_ub * manufacture[p] for p in PERIODS),
                  name='hardness_lb')
# Additions to the first problem
isUsed = m.add_variables(OILS, PERIODS, vartype=so.BIN, name='is_used')
for p in PERIODS:
    for o in VEG:
       use[o, p].set_bounds(ub=veg_ub)
    for o in NONVEG:
        use[o, p].set_bounds(ub=nonveg_ub)
m.add_constraints((use[o, p] <= use[o, p]._ub * isUsed[o, p]</pre>
                  for o in OILS for p in PERIODS), name='link')
m.add\_constraints((isUsed.sum('*', p) \le max\_num\_oils\_used))
                  for p in PERIODS), name='logical1')
m.add_constraints((use[o, p] >= min_oil_used_threshold * isUsed[o, p]
                  for o in OILS for p in PERIODS), name='logical2')
m.add_constraints((isUsed[o, p] <= isUsed['oil3', p]</pre>
                  for o in ['veg1', 'veg2'] for p in PERIODS),
                  name='logical3')
res = m.solve()
if res is not None:
    print(so.get_solution_table(buy, use, store, isUsed))
return m.get_objective_value()
```

Output

```
In [1]: from examples.food_manufacture_2 import test
In [2]: test(cas_conn)
NOTE: Initialized model food_manufacture_2.
NOTE: Added action set 'optimization'.
(continues on next page)
```

```
NOTE: Converting model food_manufacture_2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 125 variables (0 free, 10 fixed).
NOTE: The problem has 30 binary and 0 integer variables.
NOTE: The problem has 132 linear constraints (66 LE, 30 EQ, 36 GE, 0 range).
NOTE: The problem has 384 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 50 variables and 10 constraints.
NOTE: The MILP presolver removed 66 constraint coefficients.
NOTE: The MILP presolver modified 6 constraint coefficients.
NOTE: The presolved problem has 75 variables, 122 constraints, and 318 constraint,
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger BestBound
                                                               Gap
                                                                        Time
                          3 29000.0000000
               0
                       1
                                                     343250
                                                               91.55%
                                                              72.98%
               0
                        1
                              3 29000.0000000
                                                      107333
                                                                           Ω
                             3 29000.0000000
                                                                          0
               Ω
                       1
                                                     105799 72.59%
                                                                          0
               Ω
                       1
                             3 29000.0000000
                                                     105650 72.55%
               0
                       1
                             3 29000.0000000
                                                     105650 72.55%
                                                                          0
               0
                       1
                             3 29000.0000000
                                                     105650 72.55%
                                                                          Ω
               0
                       1
                             3 29000.0000000
                                                     105650 72.55%
               \cap
                       1
                             3 29000.0000000
                                                     105650 72.55%
                      1 4 44000.0000000
               0
                                                     105650 58.35%
NOTE: The MILP solver added 14 cuts with 69 cut coefficients at the root.
              42 29 5 93416.6666667 104429
                                                             10.55%
                      35
                             6 93416.6666667
                                                      104040 10.21%
              5.5
                                                                           0
                      36 7 99008.3333333
61 8 99683.3333333
57 9 99872.2222222
57 10 100214
                                                      104040
                                                               4.84%
                                                                           0
              93
                                                      104040
                                                               4.19%
                                                              3.58%
             105
                                                      103576
                                                                           1
             114
                                                      103576 3.25%
                                                                           1
                     52
                                       100214
                                                     103431 3.11%
             173
                            11
                     62
                                       100279
                                                     103192 2.82%
             2.49
                            12
                                                                          1
             417
                      0
                             12
                                       100279
                                                      100279 0.00%
                                                                          1
NOTE: Optimal.
NOTE: Objective = 100278.7037.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS (casuser)' has 18 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 125 rows and 6.
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS (casuser)' has 132 rows and 4.
⇔columns.
          buv
                  use
                           store is_used
1
   2
                    - 500.000000
oill 0
oill 1
           0
                   0 500.000000
oill 2
           0
                   0 500.000000
oill 3
           0
                   0 500.000000
                                        0
           0
                   0 500.000000
                                        0
oil1 4
                  0 500.000000
         0
oil1 5
                                        0
```

```
oil1 6
                                                                                                              0 500.000000
  oil2 0
                                                                                                                    - 500.000000

      oil2 0
      -
      -
      500.000000

      oil2 1
      0
      0
      500.000000

      oil2 2
      0
      0
      500.000000

      oil2 3
      0
      40
      460.000000

      oil2 4
      0
      230
      230.000000

      oil2 5
      0
      230
      500.000000

      oil3 0
      -
      -
      500.000000

      oil3 1
      0
      250
      250.000000

      oil3 2
      0
      250
      0.000000

      oil3 3
      770
      210
      560.000000

      oil3 4
      0
      20
      540.000000

      oil3 5
      -0
      20
      520.000000

      oil3 6
      0
      20
      500.000000

      veg1 0
      -
      -
      500.000000

      veg1 1
      0
      85.1852
      414.814815

                                                                                                                                                                                                                                0
                                                                                                                                                                                                                                Ω
                                                                                                                                                                                                                                1
                                                                                                                                                                                                                                1
                                                                                                                                                                                                                                1
                                                                                                                                                                                                                                1
                                                                                                                                                                                                                             1
                                                                                                                                                                                                                             1

    veg1 0
    -
    -
    500.000000

    veg1 1
    0
    85.1852
    414.814815

    veg1 2
    0
    85.1852
    329.629630

    veg1 3
    0
    0
    329.629630

    veg1 4
    0
    155
    174.629630

    veg1 5
    -0
    155
    19.629630

    veg1 6
    480.37
    0
    500.000000

    veg2 0
    -
    -
    500.000000

                                                                                                                                                                                                                             1
                                                                                                                                                                                                                                1
                                                                                                                                                                                                                                0
                                                                                                                                                                                                                                1

    veg2 1
    0
    114.815
    385.185185

    veg2 2
    0
    114.815
    270.370370

    veg2 3
    -0
    200
    70.370370

                                                                 0 114.815 385.185185
                                                                                                                                                                                                                               1
                                                         -0 200 70.370370
                                                                                                                                                                                                                             1

    veg2 4
    -0
    0
    70.370370

    veg2 5
    0
    0
    70.370370

    veg2 6
    629.63
    200
    500.000000

                                                                                                                                                                                                                             0
                                                                                                                                                                                                                               0
                                                                                                                                                                                                                                1
  Out[2]: 100278.703704
```

7.1.3 Factory Planning 1

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex3_toc.htm&docsetVersion=14.3&locale=en

https://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex03.html

Model

```
import sasoptpy as so
import pandas as pd

def test(cas_conn):

    m = so.Model(name='factory_planning_1', session=cas_conn)

# Input data
    product_list = ['prod{}'.format(i) for i in range(1, 8)]
    product_data = pd.DataFrame([10, 6, 8, 4, 11, 9, 3],
```

```
columns=['profit'], index=product_list)
demand_data = [
    [500, 1000, 300, 300, 800, 200, 100],
    [600, 500, 200, 0, 400, 300, 150],
    [300, 600, 0,
                      0, 500, 400, 100],
    [200, 300, 400, 500, 200, 0, 100],
           100, 500, 100, 1000, 300,
    [500, 500, 100, 300, 1100, 500, 60]]
demand_data = pd.DataFrame(
    demand_data, columns=product_list, index=range(1, 7))
machine_types_data = [
    ['grinder', 4],
    ['vdrill', 2],
    ['hdrill', 3],
    ['borer', 1],
    ['planer', 1]]
machine_types_data = pd.DataFrame(machine_types_data, columns=[
    'machine_type', 'num_machines']).set_index(['machine_type'])
machine_type_period_data = [
    ['grinder', 1, 1],
    ['hdrill', 2, 2],
    ['borer', 3, 1],
    ['vdrill', 4, 1],
    ['grinder', 5, 1],
    ['vdrill', 5, 1],
    ['planer', 6, 1],
    ['hdrill', 6, 1]]
machine_type_period_data = pd.DataFrame(machine_type_period_data, columns=[
    'machine_type', 'period', 'num_down'])
machine_type_product_data = [
    ['grinder', 0.5, 0.7, 0,
                               0,
0.3, 0, 0.0
                                       0.3, 0.2, 0.5],
    ['vdrill', 0.1, 0.2, 0, 0.3, 0, ['hdrill', 0.2, 0, 0.8, 0, 0.0]
                                              0.6, 0],
                                  0.07, 0.1, 0,
    ['planer', 0,
                            0.01, 0,
                                        0.05, 0,
                     0,
machine_type_product_data = \
    pd.DataFrame(machine_type_product_data, columns=['machine_type'] +
                 product_list).set_index(['machine_type'])
store_ub = 100
storage_cost_per_unit = 0.5
final storage = 50
num_hours_per_period = 24 * 2 * 8
# Problem definition
PRODUCTS = product_list
PERIODS = range(1, 7)
MACHINE_TYPES = machine_types_data.index.values
num_machine_per_period = pd.DataFrame()
for i in range (1, 7):
   num_machine_per_period[i] = machine_types_data['num_machines']
for _, row in machine_type_period_data.iterrows():
    num_machine_per_period.at[row['machine_type'],
                              row['period']] -= row['num_down']
make = m.add_variables(PRODUCTS, PERIODS, lb=0, name='make')
sell = m.add_variables(PRODUCTS, PERIODS, 1b=0, ub=demand_data.transpose(),
                                                                      (continues on next page)
```

```
name='sell')
store = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=store_ub, name='store')
for p in PRODUCTS:
    store[p, 6].set_bounds(lb=final_storage, ub=final_storage+1)
storageCost = storage_cost_per_unit * store.sum('*', '*')
revenue = so.quick_sum(product_data.at[p, 'profit'] * sell[p, t]
                       for p in PRODUCTS for t in PERIODS)
m.set_objective(revenue-storageCost, sense=so.MAX, name='total_profit')
production_time = machine_type_product_data
m.add_constraints((
    so.quick_sum(production_time.at[mc, p] * make[p, t] for p in PRODUCTS)
    <= num_hours_per_period * num_machine_per_period.at[mc, t]</pre>
    for mc in MACHINE_TYPES for t in PERIODS), name='machine_hours')
m.add\_constraints(((store[p, t-1] if t-1 in PERIODS else 0) + make[p, t] ==
                  sell[p, t] + store[p, t] for p in PRODUCTS
                  for t in PERIODS),
                  name='flow_balance')
res = m.solve()
if res is not None:
    print(so.get_solution_table(make, sell, store))
return m.get_objective_value()
```

Output

```
In [1]: from examples.factory_planning_1 import test
In [2]: test(cas_conn)
NOTE: Initialized model factory_planning_1.
NOTE: Added action set 'optimization'.
NOTE: Converting model factory_planning_1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 126 variables (0 free, 6 fixed).
NOTE: The problem has 72 linear constraints (30 LE, 42 EQ, 0 GE, 0 range).
NOTE: The problem has 281 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 24 variables and 23 constraints.
NOTE: The LP presolver removed 91 constraint coefficients.
NOTE: The presolved problem has 102 variables, 49 constraints, and 190 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
         Phase Iteration
                               Value
                                              Time
          D 2
                     1
                           9.501963E+04
          P 2
                      34
                          9.371518E+04
NOTE: Optimal.
NOTE: Objective = 93715.178571.
```

```
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 126 rows and 6
\hookrightarrowcolumns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 72 rows and 4 columns.
                           sell store
              make
    2.
prod1 1 500.000000 500.000000
                                 0.0
prod1 2 700.000000 600.000000 100.0
prod1 3
          0.000000 100.000000
                                 0.0
prod1 4 200.000000 200.000000
                                 0.0
prod1 5
          0.000000
                      0.000000
                                  0.0
prod1 6 550.000000
                     500.000000
                                  50.0
prod2 1 888.571429
                      888.571429
                                  0.0
prod2 2
         600.000000
                      500.000000 100.0
prod2 3
         0.000000
                      100.000000
                                  0.0
prod2 4
         300.000000
                      300.000000
                                   0.0
prod2 5
         100.000000
                      100.000000
                                   0.0
prod2 6
         550.000000
                      500.000000
                                  50.0
prod3 1 382.500000
                     300.000000
                                 82.5
prod3 2 117.500000
                     200.000000
                                 0.0
prod3 3
         0.000000
                      0.000000
                                 0.0
prod3 4 400.000000 400.000000
                                  0.0
prod3 5 600.000000
                     500.000000 100.0
prod3 6
          0.000000
                      50.000000
                                 50.0
        300.000000
                     300.000000
                                  0.0
prod4 1
prod4 2
          0.000000
                      0.000000
                                   0.0
prod4 3
           0.000000
                       0.000000
                                   0.0
prod4 4
         500.000000
                     500.000000
                                   0.0
prod4 5
         100.000000
                      100.000000
                                   0.0
prod4 6
         350.000000
                      300.000000
                                 50.0
                    800.000000
prod5 1 800.000000
                                  0.0
prod5 2 500.000000 400.000000 100.0
prod5 3
         0.000000 100.000000
                                 0.0
prod5 4 200.000000 200.000000
                                  0.0
prod5 5 1100.000000 1000.000000 100.0
prod5 6 0.000000
                     50.000000
prod6 1 200.000000
                     200.000000
                                  0.0
prod6 2 300.000000
                     300.000000
                                  0.0
                      400.000000
prod6 3
        400.000000
                                  0.0
prod6 4
                      0.000000
         0.000000
                                   0.0
prod6 5
        300.000000
                      300.000000
                                  0.0
prod6 6
         550.000000
                      500.000000
                                  50.0
prod7 1
           0.000000
                      0.000000
                                  0.0
prod7 2
         250.000000
                     150.000000 100.0
prod7 3
                                 0.0
         0.000000
                     100.000000
prod7 4 100.000000
                                  0.0
                     100.000000
prod7 5 100.000000
                      0.000000 100.0
prod7 6
           0.000000
                    50.000000
                                 50.0
Out[2]: 93715.178571
```

7.1.4 Factory Planning 2

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex4_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex04.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
    m = so.Model(name='factory_planning_2', session=cas_conn)
    # Input data
    product_list = ['prod{}'.format(i) for i in range(1, 8)]
    product_data = pd.DataFrame([10, 6, 8, 4, 11, 9, 3],
                                 columns=['profit'], index=product_list)
    demand_data = [
        [500, 1000, 300, 300, 800, 200, 100],
        [600, 500, 200, 0, 400, 300, 150],
        [300, 600, 0, 0, 500, 400, 100],
        [200, 300, 400, 500, 200,
                                     0, 100],
        [0,
              100, 500, 100, 1000, 300,
        [500, 500, 100, 300, 1100, 500, 60]]
    demand_data = pd.DataFrame(
        demand_data, columns=product_list, index=range(1, 7))
    machine_type_product_data = [
        ['grinder', 0.5, 0.7, 0, 0, 0.3]
['vdrill', 0.1, 0.2, 0, 0.3, 0,
['hdrill', 0.2, 0, 0.8, 0, 0,
                                      0, 0.3, 0.2, 0.5],
                                            0,
                                                   0.6, 01,
                                                   0, 0.6],
        ['borer', 0.05, 0.03, 0, 0.07, 0.1, 0,
        ['planer', 0, 0,
                                0.01, 0, 0.05, 0,
                                                       0.0511
    machine_type_product_data = \
        pd.DataFrame(machine_type_product_data, columns=['machine_type'] +
                     product_list).set_index(['machine_type'])
    machine_types_data = [
        ['grinder', 4, 2],
        ['vdrill', 2, 2],
        ['hdrill', 3, 3],
        ['borer', 1, 1], ['planer', 1, 1]]
    machine_types_data = pd.DataFrame(machine_types_data, columns=[
        'machine_type', 'num_machines', 'num_machines_needing_maintenance'])\
        .set_index(['machine_type'])
    store\_ub = 100
    storage_cost_per_unit = 0.5
    final_storage = 50
    num_hours_per_period = 24 * 2 * 8
```

```
# Problem definition
PRODUCTS = product_list
profit = product_data['profit']
PERIODS = range(1, 7)
MACHINE_TYPES = machine_types_data.index.tolist()
num_machines = machine_types_data['num_machines']
make = m.add_variables(PRODUCTS, PERIODS, lb=0, name='make')
sell = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=demand_data.transpose(),
                       name='sell')
store = m.add_variables(PRODUCTS, PERIODS, lb=0, ub=store_ub, name='store')
for p in PRODUCTS:
    store[p, 6].set_bounds(lb=final_storage, ub=final_storage)
storageCost = so.quick_sum(
    storage_cost_per_unit * store[p, t] for p in PRODUCTS for t in PERIODS)
revenue = so.quick_sum(profit[p] * sell[p, t]
                       for p in PRODUCTS for t in PERIODS)
m.set_objective(revenue-storageCost, sense=so.MAX, name='total_profit')
num_machines_needing_maintenance = \
    machine_types_data['num_machines_needing_maintenance']
numMachinesDown = m.add_variables(MACHINE_TYPES, PERIODS, vartype=so.INT,
                                  lb=0, name='numMachinesDown')
production_time = machine_type_product_data
m.add_constraints((
    so.quick_sum(production_time.at[mc, p] * make[p, t] for p in PRODUCTS)
    <= num_hours_per_period *
    (num_machines[mc] - numMachinesDown[mc, t])
    for mc in MACHINE_TYPES for t in PERIODS), name='machine_hours_con')
m.add_constraints((so.quick_sum(numMachinesDown[mc, t] for t in PERIODS) ==
                   num_machines_needing_maintenance[mc]
                   for mc in MACHINE_TYPES), name='maintenance_con')
m.add_constraints(((store[p, t-1] if t-1 in PERIODS else 0) + make[p, t] ==
                  sell[p, t] + store[p, t]
                  for p in PRODUCTS for t in PERIODS),
                  name='flow_balance_con')
res = m.solve()
if res is not None:
    print(so.get_solution_table(make, sell, store))
    print(so.get_solution_table(numMachinesDown).unstack(level=-1))
print(m.get_solution_summary())
print (m.get_problem_summary())
return m.get_objective_value()
```

Output

```
In [1]: from examples.factory_planning_2 import test
In [2]: test(cas_conn)
NOTE: Initialized model factory_planning_2.
NOTE: Added action set 'optimization'.
NOTE: Converting model factory_planning_2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 156 variables (0 free, 13 fixed).
NOTE: The problem has 0 binary and 30 integer variables.
NOTE: The problem has 77 linear constraints (30 LE, 47 EQ, 0 GE, 0 range).
NOTE: The problem has 341 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 27 variables and 15 constraints.
NOTE: The MILP presolver removed 63 constraint coefficients.
NOTE: The MILP presolver modified 16 constraint coefficients.
NOTE: The presolved problem has 129 variables, 62 constraints, and 278 constraint,
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger BestBound
                                                                 Gap Time
                             2 92755.0000000
                                                      116455 20.35%
               0
                       1
                              2 92755.0000000
               0
                        1
                                                       116455 20.35%
                                                               20.14%
               0
                        1
                              2 92755.0000000
                                                       116141
                            2 92755.0000000
2 92755.0000000
2 92755.0000000
2 92755.0000000
2 92755.0000000
               0
                        1
                                                       115660 19.80%
                                                                             0
               \cap
                        1
                                                       114597 19.06%
                                                                            0
               0
                       1
                                                       113265 18.11%
                                                                            0
               0
                       1
                                                      111849 17.07%
                                                                            0
               0
                       1
                                                      110679 16.19%
               0
                       1
                             2 92755.0000000
                                                      109751 15.49%
                                                                            1
               0
                       1
                             2 92755.0000000
                                                      109476 15.27%
                                                                            1
               0
                       1
                             2 92755.0000000
                                                      109039 14.93%
                                                                            1
                             2 92755.0000000
                                                       108998 14.90%
               Ω
                       1
                                                                            1
                             2 92755.0000000
               0
                       1
                                                       108924 14.84%
                              2 92755.0000000
                                                               14.82%
               0
                        1
                                                       108893
               0
                               2 92755.0000000
                                                               14.80%
                        1
                                                       108863
               0
                        1
                                         108855
                                                       108863
NOTE: The MILP solver added 46 cuts with 181 cut coefficients at the root.
NOTE: Optimal within relative gap.
NOTE: Objective = 108855.00931.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows.
⇒and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 156 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 77 rows and 4 columns.
                            sell
               make
                                       store
         500.000000 500.000000
                                    0.000000
prod1 1
prod1 2 600.000665 600.000000
                                   0.000665
```

```
prod1 3 399.999335
                    300.000000 100.000000
prod1 4 0.000544 100.000544
                               0.000000
prod1 5
         0.000000
                     0.000000
                                0.000000
       550.000000
                   500.000000
                               50.000000
prod1 6
prod2 1 1000.000000 1000.000000
                                0.000000
prod2 2
        500.000000
                    500.000000
                                 0.000000
       699.998346
prod2 3
                   599.998891
                               99.999456
                   100.001089
prod2 4
         0.003085
                                0.001452
prod2 5 100.000323 100.000000
                                0.001775
prod2 6 549.998225 500.000000 50.000000
prod3 1 300.000000 300.000000
                               0.000000
prod3 2 199.999677 199.999677
                               0.000000
prod3 3 99.999456
                    0.000000 99.999456
prod3 4 0.002178 100.001633
                               0.000000
prod3 5 500.000000 500.000000
                               0.000000
prod3 6 150.000000 100.000000
                               50.000000
prod4 1 300.000000
                    300.000000
                                0.000000
                    0.000000
prod4 2
         0.000000
                                0.000000
prod4 3
         99.999456
                                99.999456
prod4 4
         0.002722
                    100.002178
                                0.000000
prod4 5
        100.001129
                    100.000000
                                0.001129
prod4 6 349.998871
                    300.000000
                                50.000000
prod5 1 800.000000 800.000000 0.0000000
prod5 2 399.999544 399.999322
                                0.000222
prod5 3 599.998619 499.999113 99.999728
prod5 4 0.000817 100.000544 0.000000
prod5 5 1000.004598 1000.000000 0.004598
prod5 6 1149.995402 1100.000000 50.000000
                               0.000000
prod6 1 200.000000 200.000000
prod6 2 300.000000
                   300.000000
                               0.000000
                   400.000000
prod6 3
       400.000000
                               0.000000
prod6 4
         0.000000
                     0.000000
                                0.000000
prod6 5
        300.000000
                   300.000000
                                 0.000000
                               50.000000
prod6 6 550.000000 500.000000
                               0.000000
prod7 1 100.000000 100.000000
prod7 2 150.000222 150.000000
                                0.000222
prod7 3 199.999234 100.000000 99.999456
prod7 4 0.000544 100.000000
                               0.000000
prod7 5
         0.000355
                    0.000000
                               0.000355
prod7 6 109.999645 60.000000 50.000000
      numMachinesDown numMachinesDown numMachinesDown \
                   1
1
             0.000000
                           0.000000
                                          0.000002
                                                        0.999995
borer
grinder
             0.000000
                           0.000000
                                          0.000000
                                                        2.000000
hdrill
             0.999999
                           2.000001
                                          0.000000
                                                        0.000000
planer
             0.000000
                           0.000003
                                          0.000002
                                                        0.999995
vdrill
             0.000000
                           0.000000
                                          0.000000
                                                        1.999996
       numMachinesDown numMachinesDown
2
           5
1
borer
             0.000000
                           0.000003
grinder
             0.000000
                           0.000000
             0.000000
                           0.000000
hdrill
             0.000000
                           0.000000
planer
             0.000001
                            0.000003
vdrill
```

			(continued from previous page)
Solution Summary			
		Value	
Label		Value	
Solver		MILP	
Algorithm	В:	ranch and Cut	
Objective Function		total_profit	
	Optimal within	<u></u> -	
Objective Value	1	108855.00931	
Relative Gap		0.0000746796	
Absolute Gap		8.1298591363	
Primal Infeasibility		2.060574E-13	
Bound Infeasibility		0	
Integer Infeasibility		5.4448817E-6	
Best Bound		108863.13917	
Nodes		1	
Solutions Found		3	
Iterations		328	
Presolve Time		0.03	
Solution Time		1.75	
Problem Summary			
	77.7		
Label	Value		
Objective Sense	Maximization		
Objective Sense Objective Function	total_profit		
Objective Type	Linear		
0.0 J 0.0 0 1 1 1 F 0			
Number of Variables	156		
Bounded Above	0		
Bounded Below	72		
Bounded Below and Above			
Free	0		
Fixed	13		
Binary	0		
Integer	30		
Number of Constraints	77		
Linear LE (<=)	30		
Linear EQ (=)	47		
Linear GE (>=)	0		
Linear Range	0		
Constraint Coefficients	s 341		
Out[2]: 108855.009314			

7.1.5 Manpower Planning

Reference

 $http://go.documentation.sas.com/?docsetId=ormpex\&docsetTarget=ormpex_ex5_toc.htm\&docsetVersion=14.3\&locale=en$

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex05.html

Model

```
import sasoptpy as so
import pandas as pd
import math
def test(cas_conn):
    # Input data
   demand_data = pd.DataFrame([
        [0, 2000, 1500, 1000],
        [1, 1000, 1400, 1000],
        [2, 500, 2000, 1500],
        [3, 0, 2500, 2000]
        ], columns=['period', 'unskilled', 'semiskilled', 'skilled'])\
        .set_index(['period'])
    worker_data = pd.DataFrame([
        ['unskilled', 0.25, 0.10, 500, 200, 1500, 50, 500],
        ['semiskilled', 0.20, 0.05, 800, 500, 2000, 50, 400],
                      0.10, 0.05, 500, 500, 3000, 50, 400]
        ], columns=['worker', 'waste_new', 'waste_old', 'recruit_ub',
                    'redundancy_cost', 'overmanning_cost', 'shorttime_ub',
                    'shorttime_cost']).set_index(['worker'])
    retrain_data = pd.DataFrame([
        ['unskilled', 'semiskilled', 200, 400],
        ['semiskilled', 'skilled', math.inf, 500],
        ], columns=['worker1', 'worker2', 'retrain_ub', 'retrain_cost']).\
        set_index(['worker1', 'worker2'])
    downgrade_data = pd.DataFrame([
        ['semiskilled', 'unskilled'],
        ['skilled', 'semiskilled'],
        ['skilled', 'unskilled']
        ], columns=['worker1', 'worker2'])
    semiskill_retrain_frac_ub = 0.25
   downgrade_leave_frac = 0.5
   overmanning_ub = 150
   shorttime_frac = 0.5
   WORKERS = worker_data.index.tolist()
   PERIODS0 = demand_data.index.tolist()
   PERIODS = PERIODS0[1:]
    RETRAIN_PAIRS = [i for i, _ in retrain_data.iterrows()]
   DOWNGRADE_PAIRS = [(row['worker1'], row['worker2'])
                       for _, row in downgrade_data.iterrows()]
   waste_old = worker_data['waste_old']
   waste_new = worker_data['waste_new']
   redundancy_cost = worker_data['redundancy_cost']
   overmanning_cost = worker_data['overmanning_cost']
    shorttime_cost = worker_data['shorttime_cost']
    retrain_cost = retrain_data['retrain_cost'].unstack(level=-1)
    # Initialization
   m = so.Model(name='manpower_planning', session=cas_conn)
```

```
# Variables
numWorkers = m.add_variables(WORKERS, PERIODSO, name='numWorkers', 1b=0)
demand0 = demand_data.loc[0]
for w in WORKERS:
   numWorkers[w, 0].set_bounds(lb=demand0[w], ub=demand0[w])
numRecruits = m.add_variables(WORKERS, PERIODS, name='numRecruits', 1b=0)
worker_ub = worker_data['recruit_ub']
for w in WORKERS:
   for p in PERIODS:
        numRecruits[w, p].set_bounds(ub=worker_ub[w])
numRedundant = m.add_variables(WORKERS, PERIODS, name='numRedundant', 1b=0)
numShortTime = m.add_variables(WORKERS, PERIODS, name='numShortTime', 1b=0)
shorttime_ub = worker_data['shorttime_ub']
for w in WORKERS:
    for p in PERIODS:
        numShortTime.set_bounds(ub=shorttime_ub[w])
numExcess = m.add_variables(WORKERS, PERIODS, name='numExcess', 1b=0)
retrain_ub = pd.DataFrame()
for i in PERIODS:
   retrain_ub[i] = retrain_data['retrain_ub']
numRetrain = m.add_variables(RETRAIN_PAIRS, PERIODS, name='numRetrain',
                             lb=0, ub=retrain_ub)
numDowngrade = m.add_variables(DOWNGRADE_PAIRS, PERIODS,
                               name='numDowngrade', lb=0)
# Constraints
m.add_constraints((numWorkers[w, p]
                  - (1-shorttime_frac) * numShortTime[w, p]
                  - numExcess[w, p] == demand_data.loc[p, w]
                  for w in WORKERS for p in PERIODS), name='demand')
m.add_constraints((numWorkers[w, p] ==
                  (1 - waste_old[w]) * numWorkers[w, p-1]
                  + (1 - waste_new[w]) * numRecruits[w, p]
                  + (1 - waste_old[w]) * numRetrain.sum('*', w, p)
                  + (1 - downgrade_leave_frac) *
                  numDowngrade.sum('*', w, p)
                  - numRetrain.sum(w, '*', p)
                  - numDowngrade.sum(w, '*', p)
                  - numRedundant[w, p]
                  for w in WORKERS for p in PERIODS),
                  name='flow_balance')
m.add_constraints((numRetrain['semiskilled', 'skilled', p] <=</pre>
                  semiskill_retrain_frac_ub * numWorkers['skilled', p]
                  for p in PERIODS), name='semiskill_retrain')
m.add\_constraints((numExcess.sum('*', p) <= overmanning\_ub))
                  for p in PERIODS), name='overmanning')
# Objectives
redundancy = so.Expression(numRedundant.sum('*', '*'), name='redundancy')
cost = so.Expression(so.quick_sum(redundancy_cost[w] * numRedundant[w, p] +
                                  shorttime_cost[w] * numShortTime[w, p] +
                                  overmanning_cost[w] * numExcess[w, p]
                                  for w in WORKERS for p in PERIODS)
                     + so.quick_sum(
                         retrain_cost.loc[i, j] * numRetrain[i, j, p]
                         for i, j in RETRAIN_PAIRS for p in PERIODS),
                     name='cost')
```

```
m.set_objective(redundancy, sense=so.MIN, name='redundancy_obj')
res = m.solve()
if res is not None:
    print('Redundancy:', redundancy.get_value())
    print('Cost:', cost.get_value())
    print(so.get_solution_table(
        numWorkers, numRecruits, numRedundant, numShortTime, numExcess))
    print(so.get_solution_table(numRetrain))
    print(so.get_solution_table(numDowngrade))
m.set_objective(cost, sense=so.MIN, name='cost_obj')
res = m.solve()
if res is not None:
    print('Redundancy:', redundancy.get_value())
    print('Cost:', cost.get_value())
    print(so.get_solution_table(numWorkers, numRecruits, numRedundant,
                                numShortTime, numExcess))
    print(so.get_solution_table(numRetrain))
    print(so.get_solution_table(numDowngrade))
return m.get_objective_value()
```

