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MCAT® MATH - SCIENTIFIC NOTATION, EXPONENTS AND APPROXIMATION

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Introduction

Although the MCAT is a science test, not a math test, many general physics and chemistry questions on the exam require the ability to recall and solve complex mathematical equations without a calculator. However, most MCAT study guides either gloss over calculations, leaving students to wonder how, exactly, the review authors got an answer (especially without a calculator), or the study guides provide a huge list of theorems and formulas, leaving students to wonder how, exactly, to know when to apply each. This book is different. Each MCAT® Math guide covers a core mathematical concept, explaining it in detail and applying it in multiple examples from general chemistry and physics.

The three techniques presented in this guide—scientific notation, the rules of exponents and sensible approximation—are the foundation of solving math problems on the MCAT. Once you've completed this guide and worked through the practice exercises on your own, you'll feel comfortable quickly solving a variety of general chemistry and physics problems using these three techniques. You'll also be ready to tackle more advanced topics like trigonometry and logarithms. One thing to note: this guide focuses on the mathematical issues involved in solving a particular problem. It is a companion to—not a replacement for—a review of the scientific concepts covered on the exam.

Scientific Notation

Most MCAT questions are written with quantities expressed in scientific notation, and it's expected that students will be able to work with these quantities quickly and easily. Far from being an encumbrance, scientific notation is a huge help in doing complex computations without a calculator. Therefore, it's helpful to have a quick review of how scientific notation works.

Scientific notation takes advantage of the fact that, in our base-10 number system, it is easy to mentally multiply and divide by powers of 10. It takes far more mental energy to compute 3×17 than it does 3×10 or 3×100 or 3×0.001. Scientific notation represents any number—now matter how large or small—as a product involving a power of 10.

Using Scientific Notation To Represent A Number Greater than 1

Numbers in scientific notation have the form $N \times 10^m$ where $1 \leq N < 10$ and m is an integer (a whole number). N is referred to as the "significand", 10 the "base" and m the "exponent". When a number in scientific notation has a positive exponent, it indicates that the base (which is always 10) is multiplied by itself the number of times written in the exponent. 10^m is 10 multiplied by 10 m times. For example, $10^2 = 10 \times 10$ and $10^5 = 10 \times 10 \times 10 \times 10 \times 10$.

To express the number 300 in scientific notation, think of it as the product of a number between 1 and 10 and a multiple of 10: $300 = 3 \times 100 = 3 \times 10^2$. Notice that the power in the exponent term is equal to the number of places the decimal point must be moved to the left until there is only one non-zero digit remaining in the significand. For 300, the decimal point must be moved two places to the left so that the significand is 3. Therefore, the exponent is 2. The same pattern applies to numbers with more than 1 non-zero digit: $5293 = 5.293 \times 1000 = 5.293 \times 10^3$. To express a single-digit number in scientific notation, recall that any number raised to the power 0 equals 1. Therefore $10^0 = 1$ and $6 = 6 \times 1 = 6 \times 10^0$.

While scientific notation can be used to express any number, it is most useful for very large or very small quantities. Avogadro's Number $\approx 6.022 \times 10^{23}$. Written without using scientific notation it is $602,200,000,000,000,000,000,000$—a much more difficult expression to work with.

Using Scientific Notation To Represent A Number Between 0 *and* 1

Numbers between 0 and 1 written in scientific notation will also have the form $N \times 10^m$ where $1 \leq N < 10$ and m is an integer; however, instead of having a positive exponent, numbers less than 1 will have negative exponent. While a positive exponent indicates the number of times the base is multiplied by itself, a negative exponent indicates the number of times the base is divided by itself.

To express 0.008 in scientific notation, again think of it as the product of a number between 1 and 10 and a multiple of 10: $0.008 = 8 \times 0.001 = 8 \times 10^{-3}$. Similarly, $0.4235 = 4.235 \times 0.1 = 4.325 \times 10^{-1}$ and $0.00000092 = 9.2 \times 0.0000001 = 9.2 \times 10^{-7}$. Notice that the power in the exponent term is equal to the number of places the decimal point must be moved to the right until there is only one non-zero digit remaining in the significand.

Without using scientific notation, the threshold of hearing—a quantity that should be memorized for the MCAT—would be written as $0.000000000001 \frac{\mathrm{W}}{\mathrm{m}^2}$. Moving the decimal 12 places to the right yields the more familiar—and easier to manipulate—expression $1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}$.

Metric Conversions

Another benefit of using scientific notation is that it makes it easy to convert quantities to different units in the metric system. For example, a question will provide a quantity expressed in milliliters, but the appropriate formula to answer the question will require the quantity to be expressed in liters. You could memorize the number of milliliters in a liter or the number of microns in a meter; however, it's much simpler and faster to learn a single method that can be used to convert between any metric prefix and any other.

Medical students love mnemonics, and one (of many) that can be used to memorize the common prefixes in the metric system is King Henry Died (gram /meter/ liter) drinking Chocolate Milk – Move 3, 'Nother 3. The first letter of each word recalls kilo-, hecto-, deka-, (gram /meter/ liter), deci-, centi-, milli-, micro-, and nano-. Because there are two "d" prefixes, upper and lower case are used to distinguish between deka- ("D") and deci- ("d"). The entry (gram/ meter/ liter) refers to the type of measurement you are trying to convert. Typically this will be meters, liters or grams, but this method could also be used to convert other units such as joules to kilojoules.

It is always worth the time to jot down the mnemonic and use it to ensure the unit conversion has been done correctly. After a bit of practice, you will only need to write the first letter of each prefix and the value of the exponent (not a full table) to jog your memory. This will be much faster and more accurate than trying to recall, for example, how many nanograms are in a kilogram.

To convert from a larger to a smaller unit (from kilograms to grams, for example), add one to the exponent for each prefix you pass as you move along the list. To convert from a smaller to a larger unit (from millimeters to kilometers, for example), subtract one from the exponent for each prefix you pass as you move along the list.

The exception to this is converting to micrometers and nanometers, which requires adding or subtracting 3 (hence the 3 in the mnemonic) from the exponent reflecting a 10^3 rather than 10^1 change in scale.

Example 1

A problem requires you to convert 5×10^2 grams to kilograms. First, write "5×10^2" next to the entry for grams. Then subtract 1 from the exponent for each prefix passed as you move from the gram row to the kilo- row (because the desired units is kilograms). The full table is shown in the example for instructive purposes. With practice you will only need to write the first letter of each prefix and the value of the exponent.

