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MCAT® MATH - COMMON LOGARITHMS

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Introduction

Logarithms are one of the most common math topics on the MCAT, appearing in questions on the Chemical and Physical Foundations of Biological Systems portion of the exam. Briefly, there are two kinds of logarithms, the common logarithm (seen written as "log" or "$\log_{10}$") and the natural logarithm (seen written as "ln"). While they are related topics, they have different applications on the MCAT and require different techniques to solve them without a calculator. Natural logarithms are covered in the MCAT® Math—Scientific Notation, Exponents and Approximation guide.

This guide covers common logarithms—the kind easily solved using the LOG button on your calculator—if calculators were allowed on the MCAT. Common logarithms are necessary when solving general chemistry questions about pH, pOH, pK_a and pK_b. They are also used in physics questions about sound level and sound intensity. This guide will reference properties of exponents, so it is recommended that students read the MCAT® Math—Scientific Notation, Exponents and Approximation guide before beginning this one.

Common Logarithms

Common logarithms, seen in mathematical equations as "log" or "$\log_{10}$", are used to solve equations where the variable is an exponent with base 10. For example, $10^x = 100$. The log operator allows us to solve for x and write an equivalent expression as $\log 100 = x$. $10^x = 100$ is the exponential form of the expression, and $\log 100 = x$ is the logarithmic form.

Just as subtraction is the opposite (or "inverse" in mathematical language) of addition and division is the inverse of multiplication, the common logarithm is the inverse of (that is, it "undoes") exponentiation with a base of 10. Mastery of logarithms was essential for scientists and engineers in the 19th and early 20th centuries because they could be used to simplify all kinds of complex calculations. Modern computers have made those skills obsolete, but common logarithms still show up in problems, like pH calculations, where the underlying scale changes according to powers of 10.

The log operator is most useful when describing something that can be measured on a very large scale. PH measures acidity on a scale of 0 - 14 rather than the actual scale of $[H^+]$, which ranges from 0.00000000000001M (pH = 14) to 1M (pH = 0). The following table illustrates the equivalent $[H^+]$ for each value of pH.

	$[H^+]$ M	
pH	Scientific Notation	Decimal Form
0	10^0	1
1	10^{-1}	0.1
2	10^{-2}	0.01
3	10^{-3}	0.001
4	10^{-4}	0.0001
5	10^{-5}	0.00001
6	10^{-6}	0.000001
7	10^{-7}	0.0000001
8	10^{-8}	0.00000001
9	10^{-9}	0.000000001
10	10^{-10}	0.0000000001
11	10^{-11}	0.00000000001
12	10^{-12}	0.000000000001
13	10^{-13}	0.0000000000001
14	10^{-14}	0.00000000000001

PH conveys the same information without using scientific notation or lots of 0 placeholders. It also allows us to compare the acidity of two different substances in a meaningful way. Just as it makes more sense to talk about the distance between San Francisco and New York City in terms of miles rather than inches, it makes more sense to compare the acidity of different solutions in terms of pH rather than $[H^+]$.

Mastering common logarithms on the MCAT requires the use of a few basic rules: the definition of a logarithm, properties of logarithms, and an approximation method for small logarithms. Each of these topics is covered individually followed by examples of how they are used to solve pH and sound intensity problems on the MCAT.

The Definition of a Logarithm

By definition, taking a log undoes exponentiation with a base of 10: $\log 10^x = x$. For example, $\log 1000 = 3$ because $1000 = 10^3$. The same rule applies even when the exponent is not a whole number: $\log 25 \approx 1.3979$ because $10^{1.3979} \approx 25$. Note that $\log 1 = 0$ because $10^0 = 1$.

The next section illustrates two other properties of logarithms—the product rule and power rule—which are used to simplify log expressions to the point that they can be solved without using a calculator.

The Product Rule and Power Rule for Logarithms

The product rule states that the log of a product is equal to the sum of the log of each of its terms

$$\log(MN) = \log M + \log N$$

For a value expressed in scientific notation, such as 3.5×10^3, applying this rule would yield

$$\log(3.5 \times 10^3) = \log 3.5 + \log 10^3$$

Note that the second term, $\log 10^3$, can be further simplified using the definition of a logarithm $\log 10^3 = 3$.