Output

```
In [1]: from examples.manpower_planning import test
In [2]: test(cas_conn)
NOTE: Initialized model manpower_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model manpower_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 3 fixed).
NOTE: The problem has 24 linear constraints (6 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 108 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 9 constraints.
NOTE: The LP presolver removed 21 constraint coefficients.
NOTE: The presolved problem has 42 variables, 15 constraints, and 87 constraint,
→coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                              Objective
                                               Time
         Phase Iteration
                               Value
                    1
                           5.223600E+02
          D 2
          P 2
                          8.417969E+02
                      13
NOTE: Optimal.
NOTE: Objective = 841.796875.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
\rightarrowand 4 columns.
                                                                          (continues on next page)
```

```
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 24 rows and 4 columns.
Redundancy: 841.796875
Cost: 1462047.697368
              numWorkers numRecruits numRedundant numShortTime numExcess
semiskilled 0 1500.00000
semiskilled 1 1442.96875
                                 0
                                            0
                                                          50
                                                             17.9688
semiskilled 2 2000.00000 682.198
                                             0
                                                          Ω
                                                                0
                                             0
                                                          0
semiskilled 3 2500.00000 645.724
                                                                    Ω
skilled 0 1000.00000
                                 0
skilled 1 1025.00000
                                             0
                                                          50
                                                                    0
skilled 2 1525.00000
                               500
                                             0
                                                          50
skilled 3 2000.00000
                               500
                                             0
                                                          0
                                                                    0
unskilled 0 2000.00000
unskilled 1 1157.03125
                                       442.969
                                 0
                                                        50 132.031
unskilled 2 675.00000
                                 0
                                        166.328
                                                         50 150
                          0
unskilled 3 175.00000
                                                                 150
                                                        50
                                        232.5
                         numRetrain
semiskilled skilled 1 256.250000
semiskilled skilled 2 106.578947 semiskilled skilled 3 106.578947
unskilled semiskilled 1 200.000000
unskilled semiskilled 2 200.000000
unskilled semiskilled 3 200.000000
                        numDowngrade
                     3
          2
semiskilled unskilled 1
                              0.0000
                             0.0000
semiskilled unskilled 2
semiskilled unskilled 3
                               0.0000
skilled semiskilled 1
                            168.4375
skilled semiskilled 2
skilled semiskilled 3
skilled unskilled 1
                               0.0000
                              0.0000
                              0.0000
skilled unskilled 2 skilled unskilled 3
                              0.0000
                           0.0000
NOTE: Added action set 'optimization'.
NOTE: Converting model manpower_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 3 fixed).
NOTE: The problem has 24 linear constraints (6 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 108 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 30 variables and 11 constraints.
NOTE: The LP presolver removed 39 constraint coefficients.
NOTE: The presolved problem has 33 variables, 13 constraints, and 69 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                           Objective
        Phase Iteration
                             Value
                                            Time
         D 2 1
                          2.143730E+05
                                                                     (continues on next page)
```

```
D 2
                                    4.986773E+05
NOTE: Optimal.
NOTE: Objective = 498677.28532.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and_
 \rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS (casuser)' has 13 rows,
 \rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 24 rows and 4 columns.
Redundancy: 1423.718837
Cost: 498677.285319
                numWorkers numRecruits numRedundant numShortTime numExcess
semiskilled 0 1500.0 semiskilled 1 1400.0
                      1400.0
                                            0
                                                             0
                                                                               0
                                                                                            0
                       2500.0 800
1000.0 -
1000.0 55.5556
1500.0
semiskilled 2
                      2000.0
                                         800
                                                             0
                                                                               0
                                                                                            0

      semiskilled 2
      2000.0
      800

      semiskilled 3
      2500.0
      800

      skilled 0
      1000.0
      -

      skilled 1
      1000.0
      55.5556

      skilled 2
      1500.0
      500

      skilled 3
      2000.0
      -

      unskilled 1
      1000.0
      0

      unskilled 2
      500.0
      0

                                                             0
                                                                               0
                                                                                            0
                                                              0
                                                                               0
                                                              Ω
                                                                               0
                                                                                            0
                                                             0
                                                                               0
                                                                                            0
                                                       812.5
                                             0
                                                                              Ω
                                                                                            Ω
                       500.0
unskilled 2
                                            0
                                                     257.618
                                                                              0
                                                                                            0
                        0.0
unskilled 3
                                            0
                                                      353.601
                                                                             0
                                numRetrain
              2
                             3
semiskilled skilled 1
                                    0.000000
semiskilled skilled 2 105.263158
semiskilled skilled 3 131.578947
unskilled semiskilled 1
                                    0.000000
unskilled semiskilled 2 142.382271
unskilled semiskilled 3 96.398892
                                numDowngrade
                             3
              2
1
semiskilled unskilled 1
                                           25.0
semiskilled unskilled 2
                                            0.0
semiskilled unskilled 3
                                             0.0
skilled semiskilled 1
skilled semiskilled 2
skilled semiskilled 3
skilled unskilled 1
                                             0.0
                                             0.0
                                             0.0
                                             0.0
skilled unskilled 2 skilled unskilled 3
             unskilled 2
                                             0.0
                                              0.0
Out[2]: 498677.285319
```

7.1.6 Refinery Optimization

Reference

 $http://go.documentation.sas.com/?docsetId=ormpex\&docsetTarget=ormpex_ex6_toc.htm\&docsetVersion=14.3\&locale=en$

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex06.html

Model

```
import sasoptpy as so
import pandas as pd
import numpy as np
def test(cas_conn):
   m = so.Model(name='refinery_optimization', session=cas_conn)
    crude_data = pd.DataFrame([
        ['crude1', 20000],
        ['crude2', 30000]
        ], columns=['crude', 'crude_ub']).set_index(['crude'])
    arc_data = pd.DataFrame([
        ['source', 'crude1', 6],
        ['source', 'crude2', 6],
        ['crude1', 'light_naphtha', 0.1],
        ['crude1', 'medium_naphtha', 0.2],
        ['crude1', 'heavy_naphtha', 0.2],
        ['crude1', 'light_oil', 0.12],
        ['crude1', 'heavy_oil', 0.2],
        ['crude1', 'residuum', 0.13],
        ['crude2', 'light_naphtha', 0.15],
        ['crude2', 'medium_naphtha', 0.25],
        ['crude2', 'heavy_naphtha', 0.18],
        ['crude2', 'light_oil', 0.08],
        ['crude2', 'heavy_oil', 0.19],
        ['crude2', 'residuum', 0.12],
        ['light_naphtha', 'regular_petrol', np.nan],
        ['light_naphtha', 'premium_petrol', np.nan],
        ['medium_naphtha', 'regular_petrol', np.nan],
        ['medium_naphtha', 'premium_petrol', np.nan],
        ['heavy_naphtha', 'regular_petrol', np.nan],
        ['heavy_naphtha', 'premium_petrol', np.nan],
        ['light_naphtha', 'reformed_gasoline', 0.6],
        ['medium_naphtha', 'reformed_gasoline', 0.52],
        ['heavy_naphtha', 'reformed_gasoline', 0.45],
        ['light_oil', 'jet_fuel', np.nan],
        ['light_oil', 'fuel_oil', np.nan],
        ['heavy_oil', 'jet_fuel', np.nan],
        ['heavy_oil', 'fuel_oil', np.nan],
        ['light_oil', 'light_oil_cracked', 2],
        ['light_oil_cracked', 'cracked_oil', 0.68],
        ['light_oil_cracked', 'cracked_gasoline', 0.28],
        ['heavy_oil', 'heavy_oil_cracked', 2],
        ['heavy_oil_cracked', 'cracked_oil', 0.75],
        ['heavy_oil_cracked', 'cracked_gasoline', 0.2],
        ['cracked_oil', 'jet_fuel', np.nan],
        ['cracked_oil', 'fuel_oil', np.nan],
        ['reformed_gasoline', 'regular_petrol', np.nan],
        ['reformed_gasoline', 'premium_petrol', np.nan],
        ['cracked_gasoline', 'regular_petrol', np.nan],
        ['cracked_gasoline', 'premium_petrol', np.nan],
        ['residuum', 'lube_oil', 0.5],
```

```
['residuum', 'jet_fuel', np.nan],
    ['residuum', 'fuel_oil', np.nan],
    ], columns=['i', 'j', 'multiplier']).set_index(['i', 'j'])
octane_data = pd.DataFrame([
    ['light_naphtha', 90],
    ['medium_naphtha', 80],
    ['heavy_naphtha', 70],
    ['reformed_gasoline', 115],
    ['cracked_gasoline', 105],
    ], columns=['i', 'octane']).set_index(['i'])
petrol_data = pd.DataFrame([
    ['regular_petrol', 84],
    ['premium_petrol', 94],
    ], columns=['petrol', 'octane_lb']).set_index(['petrol'])
vapour_pressure_data = pd.DataFrame([
    ['light_oil', 1.0],
    ['heavy_oil', 0.6],
    ['cracked_oil', 1.5],
    ['residuum', 0.05],
    ], columns=['oil', 'vapour_pressure']).set_index(['oil'])
fuel_oil_ratio_data = pd.DataFrame([
    ['light_oil', 10],
    ['cracked_oil', 4],
    ['heavy_oil', 3],
    ['residuum', 1],
    ], columns=['oil', 'coefficient']).set_index(['oil'])
final_product_data = pd.DataFrame([
    ['premium_petrol', 700],
    ['regular_petrol', 600],
    ['jet_fuel', 400],
    ['fuel_oil', 350],
    ['lube_oil', 150],
    ], columns=['product', 'profit']).set_index(['product'])
vapour_pressure_ub = 1
crude_total_ub = 45000
naphtha_ub = 10000
cracked_oil_ub = 8000
lube\_oil\_lb = 500
lube\_oil\_ub = 1000
premium_ratio = 0.40
ARCS = arc_data.index.tolist()
arc_mult = arc_data['multiplier'].fillna(1)
FINAL_PRODUCTS = final_product_data.index.tolist()
final_product_data['profit'] = final_product_data['profit'] / 100
profit = final_product_data['profit']
ARCS = ARCS + [(i, 'sink') for i in FINAL_PRODUCTS]
flow = m.add_variables(ARCS, name='flow', lb=0)
NODES = np.unique([i for j in ARCS for i in j])
```

```
m.set_objective(so.quick_sum(profit[i] * flow[i, 'sink']
                              for i in FINAL_PRODUCTS
                              if (i, 'sink') in ARCS),
                name='totalProfit', sense=so.MAX)
m.add_constraints((so.quick_sum(flow[a] for a in ARCS if a[0] == n) ==
                  so.quick_sum(arc_mult[a] * flow[a]
                                for a in ARCS if a[1] == n)
                  for n in NODES if n not in ['source', 'sink']),
                  name='flow_balance')
CRUDES = crude_data.index.tolist()
crudeDistilled = m.add_variables(CRUDES, name='crudesDistilled', lb=0)
crudeDistilled.set_bounds(ub=crude_data['crude_ub'])
m.add_constraints((flow[i, j] == crudeDistilled[i]
                  for (i, j) in ARCS if i in CRUDES), name='distillation')
OILS = ['light_oil', 'heavy_oil']
CRACKED_OILS = [i+'_cracked' for i in OILS]
oilCracked = m.add_variables(CRACKED_OILS, name='oilCracked', lb=0)
m.add_constraints((flow[i, j] == oilCracked[i] for (i, j) in ARCS
                  if i in CRACKED_OILS), name='cracking')
octane = octane_data['octane']
PETROLS = petrol_data.index.tolist()
octane_lb = petrol_data['octane_lb']
vapour_pressure = vapour_pressure_data['vapour_pressure']
m.add_constraints((so.quick_sum(octane[a[0]] * arc_mult[a] * flow[a]
                                 for a in ARCS if a[1] == p)
                   >= octane_lb[p] *
                  so.quick_sum(arc_mult[a] * flow[a]
                                for a in ARCS if a[1] == p)
                  for p in PETROLS), name='blending_petrol')
m.add_constraint(so.quick_sum(vapour_pressure[a[0]] * arc_mult[a] * flow[a]
                               for a in ARCS if a[1] == 'jet_fuel') <=</pre>
                 vapour_pressure_ub *
                 so.quick_sum(arc_mult[a] * flow[a]
                               for a in ARCS if a[1] == 'jet_fuel'),
                 name='blending_jet_fuel')
fuel_oil_coefficient = fuel_oil_ratio_data['coefficient']
sum_fuel_oil_coefficient = sum(fuel_oil_coefficient)
m.add_constraints((sum_fuel_oil_coefficient * flow[a] ==
                  fuel_oil_coefficient[a[0]] * flow.sum('*', ['fuel_oil'])
                  for a in ARCS if a[1] == 'fuel_oil'),
                  name='blending_fuel_oil')
m.add_constraint(crudeDistilled.sum('*') <= crude_total_ub,</pre>
                 name='crude_total_ub')
m.add_constraint(so.quick_sum(flow[a] for a in ARCS
                               if a[0].find('naphtha') > -1 and
                               a[1] == 'reformed_gasoline')
                 <= naphtha_ub, name='naphtba_ub')</pre>
                                                                      (continues on next page)
```

```
m.add_constraint(so.quick_sum(flow[a] for a in ARCS if a[1] ==
                               'cracked_oil') <=</pre>
                 cracked_oil_ub, name='cracked_oil_ub')
m.add_constraint(flow['lube_oil', 'sink'] == [lube_oil_lb, lube_oil_ub],
                 name='lube_oil_range')
m.add_constraint(flow.sum('premium_petrol', '*') >= premium_ratio *
                 flow.sum('regular_petrol', '*'), name='premium_ratio')
res = m.solve()
if res is not None:
    print(so.get_solution_table(crudeDistilled))
    print(so.get_solution_table(oilCracked))
    print(so.get_solution_table(flow))
    octane_sol = []
    for p in PETROLS:
        octane_sol.append(so.quick_sum(octane[a[0]] * arc_mult[a] *
                                        flow[a].get_value() for a in ARCS
                                        if a[1] == p) /
                          sum(arc_mult[a] * flow[a].get_value()
                               for a in ARCS if a[1] == p))
    octane_sol = pd.Series(octane_sol, name='octane_sol', index=PETROLS)
    print(so.get_solution_table(octane_sol, octane_lb))
    print(so.get_solution_table(vapour_pressure))
    vapour_pressure_sol = sum(vapour_pressure[a[0]] *
                               arc_mult[a] *
                               flow[a].get_value() for a in ARCS
                              if a[1] == 'jet_fuel') /\
        sum(arc_mult[a] * flow[a].get_value() for a in ARCS
            if a[1] == 'jet_fuel')
    print('Vapour_pressure_sol: {:.4f}'.format(vapour_pressure_sol))
    num_fuel_oil_ratio_sol = [arc_mult[a] * flow[a].get_value() /
                              sum(arc_mult[b] *
                                   flow[b].get_value()
                                   for b in ARCS if b[1] == 'fuel_oil')
                              for a in ARCS if a[1] == 'fuel_oil']
    num_fuel_oil_ratio_sol = pd.Series(num_fuel_oil_ratio_sol,
                                        name='num_fuel_oil_ratio_sol',
                                        index=[a[0] for a in ARCS
                                               if a[1] == 'fuel_oil'])
    print(so.get_solution_table(fuel_oil_coefficient,
                                 num_fuel_oil_ratio_sol))
return m.get_objective_value()
```

Output

```
In [1]: from examples.refinery_optimization import test
In [2]: test(cas_conn)
NOTE: Initialized model refinery_optimization.
```

```
NOTE: Added action set 'optimization'.
NOTE: Converting model refinery_optimization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 51 variables (0 free, 0 fixed).
NOTE: The problem has 46 linear constraints (4 LE, 38 EQ, 3 GE, 1 range).
NOTE: The problem has 158 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 28 variables and 30 constraints.
NOTE: The LP presolver removed 85 constraint coefficients.
NOTE: The presolved problem has 23 variables, 16 constraints, and 73 constraint,
→coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                             Objective
         Phase Iteration
                               Value
                                              Time
         D 2
                     1
                            7.189280E+05
          P 2
                      21
                           2.113651E+05
NOTE: Optimal.
NOTE: Objective = 211365.13477.
NOTE: The Dual Simplex solve time is 0.02 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS (casuser)' has 13 rows.
→and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS (casuser)' has 51 rows and 6,
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 46 rows and 4 columns.
       crudesDistilled
crude1
               15000.0
                30000.0
crude2
                   oilCracked
heavy_oil_cracked
                      3800.0
light_oil_cracked
                       4200.0
                                             flow
cracked_gasoline premium_petrol
                                        0.000000
cracked_gasoline regular_petrol
                                      1936.000000
cracked_oil fuel_oil
                                         0.000000
                                      5706.000000
cracked_oil
                 jet_fuel
crude1
                 heavy_naphtha
                                   15000.000000
crude1
                 heavy_oil
                                     15000.000000
                 light_naphtha
crude1
                                    15000.000000
crude1
                 light_oil
                                    15000.000000
                                 15000.000000
crude1
                medium_naphtha
                residuum
                                    15000.000000
crude1
                                   30000.000000
crude2
                heavy_naphtha
crude2
                heavy_oil
                                    30000.000000
crude2
                 light_naphtha
                                    30000.000000
crude2
                 light oil
                                    30000.000000
                 medium_naphtha
                                     30000.000000
crude2
                                     30000.000000
crude2
                 residuum
                                         0.000000
fuel_oil
                  sink
                                                                        (continues on next page)
```

```
heavy_naphtha premium_petrol 1677.804016
heavy_naphtha reformed_gasoline 5406.861844
heavy_naphtha regular_petrol 1315.334140
heavy_oil fuel_oil 0.000000
heavy_oil heavy_oil_cracked 3800.000000
heavy_oil jet_fuel 4900.000000
heavy_oil_cracked cracked_gasoline 3800.000000
heavy_oil_cracked cracked_oil 3800.000000
 heavy_oil_cracked cracked_oil 3800.000000 jet_fuel sink 15156.000000
light_oil_cracked cracked_gasoline 4200.000000 light_oil_cracked cracked_oil 4200.000000 lube_oil sink 500.000000
 1ube_oilsink500.000000medium_naphthapremium_petrol0.000000medium_naphthareformed_gasoline0.000000medium_naphtharegular_petrol10500.000000premium_petrolsink6817.778853

        premium_petrol
        sink
        681/./78853

        reformed_gasoline
        premium_petrol
        2433.087830

        reformed_gasoline
        regular_petrol
        0.000000

        regular_petrol
        sink
        17044.447133

        residuum
        fuel_oil
        0.000000

        residuum
        jet_fuel
        4550.000000

        residuum
        lube_oil
        1000.000000

        source
        crude1
        15000.000000

        source
        crude2
        30000.000000

                                     octane_sol octane_lb
 premium_petrol 94.0 regular_petrol 84.0
                                                       94.0
                                                                                          94
                                                                                            84
                               vapour_pressure
 1
 cracked_oil heavy_oil
                                                           1.50
                                                            0.60
 light_oil
                                                              1.00
 residuum
 Vapour_pressure_sol: 0.7737
                    coefficient num_fuel_oil_ratio_sol
 1
                                                           4
 cracked_oil
                                                                                                                      NaN
 heavy_oil
                                                            3
                                                                                                                      NaN
 light_oil
                                                           10
                                                                                                                      NaN
 residuum
                                                             1
                                                                                                                      NaN
 Out[2]: 211365.134769
```

7.1.7 Mining Optimization

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex7_toc.htm&docsetVersion=14.3& locale=en

http://support.sas.com/documentation/onlinedoc/or/ex code/143/mpex07.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas conn):
   m = so.Model(name='mining_optimization', session=cas_conn)
    mine_data = pd.DataFrame([
        ['mine1', 5, 2, 1.0],
        ['mine2', 4, 2.5, 0.7],
        ['mine3', 4, 1.3, 1.5],
        ['mine4', 5, 3, 0.5],
        ], columns=['mine', 'cost', 'extract_ub', 'quality']).\
        set_index(['mine'])
    year_data = pd.DataFrame([
        [1, 0.9],
        [2, 0.8],
        [3, 1.2],
        [4, 0.6],
        [5, 1.0],
        ], columns=['year', 'quality_required']).set_index(['year'])
   max_num_worked_per_year = 3
    revenue_per_ton = 10
    discount_rate = 0.10
   MINES = mine_data.index.tolist()
    cost = mine_data['cost']
    extract_ub = mine_data['extract_ub']
    quality = mine_data['quality']
    YEARS = year_data.index.tolist()
    quality_required = year_data['quality_required']
    isOpen = m.add_variables(MINES, YEARS, vartype=so.BIN, name='isOpen')
    isWorked = m.add_variables(MINES, YEARS, vartype=so.BIN, name='isWorked')
    extract = m.add_variables(MINES, YEARS, 1b=0, name='extract')
    [extract[i, j].set_bounds(ub=extract_ub[i]) for i in MINES for j in YEARS]
    extractedPerYear = {j: extract.sum('*', j) for j in YEARS}
    discount = {j: 1 / (1+discount_rate) ** (j-1) for j in YEARS}
    totalRevenue = revenue_per_ton *\
        so.quick_sum(discount[j] * extractedPerYear[j] for j in YEARS)
    totalCost = so.quick_sum(discount[j] * cost[i] * isOpen[i, j]
                              for i in MINES for j in YEARS)
   m.set_objective(totalRevenue-totalCost, sense=so.MAX, name='totalProfit')
    m.add_constraints((extract[i, j] <= extract[i, j]._ub * isWorked[i, j]</pre>
                      for i in MINES for j in YEARS), name='link')
    m.add_constraints((isWorked.sum('*', j) <= max_num_worked_per_year</pre>
                                                                           (continues on next page)
```

```
for j in YEARS), name='cardinality')
m.add_constraints((isWorked[i, j] <= isOpen[i, j] for i in MINES</pre>
                  for j in YEARS), name='worked_implies_open')
m.add_constraints((isOpen[i, j] <= isOpen[i, j-1] for i in MINES</pre>
                  for j in YEARS if j != 1), name='continuity')
m.add_constraints((so.quick_sum(quality[i] * extract[i, j] for i in MINES)
                  == quality_required[j] * extractedPerYear[j]
                  for j in YEARS), name='quality_con')
res = m.solve()
if res is not None:
    print(so.get_solution_table(isOpen, isWorked, extract))
    quality_sol = {j: so.quick_sum(quality[i] * extract[i, j].get_value()
                                    for i in MINES)
                    / extractedPerYear[j].get_value() for j in YEARS}
    qs = so.dict_to_frame(quality_sol, ['quality_sol'])
    epy = so.dict_to_frame(extractedPerYear, ['extracted_per_year'])
    print(so.get_solution_table(epy, qs, quality_required))
return m.get_objective_value()
```

Output

```
In [1]: from examples.mining_optimization import test
In [2]: test(cas_conn)
NOTE: Initialized model mining_optimization.
NOTE: Added action set 'optimization'.
NOTE: Converting model mining_optimization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 60 variables (0 free, 0 fixed).
NOTE: The problem has 40 binary and 0 integer variables.
NOTE: The problem has 66 linear constraints (61 LE, 5 EQ, 0 GE, 0 range).
NOTE: The problem has 151 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 8 variables and 8 constraints.
NOTE: The MILP presolver removed 16 constraint coefficients.
NOTE: The MILP presolver modified 8 constraint coefficients.
NOTE: The presolved problem has 52 variables, 58 constraints, and 135 constraint,
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger
                                                   BestBound
                                                                 Gap
                                                                         Time
               0
                        1
                               7
                                     95.6438817
                                                   364.3638322 73.75%
                                                                             0
               0
                        1
                               7
                                     95.6438817
                                                  157.7308887 39.36%
                                                                             0
               Ω
                        1
                               7
                                     95.6438817
                                                 153.3061673 37.61%
                                                                             0
               Λ
                              7
                        1
                                    95.6438817 150.9827514 36.65%
                                                                             0
```