	Mnemonic for remembering metric prefixes	
Mnemonic	Prefix	Quantity
King (K)	kilo-	5×10^{-1}
Henry (H)	hecto-	5×10^{0}
Died (D)	deka-	5×10^{1}
gram	*no prefix*	5×10^{2}
drinking (d)	deci-	
Chocolate (C)	centi-	
Milk (M)	milli-	
Move 3 (M 3)	micro-	
'Nother 3 (N 3)	nano-	

Therefore, 5×10^2 g $= 5 \times 10^{-1}$kg.

Example 2

Convert 3.2×10^{-4} km to cm.

To accomplish this unit conversion, write 3.2×10^{-4} next to the "K" (for kilo-) entry and add 1 to the exponent for each prefix passed as you move from the "K" row to the "C" (for centi-) row.

Mnemonic	Quantity
K	3.2×10^{-4}
H	3.2×10^{-3}
D	3.2×10^{-2}
meter	3.2×10^{-1}
d	3.2×10^{0}
C	3.2×10^{1}
M	
M 3	
N 3	

Therefore 3.2×10^{-4} km $= 3.2 \times 10^{1}$ cm $= 32$ cm.

Example 3

Convert 9 nm to m.

To accomplish this unit conversion, write 9×10^0 next to the "N3" (for nano-) entry. Subtract 3 from the exponent to convert to micrometers and another 3 to convert to millimeters. Then subtract 1 from the exponent for each prefix you pass as you move from the "M" (for milli-) row to the meter row.

Mnemonic	Quantity
K	
H	
D	
meter	9×10^{-9}
d	9×10^{-8}
C	9×10^{-7}
M	9×10^{-6}
M 3	9×10^{-3}
N 3	9×10^{0}

Therefore 9 nm $= 9 \times 10^{-9}$ m.

These examples should convince you that using scientific notation is more than just busy work—something you don't need when you are already pressed for time. However, if you still need more convincing, the next sections will illustrate how understanding and using scientific notation actually speeds up and simplifies calculations by allowing the use of the rules of exponents.

The Rules of Exponents - Addition

Perhaps surprisingly, it is uncommon to have to add or subtract quantities on the MCAT. However, this type of calculation does come up in a few applications such as calculating change in volume or pressure.

The addition rule for exponents states that, to add two numbers with the same base and exponent, add the significands and keep the same base and exponent. This works regardless of if the exponents are positive or negative.

$$(a \times m^n) + (b \times m^n) = (a + b) \times m^n$$

For example,

$$(3 \times 10^8) + (2 \times 10^8) = (3 + 2) \times 10^8 = 5 \times 10^8$$
$$(6 \times 10^{-5}) + (2 \times 10^{-5}) = (6 + 2) \times 10^{-5} = 8 \times 10^{-5}$$

To add two quantities with different exponents (in scientific notation the base will always be 10) one must be re-expressed with the same exponent as the other using the rules of scientific notation.

For example, 6×10^2 kg of one substance is added to 4×10^3 kg of another. What is the resulting mass?

To add the two quantities using the addition rule, either 6×10^2 must be re-expressed as 0.6×10^3 by moving the decimal one place to the left and increasing the exponent by 1 or 4×10^3 must be re-expressed as 40×10^2 by moving the decimal one place to the right and decreasing the exponent by 1.

Either strategy will result in the correct answer; however, converting 6×10^2 kg to 0.6×10^3 kg will result in an addition problem where the solution is already in scientific notation with the correct form, saving a step at the end.

$$\begin{aligned} \text{Total mass} &= (6 \times 10^2 \text{ kg}) + (4 \times 10^3 \text{ kg}) \\ &= (0.6 \times 10^3 \text{ kg}) + (4 \times 10^3 \text{ kg}) \\ &= 4.6 \times 10^3 \text{ kg} \end{aligned}$$

The Rules of Exponents - Subtraction

The subtraction rule for exponents states that, to subtract one number from another number with the same base and exponent, subtract one significand from the other and keep the same base and exponent.

$$(a \times m^n) - (b \times m^n) = (a - b) \times m^n$$

For example,

$$(8 \times 10^9) - (2 \times 10^9) = (8 - 2) \times 10^9 = 6 \times 10^9$$
$$(4 \times 10^{-1}) - (1 \times 10^{-1}) = (4 - 1) \times 10^{-1} = 3 \times 10^{-1}$$

As when adding, sometimes the rules of scientific notation must be used to re-express one number so it has the same exponent as the other.

For example, the volume of a gas increases from 2×10^1 m^3 to 1×10^3 m^3. What is the change in the volume of the gas?

Here either 2×10^1 must be re-expressed as 0.02×10^3 by moving the decimal two places to the left and increasing the exponent by 2 or 1×10^3 must be re-expressed as 100×10^1 by moving the decimal two places to the right and decreasing the exponent by 2.

Again, either will result in the correct answer. Converting 2×10^1 m^3 to 0.02×10^3 m^3 yields

$$\begin{aligned}
\text{Change in volume} &= (1 \times 10^3 \text{ m}^3) - (2 \times 10^1 \text{ m}^3) \\
&= (1 \times 10^3 \text{ m}^3) - (0.02 \times 10^3 \text{ m}^3) \\
&= 0.98 \times 10^3 \text{ kg} \\
&= 9.8 \times 10^2 \text{ kg (using the rules of scientific notation)}
\end{aligned}$$

The Rules of Exponents - Multiplication

In contrast, the two most common types of arithmetic found on the MCAT are multiplication and division. The product rule is used to multiply two terms expressed in scientific notation.

The product rule for exponents states that, to multiply two numbers with the same base, keep the same base and add the two exponents. This works regardless of if the exponents are positive or negative.

$$a^m \times a^n = a^{(m+n)}$$

For example,

$$\begin{aligned}
10^8 \times 10^2 &= 10^{(8+2)} = 10^{10} \\
10^{-1} \times 10^{-8} &= 10^{(-1+(-8))} = 10^{-9} \\
10^3 \times 10^{-2} &= 10^{(3+(-2))} = 10^1
\end{aligned}$$

Many general chemistry and physics formulas require the multiplication of two numbers in scientific notation. Here is a simplified example to illustrate the concept: A patient has received an electrical shock that passes 10 A of current through his body for 0.01 seconds. How much charge has gone through the patient's skin?

The formula for electrical current is

$$I = \frac{Q}{\Delta t}$$

where I is the current in A, Q is the charge in C and Δt is the amount of time in s the current is applied.