Because pH calculations require taking the *negative* log of a power with a *negative* exponent, solving these problems requires use of the power rule for logarithms. The power rule states that the log of a number raised to a power can be expressed as that power times the log of the number

$$\log(M^p) = p \log M$$

This rule is used to "cancel out" the negative signs before the log operator and in the exponent by recognizing that

$$-\log M^{-1} = (-1) \times -\log M = \log M$$

Example 1

What is the pH of a 2.3×10^{-2} M HNO_3 solution?

Because HNO_3 is a strong acid, $[H^+] = 2.3 \times 10^{-2}$ M.

The equation used to calculate pH from $[H^+]$ is $\text{pH} = -\log[H^+]$. Substituting in the $[H^+]$ given in the problem yields

$$\text{pH} = -\log(2.3 \times 10^{-2})$$

To solve this expression, use the product rule (correctly distributing the leading negative sign to both parts)

$$\begin{aligned}\text{pH} &= -\log 2.3 + (-\log 10^{-2}) \\ &= -\log 2.3 - \log 10^{-2}\end{aligned}$$

The second term is simplified again by using the power rule

$$\text{pH} = -\log 2.3 + 2$$

It's convenient to rewrite this expression by moving the 2 to the front

$$\text{pH} = 2 - \log 2.3$$

To solve this problem completely, it is necessary to either calculate or approximate the value of $-\log 2.3$.

Approximating Small Logarithms

One of the most important principles of mastering MCAT math is using approximation to select the correct answer from the list of possible choices rather than calculating an exact solution. To approximate the logarithm of a small number ($1 < n < 10$)

$$\log n \approx 0.n$$

An extension of this rule can be used approximate the value of 10 raised to a power that is not an integer (a whole number). For a number with base 10 and an exponent written in the form $m - 0.n$

$$m - 0.n \approx -\log(n \times 10^{-m})$$

Example 1 - Continued

Previously, the pH of the solution described in the problem had been calculated as $2 - \log 2.3$. Using the approximation for small logarithms yields

$$\begin{aligned} \text{pH} &= 2 - \log 2.3 \\ &\approx 2 - .23 \\ &= 1.77 \end{aligned}$$

To get an sense of the accuracy of this approximation, the following table shows pH calculated using the approximation and using a calculator for concentrations between 10^{-2} M and 10^{-1} M including the concentration described in the problem.

	pH	
$[HNO_3]$	Approximation	Using a Calculator
$10^{-}1$ M	1	1
8×10^{-2} M	1.2	1.09
6×10^{-2} M	1.4	1.22
4×10^{-2} M	1.6	1.39
$\mathbf{2.3 \times 10^{-2}}$ M	**1.77**	**1.64**
2×10^{-2} M	1.8	1.70
10^{-2} M	2	2

The actual value of $\log 2.3$ is about 0.3617, so the approximation is off by about one-third. There are, of course, more accurate (and more complex) approximation methods for logarithms, but they aren't required on the MCAT. Test designers expect students to use this approximation and the answer choices provided will reflect that.

Sometimes, instead of being given a concentration and asked to find the pH, the problem will direrct you to find the concentration of H^+ ions for a solution with a given pH. In that case, the same methods can be used.

Example 2

A sample of gastric juice has a pH of 3.1. What is the concentration of hydrogen ions?.

To answer this question, begin with the definition of pH

$$\mathrm{pH} = -\log[\mathrm{H}^+]$$

This time, instead of substituting in the concentration of H^+ ions, fill in the pH

$$3.1 = -\log[\mathrm{H}^+]$$

Recall that the power rule states that the negative log of 10 raised to a negative exponent is a positive number. Therefore the correct value of the exponent will be negative (indicating a small quantity between 0 and 1). To solve this equation without a calculator, we must use the extension of the approximation for small logarithms. For a number with base 10 and an exponent written in the form $m - 0.n$,

$$m - 0.n \approx -\log(n \times 10^{-m})$$

Re-expressing the exponent -3.1 in the form $-4 - (-0.9)$

$$-4 - (-0.9) \approx -\log(9 \times 10^{-4})$$

Therefore, $[\mathrm{H}^+] \approx 9 \times 10^{-4} = 0.0009$ M. The exact $[\mathrm{H}^+] = 10^{-3.1}\,\mathrm{M} = 0.000794$ M, but the approximation would be close enough to select the correct answer from the possible choices.