```
95.6438817
                                                   146.8623445
                                                                34.88%
                                                                             0
                        1
                               8
                                   146.8619786
                                                  146.8623445
                                                                 0.00%
                                                                             0
NOTE: The MILP solver added 4 cuts with 19 cut coefficients at the root.
NOTE: Optimal within relative gap.
NOTE: Objective = 146.86197857.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 60 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 66 rows and 4 columns.
          isOpen isWorked extract
minel 1 1.000000 1.000000 2.000000
mine1 2 1.000000 0.000000 0.000000
minel 3 1.000000 1.000000 1.950000
minel 4 1.000000 1.000000 0.125000
mine1 5 1.000000 1.000000 2.000000
mine2 1 1.000000 0.000000 0.000000
mine2 2 1.000000 1.000000 2.500000
mine2 3 1.000000 0.000000 0.000000
mine2 4 1.000000 1.000000 2.500000
mine2 5 0.999998 0.999998 2.166667
mine3 1 1.000000 1.000000 1.300000
mine3 2 1.000000 1.000000 1.300000
mine3 3 1.000000 1.000000 1.300000
mine3 4 1.000000 0.000000 0.000000
mine3 5 1.000000 1.000000 1.300000
mine4 1 1.000000 1.000000 2.450000
mine4 2 1.000000 1.000000 2.200000
mine4 3 1.000000 0.000000 0.000000
mine4 4 1.000000 1.000000
                            3.000000
mine4 5 0.000000 0.000000 0.000000
   extracted_per_year quality_sol quality_required
1
1
            5.750000
                              0.9
                                                0.9
2
            6.000000
                              0.8
                                                0.8
3
            3.250000
                             1.2
                                                1.2
            5.625000
                             0.6
                                                0.6
            5.466667
                             1.0
                                                1.0
Out[2]: 146.861979
```

7.1.8 Farm Planning

Reference

 $http://go.documentation.sas.com/?docsetId=ormpex\&docsetTarget=ormpex_ex8_toc.htm\&docsetVersion=14.3\&locale=en$

http://support.sas.com/documentation/onlinedoc/or/ex code/143/mpex08.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
   m = so.Model(name='farm_planning', session=cas_conn)
    # Input Data
    cow_data_raw = []
    for age in range(12):
        if age < 2:
            row = {'age': age,
                    'init_num_cows': 10,
                   'acres_needed': 2/3.0,
                   'annual_loss': 0.05,
                   'bullock_yield': 0,
                   'heifer_yield': 0,
                   'milk_revenue': 0,
                   'grain_req': 0,
                   'sugar_beet_req': 0,
                   'labour_req': 10,
                   'other_costs': 50}
        else:
            row = {'age': age,
                    'init_num_cows': 10,
                   'acres_needed': 1,
                   'annual_loss': 0.02,
                   'bullock_yield': 1.1/2,
                   'heifer_yield': 1.1/2,
                   'milk_revenue': 370,
                   'grain_req': 0.6,
                   'sugar_beet_req': 0.7,
                   'labour_req': 42,
                   'other_costs': 100}
        cow_data_raw.append(row)
    cow_data = pd.DataFrame(cow_data_raw).set_index(['age'])
    grain_data = pd.DataFrame([
        ['group1', 20, 1.1],
        ['group2', 30, 0.9],
        ['group3', 20, 0.8],
        ['group4', 10, 0.65]
        ], columns=['group', 'acres', 'yield']).set_index(['group'])
    num\_years = 5
   num\_acres = 200
   bullock_revenue = 30
   heifer_revenue = 40
   dairy_cow_selling_age = 12
   dairy_cow_selling_revenue = 120
   max_num_cows = 130
    sugar_beet_yield = 1.5
    grain_cost = 90
    grain_revenue = 75
    grain_labour_req = 4
```

```
grain_other_costs = 15
sugar_beet_cost = 70
sugar_beet_revenue = 58
sugar_beet_labour_req = 14
sugar_beet_other_costs = 10
nominal_labour_cost = 4000
nominal_labour_hours = 5500
excess_labour_cost = 1.2
capital_outlay_unit = 200
num_loan_years = 10
annual_interest_rate = 0.15
max_decrease_ratio = 0.50
max_increase_ratio = 0.75
# Sets
AGES = cow_data.index.tolist()
init_num_cows = cow_data['init_num_cows']
acres_needed = cow_data['acres_needed']
annual_loss = cow_data['annual_loss']
bullock_yield = cow_data['bullock_yield']
heifer_yield = cow_data['heifer_yield']
milk_revenue = cow_data['milk_revenue']
grain_req = cow_data['grain_req']
sugar_beet_req = cow_data['sugar_beet_req']
cow_labour_req = cow_data['labour_req']
cow_other_costs = cow_data['other_costs']
YEARS = list(range(1, num_years+1))
YEARS0 = [0] + YEARS
# Variables
numCows = m.add_variables(AGES + [dairy_cow_selling_age], YEARSO, 1b=0,
                          name='numCows')
for age in AGES:
    numCows[age, 0].set_bounds(lb=init_num_cows[age],
                               ub=init_num_cows[age])
numCows[dairy_cow_selling_age, 0].set_bounds(lb=0, ub=0)
numBullocksSold = m.add_variables(YEARS, lb=0, name='numBullocksSold')
numHeifersSold = m.add_variables(YEARS, lb=0, name='numHeifersSold')
GROUPS = grain_data.index.tolist()
acres = grain_data['acres']
grain_yield = grain_data['yield']
grainAcres = m.add_variables(GROUPS, YEARS, lb=0, name='grainAcres')
for group in GROUPS:
    for year in YEARS:
        grainAcres[group, year].set_bounds(ub=acres[group])
grainBought = m.add_variables(YEARS, lb=0, name='grainBought')
grainSold = m.add_variables(YEARS, lb=0, name='grainSold')
sugarBeetAcres = m.add_variables(YEARS, 1b=0, name='sugarBeetAcres')
sugarBeetBought = m.add_variables(YEARS, 1b=0, name='sugarBeetBought')
sugarBeetSold = m.add_variables(YEARS, lb=0, name='sugarBeetSold')
```

```
numExcessLabourHours = m.add_variables(YEARS, 1b=0,
                                       name='numExcessLabourHours')
capitalOutlay = m.add_variables(YEARS, 1b=0, name='capitalOutlay')
yearly_loan_payment = (annual_interest_rate * capital_outlay_unit) /\
                       (1 - (1+annual_interest_rate) **(-num_loan_years))
# Objective function
revenue = {year:
           bullock_revenue * numBullocksSold[year] +
           heifer_revenue * numHeifersSold[year] +
           dairy_cow_selling_revenue * numCows[dairy_cow_selling_age,
                                                year] +
           so.quick_sum(milk_revenue[age] * numCows[age, year]
                        for age in AGES) +
           grain_revenue * grainSold[year] +
           sugar_beet_revenue * sugarBeetSold[year]
           for year in YEARS}
cost = {year:
        grain_cost * grainBought[year] +
        sugar_beet_cost * sugarBeetBought[year] +
        nominal_labour_cost +
        excess_labour_cost * numExcessLabourHours[year] +
        so.quick_sum(cow_other_costs[age] * numCows[age, year]
                     for age in AGES) +
        so.quick_sum(grain_other_costs * grainAcres[group, year]
                     for group in GROUPS) +
        sugar_beet_other_costs * sugarBeetAcres[year] +
        so.quick_sum(yearly_loan_payment * capitalOutlay[y]
                     for y in YEARS if y <= year)</pre>
        for year in YEARS}
profit = {year: revenue[year] - cost[year] for year in YEARS}
totalProfit = so.quick_sum(profit[year] -
                           yearly_loan_payment * (num_years - 1 + year) *
                           capitalOutlay[year] for year in YEARS)
m.set_objective(totalProfit, sense=so.MAX, name='totalProfit')
# Constraints
m.add_constraints((
    so.quick_sum(acres_needed[age] * numCows[age, year] for age in AGES) +
    so.quick_sum(grainAcres[group, year] for group in GROUPS) +
    sugarBeetAcres[year] <= num_acres</pre>
    for year in YEARS), name='num_acres')
m.add_constraints((
    numCows[age+1, year+1] == (1-annual_loss[age]) * numCows[age, year]
    for age in AGES if age != dairy_cow_selling_age
    for year in YEARSO if year != num_years), name='aging')
m.add_constraints((
    numBullocksSold[year] == so.quick_sum(
        bullock_yield[age] * numCows[age, year] for age in AGES)
```

```
for year in YEARS), name='numBullocksSold_def')
m.add_constraints((
    numCows[0, year] == so.quick_sum(
        heifer_yield[age] * numCows[age, year]
        for age in AGES) - numHeifersSold[year]
    for year in YEARS), name='numHeifersSold_def')
m.add_constraints((
    so.quick_sum(numCows[age, year] for age in AGES) <= max_num_cows +</pre>
    so.quick_sum(capitalOutlay[y] for y in YEARS if y <= year)</pre>
    for year in YEARS), name='max_num_cows_def')
grainGrown = {(group, year): grain_yield[group] * grainAcres[group, year]
              for group in GROUPS for year in YEARS}
m.add_constraints((
    so.quick_sum(grain_req[age] * numCows[age, year] for age in AGES) <=</pre>
    so.quick_sum(grainGrown[group, year] for group in GROUPS)
    + grainBought[year] - grainSold[year]
    for year in YEARS), name='grain_req_def')
sugarBeetGrown = {(year): sugar_beet_yield * sugarBeetAcres[year]
                  for year in YEARS}
m.add_constraints((
    so.quick_sum(sugar_beet_req[age] * numCows[age, year] for age in AGES)
    sugarBeetGrown[year] + sugarBeetBought[year] - sugarBeetSold[year]
    for year in YEARS), name='sugar_beet_req_def')
m.add_constraints((
    so.quick_sum(cow_labour_req[age] * numCows[age, year]
                 for age in AGES) +
    so.quick_sum(grain_labour_req * grainAcres[group, year]
                 for group in GROUPS) +
    sugar_beet_labour_req * sugarBeetAcres[year] <=</pre>
    nominal_labour_hours + numExcessLabourHours[year]
    for year in YEARS), name='labour_req_def')
m.add_constraints((profit[year] >= 0 for year in YEARS), name='cash_flow')
m.add_constraint(so.quick_sum(numCows[age, num_years] for age in AGES
                              if age >= 2) /
                 sum(init_num_cows[age] for age in AGES if age >= 2) ==
                 [1-max_decrease_ratio, 1+max_increase_ratio],
                 name='final_dairy_cows_range')
res = m.solve()
if res is not None:
    print(so.get_solution_table(numCows))
    revenue_df = so.dict_to_frame(revenue, cols=['revenue'])
    cost_df = so.dict_to_frame(cost, cols=['cost'])
    profit_df = so.dict_to_frame(profit, cols=['profit'])
    print(so.get_solution_table(numBullocksSold, numHeifersSold,
                                 capitalOutlay, numExcessLabourHours,
                                 revenue_df, cost_df, profit_df))
    gg_df = so.dict_to_frame(grainGrown, cols=['grainGrown'])
    print(so.get_solution_table(grainAcres, gg_df))
                                                                      (continues on next page)
```

Output

```
In [1]: from examples.farm_planning import test
In [2]: test(cas_conn)
NOTE: Initialized model farm_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model farm_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 143 variables (0 free, 13 fixed).
NOTE: The problem has 101 linear constraints (25 LE, 70 EQ, 5 GE, 1 range).
NOTE: The problem has 780 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 84 variables and 69 constraints.
NOTE: The LP presolver removed 533 constraint coefficients.
NOTE: The presolved problem has 59 variables, 32 constraints, and 247 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                             Objective
         Phase Iteration
                               Value
                                              Time
                          4.195000E+02
          D 1
                     1
                     37
                           1.744078E+05
                                                 0
         D 2
                     55
                           1.217192E+05
NOTE: Optimal.
NOTE: Objective = 121719.17286.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
→4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 143 rows and 6,
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 101 rows and 4
→columns.
       numCows
1 2
0 0 10.000000
0 1 22.800000
0 2 11.584427
                                                                         (continues on next page)
```

```
0.000000
0 4 0.000000
0 5
     0.000000
1 0 10.000000
1
  1
      9.500000
  2 21.660000
1
  3 11.005205
  4
      0.000000
1 5
     0.000000
2 0 10.000000
2 1 9.500000
2 2 9.025000
2 3 20.577000
2 4 10.454945
2 5 0.000000
3 0 10.000000
3 1
     9.800000
3 2
     9.310000
     8.844500
3
  3
3
  4
     20.165460
  5
     10.245846
  0 10.000000
4
4 1 9.800000
4 2 9.604000
4 3 9.123800
4 4 8.667610
4 5 19.762151
8 0 10.000000
8 1
     9.800000
8 2
     9.604000
      9.411920
8
  3
8
  4
      9.223682
  5
      9.039208
9
  0 10.000000
9 1 9.800000
9 2 9.604000
9 3 9.411920
9 4 9.223682
9 5 9.039208
10 0 10.000000
10 1 9.800000
10 2 9.604000
10 3 9.411920
10 4
     9.223682
10 5 9.039208
11 0 10.000000
11 1 9.800000
11 2 9.604000
11 3 9.411920
11 4 9.223682
11 5 9.039208
12 0 0.000000
12 1 9.800000
      9.604000
12 2
12 3
      9.411920
      9.223682
12 4
```

```
12 5
      9.039208
[78 rows x 1 columns]
  numBullocksSold numHeifersSold capitalOutlay numExcessLabourHours \
                                         0.0
1
        53.735000
                      30.935000
                                                             0.0
2
        52.341850
                      40.757423
                                         0.0
                                                             0.0
3
        57.435807
                      57.435807
                                         0.0
                                                             0.0
4
        56.964286
                      56.964286
                                         0.0
                                                             0.0
5
       50.853436
                      50.853436
                                        0.0
                                                             0.0
       revenue
                    cost profit
1
1 41494.530000 19588.466667 21906.063333
2 41153.336497 19264.639818 21888.696679
3 45212.490308 19396.435208 25816.055100
4 45860.056078 19034.285714 26825.770363
5 42716.941438 17434.354053 25282.587385
         grainAcres grainGrown
        20.000000 22.000000
group1 1
group1 2 20.000000 22.000000
group1 3 20.000000 22.000000
group1 4 20.000000 22.000000
group1 5 20.000000 22.000000
group2 1 0.000000 0.000000
group2 2 0.000000 0.000000
group2 3 3.134152 2.820737
group2 4 0.000000 0.000000
group2 5 0.000000 0.000000
group3 1 0.000000 0.000000
        0.000000
                   0.000000
group3 2
          0.000000 0.000000
0.000000 0.000000
group3 3
group3 4
group3 5 0.000000 0.000000
group4 1 0.000000 0.000000
group4 2 0.000000 0.000000
group4 3 0.000000 0.000000
group4 4 0.000000 0.000000
group4 5 0.000000 0.000000
  grainBought grainSold sugarBeetAcres sugerBeetGrown sugarBeetBought \
1
                  0.0
                            60.766667
                                                                0.0
1
    36.620000
                                          91.150000
2
    35.100200
                  0.0
                           62.670049
                                          94.005073
                                                               0.0
                   0.0
                           65.100304
3
    37.836507
                                          97.650456
                                                               0.0
                  0.0
                           76.428571
4
    40.142857
                                          114.642857
                                                                0.0
                  0.0
5
    33.476475
                           87.539208
                                          131.308812
                                                                0.0
  sugarBeetSold
1
1
     22.760000
2
     27.388173
3
     24.550338
4
      42.142857
5
     66.586258
  num_acres max_num_cows_def
```

```
1 200.0 130.000000
2 200.0 128.411427
3 200.0 115.433945
4 200.0 103.571429
5 200.0 92.460792
Out[2]: 121719.172861
```

7.1.9 Economic Planning

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex9_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex09.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
    m = so.Model(name='economic_planning', session=cas_conn)
    industry_data = pd.DataFrame([
        ['coal', 150, 300, 60],
        ['steel', 80, 350, 60],
        ['transport', 100, 280, 30]
        ], columns=['industry', 'init_stocks', 'init_productive_capacity',
                     'demand']).set_index(['industry'])
    production_data = pd.DataFrame([
        ['coal', 0.1, 0.5, 0.4],
        ['steel', 0.1, 0.1, 0.2],
        ['transport', 0.2, 0.1, 0.2],
        ['manpower', 0.6, 0.3, 0.2],
        ], columns=['input', 'coal',
                     'steel', 'transport']).set_index(['input'])
    productive_capacity_data = pd.DataFrame([
        ['coal', 0.0, 0.7, 0.9],
        ['steel', 0.1, 0.1, 0.2],
        ['transport', 0.2, 0.1, 0.2], ['manpower', 0.4, 0.2, 0.1],
        ], columns=['input', 'coal',
                     'steel', 'transport']).set_index(['input'])
    manpower_capacity = 470
    num_years = 5
    YEARS = list(range(1, num_years+1))
```

```
YEARS0 = [0] + list(YEARS)
INDUSTRIES = industry_data.index.tolist()
[init_stocks, init_productive_capacity, demand] = so.read_frame(
    industry_data)
# INPUTS = production_data.index.tolist()
production_coeff = so.flatten_frame(production_data)
productive_capacity_coeff = so.flatten_frame(productive_capacity_data)
static_production = m.add_variables(INDUSTRIES, lb=0,
                                    name='static_production')
m.set_objective(0, sense=so.MIN, name='Zero')
m.add_constraints((static_production[i] == demand[i] +
                   so.quick_sum(
                       production_coeff[i, j] * static_production[j]
                       for j in INDUSTRIES) for i in INDUSTRIES),
                  name='static_con')
m.solve()
print(so.get_solution_table(static_production, sort=True))
final_demand = so.get_solution_table(
    static_production, sort=True)['static_production']
# Alternative way
# final_demand = {}
# for i in INDUSTRIES:
      final_demand[i] = static_production.get_value()
production = m.add_variables(INDUSTRIES, range(0, num_years+2), lb=0,
                             name='production')
stock = m.add_variables(INDUSTRIES, range(0, num_years+2), lb=0,
                        name='stock')
extra_capacity = m.add_variables(INDUSTRIES, range(1, num_years+3), lb=0,
                                 name='extra_capacity')
productive_capacity = {}
for i in INDUSTRIES:
    for year in range(1, num_years+2):
        productive_capacity[i, year] = init_productive_capacity[i] +\
            so.quick_sum(extra_capacity[i, y] for y in range(2, year+1))
for i in INDUSTRIES:
    production[i, 0].set_bounds(ub=0)
    stock[i, 0].set_bounds(lb=init_stocks[i], ub=init_stocks[i])
total_productive_capacity = sum(productive_capacity[i, num_years]
                                for i in INDUSTRIES)
total_production = so.quick_sum(production[i, year] for i in INDUSTRIES
                                for year in [4, 5])
total_manpower = so.quick_sum(production_coeff['manpower', i] *
                              production[i, year+1] +
                              productive_capacity_coeff['manpower', i] *
                              extra_capacity[i, year+2]
                              for i in INDUSTRIES for year in YEARS)
continuity_con = m.add_constraints((
    stock[i, year] + production[i, year] ==
    (demand[i] if year in YEARS else 0) +
    so.quick_sum(production_coeff[i, j] * production[j, year+1] +
                 productive_capacity_coeff[i, j] *
```

```
extra_capacity[j, year+2] for j in INDUSTRIES) +
    stock[i, vear+1]
    for i in INDUSTRIES for year in YEARSO), name='continuity_con')
manpower_con = m.add_constraints((
    so.quick_sum(production_coeff['manpower', j] * production[j, year] +
                 productive_capacity_coeff['manpower', j] *
                 extra_capacity[j, year+1]
                 for j in INDUSTRIES)
    <= manpower_capacity for year in range(1, num_years+2)),</pre>
    name='manpower_con')
capacity_con = m.add_constraints((production[i, year] <=</pre>
                                   productive_capacity[i, year]
                                   for i in INDUSTRIES
                                   for year in range(1, num_years+2)),
                                  name='capacity_con')
for i in INDUSTRIES:
    production[i, num_years+1].set_bounds(lb=final_demand[i])
for i in INDUSTRIES:
    for year in [num_years+1, num_years+2]:
        extra_capacity[i, year].set_bounds(ub=0)
problem1 = so.Model(name='Problem1', session=cas_conn)
problem1.include (production, stock, extra_capacity,
                 continuity_con, manpower_con, capacity_con)
problem1.set_objective(total_productive_capacity, sense=so.MAX,
                       name='total_productive_capacity')
problem1.solve()
productive_capacity_fr = so.dict_to_frame(productive_capacity,
                                           cols=['productive_capacity'])
print(so.get_solution_table(production, stock, extra_capacity,
                             productive_capacity_fr, sort=True))
print(so.get_solution_table(manpower_con.get_expressions(), sort=True))
# Problem 2
problem2 = so.Model(name='Problem2', session=cas_conn)
problem2.include(problem1)
problem2.set_objective(total_production, name='total_production',
                       sense=so.MAX)
for i in INDUSTRIES:
    for year in YEARS:
        continuity_con[i, year].set_rhs(0)
problem2.solve()
print(so.get_solution_table(production, stock, extra_capacity,
                             productive_capacity, sort=True))
print (so.get_solution_table (manpower_con.get_expressions(), sort=True))
# Problem 3
problem3 = so.Model(name='Problem3', session=cas_conn)
problem3.include(production, stock, extra_capacity, continuity_con,
                 capacity_con)
problem3.set_objective(total_manpower, sense=so.MAX, name='total_manpower')
                                                                      (continues on next page)
```

Output

```
In [1]: from examples.economic_planning import test
In [2]: test(cas_conn)
NOTE: Initialized model economic_planning.
NOTE: Added action set 'optimization'.
NOTE: Converting model economic_planning to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 3 variables (0 free, 0 fixed).
NOTE: The problem has 3 linear constraints (0 LE, 3 EQ, 0 GE, 0 range).
NOTE: The problem has 9 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed all variables and constraints.
NOTE: Optimal.
NOTE: Objective = 0.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
⇒and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 3 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 3 rows and 4 columns.
           static_production
1
                 166.396761
coal
                 105.668016
steel
                  92.307692
transport
NOTE: Initialized model Problem1.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 42 linear constraints (24 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 255 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 7 constraints.
NOTE: The LP presolver removed 64 constraint coefficients.
NOTE: The presolved problem has 42 variables, 35 constraints, and 191 constraint_
⇔coefficients.
                                                                          (continues on next page)
```

```
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                        Objective
        Phase Iteration
                            Value
                                          Time
         D 2 1
                                           0
                          1.360782E+04
         P 2
                    38
                         2.141875E+03
                                             0
NOTE: Optimal.
NOTE: Objective = 2141.875197.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS (casuser)' has 13 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 42 rows and 4 columns.
          production stock extra_capacity productive_capacity
1
                  0
coal
         0
                         150
         1 260.403
                         0
coal
                                         0
                                                           300
coal
         2
             293.406
                           0
                                         0
                                                           300
              300 0
coal
coal
        3 300
4 17.9487 148.448
                                         0
                                                          300
                                  189.203
                                                      489.203
coal coal
        5 166.397 0
                                   1022.67
                                                      1511.88
                           0
                                    0
        6 166.397
                                                      1511.88
        7 – –
0 0 80
coal
                                         0
steel
        1 135.342 12.2811
steel
                                        0
                                                          350
        2 181.66 0
steel
                                         0
                                                          350
steel
        3
              193.09
                          0
                                         0
                                                          350
steel
        4 105.668
                                         0
                          0
                                                          350
steel 5
steel 6
steel 7
        5 105.668
6 105.668
                                         0
                           0
                                                          350
                           0
                                          0
                                                           350
                                          0
                 0 100
transport 0
transport 1 140.722 6.24084
                                         0
                                                          2.80
transport 2
              200.58 0
                                         0
                                                           280
transport 3 267.152
                          0
                                         0
                                                          280
transport 4 92.3077
                          0
                                         0
                                                          280
transport 5 92.3077
                          0
                                         0
                                                          280
                                         0
transport 6 92.3077
                                                          280
transport 7
  manpower_con
1
1
   224.988515
2
    270.657715
3
    367.038878
4
    470.000000
5
    150,000000
    150.000000
NOTE: Initialized model Problem2.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem2 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 42 linear constraints (24 LE, 18 EQ, 0 GE, 0 range).
                                                                   (continues on next page)
```

```
NOTE: The problem has 255 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 21 variables and 7 constraints.
NOTE: The LP presolver removed 64 constraint coefficients.
NOTE: The presolved problem has 42 variables, 35 constraints, and 191 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                       Objective
        Phase Iteration
                          Value
                                        Time
        D 2 1 9.413902E+03
         P 2
                   46 2.618579E+03
NOTE: Optimal.
NOTE: Objective = 2618.5791147.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and_
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 42 rows and 4 columns.
         production stock extra_capacity dict
1
                 0 150
coal
       0
                                 0 300
        1 184.818 31.6285
coal
        2 430.505 16.3725
coal
                                 130.505 430.505
                                   0 430.505
        3 430.505 0
coal
coal
        4 430.505
                         0
                                       0 430.505
        430.505 0
5 430.505 0
6 166.397 324.108
                                        0 430.505
coal
coal
                                        0 430.505
            _
0
                      80
        7
                                       0
coal
steel
steel
        0
       1 86.7295 11.5323
                                       0
                                              350
                                       0
steel
       2 155.337 0
                                             350
       3 182.867
                         0
                                       0
steel
                                              350
       4 359.402 0
steel
                                 9.40227 359.402
                                   0 359.402
steel
       5 359.402 176.535
        6 105.668 490.269
steel
                                       0 359.402
steel 7
                                       0
transport 0 0 100
transport 1 141.312 0
transport 2 198.388 0
                                0 280
0 280
0 280
239.383 519.383
            225.918
transport 3
transport 4
transport 5
            519.383 293.465
                                   0 519.383
transport 6 92.3077 750.54
                                       0 519.383
                                       0
transport 7
  manpower_con
1
1 217.374162
2 344.581624
3
   384.165212
4 470,000000
5
    470.000000
                                                                 (continues on next page)
```

```
150.000000
NOTE: Initialized model Problem3.
NOTE: Added action set 'optimization'.
NOTE: Converting model Problem3 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 63 variables (0 free, 12 fixed).
NOTE: The problem has 36 linear constraints (18 LE, 18 EQ, 0 GE, 0 range).
NOTE: The problem has 219 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 18 variables and 3 constraints.
NOTE: The LP presolver removed 31 constraint coefficients.
NOTE: The presolved problem has 45 variables, 33 constraints, and 188 constraint,
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                           Objective
        Phase Iteration
                           Value
                                          Time
             1
         D 2
                          4.013232E+04
         P 2
                    50
                        2.450027E+03
NOTE: Optimal.
NOTE: Objective = 2450.0266228.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
→and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 63 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 36 rows and 4 columns.
          production stock extra_capacity
        0 0 150
1 251.793 0
2 316.015 0
3 319.832 0
coal
coal
                                        0
                                  16.0152 316.015
                          0
coal
                          0
                                    3.8168 319.832
coal
             366.35
coal
        4
                          0
                                   46.5177 366.35
             859.36 0
coal
        5
                                    493.01 859.36
coal
        6
             859.36 460.208
                                        0 859.36
coal
        7
                                         0
              0 80
steel
        0
steel
        1 134.795 11.028
                                         0
                                               3.5.0
        2 175.041 0
steel
                                               350
                                         0
           224.064
steel
        3
                           0
                                         0
                                                350
           223.136
         4
                           0
                                         0
steel
                                                350
           220.044
steel
                                         0
        5
                           0
                                                350
                          0
steel
steel
        6 350
                                         0
                                                350
        7
             - - -
0 100
                                         0
transport 0
                                         _
transport 1 143.559 4.24723
                                         0
                                                280
transport 2 181.676 0
                                         0
                                                280
                                         0
transport 3
              280
                          0
                                                2.80
transport 4 279.072
                          0
                                         0
                                                280
              275.98
                           0
                                         0
                                                280
transport 5
transport 6 195.539
                           0
                                          0
                                                280
```

7.1.10 Decentralization

Reference

 $http://go.documentation.sas.com/?docsetId=ormpex\&docsetTarget=ormpex_ex10_toc.htm\&docsetVersion=14.3\&locale=en$

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex10.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
   m = so.Model(name='decentralization', session=cas_conn)
   DEPTS = ['A', 'B', 'C', 'D', 'E']
   CITIES = ['Bristol', 'Brighton', 'London']
    benefit_data = pd.DataFrame([
        ['Bristol', 10, 15, 10, 20, 5],
        ['Brighton', 10, 20, 15, 15, 15]],
        columns=['city'] + DEPTS).set_index('city')
    comm_data = pd.DataFrame([
        ['A', 'B', 0.0],
        ['A', 'C', 1.0],
        ['A', 'D', 1.5],
        ['A', 'E', 0.0],
        ['B', 'C', 1.4],
        ['B', 'D', 1.2],
        ['B', 'E', 0.0],
        ['C', 'D', 0.0],
        ['C', 'E', 2.0],
        ['D', 'E', 0.7]], columns=['i', 'j', 'comm']).set_index(['i', 'j'])
    cost_data = pd.DataFrame([
       ['Bristol', 'Bristol', 5],
        ['Bristol', 'Brighton', 14],
```

```
['Bristol', 'London', 13],
    ['Brighton', 'Brighton', 5],
    ['Brighton', 'London', 9],
    ['London', 'London', 10]], columns=['i', 'j', 'cost']).set_index(
        ['i', 'j'])
max_num_depts = 3
benefit = {}
for city in CITIES:
    for dept in DEPTS:
            benefit[dept, city] = benefit_data.loc[city, dept]
        except:
            benefit[dept, city] = 0
comm = {}
for row in comm_data.iterrows():
    (i, j) = row[0]
    comm[i, j] = row[1]['comm']
    comm[j, i] = comm[i, j]
cost = {}
for row in cost_data.iterrows():
    (i, j) = row[0]
    cost[i, j] = row[1]['cost']
    cost[j, i] = cost[i, j]
assign = m.add_variables(DEPTS, CITIES, vartype=so.BIN, name='assign')
IJKL = [(i, j, k, l)]
        for i in DEPTS for j in CITIES for k in DEPTS for l in CITIES
        if i < k]
product = m.add_variables(IJKL, vartype=so.BIN, name='product')
totalBenefit = so.quick_sum(benefit[i, j] * assign[i, j]
                            for i in DEPTS for j in CITIES)
totalCost = so.quick_sum(comm[i, k] * cost[j, l] * product[i, j, k, l]
                         for (i, j, k, l) in IJKL)
m.set_objective(totalBenefit-totalCost, name='netBenefit', sense=so.MAX)
m.add_constraints((so.quick_sum(assign[dept, city] for city in CITIES)
                  == 1 for dept in DEPTS), name='assign_dept')
m.add_constraints((so.quick_sum(assign[dept, city] for dept in DEPTS)
                  <= max_num_depts for city in CITIES), name='cardinality')</pre>
product_def1 = m.add_constraints((assign[i, j] + assign[k, 1] - 1
                                  <= product[i, j, k, l]
                                  for (i, j, k, l) in IJKL),
                                  name='product_def1')
product_def2 = m.add_constraints((product[i, j, k, 1] <= assign[i, j]</pre>
                                   for (i, j, k, l) in IJKL),
                                  name='product_def2')
```

```
product_def3 = m.add_constraints((product[i, j, k, 1] <= assign[k, 1]</pre>
                                   for (i, j, k, l) in IJKL),
                                  name='product_def3')
m.solve()
print(m.get_problem_summary())
m.drop_constraints(product_def1)
m.drop_constraints(product_def2)
m.drop_constraints(product_def3)
m.add_constraints((
    so.quick_sum(product[i, j, k, l]
                 for j in CITIES if (i, j, k, l) in IJKL) == assign[k, l]
    for i in DEPTS for k in DEPTS for l in CITIES if i < k),
    name='product_def4')
m.add_constraints((
    so.quick_sum(product[i, j, k, 1]
                 for 1 in CITIES if (i, j, k, 1) in IJKL) == assign[i, j]
    for k in DEPTS for i in DEPTS for j in CITIES if i < k),</pre>
    name='product_def4')
m.solve()
print(m.get_problem_summary())
totalBenefit.set_name('totalBenefit')
totalCost.set_name('totalCost')
print(so.get_solution_table(totalBenefit, totalCost))
print(so.get_solution_table(assign).unstack(level=-1))
return m.get_objective_value()
```

