Trajectory Optimization Through Mixed-Integer Optimization of Contact Dynamics for Switching End Effector Locomotion

Abstract— **Trajectory optimizers for legged robots typically assume fixed end effectors with constant dynamics. Robots employing point-modeled end effectors, compared to those with wheeled end effectors, often benefit in adaptability and maneuverability but at the cost of higher energy expenditure and lower speed. While current hardware supports switching between these two end-effector types, existing research has largely focused on maintaining stability during switching, with little attention to determining when each type is most effective. To our knowledge, this paper introduces the first framework that simultaneously optimizes both trajectories and end-effector contact dynamics through mixed-integer optimization. We validate our approach by solving and executing trajectories with a whole-body controller in Gazebo across a variety of terrains, including ramps and stepping stones. The results show that our framework not only handles diverse terrains but also exploits contact dynamics to reduce cost of transport and increase speed compared to foot-only locomotion.**

I. Introduction

Legged robots have emerged as promising platforms for mobility and manipulation in complex, unstructured environments. Their performance, however, can be heavily influenced by the design of the End Effector (EE) interacting with the terrain. The choice of EE directly affects locomotion efficiency, stability, and adaptability, yet no single design provides an ideal solution across all conditions. As a result, design and selection of optimal EE as well as identifying and leveraging the strengths of different end-effector types remains an open challenge in legged robotics.

For legged robots, two dominant paradigms exist: point-contact feet [1]–[4] and wheeled EEs [5]–[8]. Point-contact feet offer versatility, agility, and simpler low-dimensional control, particularly on irregular or cluttered terrain [9], [10]. By contrast, wheels excel on flat or moderately structured surfaces, where rolling contact provides significant reductions in cost of transport [5]. In practice, many real-world environments contain both structured and unstructured regions, making it difficult for a robot limited to a single end-effector type to achieve both efficiency and adaptability.

This observation has motivated initial explorations into hybrid or reconfigurable systems [11]–[13], including mechanisms that allow switching between wheels and feet. For example, [14] introduced a parallel switching mechanism with a state-machine controller. [15] introduced a design to quickly change contact dynamics by changing the knee configuration. However, while these works demonstrate the feasibility of switching, little research has addressed how to plan motion that fully exploits both modalities and optimizes across their distinct contact dynamics. This gap motivates the approach presented in this paper.

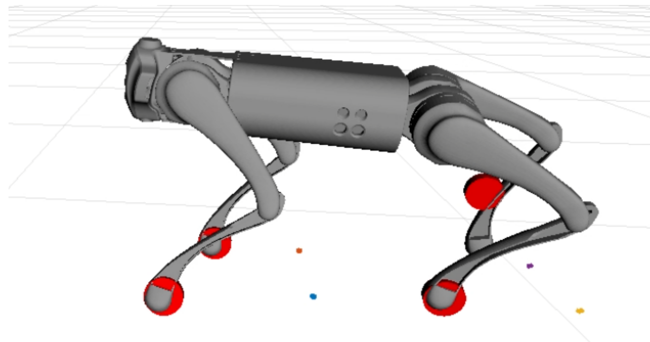

Fig. 1: Simulated Quadruped with Switching Wheel-feet EEs

Switching between EEs is inherently a discrete problem, since each EE either contributes its contact dynamics or does not. In contrast, the dominant modeling approaches for legged robots rely on trajectory optimization, which is formulated as a continuous process that drives the system from an initial state to a desired state. While most solutions use a nonlinear trajectory optimizer over a limited time horizon in a framework known as model predictive control (MPC) [16]–[18], these nonlinear optimizers are generally not tractable for handling discrete mode-switching in real time. Instead, trajectory optimization approaches based on mixed-integer formulations have emerged as a way to incorporate discrete variables, such as contact decisions, directly into the optimization process [19], [20].

[21] first introduced the problem as a footstep-planning formulation, where goal distance, footstep distance, and total number of contacts are weighted to generate a plan that progresses toward the goal while remaining constrained to distinct steppable regions. [22]–[24] extend the mixed-integer footstep planning approach by integrating it with nonlinear MPC. However, this decoupled strategy—separating footstep planning from body dynamics—can be problematic, as a fixed footstep plan may lead to failure when the system dynamics change due to external factors. [19], [20] instead extend [21] by optimizing the footstep plan jointly with the body dynamics, enabling simultaneous gait and motion planning.