Filling in the quantities from the example yields

$$10\text{A} = \frac{Q}{0.01\text{s}}$$

The first step in solving any problem is to express all of the terms in scientific notation (recall that $10 = 10^1$ and $0.01 = 10^{-2}$)

$$10^1\text{A} = \frac{Q}{10^{-2}\ \text{s}}$$

Then solve for Q by multiplying each side by 10^{-2}

$$(10^1\text{A}) \times (10^{-2}\ \text{s}) = Q$$

Finally, multiply the two terms together using the product rule by adding the exponents

$$Q = 10^{(1+-2)} = 10^{-1} = 0.1\,\text{C}$$

Returning to the original question: A patient has received an electrical shock that passes 10 A of current through his body for 0.01 seconds. How much charge has gone through the patient's skin? The answer is 0.1 C.

Modifying An Answer So It Is In The Correct Form

Sometimes after doing calculations, the final answer is almost—but not quite—in scientific notation of the form $N \times 10^m$ where $1 \leq N < 10$ and m is an integer. For example, two numbers are multiplied together yielding the answer 32×10^4 or one number is divided into another yielding 0.05×10^{-3}. The product rule will allow us to "fix" these answers to be in the correct form.

When a number has too many digits in the signifcand, move the decimal to the left until there is only one non-zero digit remaining. For each place you moved the decimal, increase the exponent by 1. For example, to convert 32.0×10^4 to the correct form, move the decimal one place to the left and increase the exponent by 1 so 32.0×10^4 is rewritten as 3.20×10^5.

The principle behind this technique is the product rule. To convert 32.0×10^4 to the correct form, rewrite 32 as 3.2×10^1. The product rule is used to calculate $32.0 \times 10^4 = (3.2 \times 10^1) \times 10^4 = 3.2 \times 10^5$. Notice that the power in the exponent term increases by 1 for each place the decimal point must be moved to the left until there is only one non-zero digit remaining in the significand. Because the decimal point was moved to the left 1 place, the exponent increased by 1.

When a number has too few digits in the signifcand, move the decimal to the right until there is only one non-zero digit remaining. For each place you moved the decimal, decrease the exponent by 1. For example, to convert 0.05×10^{-3} to the correct form, move the decimal two places to the right and decrease the exponent by 2 so 0.05×10^{-3} is rewritten as 5×10^{-5}.

Again, the principle behind this technique is the product rule. To convert 0.05×10^{-3} to the correct form, rewrite 0.05 as 5.0×10^{-2}. The product rule is used to calculate $(5.0 \times 10^{-2}) \times 10^{-3} = 5.0 \times 10^{-5}$. The power in the exponent term decreases by 1 for each place the decimal point must be moved to the right until there is only one non-zero digit remaining in the significand. Because the decimal point was moved to the right 2 places, the exponent decreased by 2.

The Rules of Exponents - Division

The quotient rule for exponents states that, to divide two numbers with the same base, keep the base but subtract the exponent in the denominator from the exponent in the numerator. Again, this works regardless of if the exponents are positive or negative.

$$\frac{a^m}{a^n} = a^{m-n}$$

For example,

$$\frac{10^3}{10^2} = 10^{(3-2)} = 10^1$$
$$\frac{10^5}{10^8} = 10^{(5-8)} = 10^{-3}$$
$$\frac{10^{-6}}{10^{-2}} = 10^{(-6-(-2))} = 10^{-4}$$

Here is a simplified example to illustrate how this technique would be used to solve a general chemistry problem: Find the NO_3^- concentration when 10^{-3} moles of $NaNO_3$ are added to 10^2 L of H_2O.

The concentration of a substance in a solution is calculated using the formula

$$\mathrm{M} = \frac{\mathrm{mol}}{\mathrm{L}}$$

Substituting in the quantities given in the problem yields

$$\mathrm{M} = \frac{10^{-3}\,\mathrm{mol}}{10^2\,\mathrm{L}}$$

The numbers are already expressed in scientific notation, saving a step. The answer is calculated using the quotient rule by subtracting the exponent in the denominator from the exponent in the numerator

$$\mathrm{M} = \frac{10^{-3}\,\mathrm{mol}}{10^2\,\mathrm{L}} = 10^{(-3-2)}\,\frac{\mathrm{mol}}{\mathrm{L}} = 10^{-5}\,\frac{\mathrm{mol}}{\mathrm{L}}$$

Returning to the problem: Find the NO_3^- concentration when 10^{-3} moles of $NaNO_3$ are added to 10^2 L of H_2O. The answer is $10^{-5}\,\frac{\mathrm{mol}}{\mathrm{L}}$.

The Rules of Exponents - Squaring Numbers

It is somewhat less common to have to find the square or square root of a number in scientific notation. However, each will be required for two important physics applications: calculating an object's kinetic energy from its velocity and vice versa. The power rule is used to square a number expressed in scientific notation.

The power rule of exponents states that, to raise a base raised to a power to another power, multiply the exponents together.

$$(a^m)^n = a^{(m \times n)}$$

For example,

$$(10^3)^2 = 10^{(3\times2)} = 10^6$$
$$(10^1)^2 = 10^{(1\times2)} = 10^2$$

This rule is nearly always used to find the square of a term written in scientific notation. Here is a simplified example calculating the kinetic energy of an object from it's velocity: A 10^4 kg boulder rolls down a frictionless mountain and comes to the bottom with a speed of $10^2\,\frac{\mathrm{m}}{\mathrm{s}}$. What is the kinetic energy of the boulder at the bottom of the mountain?

Begin with the kinetic energy formula

$$K = \frac{1}{2}mv^2$$

where K is the kinetic energy of the object, m is the mass of the object and v is its velocity.

Substituting in the quantities given in the problem yields

$$K = \frac{1}{2}(10^3)(10^2)^2\,\mathrm{J}$$

To square 10^2 in the last term, use the power rule and multiply the exponents together

$$K = \frac{1}{2}(10^3)(10^4)\,\mathrm{J}$$

Now use the product rule and add the exponents to simplify the expression to

$$K = \frac{1}{2}(10^7)\,\mathrm{J}$$

Convert the fraction to a decimal

$$K = 0.5 \times 10^7\,\mathrm{J}$$

And finally, re-express the number in correct scientific notation using the product rule by moving the decimal to the right one place and subtracting one from the exponent

$$K = 5 \times 10^6\,\mathrm{J}$$

Returning to the original question: A 10^4 kg boulder rolls down a frictionless mountain and comes to the bottom with a speed of $10^2\,\frac{\mathrm{m}}{\mathrm{s}}$. What is the kinetic energy of the boulder at the bottom of the mountain? The answer is 5×10^6 J.

The Rules of Exponents - Taking a Square Root

If instead the problem asks you to solve for the velocity of an object given its kinetic energy, two other rules of exponents are required: the rational rule and the raising a product to a power rule.