Output

```
In [1]: from examples.decentralization import test
In [2]: test(cas_conn)
NOTE: Initialized model decentralization.
NOTE: Added action set 'optimization'.
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 278 linear constraints (183 LE, 5 EQ, 90 GE, 0 range).
NOTE: The problem has 660 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 120 constraints.
NOTE: The MILP presolver removed 120 constraint coefficients.
NOTE: The MILP presolver added 120 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 158 constraints, and 540 constraint,
⇔coefficients.
                                                                          (continues on next page)
```

```
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger
                                                    BestBound
                                                                    Gap
                                                                           Time
                                    -14.9000000 135.0000000 111.04%
                             2
               Ω
                       1
                              2
                                                   67.5000000 122.07%
                0
                        1
                                    -14.9000000
                                                                              Ω
                                                    55.0000000 127.09%
                0
                        1
                               2
                                   -14.9000000
                                                   55.0000000 85.27%
                0
                        1
                               3
                                      8.1000000
                0
                        1
                              3
                                     8.1000000 48.0000000 83.12%
                                                                             0
                             3
                0
                        1
                                    8.1000000 44.8375000 81.93%
                                                                             1
               Ω
                        1
                             3
                                    8.1000000 42.0000000 80.71%
                                                                             1
                             3 3 3
                                    8.1000000 39.0666667 79.27%
8.1000000 34.7500000 76.69%
               0
                        1
                                                                             1
               Ω
                        1

      8.1000000
      33.9000000
      76.11%

      8.1000000
      29.6800000
      72.71%

      8.1000000
      28.5000000
      71.58%

                0
                        1
               0
                       1
                              3
                                                                             1
               0
                       1
                              3
                                                                             1
                                                   28.5000000 71.58%
               0
                              3
                                     8.1000000
                       1
                                                                             1
               Ω
                              3
                                                   28.5000000
                                                                 71.58%
                        1
                                      8.1000000
                                  8.1000000 28.500000 71.58%
                       1 3
               0
NOTE: The MILP solver added 31 cuts with 168 cut coefficients at the root.
                  0 4 14.9000000 14.9000000 0.00%
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 105 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS (casuser)' has 278 rows and 4,
→columns.
Problem Summary
                               Value
Label
Objective Sense
                      Maximization
Objective Function
                        netBenefit
Objective Type
                           Linear
Number of Variables
                                 105
Bounded Above
Bounded Below
                                   0
Bounded Below and Above
                                 105
Free
                                   Ω
Fixed
                                   0
Binary
                                 105
Integer
Number of Constraints
                                2.78
Linear LE (<=)
                                 183
Linear EQ (=)
                                  5
Linear GE (>=)
                                  90
Linear Range
                                  0
Constraint Coefficients
NOTE: Added action set 'optimization'.
NOTE: Converting model decentralization to OPTMODEL.
```

```
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 68 linear constraints (3 LE, 65 EQ, 0 GE, 0 range).
NOTE: The problem has 270 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 0 constraints.
NOTE: The MILP presolver removed 0 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 68 constraints, and 270 constraint.
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
             Node Active Sols BestInteger
                                                     BestBound
                           2
                              3 -16.300000 30.0000000 193.67%
4 14 000000
                                                   135.0000000 120.81%
               0
                       1
                0
                        1
                Ω
                        1
                                                                              Ω
                        1
                                                                              0
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows.
→and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 105 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 68 rows and 4 columns.
Problem Summary
                               Value
Label
Objective Sense
                       Maximization
Objective Function
                        netBenefit
Objective Type
                             Linear
Number of Variables
                                 105
Bounded Above
                                   Ω
Bounded Below
                                   0
Bounded Below and Above
                                 105
Free
                                   0
Fixed
                                   0
                                  105
Binary
Integer
                                  0
Number of Constraints
                                 68
                                  3
Linear LE (<=)
Linear EQ (=)
                                  65
Linear GE (>=)
                                  0
Linear Range
                                  0
                                 270
Constraint Coefficients
  totalBenefit totalCost
```

```
1
    80.0 65.1
    assign assign assign
2 Brighton Bristol London
1
A 0.0 1.0 0.0
B 1.0 0.0 0.0
C 1.0 0.0 0.0
D 0.0 1.0 0.0
E 1.0 0.0 0.0
Out[2]: 14.9
```

7.1.11 Optimal Wedding

Reference

SAS Blog: https://blogs.sas.com/content/operations/2014/11/10/do-you-have-an-uncle-louie-optimal-wedding-seat-assignments/

Model

```
import sasoptpy as so
import math
def test(cas_conn, num_guests=20, max_table_size=3, max_tables=None):
   m = so.Model("wedding", session=cas_conn)
    # Check max. tables
    if max_tables is None:
        max_tables = math.ceil(num_guests/max_table_size)
    # Sets
   guests = range(1, num_guests+1)
    tables = range(1, max_tables+1)
   guest_pairs = [[i, j] for i in guests for j in range(i+1, num_guests+1)]
    # Variables
    x = m.add_variables(guests, tables, vartype=so.BIN, name="x")
   unhappy = m.add_variables(tables, name="unhappy", lb=0)
    # Objective
   m.set_objective(unhappy.sum('*'), sense=so.MIN, name="obj")
    # Constraints
   m.add_constraints((x.sum(g, '*') == 1 for g in guests), name="assigncon")
   m.add_constraints((x.sum('*', t) <= max_table_size for t in tables),</pre>
                      name="tablesizecon")
    m.add\_constraints((unhappy[t] >= abs(g-h)*(x[g, t] + x[h, t] - 1)
                       for t in tables for [g, h] in guest_pairs),
                      name="measurecon")
    # Solve
```

Output

```
In [1]: from examples.sas optimal wedding import test
In [2]: test(cas_conn)
NOTE: Initialized model wedding.
NOTE: Added action set 'optimization'.
NOTE: Converting model wedding to DataFrame.
NOTE: Uploading the problem DataFrame to the server.
NOTE: Cloud Analytic Services made the uploaded file available as table TMP3CP5D7ME_
→in caslib CASUSERHDFS(casuser).
NOTE: The table TMP3CP5D7ME has been created in caslib CASUSERHDFS(casuser) from,
→binary data uploaded to Cloud Analytic Services.
NOTE: The problem wedding has 147 variables (140 binary, 0 integer, 0 free, 0 fixed).
NOTE: The problem has 1357 constraints (7 LE, 20 EQ, 1330 GE, 0 range).
NOTE: The problem has 4270 constraint coefficients.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value NONE is applied.
NOTE: The MILP solver is called.
NOTE: The Decomposition algorithm is used.
NOTE: The Decomposition algorithm is executing in the distributed computing,
→environment in single-machine mode.
NOTE: The DECOMP method value SET is applied.
NOTE: All blocks are identical and the master model is set partitioning.
NOTE: The Decomposition algorithm is using an aggregate formulation and Ryan-Foster_
⇒branching.
NOTE: The number of block threads has been reduced to 1 threads.
NOTE: The problem has a decomposable structure with 7 blocks. The largest block,
→covers 14.08% of the constraints in the problem.
NOTE: The decomposition subproblems cover 147 (100%) variables and 1337 (98.53%)
\hookrightarrowconstraints.
NOTE: The deterministic parallel mode is enabled.
NOTE: The Decomposition algorithm is using up to 32 threads.
                                              Best
                     Best Master
         Iter
                                                        LP
                                                                  IP CPU Real
                                                        Gap Gap Time Time
                    Bound Objective
                                           Integer
                   0.0000 13.0000
                                           13.0000 1.30e+01 1.30e+01 0 0
           1
                   0.0000
                              13.0000
                                           13.0000 1.30e+01 1.30e+01
                                                                             1
                   0.0000
                              13.0000
                                           13.0000 1.30e+01 1.30e+01
                                                                        7
                                                                            12
```

```
10
                 0.0000 13.0000 13.0000 1.30e+01 1.30e+01
                                                                  13
                                    13.0000 205.88% 205.88%
         18
                 4.2500
                          13.0000
                                                            20
         19
                 6.0000
                          13.0000
                                     13.0000 116.67% 116.67% 23
                                                                  33
                 6.0000
                          13.0000
                                     13.0000 116.67% 116.67% 23
                                                                34
                                                            24
         20
                 6.0000
                           13.0000
                                     13.0000 116.67% 116.67%
                                                                35
                                                            25
                           13.0000
                                      13.0000
                                             36.84%
         21
                 9.5000
                                                     36.84%
                                                                  36
                           13.0000 13.0000 0.00%
                13.0000
         23
                                                     0.00% 27
                           Best Best Gap CPU Real
          Node Active Sols
                               Integer
                                           Bound
                                                          Time Time
                  0 3 13.0000 13.0000 0.00% 27 39
            0
NOTE: The Decomposition algorithm used 32 threads.
NOTE: The Decomposition algorithm time is 39.79 seconds.
NOTE: Optimal.
NOTE: Objective = 13.
      Х
1 2
1 1 1.0
  2 0.0
1
  3 0.0
1
1
  4
    0.0
1
  5
    0.0
  6 0.0
1
  7 0.0
2
  1 1.0
2 2 0.0
2 3 0.0
2 4 0.0
2 5 0.0
2 6 0.0
2
 7 0.0
3 1 1.0
3 2 0.0
3
  3 0.0
3
  4
    0.0
3
  5 0.0
3 6 0.0
3 7 0.0
4 1 0.0
4 2 1.0
4 3 0.0
4 4 0.0
4 5 0.0
4 6 0.0
4 7 0.0
5 1 0.0
5 2 1.0
. . .
16 6 1.0
16 7 0.0
17 1 0.0
17 2 0.0
17 3 0.0
17 4 0.0
17 5 0.0
17 6 1.0
17 7 0.0
18 1 0.0
```

```
18 2 0.0
18 3 0.0
18 4 0.0
18 5 0.0
18 6 1.0
18 7 0.0
19 1 0.0
19 2 0.0
19 3 0.0
19 4 0.0
19 5 0.0
19 6 0.0
19 7 1.0
20 1 0.0
20 2 0.0
20 3 0.0
20 4 0.0
20 5 0.0
20 6 0.0
20 7 1.0
[140 rows x 1 columns]
Table 1 : [ 1 2 3 ]
Table 2 : [ 4 5 6 ]
Table 3 : [ 7 8 9 ]
Table 4 : [ 10 11 12 ]
Table 5 : [ 13 14 15 ]
Table 6 : [ 16 17 18 ]
Table 7 : [ 19 20 ]
Out[2]: 13.0
```

7.1.12 Kidney Exchange

Reference

SAS Blog: https://blogs.sas.com/content/operations/2015/02/06/the-kidney-exchange-problem/

Model

```
import sasoptpy as so
import random

def test(cas_conn):
    # Data generation
    n = 100
    p = 0.02

    random.seed(1)

ARCS = {}
    for i in range(0, n):
        for j in range(0, n):
```

```
if random.random() < p:</pre>
            ARCS[i, j] = random.random()
max\_length = 10
# Model
model = so.Model("kidney_exchange", session=cas_conn)
# Sets
NODES = set().union(*ARCS.keys())
MATCHINGS = range(1, int(len(NODES)/2)+1)
# Variables
UseNode = model.add_variables(NODES, MATCHINGS, vartype=so.BIN,
                              name="usenode")
UseArc = model.add_variables(ARCS, MATCHINGS, vartype=so.BIN,
                             name="usearc")
Slack = model.add_variables(NODES, vartype=so.BIN, name="slack")
print('Setting objective...')
# Objective
model.set_objective(so.quick_sum((ARCS[i, j] * UseArc[i, j, m]
                                   for [i, j] in ARCS for m in MATCHINGS)),
                    name="total_weight", sense=so.MAX)
print('Adding constraints...')
# Constraints
Node_Packing = model.add_constraints((UseNode.sum(i, '*') + Slack[i] == 1
                                      for i in NODES), name="node_packing")
Donate = model.add_constraints((UseArc.sum(i, '*', m) == UseNode[i, m]
                                 for i in NODES
                                 for m in MATCHINGS), name="donate")
Receive = model.add_constraints((UseArc.sum('*', j, m) == UseNode[j, m]
                                  for j in NODES
                                  for m in MATCHINGS), name="receive")
Cardinality = model.add_constraints((UseArc.sum('*', '*', m) <= max_length</pre>
                                      for m in MATCHINGS),
                                     name="cardinality")
# Solve
model.solve(options={'with': 'milp', 'maxtime': 300})
# Define decomposition blocks
for i in NODES:
    for m in MATCHINGS:
        Donate[i, m].set_block(m-1)
        Receive[i, m].set block(m-1)
for m in MATCHINGS:
    Cardinality[m].set_block(m-1)
model.solve(verbose=True, options={
    'with': 'milp', 'maxtime': 300, 'presolver': 'basic',
    'decomp': {'method': 'user'}})
return model.get_objective_value()
```

Output

```
In [1]: from examples.sas_kidney_exchange import test
In [2]: test(cas_conn)
NOTE: Initialized model kidney_exchange.
Setting objective...
Adding constraints...
NOTE: Added action set 'optimization'.
NOTE: Converting model kidney_exchange to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 15828 variables (0 free, 0 fixed).
NOTE: The problem has 15828 binary and 0 integer variables.
NOTE: The problem has 9850 linear constraints (49 LE, 9801 EQ, 0 GE, 0 range).
NOTE: The problem has 47286 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The remaining solution time after problem generation and solver initialization,
\rightarrowis 299.53 seconds.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 8223 variables and 7096 constraints.
NOTE: The MILP presolver removed 22296 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 7605 variables, 2754 constraints, and 24990
→constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 32 threads.
            Node Active Sols BestInteger
                                                    BestBound
                                                                   Gap
               0
                       1 2
                                  2.8934453
                                                   33.5480359
                                                                91.38%
NOTE: The MILP solver's symmetry detection found 531 orbits. The largest orbit,
→contains 23 variables.
               0 1
                              2
                                     2.8934453 33.5480359 91.38%
                                                                             15
                0
                        1
                              2
                                     2.8934453
                                                   33.5480359 91.38%
                        1
                              2
                                     2.8934453
                                                   33.5480359 91.38%
                                     2.8934453 33.5480359 91.38%
                             2
               0
                       1
NOTE: The MILP solver added 5 cuts with 278 cut coefficients at the root.
               2 2 2.8934453 33.5480359 91.38%
                                                                             20
                                                   33.5480359 60.76%
               3
                        3
                              3
                                    13.1640039
                                                                             22
                                  13.1640039
24.0492143
24.0492143
                                                   33.5480359 60.76%
                        7
                              3
                                                                             25
               8
                       9
                                                    33.5480359 28.31%
               10
                               4
                                                                             25
                                                    33.1832987 27.53%
              4.5
                       32
                               4
                                                                             30
              126
                       85
                              5
                                     26.1164550
                                                 32.7482323 20.25%
                                                                             32
              154
                      88
                              6 26.5523364 32.7482323 18.92%
                                                                             35
              155
                      89
                              6
                                   26.5523364 32.7482323 18.92%
                                                                             35

      26.5523364
      32.7482323
      18.92%

      28.0518710
      32.7482323
      14.34%

              262
                     154
                              6
                                                                             40
              416
                     273
                              7
                                                                             43
              421
                     242
                              7
                                    28.0518710
                                                   32.7482323 14.34%
                                                                             4.5
              597
                      357
                              7
                                    28.0518710
                                                   32.4428695 13.53%
                                                                             50
              646
                      397
                              7
                                    28.0518710
                                                    32.4428695 13.53%
                                                                             55
                              7
              748
                      433
                                    28.0518710
                                                    32.2947078 13.14%
                                                                             60
              769
                              7
                                                    32.2947078 13.14%
                       2
                                    28.0518710
                                                                             6.5
                               7
                                                    32.2947078 13.14%
              783
                       13
                                     28.0518710
                                                                             70
                       39
                               7
                                     28.0518710
              829
                                                    32.2947078
                                                                 13.14%
                                                                             75

      89
      7
      28.0518710
      32.2947078
      13.14%

      176
      7
      28.0518710
      32.2947078
      13.14%

              908
                                                                             80
             1037
                                                                             85
```

(continued from previous page) 1198 296 7 28.0518710 32.2947078 13.14% 91 1254 318 7 28.0518710 32.2947078 13.14% 1627 622 7 28.0518710 32.2947078 13.14% 100 1725 691 7 28.0518710 32.2947078 13.14% 105 739 7 32.2947078 13.14% 1815 28.0518710 110 7 32.2947078 13.14% 2148 989 28.0518710 115 2254 1081 7 28.0518710 32.2947078 13.14% 2433 1201 7 28.0518710 32.2947078 13.14% 2602 1331 7 28.0518710 32.2947078 13.14% 130 7 2829 1494 28.0518710 32.2947078 13.14% 135 3117 1700 7 28.0518710 32.2947078 13.14% 140 3182 1714 7 28.0518710 32.0012943 12.34% 145 3380 1866 7 28.0518710 32.0012943 12.34% 150 28.0518710 31.9947066 12.32% 3508 1955 7 3624 2025 7 28.0518710 31.9931742 12.32% 160 28.0518710 4176 2421 7 31.9931742 12.32% 166 7 31.8663012 11.97% 4209 2450 28.0518710 170 4333 2513 7 28.0518710 31.7554361 11.66% 175 7 2700 28.0518710 31.7554361 11.66% 4594 180 2782 7 31.7554361 4734 28.0518710 11.66% 185 31.7554361 4862 2849 7 28.0518710 11.66% 190 3081 28.0518710 5214 7 31.7554361 11.66% 196 5464 3243 7 28.0518710 31.7554361 11.66% 200 7 28.0518710 5981 3582 31.7071997 11.53% 205 7 28.0518710 6160 3679 31.6818160 11.46% 210 6260 3713 7 28.0518710 31.6317022 11.32% 215 6467 3839 7 28.0518710 31.6317022 11.32% 6576 3900 7 28.0518710 31.6317022 11.32% 225 4049 7 28.0518710 31.6234431 11.29% 230 6871 7 31.6234431 11.29% 7127 4190 28.0518710 2.35 7402 7 31.5637669 11.13% 4355 28.0518710 2.40 7570 4451 7 31.5534247 11.10% 28.0518710 245 11.10% 7794 4595 7 28.0518710 31.5534247 4743 7 31.5534247 8010 28.0518710 11.10% 28.0518710 8305 4904 7 31.5534247 11.10% 261 8570 5070 7 28.0518710 31.5534247 11.10% 265 7 8948 5290 28.0518710 31.5507961 11.09% 2.70 7 28.0518710 31.5507961 11.09% 7 28.0518710 31.4336628 10.76% 9069 5365 2.75 9400 5561 280 9482 5600 7 28.0518710 31.4336628 10.76% 28.0518710 31.4336628 10.76% 9602 5661 7 291 9887 5817 7 28.0518710 31.4336628 10.76% 295 5940 31.4336628 10.76% 7 10099 28.0518710 2.99 NOTE: Real time limit reached. NOTE: Objective of the best integer solution found = 28.051870979. NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and, \hookrightarrow 4 columns. NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows. \rightarrow and 4 columns. NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 15828 rows and 6 ⇔columns. NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 9850 rows and 4. NOTE: Response TIME LIM SOL NOTE: Added action set 'optimization'. NOTE: Converting model kidney_exchange to DataFrame. NOTE: Cloud Analytic Services made the uploaded file available as table BLOCKSTABLE. (continues on next page) →in caslib CASUSERHDFS(casuser).

```
NOTE: The table BLOCKSTABLE has been created in caslib CASUSERHDFS (casuser) from,
⇒binary data uploaded to Cloud Analytic Services.
NOTE: Uploading the problem DataFrame to the server.
NOTE: Cloud Analytic Services made the uploaded file available as table TMPOM5V148T_
→in caslib CASUSERHDFS(casuser).
NOTE: The table TMPOM5V148T has been created in caslib CASUSERHDFS (casuser) from _
⇒binary data uploaded to Cloud Analytic Services.
NOTE: The problem kidney_exchange has 15828 variables (15828 binary, 0 integer, 0...
\rightarrowfree, 0 fixed).
NOTE: The problem has 9850 constraints (49 LE, 9801 EQ, 0 GE, 0 range).
NOTE: The problem has 47286 constraint coefficients.
NOTE: The remaining solution time after solver initialization is 299.72 seconds.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value BASIC is applied.
NOTE: The MILP presolver removed 3156 variables and 2029 constraints.
NOTE: The MILP presolver removed 9526 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 12672 variables, 7821 constraints, and 37760.
→constraint coefficients.
NOTE: The MILP solver is called.
NOTE: The Decomposition algorithm is used.
NOTE: The Decomposition algorithm is executing in the distributed computing.
→environment in single-machine mode.
NOTE: The DECOMP method value USER is applied.
NOTE: All blocks are identical and the master model is set partitioning.
NOTE: The Decomposition algorithm is using an aggregate formulation and Ryan-Foster,
⇒branching.
NOTE: The number of block threads has been reduced to 1 threads.
NOTE: The problem has a decomposable structure with 49 blocks. The largest block,
→covers 2.02% of the constraints in the problem.
NOTE: The decomposition subproblems cover 12593 (99.38%) variables and 7742 (98.99%).
⇔constraints.
NOTE: The deterministic parallel mode is enabled.
NOTE: The Decomposition algorithm is using up to 32 threads.
                    Best Master Best LP

Pound Objective Integer Gap
        Iter
                                                                IP CPU Real
                                                             Gap Time Time
                Bound Objective Integer 358.5463 8.2725 8.2725
                                          8.2725 97.69% 97.69% 4 6
               350.4519

350.4519

344.6750

316.2089

15.5468

316.2089

15.5468

32.7247
                                           9.1816 97.38% 97.38% 8
           3
                                                                           12
           4
                                          15.5468 95.56% 95.56% 11 16
           5
                                          15.5468 95.49% 95.49% 14 21
           6
                                          15.5468 95.08% 95.08% 16 24
          10
                                          22.7247 95.08% 92.81% 21 32
                260.4562
                                          22.7247 91.28% 91.28% 24 36
          11
                             22.7247
                                          22.7247 91.25% 91.25% 28 43
                259.6150
          14
                                          22.7247 90.63% 90.64% 29 45
                242.7751
                             22.7554
          15
                                           22.7247 89.28% 89.64%
                             23.5204
          16
                 219.3485
                                                                      31
                                                                           48
                              23.7674
                                                    88.40%
          17
                                                             88.91%
                                                                      32
                                                                           51
                 204.8669
                                           22.7247
                                           22.7247
          18
                 155.3209
                              24.1564
                                                    84.45% 85.37%
                                                                      34
                                                                           53
                             24.1364
24.9509
24.9509
25.7096
26.0422
                155.3209
                                          22.7247 83.94% 85.37% 37
                                                                           59
                                                                     38 61
          2.0
                                          22.7247 83.08% 84.59%
               147.4931
          22
                113.1877
                                          23.7438 77.29% 79.02% 111 138
          23
                 79.4927
                                          23.7438 67.24% 70.13% 113 142
          30
                 79.4927
                             26.8911
                                          23.7438 66.17% 70.13% 126 163
          32
                 75.0523
                              26.9424
                                          23.7438 64.10% 68.36% 129 168
          38
                 74.7758
                              27.5362
                                          23.7438 63.17% 68.25% 143 191
                               27.6625
                  37.7021
                                          23.7438 26.63% 37.02% 145 194
          39
                                          27.1343 26.61% 28.03% 146 196
                  37.7021
                              27.6678
                                                                      (continues on next page)
```

```
40
                       37.7021 27.6678 27.1343 26.61% 28.03% 147 198
                                          28.0519
                                                            28.0519 25.60% 25.60% 153 207
               43
                         37.7021
                                          28.0519
28.0519
               45
                         30.7209
                                                            28.0519 8.69% 8.69% 155 210

      30.6065
      28.0519
      28.0519
      8.35%
      8.35%
      158
      214

      28.0519
      28.0519
      0.00%
      0.00%
      162
      219

               47
               50

        Node Active
        Sols
        Best Best Bound
        Gap CPU Real Time Time

        0
        0
        12
        28.0519
        28.0519
        0.00%
        162
        219

NOTE: The Decomposition algorithm used 32 threads.
NOTE: The Decomposition algorithm time is 219.86 seconds.
NOTE: Optimal within relative gap.
NOTE: Objective = 28.051870979.
Out[2]: 28.051871
```

7.2 Viya Examples / Abstract

7.2.1 Curve Fitting

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex11_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex11.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn, sols=False):
    # Upload data to server first
    xy_raw = pd.DataFrame([
        [0.0, 1.0],
        [0.5, 0.9],
        [1.0, 0.7],
        [1.5, 1.5],
        [1.9, 2.0],
        [2.5, 2.4],
        [3.0, 3.2],
        [3.5, 2.0],
        [4.0, 2.7],
        [4.5, 3.5],
        [5.0, 1.0],
        [5.5, 4.0],
        [6.0, 3.6],
        [6.6, 2.7],
        [7.0, 5.7],
        [7.6, 4.6],
        [8.5, 6.0],
```

```
[9.0, 6.8],
    [10.0, 7.3]
    ], columns=['x', 'y'])
xy_data = cas_conn.upload_frame(xy_raw, casout={'name': 'xy_data',
                                                 'replace': True})
# Read observations
POINTS, (x, y), xy_table_ref = so.read_table(xy_data, columns=['x', 'y'])
# Parameters and variables
order = so.Parameter(name='order')
beta = so.VariableGroup(so.exp_range(0, order), name='beta')
estimate = so.ImplicitVar(
    (beta[0] + so.quick_sum(beta[k] * x[i] ** k
                            for k in so.exp_range(1, order))
     for i in POINTS), name='estimate')
surplus = so.VariableGroup(POINTS, name='surplus', lb=0)
slack = so.VariableGroup(POINTS, name='slack', lb=0)
objective1 = so.Expression(
    so.quick_sum(surplus[i] + slack[i] for i in POINTS), name='objective1')
abs_dev_con = so.ConstraintGroup(
    (estimate[i] - surplus[i] + slack[i] == y[i] for i in POINTS),
    name='abs_dev_con')
minmax = so.Variable(name='minmax')
objective2 = so.Expression(minmax + 0.0, name='objective2')
minmax_con = so.ConstraintGroup(
    (minmax >= surplus[i] + slack[i] for i in POINTS), name='minmax_con')
order.set_init(1)
L1 = so.Model(name='L1', session=cas_conn)
L1.set_objective(objective1, sense=so.MIN)
L1.include(POINTS, x, y, xy_table_ref)
L1.include(order, beta, estimate, surplus, slack, abs_dev_con)
L1.add_statement('print x y estimate surplus slack;', after_solve=True)
L1.solve(verbose=True)
sol_data1 = L1.response['Print3.PrintTable'].sort_values('x')
print(so.get_solution_table(beta))
print(sol_data1.to_string())
Linf = so.Model(name='Linf', session=cas_conn)
Linf.include(L1, minmax, minmax_con)
Linf.set_objective(objective2, sense=so.MIN)
Linf.solve()
sol_data2 = Linf.response['Print3.PrintTable'].sort_values('x')
print(so.get_solution_table(beta))
print(sol_data2.to_string())
order.set_init(2)
L1.solve()
sol_data3 = L1.response['Print3.PrintTable'].sort_values('x')
print(so.get_solution_table(beta))
```

```
print(sol_data3.to_string())
Linf.solve()
sol_data4 = Linf.response['Print3.PrintTable'].sort_values('x')
print(so.get_solution_table(beta))
print(sol_data4.to_string())

if sols:
    return (sol_data1, sol_data2, sol_data3, sol_data4)
else:
    return Linf.get_objective_value()
```

Output

```
In [1]: from examples.curve_fitting import test
In [2]: (s1, s2, s3, s4) = test(cas_conn, sols=True)
NOTE: Cloud Analytic Services made the uploaded file available as table XY_DATA in.
→caslib CASUSERHDFS(casuser).
NOTE: The table XY_DATA has been created in caslib CASUSERHDFS(casuser) from binary_
→data uploaded to Cloud Analytic Services.
NOTE: Initialized model L1.
NOTE: Added action set 'optimization'.
NOTE: Converting model L1 to OPTMODEL.
set set_XY_DATA_N;
num x {set_XY_DATA_N};
num y {set_XY_DATA_N};
read data XY_DATA into set_XY_DATA_N=[_N_] x y;
num order = 1;
var beta {0..order};
impvar estimate {i_1 in set_XY_DATA_N} = beta[0] + sum {i_2 in 1..order}(beta[i_2] *_
\hookrightarrow (x[i_1]) ^ (i_2));
var surplus {set_XY_DATA_N} >= 0;
var slack {set_XY_DATA_N} >= 0;
min objective1 = sum {i_3 in set_XY_DATA_N}(surplus[i_3] + slack[i_3]);
con abs_dev_con {i_4 in set_XY_DATA_N} : y[i_4] + surplus[i_4] - slack[i_4] - __
\rightarrowestimate[i_4] = 0;
solve:
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x y estimate surplus slack;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 40 variables (2 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 19 linear constraints (0 LE, 19 EQ, 0 GE, 0 range).
NOTE: The problem has 75 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
                                                                           (continues on next page)
```