However, for switching EEs, the contact type and timing highly influences the body dynamics and cannot be decoupled as simply as fixed contact dynamics. Therefore, we build on the framework introduced in [19] to optimize not only contact timing but also contact dynamics, enabling robot traversal that is explicitly informed by how EEs interact with the terrain.

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TABLE I: Variables in Trajectory Optimization

$\mathbf{x} = [x_1, x_2, x_3]^T$	Center of mass in the world frame
$\theta = [\theta_1, \theta_2, \theta_3]^T$	Center of mass orientation
$\mathbf{p}_l = [p_{1,l}, p_{2,l}, p_{3,l}]^T$	End-effector positions in the world frame
$\lambda_l = [\lambda_{1,l}, \lambda_{2,l}, \lambda_{3,l}]^T$	Contact forces
$\mathbf{f}_l = [f_{1,l}, f_{2,l}, f_{3,l}]^T$	Frictional forces at leg l
N_l, N_k	Number of legs and knots in trajectory
N_r, N_e	Number of regions and EE types
N_j	Number of swing regions
$\mathbf{U}_l^+ = [U_{l,1}^+, U_{l,2}^+, U_{l,3}^+]^T$	The upper bound of $\ddot{\theta}$ from leg l.
$\mathbf{U}_l^- = [U_{l,1}^-, U_{l,2}^-, U_{l,3}^-]^T$	The lower bound of $\ddot{\theta}$ from leg l.

The main contribution of this paper is the formulation of a mixed-integer quadratically constrained control problem (MIQCCP) that optimizes end-effector selection and gait based on the interaction of each EE with the terrain. To our knowledge, this is the first trajectory optimization framework that enables locomotion informed by changing contact dynamics for legged robots equipped with switching EEs. In addition, we extend existing whole-body controllers (WBC) and state-estimation frameworks to support the robust execution of the planned trajectories. Together, these developments allow robots with switchable EEs to select contact dynamics in accordance with desired behaviors. We validate the proposed methodology in Gazebo simulation on the Unitree Go1 across a variety of terrains.

The rest of the paper is organized as follows. Section II presents a novel method of trajectory optimization through EE optimization followed by WBC in Section III. Section IV covers the results of our experimental simulations with a modified Unitree Go1 and Section V concludes the paper.

II. Simultaneous Trajectory and EE Optimization

In this section we introduce our trajectory optimizer for exploiting contact dynamics through the switching of EEs.

A. Linear Centroidal Dynamics

We begin with introducing the system inputs and decision variables for the movement of the robot's Center of Mass (CoM) as a floating base. As in [19], we decouple the EE states from the state of the body's CoM and discard the leg joints to allow for a more linear representation of dynamics. The dynamics of the body state are therefore expressed as

$$m\ddot{\mathbf{x}} = m\mathbf{g} + \sum_{l=1}^{N_l} \lambda_l + \mathbf{f}_l, \tag{1}$$

which is then discretely integrated using midpoint Euler integration across N_k knot points where each knot point k contains the robot's current state and inputs. Variables are transcribed as written in TABLE I.

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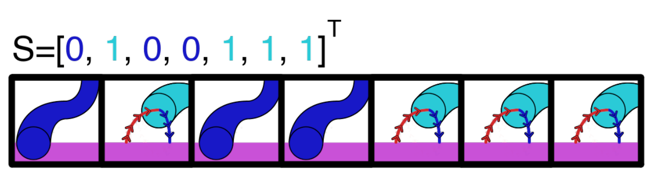

Fig. 2: The Swing Matrix Effects on Locomotion for One Leg

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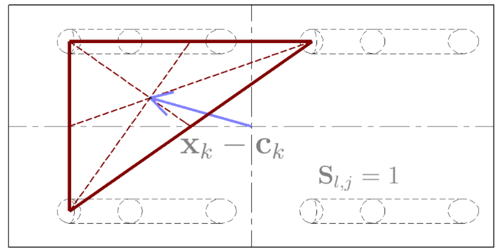

Fig. 3: Support Polygon Centroid and CoM Displacement

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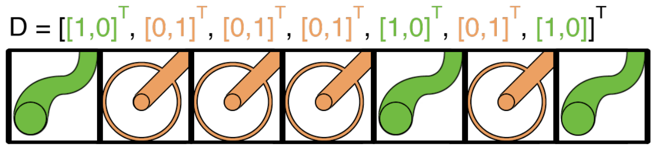