The Rational Rule

The rational rule for exponents states that taking the square root of a number is equivalent to raising that number to the $\frac{1}{2}$ power

$$\sqrt[2]{a} = a^{\frac{1}{2}}$$

For example

$$\sqrt{10} = 10^{\frac{1}{2}}$$
$$\sqrt{(10^3)} = (10^3)^{\frac{1}{2}}$$
$$\sqrt{(10^{-5})} = (10^{-5})^{\frac{1}{2}}$$

Now a simplified kinetic energy problem where the answer requires solving for the velocity: A 10^2 kg boulder rolls down a frictionless mountain and reaches the bottom with a kinetic energy of 10^6 J. What was the speed of the boulder at the bottom of the mountain in $\frac{m}{s}$?

Begin with the kinetic energy formula

$$K = \frac{1}{2}mv^2$$

where K is the kinetic energy of the object, m is the mass of the object and v is its velocity.

Substitute in the quantities given in the problem

$$10^6\,\mathrm{J} = \frac{1}{2}(10^2\,\mathrm{kg})v^2$$

The goal is to isolate the unknown v^2 on one side of the equation and solve for v.

First divide each side by 10^2 using the quotient rule

$$\begin{aligned}
\frac{10^6}{10^2}\,\mathrm{J} &= \frac{1}{2}v^2 \\
10^{(6-2)}\,\mathrm{J} &= \frac{1}{2}v^2 \\
10^4\,\mathrm{J} &= \frac{1}{2}v^2
\end{aligned}$$

Then multiply each side by 2 to cancel out the $\frac{1}{2}$

$$2 \times 10^4\,\mathrm{J} = v^2$$

Next use the rational rule to take the square root of both sides and express the left side as a quantity raised to the $\frac{1}{2}$ power.

$$\begin{aligned}
v &= \sqrt{2 \times 10^4}\,\frac{\mathrm{m}}{\mathrm{s}} \\
&= (2 \times 10^4)^{\frac{1}{2}}\,\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}$$

One final rule of exponents, the raising a product to a power rule, will be required to finish solving this equation.

The Raising a Product to a Power Rule

The raising a product to a power rule of exponents states that the product of two terms raised to a power is equal to the product of each of those individual terms raised to the same power.

$$(a \times b)^m = a^m \times b^m$$

To raise a number written in scientific notation to a power, both the significand and the base and exponent should be raised to the power.

For example,

$$\begin{aligned}
(3 \times 10^2)^2 &= 3^2 \times (10^2)^2 \\
(4 \times 10^{-6})^{\frac{1}{2}} &= 4^{\frac{1}{2}} \times (10^{-6})^{\frac{1}{2}}
\end{aligned}$$

The power rule is then used to simplify the base and exponent term of each number

$$\begin{aligned}
(3 \times 10^2)^2 &= 3^2 \times (10^2)^2 = 9 \times 10^4 \\
(4 \times 10^6)^{\frac{1}{2}} &= 4^{\frac{1}{2}} \times (10^6)^{\frac{1}{2}} = 2 \times 10^{-3}
\end{aligned}$$

Returning to the example: A 10^2 kg boulder rolls down a frictionless mountain and reaches the bottom with a kinetic energy of 10^6 J. What was the speed of the boulder at the bottom of the mountain in $\frac{m}{s}$?

Previously, using other rules of exponents, we simplified the equation to

$$v = (2 \times 10^4)^{\frac{1}{2}}\ \frac{\mathrm{m}}{\mathrm{s}}$$

Now use the raising a product to a power rule

$$v = 2^{\frac{1}{2}} \times (10^4)^{\frac{1}{2}}\ \frac{\mathrm{m}}{\mathrm{s}}$$

And the rational rule and power rule

$$v = \sqrt{2} \times 10^2\ \frac{\mathrm{m}}{\mathrm{s}}$$

The last step in solving the problem is to approximate $\sqrt{2}$. Approximation is such an important MCAT Math strategy that the next chapter is devoted to it, but one can quickly observe that, because $\sqrt{1} = 1$ and $\sqrt{4} = 2$, $\sqrt{2}$ will fall somewhere between 1 and 2, closer to 1 than 2.

A "good enough" approximation of the final answer is

$$v = 1.5 \times 10^2\ \frac{\mathrm{m}}{\mathrm{s}}$$

The answer derived using a calculator is $v = 1.4 \times 10^2\ \frac{\mathrm{m}}{\mathrm{s}}$.

Approximation

Perhaps the most important thing to keep in mind when solving math problems on the MCAT is that students are not expected to calculate the correct answer but rather to identify the correct answer from a list of choices. The authors of the test know that students will not be able to calculate an exact answer without a calculator and the answer choices will reflect that.

Students who understand the scientific concepts being tested and are confident using scientific notation and the rules of exponents covered in the previous chapters, plus one new technique introduced here—the commutative property of multiplication—will be able to select the correct answer to any math question.

The Commutative Property of Multiplication

The commutative property of multiplication states that the order in which terms are multiplied doesn't matter (unlike subtraction or division where order *does* matter).

$$(a \times b) \times (c \times d) = (a \times c) \times (b \times d)$$

In practice, this means that when two numbers expressed in scientific notation are multiplied together, the terms can be reordered from

$$y = (M \times 10^n) \times (X \times 10^z)$$

to

$$y = (M \times X) \times (10^n \times 10^z)$$

Or a division problem from

$$y = \frac{(M \times 10^n)}{(X \times 10^z)}$$

to

$$y = \frac{M}{X} \times \frac{10^n}{10^z}$$

The second term in each equation can be solved exactly using the rules of exponents, but approximation will be required to solve the first part.

It's important to keep in mind that, while the MCAT is not a math test, students *are* expected to be able to multiply and divide integers between 1 and 10. That means feeling just as comfortable recalling 7×8 now as you did when you learned the multiplication tables in elementary school. It is also helpful to recognize that there are other numbers used in daily life that can act as "anchors" for approximation when a problem requires multiplication or division of a number greater than 10.

For example, many students would struggle to solve $\frac{500}{23}$ without a calculator. However, if you are accustomed to using a currency with quarters and dollars, doing mental math with the number 25 might not be that difficult. Rather than trying to answer $\frac{500}{23}$, the question becomes "How many quarters are there in $5?" Four quarters in a dollar yields 20 quarters in 5 dollars, and therefore $\frac{500}{23} \approx 20$.