```
NOTE: The presolved problem has 40 variables, 19 constraints, and 75 constraint.
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                           Objective
        Phase Iteration
                              Value
                                            Time
         D 2
                           0.000000E+00
               1
         D 2
                     23
                          1.146625E+01
                                               0
NOTE: Optimal.
NOTE: Objective = 11.46625.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS (casuser)' has 40 rows and 6.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
→4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 19 rows and 4 columns.
      beta
1
0 0.58125
1 0.63750
   COL1
               y estimate surplus
           X
                                      slack
10 11.0 0.0 1.0 0.58125 0.00000 0.41875
18 19.0 0.5 0.9 0.90000 0.00000 0.00000
12 13.0 1.0 0.7 1.21875 0.51875 0.00000
4
    5.0 1.5 1.5 1.53750 0.03750 0.00000
\cap
    1.0 1.9 2.0 1.79250 0.00000 0.20750
15 16.0 2.5 2.4 2.17500 0.00000 0.22500
          3.0 3.2 2.49375 0.00000 0.70625
1
    2.0
         3.5 2.0 2.81250 0.81250 0.00000
11 12.0
          4.0 2.7
                    3.13125 0.43125 0.00000
5
    6.0
3
    4.0
          4.5 3.5
                    3.45000 0.00000 0.05000
    9.0
          5.0 1.0
                    3.76875 2.76875
                                      0.00000
14 15.0
         5.5 4.0 4.08750 0.08750 0.00000
13 14.0 6.0 3.6 4.40625 0.80625 0.00000
    8.0 6.6 2.7 4.78875 2.08875 0.00000
7
9
   10.0 7.0 5.7 5.04375 0.00000 0.65625
17 18.0 7.6 4.6 5.42625 0.82625 0.00000
    3.0 8.5 6.0 6.00000 0.00000 0.00000
6
    7.0 9.0 6.8 6.31875 0.00000 0.48125
16 17.0 10.0 7.3 6.95625 0.00000 0.34375
NOTE: Initialized model Linf.
NOTE: Added action set 'optimization'.
NOTE: Converting model Linf to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 41 variables (3 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 38 linear constraints (0 LE, 19 EQ, 19 GE, 0 range).
NOTE: The problem has 132 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
                                                                     (continues on next page)
```

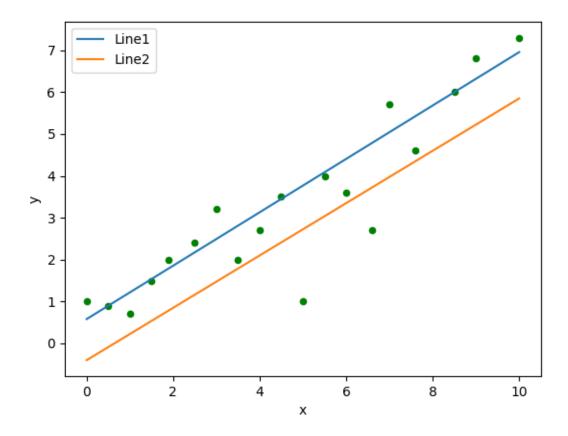
```
NOTE: The presolved problem has 41 variables, 38 constraints, and 132 constraint.
⇔coefficients.
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                           Objective
        Phase Iteration
                             Value
                                           Time
         D 2
              1
                         -5.000000E-03
                                            0
         P 2
                    26
                         1.725000E+00
                                              0
NOTE: Optimal.
NOTE: Objective = 1.725.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS (casuser)' has 41 rows and 6.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
→4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 38 rows and 4 columns.
1
0 - 0.400
1 0.625
           x y estimate surplus
   COT.1
                                     slack
10 11.0 0.0 1.0 -0.4000 0.000 1.4000
18 19.0 0.5 0.9 -0.0875 0.000 0.9875
12 13.0 1.0 0.7 0.2250
                            0.000 0.4750
4
    5.0 1.5 1.5 0.5375
                            0.000 0.9625
\cap
    1.0 1.9 2.0 0.7875
                            0.000 1.2125
15 16.0 2.5 2.4 1.1625
                             0.000 1.2375
    2.0
         3.0 3.2
                    1.4750
                             0.000 1.7250
1
                             0.000 0.2125
         3.5 2.0
11 12.0
                   1.7875
         4.0 2.7
                             0.000 0.6000
    6.0
5
                    2.1000
3
    4.0
          4.5 3.5
                     2.4125
                              0.000 1.0875
    9.0
          5.0 1.0
                     2.7250
                               1.725 0.0000
8
                    3.0375
                             0.000 0.9625
14 15.0
         5.5 4.0
13 14.0 6.0 3.6
                    3.3500 0.000 0.2500
    8.0 6.6 2.7
                    3.7250 1.025 0.0000
7
9
  10.0 7.0 5.7
                    3.9750 0.000 1.7250
17 18.0 7.6 4.6 4.3500 0.000 0.2500
2
    3.0 8.5 6.0 4.9125 0.000 1.0875
6
    7.0 9.0 6.8
                    5.2250
                            0.000 1.5750
16 17.0 10.0 7.3 5.8500
                             0.000 1.4500
NOTE: Added action set 'optimization'.
NOTE: Converting model L1 to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 41 variables (3 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 19 linear constraints (0 LE, 19 EQ, 0 GE, 0 range).
NOTE: The problem has 93 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 41 variables, 19 constraints, and 93 constraint.
                                                                    (continues on next page)
⇔coefficients.
```

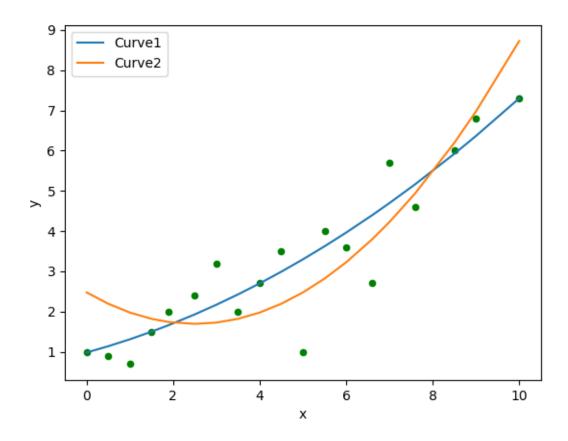
```
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                            Objective
        Phase Iteration
                              Value
                                            Time
         D 2
                     1
                           0.00000E+00
         D 2
                     20
                           1.045896E+01
                                               0
NOTE: Optimal.
NOTE: Objective = 10.458964706.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS (casuser)' has 41 rows and 6.
→columns.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 19 rows and 4 columns.
      heta
1
0 0.982353
1
  0.294510
  0.033725
                y estimate surplus
   COL1
                                          slack
           X
10 11.0 0.0 1.0 0.982353 0.000000 0.017647
18 19.0 0.5 0.9 1.138039 0.238039 0.000000
12 13.0 1.0 0.7 1.310588 0.610588 0.000000
4
    5.0 1.5 1.5 1.500000 0.000000 0.000000
0
    1.0
          1.9 2.0 1.663671 0.000000 0.336329
15 16.0
          2.5 2.4 1.929412 0.000000
                                       0.470588
    2.0
          3.0 3.2 2.169412 0.000000
                                       1.030588
1
          3.5 2.0 2.426275 0.426275
11 12.0
                                       0.000000
          4.0 2.7 2.700000 0.000000
5
    6.0
                                       0.000000
3
          4.5 3.5 2.990588 0.000000
    4.0
                                       0.509412
8
    9.0
          5.0 1.0
                    3.298039 2.298039
14 15.0
               4.0 3.622353 0.000000
          5.5
                                       0.377647
13 14.0
          6.0 3.6 3.963529 0.363529 0.000000
         6.6 2.7 4.395200 1.695200 0.000000
7
    8.0
9
   10.0
         7.0 5.7 4.696471 0.000000 1.003529
17 18.0
         7.6 4.6 5.168612 0.568612 0.000000
2
    3.0 8.5 6.0 5.922353 0.000000 0.077647
    7.0 9.0 6.8 6.364706 0.000000 0.435294
16 17.0 10.0 7.3 7.300000 0.000000 0.000000
NOTE: Added action set 'optimization'.
NOTE: Converting model Linf to OPTMODEL.
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: There were 19 rows read from table 'XY_DATA' in caslib 'CASUSERHDFS(casuser)'.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 42 variables (4 free, 0 fixed).
NOTE: The problem uses 19 implicit variables.
NOTE: The problem has 38 linear constraints (0 LE, 19 EQ, 19 GE, 0 range).
NOTE: The problem has 150 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The LP presolver value AUTOMATIC is applied.
NOTE: The LP presolver removed 0 variables and 0 constraints.
NOTE: The LP presolver removed 0 constraint coefficients.
NOTE: The presolved problem has 42 variables, 38 constraints, and 150 constraint
⇔coefficients.
```

```
NOTE: The LP solver is called.
NOTE: The Dual Simplex algorithm is used.
                          Objective
        Phase Iteration
                             Value
                                          Time
         D 2
                    1
                         -5.000000E-03
                                            0
         P 2
                    27
                         1.475000E+00
                                             0
NOTE: Optimal.
NOTE: Objective = 1.475.
NOTE: The Dual Simplex solve time is 0.01 seconds.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 13 rows_
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS (casuser)' has 42 rows and 6.
→columns.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 18 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 38 rows and 4 columns.
1
0 2.475
1 - 0.625
  0.125
   COL1
           x y estimate surplus
                                     slack
10 11.0 0.0 1.0 2.47500 1.47500 0.00000
18 19.0 0.5 0.9 2.19375 1.29375 0.00000
12 13.0 1.0 0.7 1.97500 1.27500 0.00000
4
    5.0 1.5 1.5 1.81875 0.31875 0.00000
0
    1.0 1.9 2.0 1.73875 0.00000 0.26125
15 16.0
          2.5 2.4 1.69375 0.00000 0.70625
    2.0
          3.0 3.2 1.72500 0.00000 1.47500
1
11 12.0
          3.5 2.0 1.81875 0.00000 0.18125
        4.0 2.7
                   1.97500 0.00000 0.72500
5
    6.0
3
          4.5 3.5
                   2.19375 0.00000 1.30625
    4.0
8
    9.0
          5.0 1.0
                    2.47500 1.47500 0.00000
14 15.0
                   2.81875 0.00000
          5.5
              4.0
                                    1.18125
13 14.0
         6.0 3.6 3.22500 0.00000 0.37500
7
    8.0
        6.6 2.7 3.79500 1.09500 0.00000
9
   10.0
        7.0 5.7 4.22500 0.00000 1.47500
17 18.0 7.6 4.6 4.94500 0.34500 0.00000
2
    3.0 8.5 6.0 6.19375 0.19375 0.00000
    7.0 9.0 6.8 6.97500 0.17500 0.00000
16 17.0 10.0 7.3 8.72500 1.42500 0.00000
```

```
# Plots
In [3]: import matplotlib.pyplot as plt
In [4]: p1 = s1.plot.scatter(x='x', y='y', c='g')
In [5]: s1.plot.line(ax=p1, x='x', y='estimate', label='Line1');
In [6]: s2.plot.line(ax=p1, x='x', y='estimate', label='Line2');
In [7]: p1
Out[7]: <matplotlib.axes._subplots.AxesSubplot at 0x7fc78621a550>
In [8]: p2 = s3.plot.scatter(x='x', y='y', c='g')
```

```
In [9]: s3.plot.line(ax=p2, x='x', y='estimate', label='Curve1');
In [10]: s4.plot.line(ax=p2, x='x', y='estimate', label='Curve2');
In [11]: p2
Out[11]: <matplotlib.axes._subplots.AxesSubplot at 0x7fc786114710>
```





7.2.2 Nonlinear 1

Reference

http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_examples01.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/nlpse01.html

Model

```
m.add\_constraint(1 - 0.0588*x[5]*x[7] - 0.1*x[1] >= 0, name='c1')
    m.add\_constraint(1 - 0.0588*x[6]*x[8] - 0.1*x[1] - 0.1*x[2] >= 0, name='c2')
     \texttt{m.add\_constraint} \ (1 \ - \ 4 * x \ [3] \ / \ x \ [5] \ - \ 2 \ / \ (x \ [3] \ * * 0.71 \ * \ x \ [5]) \ - \ 0.0588 * \ (x \ [7] \ / \ x \ [3] \ * * 1. 
\rightarrow3) >= 0, name='c3')
    m.add_constraint(1 - 4 \times x[4]/x[6] - 2/(x[4] \times 0.71 \times x[6]) - 0.0588 \times (x[8]/x[4] \times 1.
\rightarrow3) >= 0, name='c4')
    m.add\_constraint(f == [0.1, 4.2])
    x[1].set_init(6)
    x[2].set_init(3)
    x[3].set_init(0.4)
    x[4].set_init(0.2)
    x[5].set_init(6)
    x[6].set_init(6)
    x[7].set_init(1)
    x[8].set_init(0.5)
    m.add_statement('print x;', after_solve=True)
    m.solve(verbose=True, options={'with': 'nlp', 'algorithm': 'activeset'})
    print(m.get_problem_summary())
    print(m.get_solution_summary())
    print (m.response['Print3.PrintTable'])
    return m.get_objective_value()
```

Output

```
In [1]: from examples.nonlinear_1 import test
In [2]: test(cas_conn)
NOTE: Initialized model nlpse01.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse01 to OPTMODEL.
var x \{\{1,2,3,4,5,6,7,8\}\} >= 0.1 <= 10;
x[5] = 6;
x[6] = 6;
x[1] = 6;
x[7] = 1;
x[2] = 3;
x[8] = 0.5;
x[3] = 0.4;
x[4] = 0.2;
\min f = -x[1] + 0.4 * (((x[2]) / (x[8])) ^ (0.67)) - x[2] + 0.4 * (((x[1]) / (x[7]))_{\bullet})
\rightarrow^ (0.67)) + 10.0;
con c1 : -0.1 * x[1] - 0.0588 * x[5] * x[7] >= -1.0;
con c2 : -0.1 * x[1] - 0.1 * x[2] - 0.0588 * x[6] * x[8] >= -1.0;
con c3 : - (4 * x[3]) / (x[5]) - (2) / ((x[3]) ^ (0.71) * x[5]) - 0.0588 * ((x[7]) / (...)
\hookrightarrow ((x[3]) ^ (1.3))) >= -1.0;
con c4 : - (2) / ((x[4]) ^ (0.71) * x[6]) - (4 * x[4]) / (x[6]) - 0.0588 * ((x[8]) / _
\rightarrow ((x[4]) ^ (1.3))) >= -1.0;
con con_1 : -9.9 \leftarrow x[1] + 0.4 * (((x[1]) / (x[7])) ^ (0.67)) - x[2] + 0.4 *_
\rightarrow (((x[2]) / (x[8])) ^ (0.67)) <= -5.8;
solve with nlp / algorithm=activeset;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
```

```
print _con_.name _con_.body _con_.dual;
print x;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 8 variables (0 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 5 nonlinear constraints (0 LE, 0 EQ, 4 GE, 1 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0
→nonlinear constraints.
NOTE: Using analytic derivatives for objective.
NOTE: Using analytic derivatives for nonlinear constraints.
NOTE: The NLP solver is called.
NOTE: The Active Set algorithm is used.
                                                            Optimality
                          Objective
             Tter
                             Value
                                       Infeasibility
                                                                Error
                                         0.41664483
                Ω
                         3.65736570
                                                            0.24247905
                                          0.41664483
                1
                         3.65736570
                                                            0.24247905
                2
                         3.40486061
                                          0.10284726
                                                            0.10617988
                3
                         3.51178229
                                          0.07506389
                                                            0.10593173
                         4.23595983
                                          0.03595983
                                                            0.33749510
                5
                        4.16334906
                                                  Ω
                                                            0.26471063
                6
                        4.03168584
                                        0.00791810
                                                            0.13742971
                7
                        3.88912660
                                          0.11248991
                                                            0.06129662
                8
                        3.89579714
                                         0.09534670
                                                           0.05994916
                9
                        3.95046640
                                         0.02649207
                                                           0.06776850
               10
                        3.92833580
                                         0.03517161
                                                           0.06442935
               11
                        3.95179326
                                         0.00494247
                                                           0.05837915
               12
                        3.94741555
                                         0.00651989
                                                           0.05477333
               13
                        3.95209064
                                          0.00058609
                                                            0.05265725
               14
                        3.95058104
                                          0.00122758
                                                            0.04772557
                                          0.00099113
               15
                         3.95055959
                                                            0.04613473
               16
                         3.95141460
                                          0.00000381
                                                            0.04497006
                         3.95132211 0.0000005999371
               17
                                                            0.04260039
                         3.95114031
               18
                                          0.00000941
                                                            0.04093117
                         3.95027690
               19
                                           0.00011307
                                                            0.00020755
               2.0
                        3.95115797 0.0000007730235
                                                            0.00010507
                        3.95116558
               2.1
                                                  0
                                                            0.00001366
               22
                        3.95116364 0.000000153799
                                                            0.00000814
               23
                        3.95116355 0.0000000228326
                                                            0.00000595
               24
                        3.95116352 0.0000000257138
                                                            0.00000337
               25
                        3.95116349 0.0000000200547
                                                            0.00000132
               2.6
                         3.95116349 0.0000000192412 0.0000002015918
NOTE: Optimal.
NOTE: Objective = 3.9511634887.
NOTE: Objective of the best feasible solution found = 3.9511579677.
NOTE: The best feasible solution found is returned.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 20 rows and,
\rightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 8 rows and 6.
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 5 rows and 4 columns.
NOTE: Response BEST_FEASIBLE
Problem Summary
                                                                      (continues on next page)
```

```
Value
Label
Objective Sense Minimization
Objective Function
Objective Type
                           Nonlinear
Number of Variables
Bounded Above
Bounded Below
                                    0
Bounded Below and Above
                                    8
                                    Ω
Fixed
                                    0
Number of Constraints
                                    5
Linear LE (<=)
                                    0
Linear EQ (=)
                                    0
Linear GE (>=)
                                    0
Linear Range
                                    0
Nonlinear LE (<=)
                                    0
Nonlinear EQ (=)
                                    0
Nonlinear GE (>=)
                                    4
Nonlinear Range
                                    1
Solution Summary
                           Value
Label
Solver NLP Algorithm Active Set Objective Function f
Solver
                             NLP
Solution Status Best Feasible Objective Value 3.9511579677
Optimality Error 0.0001050714 Infeasibility 7.7302351E-7
Iterations
                              26
Presolve Time
Solution Time
                            0.00
                            0.01
 COL1 x
0 1.0 6.463315
1 2.0 2.234530
2 3.0 0.667455
3 4.0 0.595820
4 5.0 5.932980
   6.0 5.527231
    7.0 1.013787
  8.0 0.400664
Out[2]: 3.951158
```

7.2.3 Nonlinear 2

Reference

http://go.documentation.sas.com/?docsetId=ormpug&docsetTarget=ormpug_nlpsolver_examples02.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/nlpse02.html

Model

```
import sasoptpy as so
import sasoptpy.math as sm
def test(cas conn):
   m = so.Model(name='nlpse02', session=cas_conn)
   N = m.add_parameter(name='N', init=1000)
    x = m.add_variables(so.exp_range(1, N), name='x', init=1)
    m.set_objective(
        so.quick_sum(-4 \times x[i]+3 for i in so.exp_range(1, N-1)) +
        so.quick_sum((x[i]**2 + x[N]**2)**2 for i in so.exp_range(1, N-1)),
        name='f', sense=so.MIN)
    m.add_statement('print x;', after_solve=True)
    m.solve(options={'with': 'nlp'}, verbose=True)
   print (m.response['Print3.PrintTable'])
    # Model 2
   so.reset_globals()
   m = so.Model(name='nlpse02_2', session=cas_conn)
   N = m.add_parameter(name='N', init=1000)
   x = m.add_variables(so.exp_range(1, N), name='x', lb=1, ub=2)
   m.set_objective(
        so.quick_sum(sm.cos(-0.5*x[i+1] - x[i]**2) for i in so.exp_range(
            1, N-1)), name='f2', sense=so.MIN)
    m.add_statement('print x;', after_solve=True)
    m.solve(verbose=True, options={'with': 'nlp', 'algorithm': 'activeset'})
    print(m.get_solution_summary())
    return m.get_objective_value()
```

Output

```
In [1]: from examples.nonlinear_2 import test
In [2]: test(cas_conn)
NOTE: Initialized model nlpse02.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse02 to OPTMODEL.
num N = 1000;
var x {1..N} init 1;
 \min \ f = \sup \{i_2 \ in \ 1..N-1\} (((x[i_2]) \ ^ (2) \ + (x[N]) \ ^ (2)) \ ^ (2)) \ + \sup \{i_1 \ in \ 1..N-1\} (((x[i_2]) \ ^ (2) \ + (x[N]) \ ^ (2))) \ + \lim \{i_2 \ in \ 1..N-1\} (((x[i_2]) \ ^ (2) \ + (x[N]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_2 \ in \ 1..N-1\} (((x[i_2]) \ ^ (2) \ + (x[N]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_3 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_3 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2))) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ + \lim \{i_4 \ in \ 1..N-1\} ((x[i_3]) \ ^ (2) \ + (x[i_3]) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ ^ (2)) \ ^ (2)
 \rightarrow 1} (- 4 * x[i_1] + 3);
solve with nlp;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
                                                                                                                                                                                                                                                                                      (continues on next page)
```

```
NOTE: The problem has 1000 variables (1000 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0
→nonlinear constraints.
NOTE: Using analytic derivatives for objective.
NOTE: Using 2 threads for nonlinear evaluation.
NOTE: The NLP solver is called.
NOTE: The Interior Point algorithm is used.
                        Objective
                                                           Optimality
                            Value
                                      Infeasibility
             Tter
                                                            Error
               0
                    2997.00000000
                                        0
                                                     3996.00000000
                1
                     2903.33927274
                                                  0
                                                       3901.96888568
                     2720.89298022
                                                  0
                                                        3716.59858581
                3
                     2375.45256010
                                                  0
                                                        3356.96682109
                     2050.78067864
                4
                                                  0
                                                        3007.33819156
                                                       2358.96117840
                5
                     1479.51953631
                                                  0
                                                 0
                6
                     635.46851927
                                                       1297.01852837
                                                 0
                       47.88027207
                                                        263.90369702
                        0.56099667
                                                  0
                                                         25.76387053
                9
                        0.00010025
                                                  0
                                                           0.31770898
               10
                   0.0000000000556
                                                  0
                                                           0.00005787
               11
                                 Ω
                                                 0 1.9350622922E-12
NOTE: Optimal.
NOTE: Objective = 0.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows and
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows.
→and 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 1000 rows and 6
→columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS (casuser)' has 0 rows and 4 columns.
      COL1
                    X
       1.0 1.000000e+00
1
       2.0 1.000000e+00
2
       3.0 1.000000e+00
       4.0 1.000000e+00
3
      5.0 1.000000e+00
4
5
      6.0 1.000000e+00
6
      7.0 1.000000e+00
7
      8.0 1.000000e+00
8
      9.0 1.000000e+00
9
     10.0 1.000000e+00
     11.0 1.000000e+00
10
     12.0 1.000000e+00
11
12
      13.0 1.000000e+00
13
      14.0 1.000000e+00
14
      15.0 1.000000e+00
15
     16.0 1.000000e+00
     17.0 1.000000e+00
16
17
     18.0 1.000000e+00
18
     19.0 1.000000e+00
19
     20.0 1.000000e+00
20
      21.0 1.000000e+00
21
      22.0 1.000000e+00
      23.0 1.000000e+00
22
      24.0 1.000000e+00
23
```

```
24
      25.0 1.000000e+00
25
      26.0 1.000000e+00
26
      27.0 1.000000e+00
27
      28.0 1.000000e+00
28
      29.0 1.000000e+00
29
      30.0 1.000000e+00
. .
       . . .
970
     971.0 1.000000e+00
971
     972.0 1.000000e+00
972
     973.0 1.000000e+00
973
     974.0 1.000000e+00
974
     975.0 1.000000e+00
975
     976.0 1.000000e+00
976
     977.0 1.000000e+00
977
     978.0 1.000000e+00
978
     979.0 1.000000e+00
979
     980.0 1.000000e+00
     981.0 1.000000e+00
980
     982.0 1.000000e+00
981
982
     983.0 1.000000e+00
983
     984.0
            1.000000e+00
984
     985.0 1.000000e+00
985
     986.0 1.000000e+00
986
     987.0 1.000000e+00
987
     988.0 1.000000e+00
988
     989.0 1.000000e+00
989
    990.0 1.000000e+00
990
     991.0 1.000000e+00
991
     992.0 1.000000e+00
992
     993.0 1.000000e+00
993
     994.0 1.000000e+00
     995.0 1.000000e+00
994
995
     996.0 1.000000e+00
     997.0
996
            1.000000e+00
997
     998.0 1.000000e+00
     999.0 1.000000e+00
998
999 1000.0 9.684996e-16
[1000 rows x 2 columns]
NOTE: Initialized model nlpse02_2.
NOTE: Added action set 'optimization'.
NOTE: Converting model nlpse02_2 to OPTMODEL.
num N = 1000;
var x \{1..N\} >= 1 <= 2;
min f2 = sum \{i_1 in 1..N-1\} (cos(-0.5 * x[i_1 + 1] - (x[i_1]) ^ (2)));
solve with nlp / algorithm=activeset;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;
NOTE: Submitting OPTMODEL codes to CAS server.
NOTE: Problem generation will use 32 threads.
NOTE: The problem has 1000 variables (0 free, 0 fixed).
NOTE: The problem has 0 linear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver removed 0 variables, 0 linear constraints, and 0
→nonlinear constraints.
```

```
NOTE: Using analytic derivatives for objective.
NOTE: Using 3 threads for nonlinear evaluation.
NOTE: The NLP solver is called.
NOTE: The Active Set algorithm is used.
NOTE: Initial point was changed to be feasible to bounds.
                  Objective
                                                        Optimality
                        Value
                                     Infeasibility
                                                           Error
                  70.66646447
                                                       1.24686873
1.24686873
                                    0
               1
                                                0
                    -996.26893548
                                               0
                                                       0.23815533
                    -998.99328004
                                               0
                                                       0.10718277
               4
                    -998.99999439
                                               0
                                                       0.00379400
                                               0 0.00000393
                    -999.0000000
                                           0 1.7022658635E-12
                    -999.0000000
NOTE: Optimal.
NOTE: Objective = -999.
NOTE: The CAS table 'problemSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows and,
\hookrightarrow4 columns.
NOTE: The CAS table 'solutionSummary' in caslib 'CASUSERHDFS(casuser)' has 12 rows.
\rightarrowand 4 columns.
NOTE: The CAS table 'primal' in caslib 'CASUSERHDFS(casuser)' has 1000 rows and 6,
⇔columns.
NOTE: The CAS table 'dual' in caslib 'CASUSERHDFS(casuser)' has 0 rows and 4 columns.
Solution Summary
                        Value
Label
Solver
                         NLP
Algorithm Active Set
Objective Function
                          £2.
Solution Status
                     Optimal
Objective Value
                        -999
Optimality Error 1.702266E-12
Infeasibility
Iterations
                            6
                        0.00
Presolve Time
Solution Time
                        0.07
Out[2]: -999.0
```

7.3 SAS (saspy) Examples

7.3.1 Decentralization (saspy)

Reference

http://go.documentation.sas.com/?docsetId=ormpex&docsetTarget=ormpex_ex10_toc.htm&docsetVersion=14.3&locale=en

http://support.sas.com/documentation/onlinedoc/or/ex_code/143/mpex10.html

Model

```
import sasoptpy as so
import pandas as pd
def test(cas_conn):
   m = so.Model(name='decentralization', session=cas_conn)
   DEPTS = ['A', 'B', 'C', 'D', 'E']
   CITIES = ['Bristol', 'Brighton', 'London']
   benefit_data = pd.DataFrame([
        ['Bristol', 10, 15, 10, 20, 5],
        ['Brighton', 10, 20, 15, 15, 15]],
        columns=['city'] + DEPTS).set_index('city')
    comm_data = pd.DataFrame([
        ['A', 'B', 0.0],
        ['A', 'C', 1.0],
        ['A', 'D', 1.5],
        ['A', 'E', 0.0],
        ['B', 'C', 1.4],
        ['B', 'D', 1.2],
        ['B', 'E', 0.0],
        ['C', 'D', 0.0],
        ['C', 'E', 2.0],
        ['D', 'E', 0.7]], columns=['i', 'j', 'comm']).set_index(['i', 'j'])
   cost_data = pd.DataFrame([
       ['Bristol', 'Bristol', 5],
        ['Bristol', 'Brighton', 14],
        ['Bristol', 'London', 13],
        ['Brighton', 'Brighton', 5],
        ['Brighton', 'London', 9],
        ['London', 'London', 10]], columns=['i', 'j', 'cost']).set_index(
            ['i', 'j'])
   max_num_depts = 3
   benefit = {}
    for city in CITIES:
        for dept in DEPTS:
                benefit[dept, city] = benefit_data.loc[city, dept]
            except:
                benefit [dept, city] = 0
   comm = { } { }
    for row in comm_data.iterrows():
        (i, j) = row[0]
        comm[i, j] = row[1]['comm']
        comm[j, i] = comm[i, j]
    cost = {}
    for row in cost_data.iterrows():
```