Fig. 4: The Dynamics Matrix Effects on Locomotion with One Region, One Leg, and Two EE Types

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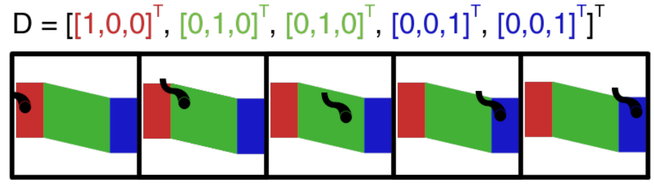

Fig. 5: The Dynamics Matrix Effects on Locomotion with Three Regions, One Leg, and One EE Type

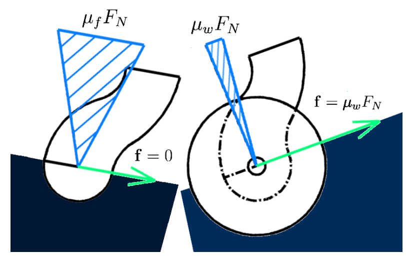

Fig. 6: Friction Cone and Frictional Force for Legs vs Wheel

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IV. Experimental Results

Our approach was validated on the Unitree Go1 quadruped robot by performing simulations in Gazebo multi-robot simulator version 11.15.1 using the Open Dynamics Engine with an artificial 9 millisecond delay between commanded torques and simulated actuation. We employ Gurobi version 12 as the mixed-integer optimizer. All computation benchmarks are performed on an *Intel Core Ultra 7 115H* processor. The method was tested on a variety of terrains as shown in Fig. 7.

A. End Effector and Gait Selection

We demonstrate in our experiments the ability to optimize gait and EE selection to minimize the associated cost functions and remain within region limits. Our proposed method allows the robot to traverse multiple terrains according to the interactions of the EEs with the terrain.

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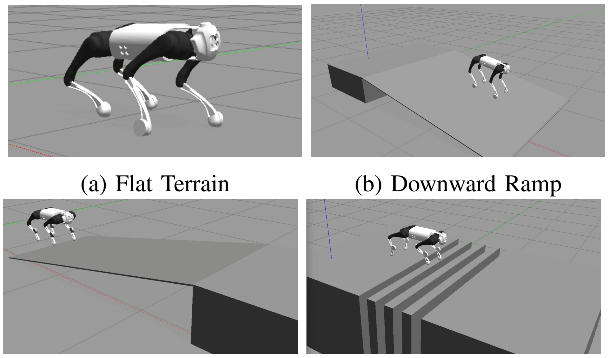

(a) Flat Terrain (b) Downward Ramp

(c) Upward Ramp (d) Stepping Stones

Fig. 7: Experimental Setups: Different Terrains for Testing

1) Flat Ground without Friction: On flat ground, our method demonstrates the ability to minimize swings, contact force, and time to reach goal position by optimizing walking in favor of wheels. Starting with feet, the optimizer switches all legs to wheels early in the trajectory, minimizing contact force. Once the rolling has brought the robot to its goal state, one leg is returned to a foot to bring the robot to a stop, minimizing swing phases. The three wheeled EEs also experimentally improve the robot's ability to adjust its position toward the goal state through the WBC, with the single foot serving as a base to correct for unplanned differences in rolling velocity. Fig. 8 shows the gait diagram for a 4.5 second trajectory on flat ground with green representing the foot contact and orange representing the wheel contact.

2) Flat Ground with Friction: On the flat ground with friction, we were able to validate both our friction model and its effects on the ability to minimize swings. The gradual decrease in velocity due to friction while rolling requires the robot to make more footed contacts to generate more force through a push leg. Fig. 9 demonstrates this by showing more foot contacts relative to Fig. 8 throughout the trajectory while most contacts remain wheels.

3) Downward Ramp: It would be expected that a contact dynamics optimizer would prefer the use of passive wheels on terrain that is slanted downward because it allows the effortless continued rolling, even with rolling friction modeled. Our results agree with this expectation. The robot approaches the ramp, switches to wheels to roll down the ramp, and then switches back at the end of the ramp to come to a complete stop using a foot. Fig. 10 demonstrates the behavior when the wheels are able to *roll* across the convex planes belonging to the same continuous group as in eq. (24). Without the grouping of convex planes, the gait would be forced to execute a *swing* phase to cross from the platform to the ramp.