If, instead, multiples of 20 are more natural to work with, the previous problem could have been approximated using 20 rather than 25. Other potential anchors are 12 (a dozen, 12 hours in a day) and 15 (four 15-minute periods in an hour). However, taking the time to convert numbers to scientific notation of the form $N \times 10^m$ where $1 \leq N < 10$ and m is an integer, means rarely having to multiply or divide numbers greater than 10.

When a problem requires multiplying or dividing two numbers in scientific notation, the most efficient way to approximate the answer is to round the significands to the nearest whole number less than 10—or another anchor like 12, 15, 20 or 25—-and then perform the computation. This will yield an solution that is close enough to select the correct answer.

Example

Find the NO_3^- concentration when 0.0035 moles of $NaNO_3$ are added to 200 L of H_2O.

a) 1.75×10^{-5} M
b) 2.63×10^{-2} M
c) 6.75×10^{-1} M
d) 7.00×10^{-1} M

The concentration of a substance in a solution is found using the formula

$$M = \frac{\text{mol}}{\text{L}}$$

Convert the quantities given in the problem to scientific notation and substitute them into the equation ($0.0035 = 3.5 \times 10^{-3}$ and $200 = 2.00 \times 10^{2}$)

$$M = \frac{3.5 \times 10^{-3}\,\text{mol}}{2.00 \times 10^{2}\,\text{L}}$$

Use the commutative property of multiplication rewrite the equation as the product of two fractions so that the significand portion of each number is in one fraction and the base and exponent portion of each number is in the other fraction

$$M = \frac{3.5}{2.00} \times \frac{10^{-3}}{10^{2}} \frac{\text{mol}}{\text{L}}$$

Approximate 3.5 with 4 in the first fraction

$$M \approx \frac{4}{2.00} \times \frac{10^{-3}}{10^{2}} \frac{\text{mol}}{\text{L}}$$
$$M \approx 2 \times \frac{10^{-3}}{10^{2}} \frac{\text{mol}}{\text{L}}$$

Use the quotient rule to simplify the second fraction by subtracting the exponent in the denominator from the exponent in the numerator

$$M \approx 2 \times \frac{10^{-3}}{10^{2}} \frac{\text{mol}}{\text{L}}$$
$$M \approx 2 \times 10^{(-3-2)} \frac{\text{mol}}{\text{L}}$$
$$M \approx 2 \times 10^{-5} \frac{\text{mol}}{\text{L}}$$

This closest answer to this by far is a) 1.75×10^{-5} M, which is the correct answer.

Additional Examples

E1: Recall that e is the fundamental unit of charge equal to 1.6×10^{-19} C, and k is the electrostatic constant equal to $8.99 \times 10^{9}\ \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$ What is the potential energy of a system if a charge of -3e and +4e are separated by a distance of 2 nm?:

a) -9.15×10^{-38} J
b) -6.90×10^{-10} J
c) 6.90×10^{-10} J
d) 9.15×10^{-38} J

The equation for potential energy is

$$U = \frac{kQq}{r^2}$$

where Q and q are the charges of the two particles, r is the distance they are separated in meters, and k is the electrostatic constant as described in the problem.

Note that r is given in nm but should be expressed in m in the equation. The mnemonic introduced in the scientific notation section shows that 2 nm = 2×10^{-9} m.

Mnemonic	Quantity
K	
H	
D	
meter	2×10^{-9}
d	2×10^{-8}
C	2×10^{-7}
M	2×10^{-6}
M 3	2×10^{-3}
N 3	2×10^{0}

Substituting in the quantities given in the problem, including the value of r which was converted to 2×10^{-9} m, yields

$$U = \frac{(8.99 \times 10^{9}\frac{\mathrm{Nm}^2}{\mathrm{C}^2})(4 \times 1.6 \times 10^{-19}\mathrm{C})(-3 \times\ 1.6 \times 10^{-19}\mathrm{C})}{(2 \times 10^{-9})^2\mathrm{m}}$$

Use the raising a product to a power rule to square the denominator

$$U = \frac{(8.99 \times 10^{9}\frac{\mathrm{Nm}^2}{\mathrm{C}^2})(4 \times 1.6 \times 10^{-19}\mathrm{C})(-3 \times\ 1.6 \times 10^{-19}\mathrm{C})}{4 \times 10^{-18}\mathrm{m}}$$

Use the commutative property of multiplication to group the significands together and the bases and exponents together

$$U = \frac{8.99 \times 4 \times 1.6 \times -3 \times 1.6}{4} \times \frac{10^9 \times 10^{-19} \times 10^{-19}}{10^{-18}} \text{ J}$$

To simplify the base and exponent portion of the equation, use the product and quotient rules to multiply and divide by adding and subtracting the exponents

$$U = \frac{8.99 \times 4 \times 1.6 \times -3 \times 1.6}{4} \times (10^{9+(-19)+(-19)-(-18)}) \text{ J}$$

The base and exponent portion of the equation can now be simplified by adding together the exponents. Note that $-19 + 9 = -10$ and $-19 + 18 = -1$

$$U = \frac{8.99 \times 4 \times 1.6 \times -3 \times 1.6}{4} \times (10^{-11}) \text{ J}$$

Now round each quantity in the first fraction to the nearest whole number to approximate the significand

$$U = \frac{9 \times 4 \times 2 \times -3 \times 2}{4} \times (10^{-11}) \text{ J}$$

Note that $\frac{4}{4} = 1$ so the denominator can be canceled in the significand

$$U = (9 \times 2 \times -3 \times 2) \times (10^{-11}) \text{ J}$$

Multiply $9 \times 2 \times -3 \times 2$ to get

$$U = -108 \times (10^{-11}) \text{ J}$$

Finally, using the rules of scientific notation, the answer is put in the correct form by moving the decimal to the left two places and increasing the exponent by 2.

$$U = -1.08 \times 10^{-9} \text{ J}$$

Which is closest to answer b) -6.90×10^{-10} J. Note that c) 6.90×10^{-10} is incorrect because the two particles are oppositely charged, resulting in a "negative" answer, which indicates an attractive force between the two particles.

E2 At time t = 0 there is a 7 mole sample of a radioactive isotope that is decaying at a rate of 3 hr^{-1}. How many nuclei remain after 30 minutes? (Hint: $e^{-1.5} \approx 0.22$)

a) 1.54×10^0
b) 4.62×10^1
c) 9.27×10^{23}
d) 2.78×10^{31}

The equation for exponential decay is

$$n = n_0 e^{-\lambda t}$$

where n is the amount remaining at time t, n_0 is the initial quantity, λ is the decay rate and t is the time elapsed.