```
(i, j) = row[0]
    cost[i, j] = row[1]['cost']
    cost[j, i] = cost[i, j]
assign = m.add_variables(DEPTS, CITIES, vartype=so.BIN, name='assign')
IJKL = [(i, j, k, 1)]
        for i in DEPTS for j in CITIES for k in DEPTS for 1 in CITIES
product = m.add_variables(IJKL, vartype=so.BIN, name='product')
totalBenefit = so.quick_sum(benefit[i, j] * assign[i, j]
                             for i in DEPTS for j in CITIES)
totalCost = so.quick_sum(comm[i, k] * cost[j, l] * product[i, j, k, l]
                          for (i, j, k, l) in IJKL)
m.set_objective(totalBenefit-totalCost, name='netBenefit', sense=so.MAX)
m.add_constraints((so.quick_sum(assign[dept, city] for city in CITIES)
                  == 1 for dept in DEPTS), name='assign_dept')
m.add_constraints((so.quick_sum(assign[dept, city] for dept in DEPTS)
                  <= max_num_depts for city in CITIES), name='cardinality')</pre>
product_def1 = m.add_constraints((assign[i, j] + assign[k, l] - 1
                                  <= product[i, j, k, l]
                                  for (i, j, k, l) in IJKL),
                                  name='product_def1')
product_def2 = m.add_constraints((product[i, j, k, 1] <= assign[i, j]</pre>
                                   for (i, j, k, l) in IJKL),
                                  name='product_def2')
product_def3 = m.add_constraints((product[i, j, k, 1] <= assign[k, 1]</pre>
                                   for (i, j, k, l) in IJKL),
                                  name='product_def3')
m.solve()
print (m.get_problem_summary())
m.drop_constraints(product_def1)
m.drop_constraints(product_def2)
m.drop_constraints(product_def3)
m.add_constraints((
    so.quick_sum(product[i, j, k, l]
                 for j in CITIES if (i, j, k, l) in IJKL) == assign[k, l]
    for i in DEPTS for k in DEPTS for l in CITIES if i < k),
    name='product_def4')
m.add_constraints((
    so.quick_sum(product[i, j, k, l]
                 for 1 in CITIES if (i, j, k, 1) in IJKL) == assign[i, j]
    for k in DEPTS for i in DEPTS for j in CITIES if i < k),
    name='product_def4')
m.solve()
                                                                       (continues on next page)
```

```
print(m.get_problem_summary())
totalBenefit.set_name('totalBenefit')
totalCost.set_name('totalCost')
print(so.get_solution_table(totalBenefit, totalCost))
print(so.get_solution_table(assign).unstack(level=-1))

return m.get_objective_value()
```

Output

```
>>> from examples.decentralization import test
>>> sas_session = saspy.SASsession(cfgname='winlocal')
>>> test(sas_session)
SAS Connection established. Subprocess id is 18384
NOTE: Initialized model decentralization.
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to SAS server.
NOTE: Writing HTML5(SASPY_INTERNAL) Body file: _TOMODS1
NOTE: Problem generation will use 4 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 278 linear constraints (183 LE, 5 EQ, 90 GE, 0 range).
NOTE: The problem has 660 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EQ, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 120 constraints.
NOTE: The MILP presolver removed 120 constraint coefficients.
NOTE: The MILP presolver added 120 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 158 constraints, and 540 constraint,
→coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 4 threads.
         Node Active Sols BestInteger BestBound
                                                                   Time
                                                             Gap
            0
                 1 3
                               -14.9000000 135.0000000 111.04%
                    1
            0
                          3
                               -14.9000000 67.5000000 122.07%
                                                                       0
                    1
            0
                          3
                               -14.9000000
                                             55.0000000 127.09%
            0
                    1
                          4
                               8.1000000
                                             48.0000000 83.12%
                                8.1000000 44.8375000 81.93%
            0
                    1
                          4
            0
                    1
                          4
                                8.1000000
                                             42.0000000 80.71%
            0
                    1
                          4
                                              39.0666667 79.27%
                                                                      0
                                8.1000000
            0
                   1
                          4
                                              34.7500000 76.69%
                                                                      0
                                8.1000000
                                             33.9000000
                                                          76.11%
            0
                    1
                          4
                                 8.1000000
                                                                      0
                          4
                                              29.6800000
            0
                    1
                                 8.1000000
                                                           72.71%
                                                                       0
            0
                    1
                          4
                                 8.1000000
                                              28.5000000
                                                           71.58%
                                                                       0
            0
                    1
                           4
                                 8.1000000
                                              28.5000000
                                                           71.58%
            0
                           4
                                 8.1000000
                                              28.5000000 71.58%
                                                                       Ω
                    1
            0
                    1
                          4
                                8.1000000
                                              28.5000000 71.58%
NOTE: The MILP solver added 31 cuts with 168 cut coefficients at the root.
                0
                       5 14.9000000 14.9000000 0.00%
                                                                       0
```

```
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The data set WORK.PROB_SUMMARY has 20 observations and 3 variables.
NOTE: The data set WORK.SOL_SUMMARY has 18 observations and 3 variables.
NOTE: The data set WORK.PRIMAL_OUT has 105 observations and 6 variables.
NOTE: The data set WORK.DUAL_OUT has 278 observations and 4 variables.
NOTE: PROCEDURE OPTMODEL used (Total process time):
     real time
                         0.34 seconds
     cpu time
                         0.29 seconds
                               Value
Label
Objective Sense
                       Maximization
Objective Function
                        netBenefit
Objective Type
                             Linear
Number of Variables
                                 105
Bounded Above
                                   Ω
Bounded Below
                                   0
Bounded Below and Above
                                 105
Free
                                   0
Fixed
                                   Ω
                                 105
Binary
Integer
                                  0
Number of Constraints
                                2.78
Linear LE (<=)
                                 183
Linear EO (=)
                                  5
Linear GE (>=)
                                  90
                                   0
Linear Range
Constraint Coefficients
                                 660
NOTE: Converting model decentralization to OPTMODEL.
NOTE: Submitting OPTMODEL codes to SAS server.
NOTE: Writing HTML5 (SASPY_INTERNAL) Body file: _TOMODS1
NOTE: Problem generation will use 4 threads.
NOTE: The problem has 105 variables (0 free, 0 fixed).
NOTE: The problem has 105 binary and 0 integer variables.
NOTE: The problem has 68 linear constraints (3 LE, 65 EQ, 0 GE, 0 range).
NOTE: The problem has 270 linear constraint coefficients.
NOTE: The problem has 0 nonlinear constraints (0 LE, 0 EO, 0 GE, 0 range).
NOTE: The OPTMODEL presolver is disabled for linear problems.
NOTE: The initial MILP heuristics are applied.
NOTE: The MILP presolver value AUTOMATIC is applied.
NOTE: The MILP presolver removed 0 variables and 0 constraints.
NOTE: The MILP presolver removed 0 constraint coefficients.
NOTE: The MILP presolver modified 0 constraint coefficients.
NOTE: The presolved problem has 105 variables, 68 constraints, and 270 constraint,
⇔coefficients.
NOTE: The MILP solver is called.
NOTE: The parallel Branch and Cut algorithm is used.
NOTE: The Branch and Cut algorithm is using up to 4 threads.
         Node Active Sols BestInteger
                                                BestBound
                                                                        Time
                                -28.1000000 135.0000000 120.81%
            0
                    1
                           3
                                               30.0000000 193.67%
            0
                     1
                           3
                                 -28.1000000
                                                                           0
            0
                           4 -16.3000000
                                                30.0000000 154.33%
                                                                           0
                     1
                           5 14.9000000
            0
                                              14.9000000
                                                             0.00%
                                                                          0
                                                                       (continues on next page)
```

```
NOTE: Optimal.
NOTE: Objective = 14.9.
NOTE: The data set WORK.PROB_SUMMARY has 20 observations and 3 variables.
NOTE: The data set WORK.SOL_SUMMARY has 18 observations and 3 variables.
NOTE: The data set WORK.PRIMAL_OUT has 105 observations and 6 variables.
NOTE: The data set WORK.DUAL_OUT has 68 observations and 4 variables.
NOTE: PROCEDURE OPTMODEL used (Total process time):
     real time
                       0.19 seconds
     cpu time
                       0.14 seconds
                             Value
Label
Objective Sense
                     Maximization
Objective Function
                      netBenefit
Objective Type
                          Linear
Number of Variables
                               105
Bounded Above
                                0
Bounded Below
                                 0
Bounded Below and Above
                               105
Free
                                0
Fixed
                                 0
Binary
                               105
Integer
                               0
Number of Constraints
                               68
Linear LE (<=)
                                3
Linear EQ (=)
                               65
                                0
Linear GE (>=)
Linear Range
                                0
                              270
Constraint Coefficients
  totalBenefit totalCost
         80.0
               65.1
   assign assign assign
2 Brighton Bristol London
1
Α
     0.0 1.0 0.0
     1.0 0.0 0.0
В
С
     1.0
            0.0 0.0
     0.0 1.0
                  0.0
D
Ε
      1.0
            0.0
                  0.0
```

CHAPTER

EIGHT

API REFERENCE

8.1 Model

8.1.1 Constructor

Mode I (name[, session]) Creates an optimization model
--

sasoptpy.Model

```
class sasoptpy.Model(name, session=None)
    Bases: object
```

Creates an optimization model

Parameters name: string

Name of the model

 $\pmb{\textit{session}}: \texttt{swat.cas.connection.CAS} \ \pmb{\textit{object or}} \ \texttt{saspy.SASsession} \ \pmb{\textit{object, optional}}$

CAS or SAS Session object

Examples

```
>>> from swat import CAS
>>> import sasoptpy as so
>>> s = CAS('cas.server.address', port=12345)
>>> m = so.Model(name='my_model', session=s)
NOTE: Initialized model my_model

>>> mip = so.Model(name='mip')
NOTE: Initialized model mip
```

8.1.2 Components

Model.set_session(session)	Sets the CAS session for model
Model.add_constraint(c[, name])	Adds a single constraint to the model
<pre>Model.add_constraints(argv[, cg, name])</pre>	Adds a set of constraints to the model
	0 11 1

Continued on next page

Table 2 – continued from previous page

Model.add_variable([var, vartype, name, lb,	Adds a new variable to the model
])	
Model.add_variables(*argv[, vg, name,])	Adds a group of variables to the model
Model.add_implicit_variable([argv, name])	Adds an implicit variable to the model
Model.add_set(name[, init, settype])	Adds a set to the model
Model.add_parameter(*argv[, name, init,	Adds a parameter to the model
p_type])	
<pre>Model.add_statement(statement[, after_solve])</pre>	Adds a PROC OPTMODEL statement to the model
Model.set_objective(expression[, sense,	Sets the objective function for the model
name])	
Model.set_coef(var, con, value)	Updates the coefficient of a variable inside constraints
Model.drop_constraint(constraint)	Drops a constraint from the model
Model.drop_constraints(constraints)	Drops a constraint group from the model
Model.drop_variable(variable)	Drops a variable from the model
Model.drop_variables(variables)	Drops a variable group from the model
Model.get_constraint(name)	Returns the reference to a constraint in the model
Model.get_constraints()	Returns a list of constraints in the model
Model.get_variable(name)	Returns the reference to a variable in the model
Model.get_variables()	Returns a list of variables
Model.get_objective()	Returns the objective function as an Expression ob-
	ject
Model.get_variable_coef(var)	Returns the objective value coefficient of a variable
Model.read_data(table, key_set[, key_cols,])	Reads a CASTable into PROC OPTMODEL and adds it
	to the model
Model.read_table(table[, key, columns,])	Reads a CAS Table or pandas DataFrame into the model
Model.include(*argv)	Adds existing variables and constraints to a model

sasoptpy.Model.set_session

Model.set_session (session)
Sets the CAS session for model

Parameters session: swat.cas.connection.CAS or saspy.SASsession objects

CAS or SAS Session object

Notes

• Session of a model can be set at initialization. See Model.

sasoptpy.Model.add_constraint

Model.add_constraint(c, name=None)

Adds a single constraint to the model

Parameters c : Constraint

Constraint to be added to the model

name : string, optional

Name of the constraint

Returns Constraint object

See also:

```
Constraint, Model.include()
```

Examples

```
>>> x = m.add_variable(name='x', vartype=so.INT, lb=0, ub=5)
>>> y = m.add_variables(3, name='y', vartype=so.CONT, lb=0, ub=10)
>>> c1 = m.add_constraint(x + y[0] >= 3, name='c1')
>>> print(c1)
x + y[0] >= 3
```

```
>>> c2 = m.add_constraint(x - y[2] == [4, 10], name='c2')
>>> print(c2)
- y[2] + x = [4, 10]
```

sasoptpy.Model.add constraints

```
Model.add_constraints (argv, cg=None, name=None)
```

Adds a set of constraints to the model

Parameters argv: Generator type objects

List of constraints as a Generator-type object

cg: ConstraintGroup object, optional

An existing list of constraints if an existing group is being added

name: string, optional

Name for the constraint group and individual constraint prefix

Returns ConstraintGroup object

A group object for all constraints aded

See also:

```
ConstraintGroup, Model.include()
```

Examples

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```
>>> t = m.add_variables(3, 4, name='t')
>>> ct = m.add_constraints((t[i, j] <= x for i in range(3)
                        for j in range(4)), name='ct')
>>> print(ct)
Constraint Group (ct) [
 [(0, 0): -x + t[0, 0] <=
 [(0, 1): t[0, 1] - x <= 0]
 [(0, 2): -x + t[0, 2] \le 0]
 [(0, 3): t[0, 3] - x <= 0]
 [(1, 0): t[1, 0] - x <= 0]
                  - x <= 0]
 [(1, 1): t[1, 1]
          - x + t[1, 2] \ll 0
 [(1, 2):
             x +
 [(1, 3):
                  t[1, 3]
 [(2, 0): -x +
                  t[2, 0]
 [(2, 1): t[2, 1]
                     X
 [(2, 2): t[2, 2] -
                     x <= 0]
 [(2, 3): t[2, 3] - x \le 0]
```

sasoptpy.Model.add_variable

```
Model.add_variable (var=None, vartype='CONT', name=None, lb=-inf, ub=inf, init=None) Adds a new variable to the model
```

New variables can be created via this method or existing variables can be added to the model.

Parameters var: Variable object, optional

Existing variable to be added to the problem

vartype: string, optional

Type of the variable, either 'BIN', 'INT' or 'CONT'

name: string, optional

Name of the variable to be created

lb: float, optional

Lower bound of the variable

ub: float, optional

Upper bound of the variable

init: float, optional

Initial value of the variable

Returns Variable object

Variable that is added to the model

See also:

```
Variable, Model.include()
```

Notes

• If argument var is not None, then all other arguments are ignored.

• A generic variable name is generated if name argument is None.

Examples

Adding a variable on the fly

```
>>> m = so.Model(name='demo')
>>> x = m.add_variable(name='x', vartype=so.INT, ub=10, init=2)
>>> print(repr(x))
NOTE: Initialized model demo
sasoptpy.Variable(name='x', lb=0, ub=10, init=2, vartype='INT')
```

Adding an existing variable to a model

```
>>> y = so.Variable(name='y', vartype=so.BIN)
>>> m = so.Model(name='demo')
>>> m.add_variable(var=y)
```

sasoptpy.Model.add_variables

```
Model.add_variables (*argv, vg=None, name=None, vartype='CONT', lb=None, ub=None, init=None, abstract=None)

Adds a group of variables to the model
```

```
Parameters argv: list, dict, pandas.Index
```

Loop index for variable group

vg: VariableGroup object, optional

An existing object if it is being added to the model

name : string, optional

Name of the variables

vartype: string, optional

Type of variables, BIN, INT, or CONT

lb: list, dict, pandas. Series

Lower bounds of variables

ub: list, dict, pandas. Series

Upper bounds of variables

init : list, dict, pandas . Series

Initial values of variables

See also:

```
VariableGroup, Model.include()
```

Notes

If vg argument is passed, all other arguments are ignored.

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Examples

sasoptpy.Model.add_implicit_variable

Model.add_implicit_variable (argv=None, name=None)
Adds an implicit variable to the model

Parameters argv: Generator type object

Generator object where each item is an entry

name: string, optional

Name of the implicit variable

Notes

• Based on whether generated by a regular expression or an abstract one, implicit variables may appear in generated OPTMODEL codes.

Examples

```
>>> x = m.add_variables(range(5), name='x')
>>> y = m.add_implicit_variable((
>>> x[i] + 2 * x[i+1] for i in range(4)), name='y')
>>> print(y[2])
x[2] + 2 * x[3]

>>> I = m.add_set(name='I')
>>> z = m.add_implicit_variable((x[i] * 2 + 2 for i in I), name='z')
>>> print(z._defn())
impvar z {i_1 in I} = 2 * x[i_1] + 2;
```

sasoptpy.Model.add_set

```
Model.add_set (name, init=None, settype=['num'])
Adds a set to the model
```

Parameters name: string, optional

Name of the set

```
init : Set, optional
    Initial value of the set
settype : list, optional
    Types of the set, a list consists of 'num' and 'str' values
```

Examples

```
>>> I = m.add_set(name='I')
>>> print(I._defn())
set I;

>>> J = m.add_set(name='J', settype=['str'])
>>> print(J._defn())
set <str> J;

>>> N = m.add_parameter(name='N', init=4)
>>> K = m.add_set(name='K', init=so.exp_range(1, N))
>>> print(K._defn())
set K = 1..N;
```

sasoptpy.Model.add_parameter

```
Model.add_parameter (*argv, name=None, init=None, p_type=None)
Adds a parameter to the model

Parameters argv: sasoptpy.data.Set object, optional
```

Key set of the parameter

name: string, optional

Name of the parameter

init: float or expression, optional

Initial value of the parameter

p_type: string, optional

Type of the parameter, 'num' for floats or 'str' for strings

Examples

```
>>> I = m.add_set(name='I')
>>> a = m.add_parameter(I, name='a', init=5)
>>> print(a._defn())
num a {I} init 5;
```

sasoptpy.Model.add_statement

```
Model.add_statement (statement, after_solve=False)
Adds a PROC OPTMODEL statement to the model
```

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```
Parameters statement: Statement, Expression or string
Statement object
after_solve: boolean, optional
Option for putting the statement after 'solve' declaration
```

Notes

• If the statement string includes 'print', then it is automatically placed after solve.

Examples

```
>>> I = m.add_set(name='I')
>>> x = m.add_variables(I, name='x', vartype=so.INT)
>>> a = m.add_parameter(I, name='a')
>>> c = m.add_constraints((x[i] <= 2 * a[i] for i in I), name='c')
>>> m.add_statement('print x;', after_solve=True)
>>> print(m.to_optmodel())
proc optmodel;
min m_obj = 0;
set I;
var x \{I\} integer >= 0;
num a {I};
con c \{i_1 in I\} : x[i_1] - 2.0 * a[i_1] <= 0;
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
print x;
quit;
```

sasoptpy.Model.set_objective

```
Model.set_objective (expression, sense=None, name=None)
Sets the objective function for the model
```

Parameters expression: Expression object

The objective function as an Expression

sense: string, optional

Objective value direction, 'MIN' or 'MAX'

name: string, optional

Name of the objective value

Returns Expression

Objective function as an Expression object

Notes

• Default objective sense is minimization (MIN)

Examples

```
>>> profit = so.Expression(5 * sales - 2 * material, name='profit')
>>> m.set_objective(profit, so.MAX)
>>> print(m.get_objective())
- 2.0 * material + 5.0 * sales

>>> m.set_objective(4 * x - 5 * y, name='obj')
>>> print(repr(m.get_objective()))
sasoptpy.Expression(exp = 4.0 * x - 5.0 * y , name='obj')
```

sasoptpy.Model.set_coef

```
Model.set_coef(var, con, value)
```

Updates the coefficient of a variable inside constraints

Parameters var: Variable object

Variable whose coefficient will be updated

con : Constraint object

Constraint where the coefficient will be updated

value: float

The new value for the coefficient of the variable

See also:

```
Constraint.update var coef()
```

Notes

Variable coefficient inside the constraint is replaced in-place.

Examples

```
>>> c1 = m.add_constraint(x + y >= 1, name='c1')
>>> print(c1)
y + x >= 1
>>> m.set_coef(x, c1, 3)
>>> print(c1)
y + 3.0 * x >= 1
```

sasoptpy.Model.drop constraint

```
Model.drop_constraint(constraint)
```

Drops a constraint from the model

Parameters constraint: Constraint object

The constraint to be dropped from the model

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See also:

```
Model.drop_constraints(), Model.drop_variable(), Model.drop_variables()
```

Examples

```
>>> c1 = m.add_constraint(2 * x + y <= 15, name='c1')
>>> print(m.get_constraint('c1'))
2.0 * x + y <= 15
>>> m.drop_constraint(c1)
>>> print(m.get_constraint('c1'))
None
```

sasoptpy.Model.drop_constraints

```
Model.drop_constraints (constraints)
```

Drops a constraint group from the model

Parameters constraints: ConstraintGroup object

The constraint group to be dropped from the model

See also:

```
Model.drop_constraints(), Model.drop_variable(), Model.drop_variables()
```

Examples

```
>>> c1 = m.add_constraints((x[i] + y <= 15 for i in [0, 1]), name='c1')
>>> print(m.get_constraints())
[sasoptpy.Constraint(x[0] + y <= 15, name='c1_0'),
sasoptpy.Constraint(x[1] + y <= 15, name='c1_1')]
>>> m.drop_constraints(c1)
>>> print(m.get_constraints())
[]
```

sasoptpy.Model.drop_variable

```
Model.drop_variable(variable)
```

Drops a variable from the model

Parameters variable: Variable object

The variable to be dropped from the model

See also:

```
Model.drop_variables(), Model.drop_constraint(), Model.drop_constraints()
```

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> print(m.get_variable('x'))
x
>>> m.drop_variable(x)
>>> print(m.get_variable('x'))
None
```

sasoptpy.Model.drop_variables

```
Model.drop_variables (variables)
```

Drops a variable group from the model

Parameters variables: VariableGroup object

The variable group to be dropped from the model

See also:

```
Model.drop_variable(), Model.drop_constraint(), Model.drop_constraints()
```

Examples

```
>>> x = m.add_variables(3, name='x')
>>> print(m.get_variables())
[sasoptpy.Variable(name='x_0', vartype='CONT'),
    sasoptpy.Variable(name='x_1', vartype='CONT')]
>>> m.drop_variables(x)
>>> print(m.get_variables())
[]
```

sasoptpy.Model.get_constraint

```
Model.get_constraint(name)
```

Returns the reference to a constraint in the model

Parameters name: string

Name of the constraint requested

Returns Constraint object

Examples

```
>>> m.add_constraint(2 * x + y <= 15, name='c1')
>>> print(m.get_constraint('c1'))
2.0 * x + y <= 15</pre>
```

sasoptpy.Model.get_constraints

```
Model.get_constraints()
```

Returns a list of constraints in the model

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Returns list: A list of Constraint objects

Examples

```
>>> m.add_constraint(x[0] + y <= 15, name='c1')
>>> m.add_constraints((2 * x[i] - y >= 1 for i in [0, 1]), name='c2')
>>> print(m.get_constraints())
[sasoptpy.Constraint(x[0] + y <= 15, name='c1'),
    sasoptpy.Constraint(2.0 * x[0] - y >= 1, name='c2_0'),
    sasoptpy.Constraint(2.0 * x[1] - y >= 1, name='c2_1')]
```

sasoptpy.Model.get_variable

```
Model.get_variable(name)
```

Returns the reference to a variable in the model

Parameters name: string

Name or key of the variable requested

Returns Variable object

Examples

```
>>> m.add_variable(name='x', vartype=so.INT, lb=3, ub=5)
>>> var1 = m.get_variable('x')
>>> print(repr(var1))
sasoptpy.Variable(name='x', lb=3, ub=5, vartype='INT')
```

sasoptpy.Model.get_variables

```
Model.get_variables()
```

Returns a list of variables

Returns list: A list of *Variable* objects

Examples

```
>>> x = m.add_variables(2, name='x')
>>> y = m.add_variable(name='y')
>>> print(m.get_variables())
[sasoptpy.Variable(name='x_0', vartype='CONT'),
    sasoptpy.Variable(name='x_1', vartype='CONT'),
    sasoptpy.Variable(name='y', vartype='CONT')]
```

sasoptpy.Model.get objective

```
Model.get_objective()
```

Returns the objective function as an Expression object

Returns Expression object

Objective function

Examples

```
>>> m.set_objective(4 * x - 5 * y, name='obj')
>>> print(repr(m.get_objective()))
sasoptpy.Expression(exp = 4.0 * x - 5.0 * y , name='obj')
```

sasoptpy.Model.get_variable_coef

```
Model.get_variable_coef(var)
```

Returns the objective value coefficient of a variable

Parameters var: Variable object or string

Variable whose objective value is requested or its name

Returns float

Objective value coefficient of the given variable

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> m.set_objective(4 * x - 5 * y, name='obj', sense=so.MAX)
>>> print(m.get_variable_coef(x))
4.0
>>> print(m.get_variable_coef('y'))
-5.0
```

sasoptpy.Model.read_data

Model.read_data(table, key_set, key_cols=None, option=", params=None)
Reads a CASTable into PROC OPTMODEL and adds it to the model

Parameters table: CASTable

The CAS table to be read to sets and parameters

 $key_set: Set$

Set object to be read as the key (index)

key_cols: list or string, optional

Column names of the key columns

option: string, optional

Additional options for read data command

params: list, optional

A list of dictionaries where each dictionary represent parameters

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See also:

```
sasoptpy.utils.read_data()
```

Notes

- This function is intended to be used internally.
- It imitates the read data statement of PROC OPTMODEL.
- This function is still under development and subject to change.
- key_cols parameters should be a list. When passing a single item, string type can be used instead.
- Values inside each dictionary in params list should be as follows:
 - param: Parameter object

Paramter object, whose index is the same as table key

- column : string, optional

Column name to be read

- index : list, optional

List of sets if the parameter has to be read in a loop

Examples

```
>>> table = session.upload_frame(df, casout='df')
>>> item = m.add_set(name='set_item')
>>> value = m.add_parameter(item, name='value')
>>> m.read_data(table, key_set=item, key_cols=['items'],params=[{'param': value, column': 'value'}])
>>> print(m.to_optmodel())
proc optmodel;
min m_obj = 0;
set set_item;
num value {set_item};
read data df into set_item=[items] value;
solve;
print _var_.name _var_.lb _var_.ub _var__var_.rc;
print _con_.name _con_.body _con_.dual;
quit;
```

sasoptpy.Model.read_table

```
Model.read_table (table, key=['_N_'], columns=None, key_type=['num'], col_types=None, up-load=False, casout=None)

Reads a CAS Table or pandas DataFrame into the model
```

Parameters table: swat.cas.table.CASTable, pandas.DataFrame object or string

Pointer to CAS Table (server data, CASTable), DataFrame (local data) or the name of the table at execution (server data, string)

key: list, optional

List of key columns (for CASTable) or index columns (for DataFrame)

columns: list, optional

List of columns to read into parameters

key_type: list or string, optional

A list of column types consists of 'num' or 'str' values

col_types: dict, optional

Dictionary of column types

upload: boolean, optional

Option for uploading a local data to CAS server first

casout: string or dict, optional

Casout options if data is uploaded

Returns tuple

A tuple where first element is the key (index) and second element is a list of requested columns

See also:

```
Model.read_data(), Model.add_parameter(), Model.add_set()
```

Notes

- This method can take either a swat.cas.table.CASTable, a pandas.DataFrame or name of the data set as a string as the first argument.
- If the model is running in saspy or MPS mode, then the data is read to client from the CAS server.
- If the model is running in OPTMODEL mode, then this method generates the corresponding optmodel code.
- When table is a CASTable object, since the actual data is stored on the CAS server, some of the functionalities may be limited.
- For the local data, upload argument can be passed for performance improvement.
- See swat.CAS.upload_frame() and table.loadtable CAS action for casout options.

Examples

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sasoptpy.Model.include

```
Model.include(*argv)
```

Adds existing variables and constraints to a model

```
Parameters argv : Model, Variable, Constraint, VariableGroup, ConstraintGroup, Set, Parameter, Statement, ImplicitVar
```

Objects to be included in the model

Notes

• Including a model causes all variables and constraints inside the original model to be included.

Examples

Adding an existing variable

```
>>> x = so.Variable(name='x', vartype=so.CONT)
>>> m.include(x)
```

Adding an existing constraint

```
>>> c1 = so.Constraint(x + y <= 5, name='c1')
>>> m.include(c1)
```

Adding an existing set of variables

```
>>> z = so.VariableGroup(3, 5, name='z', ub=10)
>>> m.include(z)
```

Adding an existing set of constraints

Adding an existing model (including all of its elements)

```
>>> new_model = so.Model(name='new_model')
>>> new_model.include(m)
```

8.1.3 Solver calls

Model.solve([options, submit, name, frame,])	Solves the model by calling CAS or SAS optimization
	solvers
Model.solve_on_cas(session, options, submit,	Solves the optimization problem on CAS Servers
)	
Model.solve_on_mva(session, options, submit,	Solves the optimization problem on SAS Clients
)	
<pre>Model.get_solution([vtype, solution, pivot])</pre>	Returns the solution details associated with the primal
	or dual solution
	Continued on next page

Table 3 – continued from previous page

Model.get_variable_value([var, name])	Returns the value of a variable.
Model.get_objective_value()	Returns the optimal objective value, if it exists
Model.get_solution_summary()	Returns the solution summary table to the user
Model.get_problem_summary()	Returns the problem summary table to the user
Model.print_solution()	Prints the current values of the variables
Model.upload_user_blocks()	Uploads user-defined decomposition blocks to the CAS
	server

sasoptpy.Model.solve

Model.solve (options=None, submit=True, name=None, frame=False, drop=False, replace=True, prima-lin=False, milp=None, lp=None, verbose=False)
Solves the model by calling CAS or SAS optimization solvers

Parameters options: dict, optional

A dictionary solver options

submit: boolean, optional

Switch for calling the solver instantly

name: string, optional

Name of the table name

frame: boolean, optional

Switch for uploading problem as a MPS DataFrame format

drop: boolean, optional

Switch for dropping the MPS table after solve (only CAS)

replace: boolean, optional

Switch for replacing an existing MPS table (only CAS and MPS)

primalin: boolean, optional

Switch for using initial values (only MILP)

verbose: boolean, optional (experimental)

Switch for printing generated OPTMODEL code

Returns pandas.DataFrame object

Solution of the optimization model

See also:

```
Model.solve_on_cas(), Model.solve_on_mva()
```

Notes

- This method is essentially a wrapper for two other methods.
- Some of the options listed under options argument may not be passed based on which CAS Action is being used.
- The option argument should be a dictionary, where keys are option names. For example, m. solve(options={'maxtime': 600}) limits the solution time to 600 seconds.

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• See Solver Options for a list of solver options.

Examples

```
>>> m.solve()
NOTE: Initialized model food_manufacture_1
NOTE: Converting model food_manufacture_1 to DataFrame
NOTE: Added action set 'optimization'.
...
NOTE: Optimal.
NOTE: Objective = 107842.59259.
NOTE: The Dual Simplex solve time is 0.01 seconds.