4) Upward Ramp: Unlike the downward ramp, it cannot be expected passive wheels would be optimal on an upward ramp because wheels cannot apply force up the ramp. Our results confirm this expectation but also show that it is optimal in some cases, particularly at the beginning of the ramp, to use a combination of feet and wheels to gain speed and switch to all feet once wheels lose forward velocity.

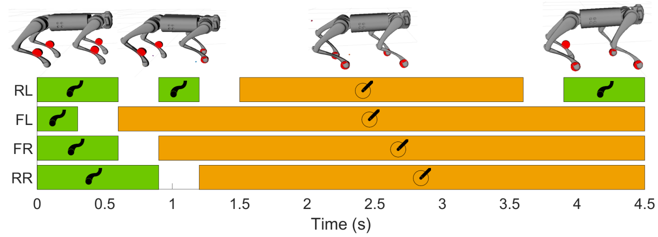

Fig. 8: Gait and EE Selection on Flat Ground with no Friction

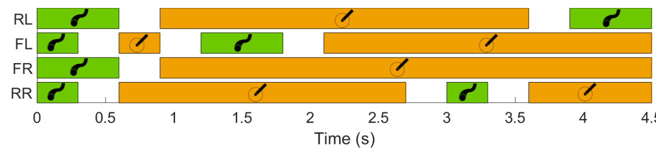

Fig. 9: Gait and EE Selection on Flat Ground with Friction

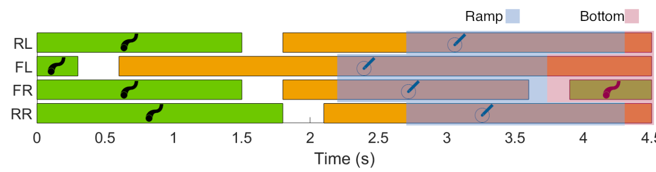

Fig. 10: Gait and EE Selection on a Downward Ramp

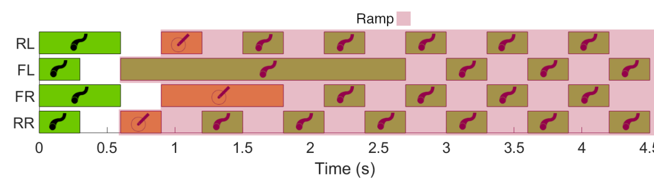

Fig. 11: Gait and EE Selection on an Upward Ramp

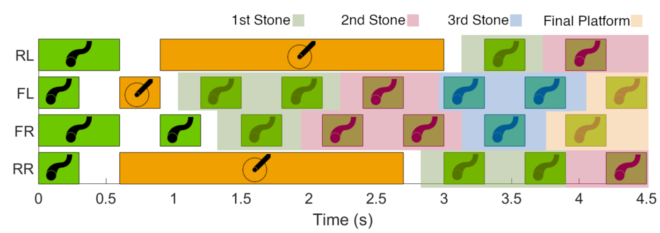

Fig. 12: Gait and EE Selection across Stepping Stones

Fig. 11 demonstrates an trajectory where this is seen.

5) Stepping Stones: For the stepping stones, the contacting wheels would roll onto the gap because they move with the body's velocity. Therefore, the optimizer selects to use feet on the areas where rollable space is limited to prevent wheels from rolling into unpermitted terrain. This is demonstrated in Fig. 12 which shows the gait across the stepping stones spaced 20cm apart. The robot's maximum stepping distance is 50cm.

B. Trajectory Generation and Speed

A key advantage of wheels over feet is their potential for faster locomotion through extended rolling contact, which reduces the need for frequent stepping discontinuities (see

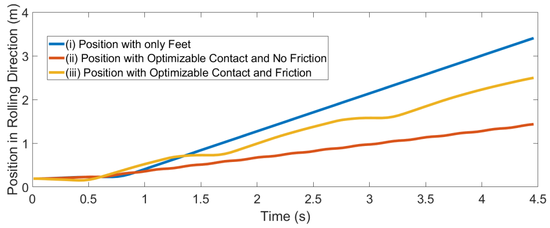

Fig. 13: Desired Trajectory on Flat Ground for Fixed Dynamics and Optimizable Dynamics (with and without Friction).