Substituting in quantities from the problem yields

$$n = 7e^{-(\frac{3}{hr})(.5hr)}$$
$$n = 7e^{-1.5}$$

Computations with base e and computing natural logarithms are beyond the scope of the MCAT, which is why the question provides a hint. Substituting in $e^{-1.5} = 0.22$ yields

$$n = 7 \times 0.22\,\text{mol}$$

Use the anchor 25 (think of 7 quarters as being 1 less than 8 quarters, which is 25 cents less than 2 dollars) to approximate

$$n = 7 \times 0.22 \approx 7 \times 0.25 = 1.75\,\text{mol}$$

To convert from mol to the number of nuclei, multiply by Avogadro's number

$$1.75 \text{ mol} \times 6.02 \times 10^{23} \frac{\text{nuclei}}{\text{mol}}$$

Rounding 1.75 to 2 and 6.02 to 6 yields

$$2 \text{ mol} \times 6 \times 10^{23} \frac{\text{nuclei}}{\text{mol}} = 12 \times 10^{23}\,\text{nuclei}$$

Moving the decimal to the left one place and increasing the exponent by 1 yields the final answer 1.2×10^{24}, which is closest to c) 9.27×10^{23} nuclei.

E3: What is the magnitude of the energy change when an electron returns from an excited state to its ground state, emitting a photon at $\lambda = 300$ nm? (Hint: $c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$ and $h = 6.626 \times 10^{-34}$ J * s)

a) 2.341×10^{-22} J
b) 6.626×10^{-19} J
c) 1.687×10^{-16} J
d) 3.402×10^{-10} J

The equation to determine the energy of one photon is

$$E = \frac{hc}{\lambda}$$

where h is Planck's constant, c = the speed of light and λ is the wavelength of light in meters.

First, $\lambda = 300$ nm must be converted to meters using the mnemonic

Mnemonic	Quantity
K	
H	
D	
meter	3×10^{-7}
d	3×10^{-6}
C	3×10^{-5}
M	3×10^{-4}
M 3	3×10^{-1}
N 3	3×10^{2}

Substituting in the appropriate quantities from the question yields

$$E = \frac{(6.626 \times 10^{-34}\, J * s) \times (3.00 \times 10^8)\frac{\text{m}}{\text{s}}}{3 \times 10^{-7}\text{m}}$$

Using the commutative property of multiplication, the expression is rearranged as

$$E = \frac{(6.626 \times 3.00)}{3} \times \frac{(10^{-34} \times 10^8)}{10^{-7}}\,\text{J}$$

The 3 in the numerator and denominator cancel each other in the first fraction. Use the product and quotient rules to multiply and divide the terms in the second fraction by adding and subtracting the exponents

$$E = 6.626 \times 10^{(-34+8-(-7))}\, J = 6.626 \times 10^{-19}\,\text{J}$$

Adding the exponents yields the exact answer, b) 6.626×10^{-19} J.

E4: Find the gravitational force between a proton and an electron that are 2 μm apart. (Hint: The mass of a proton is 1.67×10^{-15} ng and the mass of an electron is 9.11×10^{-19} ng.)

a) 2.54×10^{-56} N
b) 3.61×10^{-37} N
c) 6.19×10^{-30} N
d) 3.03×10^{2} N

Using Newton's Law of Gravitation

$$F_g = \frac{Gm_1m_2}{r^2}$$

where G = the force of gravity = $6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$, m_1 and m_2 are the mass of the proton and electron in kg, and r is the distance between the particles.

Because the force of gravity is expressed in $\frac{\text{Nm}^2}{\text{kg}^2}$, both the mass of the proton and electron need to be expressed in kg rather than ng and the distance needs to be expressed in meters rather than μm using the mnemonic.

Now that this is a familiar concept, the same table can be used to convert all three quantities to the correct units.

Mnemonic	Proton Mass	Electron Mass	Distance
K	1.67×10^{-27}	9.11×10^{-31}	
H	1.67×10^{-26}	9.11×10^{-30}	
D	1.67×10^{-25}	9.11×10^{-29}	
gram/ meter	1.67×10^{-24}	9.11×10^{-28}	2×10^{-6}
d	1.67×10^{-23}	9.11×10^{-27}	2×10^{-5}
C	1.67×10^{-22}	9.11×10^{-26}	2×10^{-4}
M	1.67×10^{-21}	9.11×10^{-25}	2×10^{-3}
M 3	1.67×10^{-18}	9.11×10^{-22}	2×10^{0}
N 3	1.67×10^{-15}	9.11×10^{-19}	

The mass of a proton is 1.67×10^{-27} kg, the mass of an electron is 9.11×10^{-31} kg, and the distance between the particles is 2×10^{-6} m.

Substituting these quantities into Newton's Law of Gravitation

$$F_g = \frac{(6.67 \times 10^{-11}\frac{\text{Nm}^2}{\text{kg}^2})(9.11 \times 10^{-31}\text{kg})(1.67 \times 10^{-27}\text{kg})}{(2 \times 10^{-6}\text{m})^2}$$

Applying the raising a product to a power rule to the denominator yields

$$F_g = \frac{(6.67 \times 10^{-11} \frac{\mathrm{Nm}^2}{\mathrm{kg}^2})(9.11 \times 10^{-31}\mathrm{kg})(1.67 \times 10^{-27}\mathrm{kg})}{(4 \times 10^{-12}\ \mathrm{m}^2)}$$

Now use the commutative property of multiplication to collect the significands and the base and exponent terms

$$F_g = \frac{6.67 \times 9.11 \times 1.67}{4} \times \frac{10^{-11} \times 10^{-31} \times 10^{-27}}{10^{-12}}\ \mathrm{N}$$

Round terms in the first fraction to whole numbers and use the product and quotient rules to simplify the second fraction by adding and subtracting the exponents.

$$F_g = \frac{7 \times 9 \times 2}{4} \times 10^{(-11+(-31)+(-27)-(-12))}\ \mathrm{N}$$

Note that $\frac{2}{4}$ in the first fraction can be simplified to $\frac{1}{2}$.

$$F_g = \frac{63}{2} \times 10^{-57}\ \mathrm{N}$$

Simplify the significand completely. Move the decimal point one place to the left and increase the exponent by 1 to put the answer in the proper form

$$F_g = 31.5 \times 10^{-57}\ \mathrm{N}$$
$$F_g = 3.15 \times 10^{-56}\ \mathrm{N}$$

which is closest to answer a) 2.54×10^{-56} N

E5: A nuclear fusion process converts 9.0×10^{-10} ng to energy. How much energy was released?

a) 9.3×10^{-20} J
b) 8.4×10^{-14} J
c) 1.8×10^{-10} J
d) 2.7×10^{-5} J

The amount of energy produced can be calculated using Einstein's equation: $E = mc^2$ where c, the speed of light, is $3.0 \times 10^8 \frac{\text{m}}{\text{s}}$ and m is the amount of mass converted to energy.