>>> m.solve(options={'maxtime': 600})

>>> m.solve(options={'algorithm': 'ipm'})
```

sasoptpy.Model.solve_on_cas

Model.solve_on_cas (session, options, submit, name, frame, drop, replace, primalin, verbose) Solves the optimization problem on CAS Servers

See also:

```
Model.solve()
```

Notes

• This function is not supposed to be used directly. Instead, use the swat.cas.CAS type of session for Model objects and use Model.solve().

sasoptpy.Model.solve_on_mva

Model.solve_on_mva (session, options, submit, name, frame, drop, replace, primalin, verbose) Solves the optimization problem on SAS Clients

See also:

```
Model.solve()
```

Notes

• This function is not supposed to be used directly. Instead, use the <code>saspy.SASsession</code> type of session for <code>Model</code> objects and use <code>Model.solve()</code>.

sasoptpy.Model.get_solution

```
Model.get_solution (vtype='Primal', solution=None, pivot=False)
Returns the solution details associated with the primal or dual solution
```

Parameters vtype: string, optional

```
'Primal' or 'Dual'
```

solution: integer, optional

Solution number to be returned (for MILP)

pivot: boolean, optional

Switch for returning multiple solutions in columns as a pivot table

Returns pandas. DataFrame object

Primal or dual solution table returned from the CAS Action

Notes

• If <code>Model.solve()</code> method is used with frame=True option, MILP solver returns multiple solutions. You can obtain different results using solution parameter.

Examples

```
>>> m.solve()
>>> print (m.get_solution('Primal'))
           var lb ub value solution
       x[clock] 0.0 1.797693e+308 0.0
                                         1.0
1
        x[pc] 0.0 1.797693e+308
                                    5.0
                                              1.0
2
                                             1.0
   x[headphone] 0.0 1.797693e+308 2.0
     [headphone] 0.0 1.797693e+308 2.0 x[mug] 0.0 1.797693e+308 0.0 x[book] 0.0 1.797693e+308 0.0
                                             1.0
3
                                             1.0
4
       x[book] 0.0 1.797693e+308 0.0
5
        x[pen] 0.0 1.797693e+308 1.0
                                             1.0
      x[clock] 0.0 1.797693e+308 0.0
6
                                             2.0
7
         x[pc] 0.0 1.797693e+308 5.0
                                             2.0
8
  x[headphone] 0.0 1.797693e+308 2.0
                                             2.0
        x[mug] 0.0 1.797693e+308 0.0
                                             2.0
10
       x[book] 0.0 1.797693e+308
                                  0.0
                                             2.0
        x[pen] 0.0 1.797693e+308
11
                                    0.0
                                             2.0
       x[clock] 0.0 1.797693e+308
12
                                    1.0
                                              3.0
          x[pc] 0.0 1.797693e+308
13
                                     4.0
                                              3.0
. . .
```

```
>>> print (m.get_solution('Primal', solution=2))
           var lb
                             ub value solution
6
      x[clock] 0.0 1.797693e+308 0.0 2.0
7
         x[pc] 0.0 1.797693e+308 5.0
                                           2.0
8
  x[headphone] 0.0 1.797693e+308 2.0
                                          2.0
9
       x[mug] 0.0 1.797693e+308 0.0
                                           2.0
10
       x[book] 0.0 1.797693e+308
                                  0.0
                                           2.0
        x[pen] 0.0 1.797693e+308
                                  0.0
11
                                           2.0
```

(continues on next page)

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```
x[pc] 5.0 5.0 4.0 1.0 0.0
x[pen] 1.0 0.0 0.0 1.0 0.0
```

```
>>> print (m.get_solution('Dual'))
                  con value solution
            weight_con 20.0
                             1.0
0
       limit_con[clock] 0.0 limit_con[pc] 5.0
1
                                 1.0
2
                                1.0
   limit_con[headphone]
                       2.0
3
                                 1.0
        limit_con[mug]
                                1.0
                       0.0
4
                                1.0
5
       limit_con[book] 0.0
6
                        1.0
                                1.0
       limit_con[pen]
7
            weight_con 19.0
                                2.0
8
      limit_con[clock] 0.0
                                2.0
9
        limit_con[pc]
                       5.0
                                2.0
10 limit_con[headphone] 2.0
                                2.0
11
       limit_con[mug] 0.0
                                2.0
12
       limit_con[book] 0.0
                                 2.0
        limit_con[pen]
13
                        0.0
                                 2.0
```

```
>>> print (m.get_solution('dual', pivot=True))
          1.0
solution
                        2.0
                              3.0
                                   4.0
con
limit_con[book] 0.0 0.0 0.0 1.0 0.0 limit_con[clock] 0.0 0.0 1.0 1.0 0.0
limit_con[headphone] 2.0 2.0 1.0 1.0 0.0
limit_con[mug] 0.0 0.0 0.0 1.0 0.0
limit_con[pc]
                   5.0 5.0 4.0 1.0 0.0
                   1.0 0.0 0.0 1.0 0.0
limit_con[pen]
weight_con
                   20.0 19.0 20.0 19.0 0.0
```

sasoptpy.Model.get_variable_value

Model.get_variable_value(var=None, name=None)

Returns the value of a variable.

Parameters var: Variable object, optional

Variable object

name: string, optional

Name of the variable

Notes

- It is possible to get a variable's value using Variable.get_value() method, if the variable is not abstract
- This method is a wrapper around <code>Variable.get_value()</code> and an overlook function for model components

sasoptpy.Model.get_objective_value

```
Model.get_objective_value()
```

Returns the optimal objective value, if it exists

Returns float: Objective value at current solution

Notes

• This method should be used for getting the objective value after solve. Using m.get_objective(). get_value() actually evaluates the expression using optimal variable values. This may not be available for nonlinear expressions.

Examples

```
>>> m.solve()
>>> print(m.get_objective_value())
42.0
```

sasoptpy.Model.get_solution_summary

```
Model.get_solution_summary()
```

Returns the solution summary table to the user

Returns swat.dataframe.SASDataFrame object

Solution summary obtained after solve

Examples

```
>>> m.solve()
>>> soln = m.get_solution_summary()
>>> print(type(soln))
<class 'swat.dataframe.SASDataFrame'>
```

```
>>> print(soln)
Solution Summary
                             Value
Label
Solver
                                T.P
Algorithm Dual Simplex
                     obj
Objective Function
Solution Status
Objective Value
                          Optimal
                               10
Primal Infeasibility
                               0
                               0
Dual Infeasibility
Bound Infeasibility
                               0
                                 2
Iterations
Presolve Time
                              0.00
Solution Time
                              0.01
```

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```
>>> print(soln.loc['Solution Status', 'Value'])
Optimal
```

sasoptpy.Model.get_problem_summary

```
Model.get_problem_summary()
```

Returns the problem summary table to the user

Returns swat.dataframe.SASDataFrame object

Problem summary obtained after Model.solve()

Examples

```
>>> m.solve()
>>> ps = m.get_problem_summary()
>>> print(type(ps))
<class 'swat.dataframe.SASDataFrame'>
```

```
>>> print(ps)
Problem Summary
                             Value
Label
Problem Name
                            model1
Objective Sense Maximization
Objective Function
                              obj
RHS
                               RHS
Number of Variables
Bounded Above
                                 0
Bounded Below
                                 2
Bounded Above and Below
                                0
                                 0
Free
Fixed
                                 0
Number of Constraints
                                 2
LE (<=)
                                 1
EQ (=)
                                 0
GE (>=)
                                 1
                                 0
Range
Constraint Coefficients
                                 4
```

```
>>> print(ps.index)
Index(['Problem Name', 'Objective Sense', 'Objective Function', 'RHS',
'', 'Number of Variables', 'Bounded Above', 'Bounded Below',
'Bounded Above and Below', 'Free', 'Fixed', '',
'Number of Constraints', 'LE (<=)', 'EQ (=)', 'GE (>=)', 'Range', '',
'Constraint Coefficients'],
dtype='object', name='Label')
```

sasoptpy.Model.print_solution

```
Model.print_solution()
```

Prints the current values of the variables

See also:

```
Model.get_solution()
```

Notes

• This function may not work for abstract variables and nonlinear models.

Examples

```
>>> m.solve()
>>> m.print_solution()
x: 2.0
y: 0.0
```

sasoptpy.Model.upload_user_blocks

```
Model.upload_user_blocks()
```

Uploads user-defined decomposition blocks to the CAS server

Returns string

CAS table name of the user-defined decomposition blocks

Examples

```
>>> userblocks = m.upload_user_blocks()
>>> m.solve(milp={'decomp': {'blocks': userblocks}})
```

8.1.4 Export

Model.to_frame([constant])			Converts the Python model into a DataFrame object in MPS format
Model.to_optmodel([header,	expand,	ordered,	Generates the equivalent PROC OPTMODEL code for
])			the model.

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sasoptpy.Model.to frame

```
Model.to_frame (constant=False)
```

Converts the Python model into a DataFrame object in MPS format

Parameters constant: boolean, optional

Switching for using objConstant argument for solveMilp, solveLp. Adds the constant as an auxiliary variable if value is True.

Returns pandas.DataFrame object

Problem in strict MPS format

Notes

• This method is called inside Model.solve().

Examples

```
>>> df = m.to_frame()
>>> print(df)
    Field1 Field2 Field3 Field4 Field5 Field6 _id_
     NAME model1 0
0
1
     ROWS
2
      MAX obj
                                            3
3
                                            4
       L c1
4
  COLUMNS
                                            5
5
                 obj
                          4
6
                   c1
                           3
                                            7
              X
7
              У
                  obj
                           -5
                                            8
8
                                            9
                   с1
                           1
              У
9
                                            10
      RHS
10
             RHS
                                            11
                    с1
                            6
                                            12
11
    RANGES
12
    BOUNDS
                                            13
13
    ENDATA
                            0
                                            14
```

sasoptpy.Model.to_optmodel

Model.to_optmodel(header=True, expand=False, ordered=False, ods=False, options={})
Generates the equivalent PROC OPTMODEL code for the model.

Parameters header: boolean, optional

Option to include PROC headers

expand: boolean, optional

Option to include 'expand' command to OPTMODEL code

ordered: boolean, optional

Option to generate OPTMODEL code in a specific order (True) or in creation order (False)

options: dict, optional

Solver options for the OPTMODEL solve command

Returns string

PROC OPTMODEL representation of the model

Notes

• This method is called inside Model.solve().

Examples

```
>>> print(m.to_optmodel())
proc optmodel;
var get {{'clock','mug','headphone','book','pen'}} integer >= 0;
get['clock'] = 3.0;
get['mug'] = 4.0;
get['headphone'] = 2.0;
get['book'] = -0.0;
get['pen'] = 5.0;
con limit_con_clock : get['clock'] <= 3;</pre>
con limit_con_mug : get['mug'] <= 5;</pre>
con limit_con_headphone : get['headphone'] <= 2;</pre>
con limit_con_book : get['book'] <= 10;</pre>
con limit_con_pen : get['pen'] <= 15;</pre>
con weight_con : 4 * get['clock'] + 6 * get['mug'] + 7 * get['headphone'] + 12 *_
→get['book'] + get['pen'] <= 55;</pre>
max total_value = 8 * get['clock'] + 10 * get['mug'] + 15 * get['headphone'] + 20,
→* get['book'] + get['pen'];
solve;
print _var_.name _var_.lb _var_.ub _var_ _var_.rc;
print _con_.name _con_.body _con_.dual;
```

8.1.5 Internal functions

<pre>Model.upload_model([name, replace, constant])</pre>	Converts internal model to MPS table and upload to
	CAS session
Model.test_session()	Tests if the model session is defined and still active
Modelis_linear()	Checks if the model can be written as a linear model (in
	MPS format)

sasoptpy.Model.upload_model

Model.upload_model (name=None, replace=True, constant=False)
Converts internal model to MPS table and upload to CAS session

Parameters name: string, optional

Desired name of the MPS table on the server

replace: boolean, optional

Option to replace the existing MPS table

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Returns swat.cas.table.CASTable object

Reference to the uploaded CAS Table

Notes

- This method returns None if the model session is not valid.
- Name of the table is randomly assigned if name argument is None or not given.
- This method should not be used if <code>Model.solve()</code> is going to be used. <code>Model.solve()</code> calls this method internally.

sasoptpy.Model.test_session

```
Model.test_session()
```

Tests if the model session is defined and still active

Returns string

'CAS' for CAS sessions, 'SAS' for SAS sessions, None otherwise

sasoptpy.Model._is_linear

```
Model._is_linear()
```

Checks if the model can be written as a linear model (in MPS format)

Returns boolean

True if model does not have any nonlinear components or abstract operations, False otherwise

8.2 Expression

8.2.1 Constructor

<pre>Expression([exp, name, temp])</pre>	Creates a mathematical expression to represent model
	components

sasoptpy.Expression

```
class sasoptpy.Expression(exp=None, name=None, temp=False)
```

Bases: object

Creates a mathematical expression to represent model components

Parameters exp: Expression object, optional

An existing expression where arguments are being passed

name: string, optional

A local name for the expression

temp: boolean, optional

A boolean shows whether expression is temporary or permanent

Notes

- Two other classes (Variable and Constraint) are subclasses of this class.
- Expressions are created automatically after linear math operations with variables.
- An expression object can be called when defining constraints and other expressions.

Examples

```
>>> sales = so.Variable(name='sales')
>>> material = so.Variable(name='material')
>>> profit = 5 * sales - 3 * material
>>> print(profit)
5.0 * sales - 3.0 * material
>>> print(repr(profit))
sasoptpy.Expression(exp = 5.0 * sales - 3.0 * material , name=None)
```

```
>>> import sasoptpy.math as sm
>>> f = sm.sin(x) + sm.min(y[1],1) ** 2
>>> print(type(f))
<class 'sasoptpy.components.Expression'>
>>> print(f)
sin(x) + (min(y[1], 1)) ** (2)
```

8.2.2 Methods

Combines two expressions and produces a new one
Returns a copy of the Expression object
Returns the name of the expression
Calculates and returns the value of the linear expression
Multiplies the Expression with a scalar value
Sets the name of the expression
Converts a temporary expression into a permanent one

sasoptpy.Expression.add

```
Expression.add(other, sign=1)
```

Combines two expressions and produces a new one

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Parameters other: float or Expression object

Second expression or constant value to be added

sign: int, optional

Sign of the addition, 1 or -1

in_place: boolean, optional

Whether the addition will be performed in place or not

Returns Expression object

Notes

- This method is mainly for internal use.
- Adding an expression is equivalent to calling this method: (x-y)+(3*x-2*y) and (x-y).add(3*x-2*y) are interchangeable.

sasoptpy.Expression.copy

```
Expression.copy (name=None)
```

Returns a copy of the Expression object

Parameters name: string, optional

Name for the copy

Returns Expression object

Copy of the object

Examples

```
>>> e = so.Expression(7 * x - y[0], name='e')
>>> print(repr(e))
sasoptpy.Expression(exp = - y[0] + 7.0 * x , name='e')
>>> f = e.copy(name='f')
>>> print(repr(f))
sasoptpy.Expression(exp = - y[0] + 7.0 * x , name='f')
```

sasoptpy.Expression.get_name

```
Expression.get_name()
```

Returns the name of the expression

Returns string

Name of the expression

Examples

```
>>> var1 = m.add_variables(name='x')
>>> print(var1.get_name())
x
```

sasoptpy.Expression.get_value

```
Expression.get_value()
```

Calculates and returns the value of the linear expression

Returns float

Value of the expression

Notes

• Nonlinear expressions may not be evaluated.

Examples

```
>>> profit = so.Expression(5 * sales - 3 * material)
>>> m.solve()
>>> print(profit.get_value())
41.0
```

sasoptpy.Expression.mult

Expression.mult(other)

Multiplies the Expression with a scalar value

Parameters other: Expression or int

Second expression to be multiplied

Returns Expression object

A new ${\it Expression}$ that represents the multiplication

Notes

- This method is mainly for internal use.
- Multiplying an expression is equivalent to calling this method: 3*(x-y) and (x-y).mult(3) are interchangeable.

sasoptpy.Expression.set_name

```
Expression.set_name(name=None)
Sets the name of the expression
```

Parameters name: string

Name of the expression

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Returns string

Name of the expression after resolving conflicts

Examples

```
>>> e = x + 2*y
>>> e.set_name('objective')
```

sasoptpy.Expression.set_permanent

```
Expression.set_permanent (name=None)
```

Converts a temporary expression into a permanent one

Parameters name: string, optional

Name of the expression

Returns string

Name of the expression in the namespace

8.2.3 Private Methods

Expressionexpr()	Generates the OPTMODEL compatible string representation of the object.
Expressionis_linear()	Checks if the expression is composed of linear compo-
	nents
<pre>Expressionrelational(other, direction_)</pre>	Creates a logical relation between Expression ob-
	jects
Expressionrepr()	Returns a string representation of the object.
Expressionstr()	Generates a representation string that is Python compatible

sasoptpy.Expression._expr

```
Expression._expr()
```

Generates the OPTMODEL compatible string representation of the object.

Examples

```
>>> f = x + y ** 2

>>> print(f)

x + (y) ** (2)

>>> print(f._expr())

x + (y) ^ (2)
```

sasoptpy.Expression._is_linear

```
Expression._is_linear()
```

Checks if the expression is composed of linear components

Returns boolean

True if the expression is linear, False otherwise

Examples

```
>>> x = so.Variable()
>>> e = x*x
>>> print(e.is_linear())
False

>>> f = x*x + x*x - 2*x*x + 5
>>> print(f.is_linear())
True
```

sasoptpy.Expression._relational

```
Expression._relational (other, direction_)
```

Creates a logical relation between Expression objects

Parameters other: Expression object

Expression on the other side of the relation wrt self

direction_: string

Direction of the logical relation, either 'E', 'L', or 'G'

Returns Constraint

Constraint generated as a result of linear relation

sasoptpy.Expression.__repr__

```
Expression.__repr__()
```

Returns a string representation of the object.

Examples

```
>>> f = x + y ** 2
>>> print(repr(f))
sasoptpy.Expression(exp = x + (y) ** (2), name=None)
```

sasoptpy.Expression.__str__

```
Expression.__str__()
```

Generates a representation string that is Python compatible

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Examples

```
>>> f = x + y ** 2
>>> print(str(f))
x + (y) ** (2)
```

8.3 Variable

8.3.1 Constructor

```
Variable(name[, vartype, lb, ub, init, ...]) Creates an optimization variable to be used inside models
```

sasoptpy.Variable

```
class sasoptpy. Variable (name, vartype='CONT', lb=-inf, ub=inf, init=None, abstract=False, shadow=False, key=None)
```

 $Bases: \verb|sasoptpy.components.Expression| \\$

Creates an optimization variable to be used inside models

Parameters name: string

Name of the variable

vartype : string, optional

Type of the variable

lb: float, optional

Lower bound of the variable

ub: float, optional

Upper bound of the variable

init: float, optional

Initial value of the variable

abstract: boolean, optional

Indicator of whether the variable is abstract or not

shadow: boolean, optional

Indicator of whether the variable is shadow or not Used for internal purposes

See also:

```
sasoptpy.Model.add_variable()
```

Examples

```
>>> x = so.Variable(name='x', lb=0, ub=20, vartype=so.CONT)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')

>>> y = so.Variable(name='y', init=1, vartype=so.INT)
>>> print(repr(y))
sasoptpy.Variable(name='y', lb=0, ub=inf, init=1, vartype='INT')
```

8.3.2 Methods

Variable.set_bounds([lb, ub])	Changes bounds on a variable
Variable.set_init([init])	Changes initial value of a variable

sasoptpy. Variable.set bounds

Variable.set_bounds(lb=None, ub=None)

Changes bounds on a variable

Parameters lb: float

Lower bound of the variable

ub: float

Upper bound of the variable

Examples

```
>>> x = so.Variable(name='x', lb=0, ub=20)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=0, ub=20, vartype='CONT')
>>> x.set_bounds(lb=5, ub=15)
>>> print(repr(x))
sasoptpy.Variable(name='x', lb=5, ub=15, vartype='CONT')
```

sasoptpy. Variable.set init

Variable.set_init(init=None)

Changes initial value of a variable

Parameters init: float or None

Initial value of the variable

Examples

```
>>> x = so.Variable(name='x')
>>> x.set_init(5)

>>> y = so.Variable(name='y', init=3)
>>> y.set_init()
```

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8.3.3 Inherited Methods

Variable.add(other[, sign])	Combines two expressions and produces a new one
Variable.copy([name])	Returns a copy of the Expression object
Variable.get_dual()	Returns the dual value
Variable.get_name()	Returns the name of the expression
Variable.get_value()	Calculates and returns the value of the linear expression

sasoptpy.Variable.add

Variable.add(other, sign=1)

Combines two expressions and produces a new one

Parameters other: float or Expression object

Second expression or constant value to be added

sign: int, optional

Sign of the addition, 1 or -1

in_place: boolean, optional

Whether the addition will be performed in place or not

Returns Expression object

Notes

- This method is mainly for internal use.
- Adding an expression is equivalent to calling this method: (x-y)+(3*x-2*y) and (x-y).add(3*x-2*y) are interchangeable.

sasoptpy. Variable.copy

Variable.copy (name=None)

Returns a copy of the Expression object

Parameters name: string, optional

Name for the copy

Returns Expression object

Copy of the object

Examples

```
>>> e = so.Expression(7 * x - y[0], name='e')
>>> print(repr(e))
sasoptpy.Expression(exp = - y[0] + 7.0 * x , name='e')
>>> f = e.copy(name='f')
>>> print(repr(f))
sasoptpy.Expression(exp = - y[0] + 7.0 * x , name='f')
```

sasoptpy.Variable.get_dual

```
Variable.get_dual()
Returns the dual value
```

Returns float

Dual value of the variable

sasoptpy.Variable.get_name

```
Variable.get_name()
Returns the name of the expression
```

Returns string

Name of the expression

Examples

```
>>> var1 = m.add_variables(name='x')
>>> print(var1.get_name())
x
```

sasoptpy.Variable.get_value

```
Variable.get_value()
```

Calculates and returns the value of the linear expression

Returns float

Value of the expression

Notes

• Nonlinear expressions may not be evaluated.

Examples

```
>>> profit = so.Expression(5 * sales - 3 * material)
>>> m.solve()
>>> print(profit.get_value())
41.0
```

8.4 Variable Group

8.4.1 Constructor

8.4. Variable Group 163

VariableGroup(*argv, name[, vartype, lb, ...])

Creates a group of Variable objects

sasoptpy.VariableGroup

```
class sasoptpy. VariableGroup (*argv, name, vartype='CONT', lb=-inf, ub=-inf, init=None, ab-
                                       stract=False)
     Bases: object
     Creates a group of Variable objects
          Parameters argy: list, dict, int, pandas. Index
                  Loop index for variable group
              name : string, optional
                  Name (prefix) of the variables
              vartype: string, optional
                  Type of variables, BIN, INT, or CONT
              lb: list, dict, pandas. Series, optional
                  Lower bounds of variables
              ub: list, dict, pandas. Series, optional
                  Upper bounds of variables
              init: float, optional
                  Initial values of variables
     See also:
     sasoptpy.Model.add_variables(), sasoptpy.Model.include()
```

Notes

- When working with a single model, use the sasoptpy. Model.add_variables() method.
- If a variable group object is created, it can be added to a model using the <code>sasoptpy.Model.include()</code> method.
- An individual variable inside the group can be accessed using indices.

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', 1b=0, ub=10)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', 1b=0, ub=10, vartype='CONT')
```

Examples

```
[Period2: production['Period2']]
  [Period3: production['Period3']]
>>> x = so.VariableGroup(4, vartype=so.BIN, name='x')
>>> print(x)
Variable Group (x) [
 [0: x[0]]
  [1: x[1]]
 [2: x[2]]
 [3: x[3]]
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z')
>>> print(z)
Variable Group (z) [
 [(0, 'a'): z[0, 'a']]
 [(0, 'b'): z[0, 'b']]
 [(0, 'c'): z[0, 'c']]
 [(1, 'a'): z[1, 'a']]
  [(1, 'b'): z[1, 'b']]
 [(1, 'c'): z[1, 'c']]
>>> print(repr(z))
sasoptpy.VariableGroup([0, 1], ['a', 'b', 'c'], name='z')
```

8.4.2 Methods

VariableGroup.get_name()	Returns the name of the variable group
VariableGroup.set_bounds([lb, ub])	Sets / updates bounds for the given variable
VariableGroup.set_init(init)	Sets / updates initial value for the given variable
VariableGroup.mult(vector)	Quick multiplication method for the variable groups
VariableGroup.sum(*argv)	Quick sum method for the variable groups

sasoptpy.VariableGroup.get_name

```
VariableGroup.get_name()
```

Returns the name of the variable group

Returns string

Name of the variable group

Examples

```
>>> var1 = m.add_variables(4, name='x')
>>> print(var1.get_name())
x
```

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sasoptpy.VariableGroup.set_bounds

```
VariableGroup.set_bounds (lb=None, ub=None)
Sets / updates bounds for the given variable

Parameters lb: float, pandas.Series, optional
Lower bound
ub: float, pandas.Series, optional
Upper bound
```

Examples

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', lb=0, ub=10, vartype='CONT')
>>> z.set_bounds(lb=3, ub=5)
>>> print(repr(z[0, 'a']))
sasoptpy.Variable(name='z_0_a', lb=3, ub=5, vartype='CONT')

>>> u = so.VariableGroup(['a', 'b', 'c', 'd'], name='u')
>>> lb_vals = pd.Series([1, 4, 0, -1], index=['a', 'b', 'c', 'd'])
>>> u.set_bounds(lb=lb_vals)
>>> print(repr(u['b']))
sasoptpy.Variable(name='u_b', lb=4, ub=inf, vartype='CONT')
```

sasoptpy.VariableGroup.set_init

```
VariableGroup.set_init (init)
```

Sets / updates initial value for the given variable

Parameters init: float, list, dict, pandas. Series

Initial value of the variables

Examples

```
>>> y = m.add_variables(3, name='y')
>>> print(y._defn())
var y {{0,1,2}};
>>> y.set_init(5)
>>> print(y._defn())
var y {{0,1,2}} init 5;
```

sasoptpy.VariableGroup.mult

```
VariableGroup.mult (vector)
```

Quick multiplication method for the variable groups

Parameters vector: list, dictionary, pandas. Series object, or pandas. DataFrame object

Vector to be multiplied with the variable group

Returns Expression object

An expression that is the product of the variable group with the given vector

Examples

Multiplying with a list

