Homework 3

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Problem 1

(a) $f(x) = L^3 - 600$

(b) $x_(n+1) = (2x_n^3 + 600)/3x^2$
The minimum number of iterations required to compute the function is 7.

(c)

Iteration	x-value (cm)	Volume (cm^3)
x_0	8	512.0
x_1	8.45833	605.1379
x_2	8.43439	600.01452

(d)

(1) The error found using error propagation method (1.72e-3 cm^3) is smaller than the 'real error' of (1.45e-2 cm^3).

$$\sigma V = 3(\sigma L)L^2 \text{ (Equation 1)}$$

$$\sigma L = 6.81e-5$$

$$L = 8.43$$

$$\sigma V = 1.72e-3$$

(2) An appropriate value of the tolerance of L would be 3.95e-3 cm. This is the value of σL when it is solved for in equation 1 using $\sigma V = 0.1cm^3$.

(3) If the root were to be computed using bisection search method instead of Newton-Raphson, it would take 9 iterations to reach the correct tolerance. This is 7 more iterations than Newton-Raphson requires as the value of error is exponentially smaller for each Newton-Raphson iteration whereas the error only decreases by a factor of two for each iteration of the bisection search.

(e) As demonstrated in the tables below: the Newton-Raphson method of root finding required fewer iterations in order to find the desired root within the given tolerance. The top table displays the operations per step and the lower chart totals them.

Bisection		Newton-Raphson	
Math	Operation Count	Math	Operation Count
$x = (xlo+xhi)/2$	2	$x = (2*x*x*x+600)/(3*x*x)$	7
$x*x*x-600$	3	$x*x*x-600$	3
Total	5	Total	10

Bisection		Newton-Raphson	
Iteration	Operation Count	Iteration	Operation Count
1	5	1	10
2	5	2	10
3	5	Total	20
4	5		
5	5		
6	5		
7	5		
8	5		
9	5		
Total	45		

Problem 2

(a) The function: $f(x) = x^5 - 3x^3 + 15x^2 + 27x + 9$ has 3 real roots. The roots are located at approximately x= -2.25, -1.2, and -0.5.

Problem 4

(b) After 7 iterations, the error in the value of x was effectively zero.

(d) The value of the order of convergence for the Newton-Raphson fixed-point root-finding method is 2. This order of convergence is not exact; However, it is more accurate than the 1 or 1.5. This is what I expected as the order of convergence for the Newton-Raphson function is generally 2.

Extra Credit

The function required 9 iterations to find the root of the function. This is 7 more than the Newton-Raphson function. The two root-finding methods both returned the same answer to 4 significant figures, the root found using the Newton-Raphson method is slightly more precise than the bisection root. The roots found using bisection and Newton-Raphson are: 8.4345703125 and 8.434394724992652629819268175 respectively.