Because J = $\frac{\text{kg*m}^2}{\text{s}^2}$ the mass must be converted from nm to kg using the mnemonic before it can be substituted into the equation.

Mnemonic	Quantity
K	3×10^{-22}
H	3×10^{-21}
D	3×10^{-20}
gram	3×10^{-19}
d	3×10^{-18}
C	9.0×10^{-17}
M	9.0×10^{-16}
M 3	9.0×10^{-13}
N 3	9.0×10^{-10}

Substituting $c = 3.0 \times 10^8 \frac{\text{m}}{\text{s}}$ and $m = 3 \times 10^{-22}$ kg into Einstein's equation

$$E = (3 \times 10^{-22}\text{kg})(3.0 \times 10^8 \frac{\text{m}}{\text{s}})^2$$

Use the raising a product to a power rule to square the speed of light

$$E = (3 \times 10^{-22}\text{kg})(9.0 \times 10^{16} \frac{\text{m}^2}{\text{s}^2})$$

Then use the commutative property of multiplication to group significands and base and exponent terms

$$E = (3 \times 9.0) \times (10^{-22} \times 10^{16})\,\text{J}$$

Multiply the significands and base and exponent terms

$$E = 27 \times 10^{-6}\,\text{J}$$

Finally, move the decimal point left one place and increase the exponent by 1 to put the answer in the proper form $E = 2.7 \times 10^{-5}$ J, which is answer d).

Practice Problems

Q1: A photon has an energy of 5 eV, what is its frequency? (Hint: h = 4.1×10^{-15} eV*s)

a) 1.22×10^{15} Hz
b) 2.05×10^{16} Hz
c) 1.25×10^{18} Hz
d) 6.67×10^{22} Hz

Q2: Is it possible for a human to see a light ray with a frequency of 6×10^{14} Hz?

a) No, this is in the X-ray portion of the electromagnetic spectrum
b) No, this is in the ultraviolet portion of the electromagnetic spectrum
c) Yes, this is in the visible portion of the electromagnetic spectrum
d) No, this is in the microwave portion of the electromagnetic spectrum

Q3: Thirty minutes after the start of an experiment there remains 3 mol of a radioactive substance with a constant decay rate of 2 hr^{-1}. How many mol of the substance were present at the beginning of the experiment? (Hint: $e^{-1} \approx 0.37$)

a) 5.10 mol
b) 8.11 mol
c) 9.03 mol
d) 13.26 mol

Q4: A penny weighing 2.5 g rolls down a giant frictionless slide and comes to the bottom with a speed of $12\frac{\text{m}}{\text{s}}$. What is the kinetic energy of the penny at the bottom of the slide?

a) 0.18 J
b) 0.36 J
c) 93.75 J
d) 180.00 J

Q5: Gas in a cylinder expands when 300 kJ of heat are applied to it. The gas is kept at a constant pressure of 2.9×10^7 Pa, and during the heating process it expands from 2.1 m^3 to 3.2 m^3. How much work is done by the gas?

a) 7.6×10^2 J
b) 9.8×10^2 J
c) 3.2×10^7 J
d) 1.9×10^8 J

Practice Problem Solutions

Q1: A photon has an energy of 5 eV, what is its frequency? (Hint: h = 4.1×10^{-15} eV*s)

a) 1.22×10^{15} Hz
b) 2.05×10^{16} Hz
c) 1.25×10^{18} Hz
d) 6.67×10^{22} Hz

A1: The equation relating the energy of a photon and its frequency is

$$E = hf$$

where E is the energy of the photon, h is Planck's constant and f is the frequency of the light.

Substituting in E and h from the problem yields

$$5\,\text{eV} = (4.1 \times 10^{-15}\text{eV*s}) \times f$$

To solve for f, divide each side by Planck's constant

$$\frac{5\,\text{eV}}{4.1 \times 10^{-15}\text{eV*s}} = f$$

Rewrite 5 as 5×10^0 and use the commutative property of multiplication group the significands and bases and exponents

$$\frac{5}{4.1} \times \frac{10^0}{10^{-15}}\ \text{Hz} = f$$

Simplify the first fraction by recognizing that 5 is a little more than 4, so $\frac{5}{4.1}$ is a little more than 1 or $\frac{5}{4.1} \approx 1.25$. Use the quotient rule to divide the second term by subtracting the exponents

$$1.25 \times 10^{0-(-15)}\ \text{Hz} = f$$
$$1.25 \times 10^{15}\ \text{Hz} = f$$

which is closest to answer a) 1.22×10^{15} Hz.

Q2: Is it possible for a human to see a light ray with a frequency of 6×10^{14} Hz?

a) No, this is in the X-ray portion of the electromagnetic spectrum
b) No, this is in the ultraviolet portion of the electromagnetic spectrum
c) Yes, this is in the visible portion of the electromagnetic spectrum
d) No, this is in the microwave portion of the electromagnetic spectrum

A2: The equation relating the frequency and wavelength of a light ray is

$$c = f\lambda$$

where c, the speed of light, is $3 \times 10^8 \frac{\text{m}}{\text{s}}$, f is the frequency and λ is the wavelength of the light ray.

Replacing c and λ with quantities from the problem yields

$$3 \times 10^8 \,\frac{\text{m}}{\text{s}} = f \times (6 \times 10^{14})\,\text{Hz}$$

Divide each side by $6 \times 10^{14}\,\text{Hz}$ to solve for f

$$\frac{3 \times 10^8 \,\frac{\text{m}}{\text{s}}}{6 \times 10^{14}\,\text{Hz}} = f$$

Use the commutative property of multiplication to group the signifcand and base and exponent terms

$$\frac{3}{6} \times \frac{10^8}{10^{14}}\,\text{m} = f$$

Divide $\frac{3}{6} = 0.5$ and use the quotient rule to divide the second fraction by subtracting the exponents

$$0.5 \times 10^{(8-14)}\,\text{m} = f$$

Which further simplifies to

$$0.5 \times 10^{-6}\,\text{m} = f$$

Move the decimal one place to the right and decrease the exponent by 1

$$5.0 \times 10^{-7}\,\text{m} = f$$

Finally, use the mnemonic to convert from m to nm.

Mnemonic	Quantity
K	
H	
D	
meter	5.0×10^{-7}
d	5.0×10^{-6}
C	5.0×10^{-5}
M	5.0×10^{-4}
M 3	5.0×10^{-1}
N 3	5.0×10^{2}

5.0×10^2 nm $= 500$ nm. Wavelengths of the visible spectrum run from about $400 - 700$ nm, so the answer is c).

