

Q1.

- a.

```
for(i=0; i<512; i++)
  for (k=0; k<i; k++)
    for (j=0; j<512; j++)
      C[i][j] += A[i][k]*B[k][j];
```
- b.

```
for(k=0; k<512; k++)
  for (i=k; i<512; i++)
    for (j=0; j<512; j++)
      C[i][j] += A[i][k]*B[k][j];
```

Q2.

- a.

```
for (i=0; i<512; i++)
  for (j=0; j<512; j++)
    for(kr=0; kr < j mod 3;kr++)
      C[i][j] += A[i][kr]*B[kr][j];
  for (k=3; k<j-3; k+=3){
    C[i][j] += A[i][k]*B[k][j];
    C[i][j] += A[i][k+1]*B[k+1][j];
    C[i][j] += A[i][k+2]*B[k+2][j];}
```
- b.

```
for (i=0; i<512; i++)
  for(jr=0; jr < 512 mod 3;jr++)
    for (k=0; k<jr; k++)
      C[i][jr] += A[i][k]*B[k][jr];
  for (j=3; j<512; j+=3)
    for(kr=0; kr < j mod 3;kr++)
      C[i][j] += A[i][kr]*B[kr][j];
  for (k=3; k<j-3; k+=3){
    C[i][j] += A[i][k]*B[k][j];
    C[i][j] += A[i][k+1]*B[k+1][j];
    C[i][j] += A[i][k+2]*B[k+2][j];
    C[i][j+1] += A[i][k]*B[k][j+1];
    C[i][j+1] += A[i][k+1]*B[k+1][j+1];
    C[i][j+1] += A[i][k+2]*B[k+2][j+1];
    C[i][j+2] += A[i][k]*B[k][j+2];
    C[i][j+2] += A[i][k+1]*B[k+1][j+2];
    C[i][j+2] += A[i][k+2]*B[k+2][j+2];}
```

Q3.

- a. j and k loops can be unrolled but no others as there would make the dependency graph lexicographically negative.
- b. The valid permutations of tijk are: tijk, tikj and tkij as other wise one of the dependency graphs will be lexicographically negative.
- c. Full tiling is not valid but partial tiling is possible with the j and k loops.
- d. The j and k loops can be parallelized. The others can not because that would lead to a data dependency error as they carry the dependency.

Q4.

- a. Suppose there are two ways to write to the same element. (i_1, j_1) and (i_2, j_2) . By the code we know that the loop will access are $A[i_1+1][j_1-1]$ and $A[i_2+1][j_2-1]$. Thus to access the same element $i_1+1 = i_2+1$ and $j_1-1 = j_2-1$. Simplified $i_1 = i_2$ and $j_1 = j_2$ thus a contradiction so we have no output dependencies.
- b. Consider the first possible vector let the write in each iteration be (w_i, w_j) and the read to be (r_i, r_j) . For the same element to be accessed $i_w+1 = r_i$ and $w_j-1 = r_j$. For a flow dependency the vector is $(r_i - w_i, r_j - w_j) = (i_w+1 - i_w, w_j-1 - w_j) = (1, -1)$. For the other possible vector let the write in each iteration be (w_i, w_j) and the read to be (r_i, r_j) . For the same element to be accessed $i_w+1 = r_i+1$ and $w_j-1 = r_j-2$. For a flow dependency the vector is $(r_i - w_i, r_j - w_j) = (w_i - w_i, w_j+1 - w_j) = (0, 1)$. So there are two flow dependencies of $(1, -1)$ and $(0, 1)$
- c. Consider the first possible vector let the write in each iteration be (w_i, w_j) and the read to be (r_i, r_j) . For the same element to be accessed $i_w+1 = r_i$ and $w_j-1 = r_j$. For an anti-dependency the vector is $(w_i - r_i, w_j - r_j) = (i_w - i_w - 1, w_j - w_j + 1) = (-1, 1)$. For the other possible vector let the write in each iteration be (w_i, w_j) and the read to be (r_i, r_j) . For the same element to be accessed $i_w+1 = r_i+1$ and $w_j-1 = r_j-2$. For an anti-dependency the vector is $(w_i - r_i, w_j - r_j) = (w_i - w_i, w_j - w_j - 1) = (0, -1)$. So there are no anti-dependencies both possible vectors are negative.