```
>>> x = so.VariableGroup(4, vartype=so.BIN, name='x')
>>> e1 = x.mult([1, 5, 6, 10])
>>> print(e1)
10.0 * x[3] + 6.0 * x[2] + x[0] + 5.0 * x[1]
```

Multiplying with a dictionary

```
>>> y = so.VariableGroup([0, 1], ['a', 'b'], name='y', lb=0, ub=10)
>>> dvals = {(0, 'a'): 1, (0, 'b'): 2, (1, 'a'): -1, (1, 'b'): 5}
>>> e2 = y.mult(dvals)
>>> print(e2)
2.0 * y[0, 'b'] - y[1, 'a'] + y[0, 'a'] + 5.0 * y[1, 'b']
```

Multiplying with a pandas. Series object

```
>>> u = so.VariableGroup(['a', 'b', 'c', 'd'], name='u')
>>> ps = pd.Series([0.1, 1.5, -0.2, 0.3], index=['a', 'b', 'c', 'd'])
>>> e3 = u.mult(ps)
>>> print(e3)
1.5 * u['b'] + 0.1 * u['a'] - 0.2 * u['c'] + 0.3 * u['d']
```

Multiplying with a pandas.DataFrame object

```
>>> data = np.random.rand(3, 3)
>>> df = pd.DataFrame(data, columns=['a', 'b', 'c'])
>>> print(df)
>>> NOTE: Initialized model model1
         а
               b
0 0.966524 0.237081 0.944630
1 0.821356 0.074753 0.345596
2 0.065229 0.037212 0.136644
>>> y = m.add_variables(3, ['a', 'b', 'c'], name='y')
>>> e = y.mult(df)
>>> print(e)
0.9665237354418064 * y[0, 'a'] + 0.23708064143289442 * y[0, 'b'] +
0.944629500537536 * y[0, 'c'] + 0.8213562592159828 * y[1, 'a'] +
0.07475256894157478 * y[1, 'b'] + 0.3455957019116668 * y[1, 'c'] +
0.06522945752546017 * y[2, 'a'] + 0.03721153533250843 * y[2, 'b'] +
0.13664422498043194 * v[2, 'c']
```

sasoptpy.VariableGroup.sum

VariableGroup.sum(*argv)

Quick sum method for the variable groups

Parameters argv : Arguments

List of indices for the sum

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Returns Expression object

Expression that represents the sum of all variables in the group

Examples

```
>>> z = so.VariableGroup(2, ['a', 'b', 'c'], name='z', lb=0, ub=10)
>>> e1 = z.sum('*', '*')
>>> print(e1)
z[1, 'c'] + z[1, 'a'] + z[1, 'b'] + z[0, 'a'] + z[0, 'b'] +
z[0, 'c']
>>> e2 = z.sum('*', 'a')
>>> print (e2)
z[1, 'a'] + z[0, 'a']
>>> e3 = z.sum('*', ['a', 'b'])
>>> print(e3)
 z[1, 'a'] + z[0, 'b'] + z[1, 'b'] + z[0, 'a']
```

8.5 Constraint

8.5.1 Constructor

Constraint(exp[, direction, name, crange])

Creates a linear or quadratic constraint for optimization models

sasoptpy.Constraint

```
class sasoptpy.Constraint (exp, direction=None, name=None, crange=0)
     Bases: \verb|sasoptpy.components.Expression| \\
```

Creates a linear or quadratic constraint for optimization models

Constraints should be created by adding logical relations to Expression objects.

Parameters exp: Expression

A logical expression that forms the constraint

direction: string

Direction of the logical expression, 'E' (=), 'L' (<=) or 'G' (>=)

name: string, optional

Name of the constraint object

crange: float, optional

Range for ranged constraints

See also:

```
sasoptpy.Model.add constraint()
```

Notes

- A constraint can be generated in multiple ways:
 - 1. Using the sasoptpy.Model.add_constraint() method

```
>>> c1 = m.add_constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')</pre>
```

2. Using the constructor

```
>>> c1 = sasoptpy.Constraint(3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')
```

• The same constraint can be included into other models using the Model.include() method.

Examples

```
>>> c1 = so.Constraint( 3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( - 5.0 * y + 3.0 * x <= 10, name='c1')

>>> c2 = so.Constraint( - x + 2 * y - 5, direction='L', name='c2')
sasoptpy.Constraint( - x + 2.0 * y <= 5, name='c2')</pre>
```

8.5.2 Methods

Constraint.get_value([rhs])	Returns the current value of the constraint
Constraint.set_block(block_number)	Sets the decomposition block number for a constraint
Constraint.set_direction(direction)	Changes the direction of a constraint
Constraint.set_rhs(value)	Changes the RHS of a constraint

sasoptpy.Constraint.get value

```
Constraint.get_value(rhs=False)
```

Returns the current value of the constraint

Parameters rhs: boolean, optional

Whether constant values (RHS) will be included in the value or not. Default is false

Examples

```
>>> m.solve()
>>> print(c1.get_value())
6.0
>>> print(c1.get_value(rhs=True))
0.0
```

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sasoptpy.Constraint.set_block

```
Constraint.set_block(block_number)
```

Sets the decomposition block number for a constraint

Parameters block_number: int

Block number of the constraint

Examples

sasoptpy.Constraint.set_direction

```
Constraint.set_direction(direction)
```

Changes the direction of a constraint

Parameters direction: string

Direction of the constraint, 'E', 'L', or 'G' for equal to, less than or equal to, and greater than or equal to, respectively

Examples

```
>>> c1 = so.Constraint(exp=3 * x - 5 * y <= 10, name='c1')
>>> print(repr(c1))
sasoptpy.Constraint( 3.0 * x - 5.0 * y <= 10, name='c1')
>>> c1.set_direction('G')
>>> print(repr(c1))
sasoptpy.Constraint( 3.0 * x - 5.0 * y >= 10, name='c1')
```

sasoptpy.Constraint.set rhs

```
Constraint.set_rhs(value)
```

Changes the RHS of a constraint

Parameters value: float

New RHS value for the constraint

Examples

```
>>> x = m.add_variable(name='x')
>>> y = m.add_variable(name='y')
>>> c = m.add_constraint(x + 3*y <= 10, name='con_1')
>>> print(c)
x + 3.0 * y <= 10</pre>
```

```
>>> c.set_rhs(5)
>>> print(c)
x + 3.0 * y <= 5
```

8.6 Constraint Group

8.6.1 Constructor

ConstraintGroup(argv, name)

Creates a group of Constraint objects

sasoptpy.ConstraintGroup

```
class sasoptpy.ConstraintGroup(argv, name)
    Bases: object
```

Creates a group of Constraint objects

Parameters argv : GeneratorType object

A Python generator that includes sasoptpy. Expression objects

name : string, optional

Name (prefix) of the constraints

See also:

```
sasoptpy.Model.add_constraints(), sasoptpy.Model.include()
```

Notes

Use sasoptpy.Model.add_constraints() when working with a single model.

Examples

```
>>> print(cg2)
Constraint Group (cg2) [
  [(1, 'a'): 3.0 * z[0, 'a'] + 2.0 * z[1, 'a'] >= 2]
  [(1, 'b'): 2.0 * z[1, 'b'] + 3.0 * z[0, 'b'] >= 2]
  [(1, 'c'): 2.0 * z[1, 'c'] + 3.0 * z[0, 'c'] >= 2]
]
```

8.6.2 Methods

ConstraintGroup.get_name()	Returns the name of the constraint group
ConstraintGroup.get_expressions([rhs])	Returns constraints as a list of expressions

sasoptpy.ConstraintGroup.get_name

```
ConstraintGroup.get_name()
```

Returns the name of the constraint group

Returns string

Name of the constraint group

Examples

sasoptpy.ConstraintGroup.get_expressions

ConstraintGroup.get_expressions(rhs=False)

Returns constraints as a list of expressions

Parameters rhs: boolean, optional

Whether to pass the constant part (rhs) of the constraint or not

Returns pandas.DataFrame

Returns a DataFrame consisting of constraints as expressions

Examples

```
d u['d'] + 2.0 * t
a u['a'] + 2.0 * t
>>> ce_rhs = cg.get_expressions(rhs=True)
>>> print(ce_rhs)

cg
b u['b'] - 5 + 2.0 * t
c - 5 + u['c'] + 2.0 * t
d - 5 + u['d'] + 2.0 * t
a - 5 + 2.0 * t + u['a']
```

8.7 Others

8.7.1 Constructors

ExpressionDict([name])	Creates a dictionary of Expression objects
ImplicitVar([argv, name])	Creates an implicit variable
Set(name[, init, settype])	Creates an index set to be represented inside PROC OPTMODEL
SetIterator(initset[, conditions, datatype,])	Creates an iterator object for a given Set
<pre>Parameter(name[, keys, order, init, p_type])</pre>	Creates a parameter to be represented inside PROC OPTMODEL
ParameterValue(param[, key, prefix, suffix])	Represents a single value of a parameter

sasoptpy.ExpressionDict

```
class sasoptpy.ExpressionDict (name=None)
```

Bases: object

Creates a dictionary of Expression objects

Parameters name: string

Name of the object

Notes

- ExpressionDict is the underlying class for ImplicitVar.
- It behaves as a regular dictionary for client-side models.

Examples

```
>>> e[0] = x + 2*y

>>> e[1] = 2*x + y**2

>>> print(e.get_keys())

>>> for i in e:

>>> print(i, e[i])

(0,) x + 2 * y

(1,) 2 * x + (y) ** (2)
```

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sasoptpy.ImplicitVar

```
class sasoptpy.ImplicitVar (argv=None, name=None)
Bases: sasoptpy.data.ExpressionDict
Creates an implicit variable
Parameters argv: Generator, optional
Generator object for the implicit variable
name: string, optional
Name of the implicit variable
```

Notes

• If the loop inside generator is over an abstract object, a definition for the object will be created inside Model.to_optmodel() method.

Examples

Regular Implicit Variable

```
>>> I = range(5)
>>> x = so.Variable(name='x')
>>> y = so.VariableGroup(I, name='y')
>>> z = so.ImplicitVar((x + i * y[i] for i in I), name='z')
>>> for i in z:
>>> print(i, z[i])
(0,) x
(1,) x + y[1]
(2,) x + 2 * y[2]
(3,) x + 3 * y[3]
(4,) x + 4 * y[4]
```

Abstract Implicit Variable

```
>>> I = so.Set(name='I')
>>> x = so.Variable(name='x')
>>> y = so.VariableGroup(I, name='y')
>>> z = so.ImplicitVar((x + i * y[i] for i in I), name='z')
>>> print(z._defn())
impvar z {i_1 in I} = x + i_1 * y[i_1];
>>> for i in z:
>>> print(i, z[i])
(sasoptpy.data.SetIterator(name=i_1, ...),) x + i_1 * y[i_1]
```

sasoptpy.Set

```
class sasoptpy.Set (name, init=None, settype=['num'])
    Bases: sasoptpy.components.Expression
```

Creates an index set to be represented inside PROC OPTMODEL

Parameters name: string

```
Name of the parameter
```

init: Expression, optional

Initial value expression of the parameter

settype: list, optional

List of types for the set, consisting of 'num' and 'str' values

Examples

```
>>> I = so.Set('I')
>>> print(I._defn())
set I;

>>> J = so.Set('J', settype=['num', 'str'])
>>> print(J._defn())
set <num, str> J;

>>> N = so.Parameter(name='N', init=5)
>>> K = so.Set('K', init=so.exp_range(1,N))
>>> print(K._defn())
set K = 1..N;
```

sasoptpy.SetIterator

 $Bases: \verb|sasoptpy.components.Expression| \\$

Creates an iterator object for a given Set

Parameters initset: Set

Set to be iterated on

conditions: list, optional

List of conditions on the iterator

datatype: string, optional

Type of the iterator

group: dict, optional

Dictionary representing the order of iterator inside multi-index sets

multi_index : boolean, optional

Switch for representing multi-index iterators

Notes

- SetIterator objects are automatically created when looping over a Set.
- This class is mainly intended for internal use.

8.7. Others 175

- The group parameter consists of following keys
 - order: int Order of the parameter inside the group
 - outof: int Total number of indices inside the group
 - id: int ID number assigned to group by Python

sasoptpy.Parameter

```
class sasoptpy.Parameter (name, keys=None, order=1, init=None, p_type=None)
    Bases: object
```

Creates a parameter to be represented inside PROC OPTMODEL

Parameters name: string

Name of the parameter

keys: list, optional

List of Set to be used as keys for multi-index parameters

init: Expression, optional

Initial value expression of the parameter

p_type : string, optional

Type of the parameter, 'num' or 'str'

See also:

```
read_table(), Model.read_table()
```

Examples

```
>>> p = so.Parameter('p', init=x + 2*y)
>>> print(p._defn())
num p = x + 2 * y;
```

```
>>> I = so.Set('I')
>>> r = so.Parameter('r', keys=I, p_type='str')
>>> print(r._defn())
str r {I};
```

sasoptpy.ParameterValue

```
class sasoptpy.ParameterValue(param, key=None, prefix=", suffix=")
Bases: sasoptpy.components.Expression
```

Represents a single value of a parameter

Parameters param: Parameter

Parameter that the value belongs to

key: tuple, optional

Key of the parameter value in the multi-index parameter

```
prefix : string
```

Prefix of the parameter

suffix: string

Suffix of the parameter, such as .1b and .ub

Notes

• Parameter values are mainly used in abstract expressions

8.7.2 Methods

	ParameterValue.set_init(val)	Sets the initial value of the parameter
--	------------------------------	---

sasoptpy.ParameterValue.set_init

```
ParameterValue.set_init (val)
Sets the initial value of the parameter
Parameters val: Expression
```

Initial value

Notes

• This method is only available for parameters without index/key.

Examples

```
>>> p = so.Parameter(name='p')
>>> print(p._defn())
num p;
>>> p.set_init(10)
>>> print(p._defn())
num p = 10;
```

8.8 Functions

8.8.1 Utility Functions

check_name(name[, ctype])	Checks if a name is in valid and returns a random string
	if not
dict_to_frame(dictobj[, cols])	Converts dictionaries to DataFrame objects for pretty
	printing
exp_range(start, stop[, step])	Creates a set within given range
	Continued on next page

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<pre>extract_list_value(tuplist, listname)</pre>	Extracts values inside various object types
flatten_frame(df[, swap])	Converts a pandas.DataFrame object into
	pandas.Series
flatten_tuple(tp)	Flattens nested tuples
<pre>get_counter(ctrtype)</pre>	Returns and increments the list counter for naming
get_len(i)	Safe wrapper of len() function
get_mutable(exp)	Returns a mutable copy of the given expression if it is
	immutable
get_namespace()	Prints details of components registered to the global
	name dictionary
<pre>get_solution_table(*argv[, key, sort, rhs])</pre>	Returns the requested variable names as a DataFrame
	table
list_length(listobj)	Returns the length of an object if it is a list, tuple or dict
list_pack(obj)	Converts a given object to a list
<pre>print_model_mps(model)</pre>	Prints the MPS representation of the model
quick_sum(argv)	Quick summation function for Expression objects
read_data(table, key_set[, key_cols,])	(Experimental) Reads a CASTable into PROC OPT-
	MODEL sets
read_frame(df[, cols])	Reads each column in pandas.DataFrame into a list
	of pandas.Series objects
read_table(table[, session, key, columns,])	Reads a CAS Table or pandas DataFrame
recursive_walk(obj, func[, attr, alt])	Calls a given method recursively for given objects
register_name(name, obj)	Adds the name and order of a component into the global
	reference list
reset_globals()	Deletes the references inside the global dictionary and
	restarts counters
tuple_pack(obj)	Converts a given object to a tuple object
tuple_unpack(tp)	Grabs the first element in a tuple, if a tuple is given as
	argument
union(*args)	Returns a union of Set, list or set objects
wrap(e[, abstract])	Wraps expression inside another expression

sasoptpy.check_name

sasoptpy.check_name (name, ctype=None)

Checks if a name is in valid and returns a random string if not

Parameters name: str

Name to be checked if unique

Returns str: The given name if valid, a random string otherwise

sasoptpy.dict_to_frame

 $\verb|sasoptpy.dict_to_frame|| (\textit{dictobj}, \textit{cols=None})|$

Converts dictionaries to DataFrame objects for pretty printing

Parameters dictobj : dict

Dictionary to be converted

cols: list, optional Column names Returns DataFrame object

DataFrame representation of the dictionary

Examples

sasoptpy.exp_range

```
sasoptpy.exp_range (start, stop, step=1)
Creates a set within given range
```

Parameters start: Expression

First value of the range

stop: Expression

Last value of the range

step: Expression, optional

Step size of the range

Returns Set

Set that represents the range

Examples

```
>>> N = so.Parameter(name='N')
>>> p = so.exp_range(1, N)
>>> print(p._defn())
set 1..N;
```

sasoptpy.extract_list_value

```
sasoptpy.extract_list_value(tuplist, listname)
```

Extracts values inside various object types

Parameters tuplist: tuple

Key combination to be extracted

listname: dict or list or int or float or DataFrame or Series object

List where the value will be extracted

Returns object

Corresponding value inside listname

sasoptpy.flatten_frame

```
sasoptpy.flatten_frame (df, swap=False)
```

Converts a pandas. DataFrame object into pandas. Series

Parameters df: pandas.DataFrame object

DataFrame object to be flattened

swap: boolean, optional

Option to use columns as first index

Returns pandas.DataFrame object

A new DataFrame where indices consist of index and columns names as tuples

Examples

```
>>> price = pd.DataFrame([
>>> [1, 5, 7],
       [8, 4, 3],
>>> [5, 7, 9]], columns=['period1', 'period2', 'period3']).\
>>> set_index([['coal', 'steel', 'copper']])
>>> print('Price data: \n{}'.format(price))
>>> price_f = so.flatten_frame(price)
>>> print('Price data: \n{}'.format(price_f))
Price data:
    period1 period2 period3

    coal
    1
    5
    7

    steel
    8
    4
    3

             8 4
copper 5
                       7
                                 9
Price data:
(coal, period1) 1
(coal, period2) 5
(coal, period3)
(steel, period1)
(steel, period2)
(steel, period3)
(copper, period1) 5
(copper, period2)
(copper, period3)
dtype: int64
```

sasoptpy.flatten tuple

```
sasoptpy.flatten_tuple(tp)
```

Flattens nested tuples

Parameters tp: tuple

Nested tuple to be flattened

Returns Generator

A generator object representing the flat tuple

Examples

```
>>> tp = (3, 4, (5, (1, 0), 2))
>>> print(list(so.flatten_tuple(tp)))
[3, 4, 5, 1, 0, 2]
```

sasoptpy.get_counter

```
sasoptpy.get_counter(ctrtype)
```

Returns and increments the list counter for naming

Parameters ctrtype: string

Type of the counter, 'obj', 'var', 'con' or 'expr'

Returns int

Current value of the counter

sasoptpy.get_len

```
sasoptpy.get_len (i)
Safe wrapper of len() function
```

Returns int

len(i) if parameter i has len() function defined, othwerwise 1

sasoptpy.get_mutable

```
sasoptpy.get_mutable(exp)
```

Returns a mutable copy of the given expression if it is immutable

Parameters exp: Variable or Expression

Object to be wrapped

Returns Expression

Mutable copy of the expression, if the original is immutable

sasoptpy.get_namespace

```
sasoptpy.get_namespace()
```

Prints details of components registered to the global name dictionary

The list includes models, variables, constraints and expressions

Returns string

A string representation of the namespace

sasoptpy.get_solution_table

 $\verb|sasoptpy.get_solution_table| (*argv, key=None, sort=True, rhs=False)|$

Returns the requested variable names as a DataFrame table

Parameters key: list, optional

Keys for objects

sort: bool, optional

Option for sorting the keys

rhs: bool, optional

Option for including constant values

Returns pandas.DataFrame

DataFrame object that holds keys and values

sasoptpy.list_length

```
sasoptpy.list_length(listobj)
```

Returns the length of an object if it is a list, tuple or dict

Parameters listobj: list, tuple or dict

Object whose length will be returned

Returns int

Length of the list, tuple or dict

sasoptpy.list pack

```
sasoptpy.list_pack(obj)
```

Converts a given object to a list

If the object is already a list, the function returns the input, otherwise creates a list

Parameters obj : Object

Object that is converted to a list

Returns list

List that includes the original object

sasoptpy.print_model_mps

```
sasoptpy.print_model_mps (model)
```

Prints the MPS representation of the model

Parameters model: Model

Model whose MPS format will be printed

See also:

```
sasoptpy.Model.to_frame()
```

Examples

```
>>> m = so.Model(name='print_example', session=s)
>>> x = m.add_variable(lb=1, name='x')
>>> y = m.add_variables(2, name='y', ub=3, vartype=so.INT)
>>> m.add_constraint(x + y.sum('*') <= 9, name='c1')
>>> m.add_constraints((x + y[i] >= 2 for i in [0, 1]), name='c2')
>>> m.set_objective(x+3*y[0], sense=so.MAX, name='obj')
>>> so.print_model_mps(m)
NOTE: Initialized model print_example
    Field1 Field2 Field3 Field4 Field5 Field6 _id_
0
     NAME
                   print_example 0
1
     ROWS
2
      MAX
              obj
3
       L
               c1
              c2_0
4
        G
5
        G
              c2_1
6
   COLUMNS
7
                            obj
                                                          8
8
                  Х
                              с1
                                                          9
9
                  X
                            c2 0
                                      1
                                                         10
                            c2_1
10
                                                         11
                 X
          MARK0000
                                        'INTORG'
11
                        'MARKER'
                                                         12
                         obj
                                     3
12
                                                         13
            У_0
13
                у_0
                             c1
                                     1
                                                        14
14
                y_0
                            c2_0
                                     1
                                                        15
15
                у_1
                             c1
                y_1
16
                            c2 1
                                                        17
           MARK0001 'MARKER'
17
                                         'INTEND'
                                                        18
18
       RHS
                                                         19
19
                                                         20
                RHS
                             c1
20
                RHS
                             c2_0
                                      2
                                                         21
21
                RHS
                            c2_1
                                      2
                                                         22
22
   RANGES
                                                         23
    BOUNDS
                                                         24
23
   LO
24
                BND
                                      1
                                                         2.5
                               X
25
                BND
                                      3
        UP
                             у_0
                                                         26
                BND
                                      0
                                                         27
26
       LO
                             у_0
27
        UP
                BND
                             y_1
                                      3
                                                         28
28
                BND
                                      0
                                                         29
        LO
                              у_1
29
    ENDATA
                                                         30
```

sasoptpy.quick_sum

```
sasoptpy.quick_sum(argv)
```

Quick summation function for Expression objects

Returns Expression object

Sum of given arguments

Notes

This function is faster for expressions compared to Python's native sum() function.

Examples

```
>>> x = so.VariableGroup(10000, name='x')
>>> y = so.quick_sum(2*x[i] for i in range(10000))
```

sasoptpy.read data

```
sasoptpy.read_data(table, key_set, key_cols=None, option=", params=None) (Experimental) Reads a CASTable into PROC OPTMODEL sets
```

Parameters table: CASTable

The CAS table to be read to sets and parameters

key_set: sasoptpy.data.Set

Set object to be read as the key (index)

key_cols: list or string, optional

Column names of the key columns

option: string, optional

Additional options for read data command

params: list, optional

A list of dictionaries where each dictionary represent parameters

Notes

• *key_set* and *key_cols* parameters should be a list. When passing a single item, string type can be used instead.

sasoptpy.read_frame

```
sasoptpy.read_frame (df, cols=None)
Reads each column in pandas.DataFrame into a list of pandas.Series objects

Parameters df: pandas.DataFrame object

DataFrame to be read

cols: list of strings, optional

Column names to be read. By default, it reads all columns

Returns list

List of pandas.Series objects
```

Examples

sasoptpy.read_table

```
sasoptpy.read_table(table, session=None,
                                                     key=['_N_'], columns=None,
                                                                                       key\_type=['num'],
                            col_types=None, upload=False, casout=None, ref=True)
     Reads a CAS Table or pandas DataFrame
           Parameters table: swat.cas.table.CASTable, pandas.DataFrame object or string
                   Pointer to CAS Table (server data, CASTable), DataFrame (local data) or the name of
                   the table at execution (server data, string)
               session: swat.CAS or saspy.SASsession object
                   Session object if the table will be uploaded
               key: list, optional
                   List of key columns (for CASTable) or index columns (for DataFrame)
               columns: list, optional
                   List of columns to read into parameters
               key_type: list or string, optional
                   A list of column types consists of 'num' or 'str' values
               col_types : dict, optional
                   Dictionary of column types
               upload: boolean, optional
                   Option for uploading a local data to CAS server first
               casout: string or dict, optional
                   Casout options if data is uploaded
               ref: boolean, optional
                   Switch for returning the read data statement generated by the function
```

Returns tuple

A tuple where first element is the key (index), second element is a list of requested columns and the last element is reference to the original

See also:

```
Model.read table(), Model.read data()
```

sasoptpy.recursive walk

```
sasoptpy.recursive_walk (obj, func, attr=None, alt=None)
     Calls a given method recursively for given objects
```

Parameters func: string

Name of the method / function be called

attr: string, optional

An attribute which triggers an alternative method to be called if exists

alt: string, optional

Name of the alternative method / function to be called if passed attr exists for given objects

Notes

• This function is for internal consumption.

sasoptpy.register name

```
sasoptpy.register_name (name, obj)
```

Adds the name and order of a component into the global reference list

Parameters name: string

Name of the object

obj : object

Object to be registered to the global name dictionary

Returns int

Unique object number to represent creation order

sasoptpy.reset globals

```
sasoptpy.reset_globals()
```

Deletes the references inside the global dictionary and restarts counters

See also:

```
get_namespace()
```

Examples

```
>>> import sasoptpy as so
>>> m = so.Model(name='my_model')
>>> print(so.get_namespace())
Global namespace:
    Model
           0 my_model <class 'sasoptpy.model.Model'>,
                                                                      sasoptpy.
→Model(name='my_model', session=None)
```

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```
VariableGroup
ConstraintGroup
Expression
Variable
Constraint
>>> so.reset_globals()
>>> print(so.get_namespace())
Global namespace:
    Model
    VariableGroup
ConstraintGroup
Expression
Variable
Constraint
```

sasoptpy.tuple_pack

```
sasoptpy.tuple_pack(obj)
```

Converts a given object to a tuple object

If the object is a tuple, the function returns the input, otherwise creates a single dimensional tuple

Parameters obj : Object

Object that is converted to a tuple

Returns tuple

Tuple that includes the original object

sasoptpy.tuple unpack

```
sasoptpy.tuple_unpack(tp)
```

Grabs the first element in a tuple, if a tuple is given as argument

Parameters tp: tuple

Returns object

The first object inside the tuple.

sasoptpy.union

```
sasoptpy.union(*args)
Returns a union of Set, list or set objects
```

sasoptpy.wrap

```
sasoptpy.wrap(e, abstract=False)
Wraps expression inside another expression
```

8.8.2 Math Functions

math.math_func(exp, op, *args)	Function wrapper for math functions
math.abs(exp)	Absolute value function
math.log(exp)	Natural logarithm function
math.log2(exp)	Logarithm function to the base 2
math.log10(exp)	Logarithm function to the base 10
math.exp(exp)	Exponential function
math.sqrt(exp)	Square root function
math.mod(exp, divisor)	Modulo function
math.int(exp)	Integer value function
math.sign(exp)	Sign value function
math.max(exp, *args)	Largest value function
math.min(exp, *args)	Smallest value function
math.sin(exp)	Sine function
math.cos(exp)	Cosine function
math.tan(exp)	Tangent function

sasoptpy.math.math_func

sasoptpy.math.math_func(exp, op, *args)
Function wrapper for math functions

Parameters exp: Expression

Expression where the math func will be applied

op: string

String representation of the math function

args: float, optional

Additional arguments

sasoptpy.math.abs

sasoptpy.math.abs(*exp*)
Absolute value function

sasoptpy.math.log

sasoptpy.math.log(exp)
Natural logarithm function

sasoptpy.math.log2

sasoptpy.math.log2 (*exp*)

Logarithm function to the base 2

sasoptpy.math.log10

sasoptpy.math.log10 (exp)

Logarithm function to the base 10

sasoptpy.math.exp

```
sasoptpy.math.exp (exp)
Exponential function
```

sasoptpy.math.sqrt

```
sasoptpy.math.sqrt (exp)
Square root function
```

sasoptpy.math.mod

```
sasoptpy.math.mod(exp, divisor)
     Modulo function
```

Parameters exp: Expression

Dividend

divisor: Expression

Divisor

sasoptpy.math.int

```
sasoptpy.math.int(exp)
Integer value function
```

sasoptpy.math.sign

```
sasoptpy.math.sign(exp)
Sign value function
```

sasoptpy.math.max

```
sasoptpy.math.max(exp, *args)
Largest value function
```

sasoptpy.math.min

```
sasoptpy.math.min(exp, *args)
Smallest value function
```

sasoptpy.math.sin

```
sasoptpy.math.sin(exp)
Sine function
```

sasoptpy.math.cos

sasoptpy.math.cos(exp)
Cosine function

sasoptpy.math.tan

sasoptpy.math.tan(exp)
Tangent function

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