Fig. 8 for example). We evaluate our framework's ability to exploit this advantage in gait generation under three flat-ground conditions: (i) non-optimizable EEs restricted to feet, (ii) optimizable EEs without friction modeling (Fig. 8), and (iii) optimizable EEs with friction modeling Fig. 9. All cases are assigned the same unreachable goal state and optimize a trajectory over 4.5 seconds. Fig. 13 shows the resulting trajectories. We find that, regardless of friction modeling, our method exploits the speed advantage of wheels: in 4.5 seconds, the foot-only robot covers 1.43 m, whereas with feet and wheels it reaches 2.5 m with friction and 3.4 m without friction. This indicates that the robot with wheels reached average speeds of 0.56 m/s (74% faster) with friction modeled and 0.76 m/s (137% faster) without friction, compared to 0.32 m/s for foot-only locomotion.

C. Cost of Transport

One of the major constraints of legged robots is their power consumption. Previous works have demonstrated the ability of wheel-legged robots to decrease power consumption often using cost of transport, a unitless measure of performance, as a surrogate. We employ the method from [5] to calculate the cost of transport. Our measurements on flat ground demonstrate that, through minimization of contact forces and CoM accelerations, our optimizer is able to select EEs that minimize cost of transport. For a 9-second trajectory on flat ground, our optimizer selects EEs and gaits that yield a cost of transport of 0.105. Under the same conditions, but with optimization restricted to feet, the cost of transport increases to 0.237. This corresponds to a 55.7% reduction (≈2.3× improvement). Thus, our method, which minimizes applied force, also achieves lower cost of transport. We opt not to compare this against other methods because no methods we know of are capable of optimizing contact dynamics to generate these types of gates.

D. Computation Benchmarking

We ran experiments across a variety of terrains and measured computation times. TABLE II reports results for flat and downhill terrains without rolling friction or multi-region constraints. As expected, because mixed-integer optimization is NP-hard, computation time increases significantly with the number of binary variables N_r, N_e, and N_j. Additional experiments revealed further increases in computation time when rolling friction or multi-region constraints were included.

TABLE II: Average Computation Time without Friction or Multi-Region Constraint

Seconds	Terrain	N_r	N_e	N_j	Computation Time (s)
1.5	Flat	1	1	5	0.714
3	Flat	1	1	10	4.637
1.5	Flat	1	2	5	1.723
3	Flat	1	2	10	673.58
1.5	Down Ramp	3	1	5	0.977
3	Down Ramp	3	1	10	204.37
1.5	Down Ramp	3	2	5	3.887

Although our method demonstrated robustness across different terrains, the resulting computation times present challenges for deployment in an MPC context, where trajectories must be recalculated rapidly. However, as shown in the first row of TABLE II, our method computed a 1.5-second trajectory with five swing slots in 0.714 seconds, suggesting that for certain tasks, such as flat-terrain locomotion, our approach may still be feasible within an MPC framework. Nonetheless, the results demonstrate that our method can explicitly handle switching contact dynamics which cannot be executed using traditional nonlinear optimization methods.

V. Conclusions

We introduced and evaluated a novel mixed-integer optimization framework that simultaneously optimizes trajectories and contact dynamics, enabling robots to generate gaits that strategically exploit terrain interactions. To our knowledge, this is the first optimization approach to integrate end-effector selection into trajectory generation. We validated our approach through a variety of terrains. Experiments across flat ground, slopes, and stepping-stone terrains demonstrated the robustness and versatility of the optimizer, which consistently selected effective end-effector combinations to ensure stable execution. On flat ground, our framework leveraged the speed advantage of wheels: robots with switching end effector showed 74% increase in speed with friction modeled and 137% increase without friction, compared to the robot speed for foot-only locomotion. In addition, our method achieved a 57% reduction in cost of transport compared to conventional legged locomotion, underscoring its potential for improved energy efficiency. Although computational demands increased in more complex terrains due to the need to manage binary contact switches, this trade-off reflects the richer modeling of contact dynamics that traditional nonlinear trajectory optimization methods cannot capture.

In future work, we aim to extend our framework to hardware implementation, addressing both hardware design considerations and the execution of optimized trajectories on a physical quadruped. Another direction is to use the trajectories generated and realized by the WBC as baselines for learning-based methods. Recent studies, such as [31] and [32], highlight the promise of neural networks for solving mixed-integer programming problems, while [33] proposes warm-start strategies to reduce solve times. Our results on linear trajectories, where up to five swing segments can be generated in under 500 milliseconds, suggest that extending this approach toward real-time MPC may be feasible even without learning methods. Finally, consistent with the assessment in [19], we recognize the need for improved convex formulations of angular dynamics and intend to explore this in future work.

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