Q3: Thirty minutes after the start of an experiment there remains 3 mol of a radioactive substance with a constant decay rate of 2 hr^{-1}. How many mol of the substance were present at the beginning of the experiment? (Hint: $e^{-1} \approx 0.37$)

a) 5.10 mol
b) 8.11 mol
c) 9.03 mol
d) 13.26 mol

A3: The exponential decay equation is

$$n = n_0 e^{-\lambda t}$$

where n is the amount remaining at time t, n_0 is the initial quantity, λ is the decay rate and t is the time elapsed.

Filling in the unknowns in the equation using values from the question—including the hint—yields

$$\begin{aligned} 3 \text{ mol} &= n_0 e^{-(\frac{2}{hr})(.5hr)} \\ 3 \text{ mol} &= n_0 e^{-1} \\ 3 \text{ mol} &= n_0 (.37) \end{aligned}$$

Divide each side by 0.37 to solve for n_0

$$\frac{3}{.37} \text{ mol} = n_0$$

To simplify the division problem, express both the numerator and denominator in scientific notation

$$\frac{3 \times 10^0}{3.7 \times 10^{-1}} \text{ mol} = n_0$$

Group the significands and base and exponent pairs

$$\frac{3}{3.7} \times \frac{10^0}{10^{-1}} \text{ mol} = n_0$$

Approximate 3.7 as 4 in the first fraction and use the quotient rule to solve the second fraction

$$\frac{3}{4} \times 10^1 \text{ mol} = n_0$$

Express $\frac{3}{4}$ as a decimal and use the rules of scientific notation to put the number in proper form

$$\begin{aligned} 0.75 \times 10^1 \text{ mol} &= n_0 \\ 7.5 \times 10^0 \text{ mol} &= n_0 \\ 7.5 \text{ mol} &= n_0 \end{aligned}$$

Which is closest to answer b) 8.11 mol.

Q4: A penny weighing 2.5 g rolls down a giant frictionless slide and comes to the bottom with a speed of $12\frac{\text{m}}{\text{s}}$. What is the kinetic energy of the penny at the bottom of the slide?

a) 0.18 J
b) 0.36 J
c) 93.75 J
d) 180.00 J

A4: The formula to calculate kinetic energy is

$$K = \frac{1}{2}mv^2$$

where m is the mass of the object in kg and v is the velocity in $\frac{\text{m}}{\text{s}}$.

First, 2.5 g must be converted to 2.5×10^{-3} kg using the mnemonic

Mnemonic	Quantity
K	2.5×10^{-3}
H	2.5×10^{-2}
D	2.5×10^{-1}
gram	2.5×10^{0}
d	
C	
M	
M 3	
N 3	

Then substitute the values of m and v from the problem into the formula and express $\frac{1}{2}$ in scientific notation. Note that $\frac{1}{2} = 0.5 = 5 \times 10^{-1}$. In general, this would be the point to re-express the velocity in scientific notation; however, 12 is a typical anchor and most students find it easier to calculate $12^2 = 144$ than $(1.2 \times 10^1)^2 = 1.44 \times 10^2$.

$$K = (5 \times 10^{-1})(2.5 \times 10^{-3})(12)^2\,\mathrm{J}$$

Square the velocity term and express it in scientific notation

$$\begin{aligned} K &= (5 \times 10^{-1})(2.5 \times 10^{-3})(144)\,\mathrm{J} \\ &= (5 \times 10^{-1})(2.5 \times 10^{-3})(1.44 \times 10^{2})\,\mathrm{J} \end{aligned}$$

Use the commutative property of multiplication to group the significand and base and exponent pairs

$$K = (5 \times 2.5 \times 1.44)(10^{-1} \times 10^{-3} \times 10^{2})\,\mathrm{J}$$

Use the product rule to simplify the second term

$$K = (5 \times 2.5 \times 1.44)(10^{-2})\,\mathrm{J}$$

Use the anchor of 25 to recognize that 5 quarters equals 125 cents and therefore $5 \times 2.5 = 12.5$. Note also that 5×2.5 is a bit more than $5 \times 2 = 10$, making 12.5 a realistic answer

$$K = (12.5 \times 1.44) \times 10^{-2}\ \mathrm{J}$$

Approximate 12.5 as 12 and 1.44 as 1.5

$$K = (12 \times 1.5) \times 10^{-2}\,\mathrm{J}$$

A quick calculation yields

$$K = 18 \times 10^{-2}\,\mathrm{J}$$

The answer choices are not written in scientific notation, so the final step is to express the solution as a decimal.

$$K = 0.18\,\mathrm{J}$$

Which, (although approximation was used in the calculations) is exactly answer choice a).

Q5: Gas in a cylinder expands when 300 kJ of heat are applied to it. The gas is kept at a constant pressure of 2.9×10^7 Pa, and during the heating process it expands from 2.1 m^3 to 3.2 m^3. How much work is done by the gas?

a) 7.6×10^2 J
b) 9.8×10^2 J
c) 3.2×10^7 J
d) 1.9×10^8 J

A5: Because this is an isobaric process, the work done by the gas can be found using the equation

$$\mathrm{W} = \mathrm{P}\Delta\mathrm{V}$$

Where W is the work done in J, P is the pressure in the system and ΔV is the change in the volume of the gas.

Substituting the quantities given in the problem for P and ΔV yields

$$\mathrm{W} = (2.9 \times 10^7) \times (3.2 - 2.1)\mathrm{J}$$

Solve for ΔV and express it in scientific notation

$$\mathrm{W} = (2.9 \times 10^7) \times (1.1 \times 10^0)\mathrm{J}$$

Use the commutative property of multiplication to group the significands and bases and exponents

$$\mathrm{W} = (2.9 \times 1.1) \times (10^7 \times 10^0)\mathrm{J}$$

Approximate 1.1 with 1 and use the product rule to multiply the second term by adding the exponents.

$$\mathrm{W} = 2.9 \times 10^7\mathrm{J}$$

which is closest to answer c) 3.2×10^7 J.

Best of luck on your upcoming MCAT!

About the Author

Chelsea Myers has enjoyed a decade-long career as a research biostatistician in the Department of Ophthalmology and Visual Sciences at the University of Wisconsin - Madison. In addition, she has taught mathematics, statistics and biostatistics to numerous aspiring doctors, nurses, physical and occupational therapists—including many non-traditional students—-at Rollins College and Valencia College in Florida. She lives with her husband and three sons in the Orlando area.

Final Reminder