Now an example from physics using the positive log operator.

Example 3

What is the sound level of a noise so loud that it is painful and has an intensity of $1 \times 10^1 \frac{\mathrm{W}}{\mathrm{m}^2}$?

Humans are able to hear over a wide range of sound intensities from the threshold of hearing—$1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}$ (a number that students are expected to memorize for the MCAT)—to a sound so loud it could perforate an ear drum—$1 \times 10^4 \frac{\mathrm{W}}{\mathrm{m}^2}$.

As seen in the general chemistry application, using a log scale rather than a linear scale is helpful to describe something that is measured on a very large scale and the underlying scale changes according to powers of 10. Sound level measures the loudness of a sound on a scale of 0 - 160 dβ rather than the scale of $\frac{\mathrm{W}}{\mathrm{m}^2}$, which ranges from $0.000000000001 \frac{\mathrm{W}}{\mathrm{m}^2}$ (0 dβ) to $10000 \frac{\mathrm{W}}{\mathrm{m}^2}$ (160 dβ).

The following table showing the equivalent $\frac{\mathrm{W}}{\mathrm{m}^2}$ for each value of dβ illustrates this point.

Sound source	Sound Level dβ	Sound Intensity $\frac{\mathrm{W}}{\mathrm{m}^2}$
(Threshold of human hearing)	0	1×10^{-12}
Calm breathing	10	1×10^{-11}
Whisper	20	1×10^{-10}
Quiet room at night	30	1×10^{-9}
Refrigerator running	40	1×10^{-8}
Moderate rainfall	50	1×10^{-7}
Conversational speech	60	1×10^{-6}
Vacuum cleaner	70	1×10^{-5}
City traffic	80	1×10^{-4}
Lawn mower	90	1×10^{-3}
Jackhammer	100	1×10^{-2}
Chainsaw	110	1×10^{-1}
Thunder	120	1×10^{0}
(Threshold of pain)	130	1×10^{1}
Fireworks	140	1×10^{2}
Jet engine	150	1×10^{3}
(Eardrum perforation)	160	1×10^{4}

The equation for sound intensity is

$$\beta = 10 \times \log \frac{I}{I_0}$$

Where I is the intensity of the sound wave of interest and I_0 is the intensity of the reference sound wave. The question describes the sound wave of interest as a loud and painful noise with an intensity equal to $1 \times 10^{1} \frac{\mathrm{W}}{\mathrm{m}^2}$. I_0 is equal to $1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}$

Substituting in values for I and I_0 yields

$$\beta = 10 \times \log \frac{1 \times 10^{1} \frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}}$$

At this point there are two ways to approach simplifying the expression.

The first is to use rules of exponents and recognize that, because both the numerator and denominator have the same base, the exponent in the denominator is subtracted from the exponent in the numerator $(1 - (-12) = 1 + 12 = 13)$

$$\beta = 10 \times \log\,(1 \times 10^{13}\,\frac{\mathrm{W}}{\mathrm{m}^2})$$

Using the definition of a logarithm

$$\log\,(1 \times 10^{13}) = 13 \quad (\text{because } 1 \times 10^{13} = 10^{13})$$

And therefore

$$\begin{aligned} \beta &= 10 \times \log\,(1 \times 10^{13})\,\mathrm{d}\beta \\ &= 10 \times 13\,\mathrm{d}\beta \\ &= 130\,\mathrm{d}\beta \end{aligned}$$

Alternatively, another property of logarithms, the quotient rule, could be used to simplify the expression.

The Quotient Rule for Logarithms

The quotient rule states that the log of a quotient is equal to the log of the numerator minus the log of the denominator

$$\log\frac{M}{N} = \log M - \log N$$

For example

$$\log\frac{2}{3} = \log 2 - \log 3$$

Example 3 - Continued

The quotient rule can be used to simplify the fraction in the sound intensity example from

$$\beta = 10 \times \log\frac{1 \times 10^{1}\,\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2}}$$

to

$$\beta = 10 \times [\log(1 \times 10^{1}\,\frac{\mathrm{W}}{\mathrm{m}^2}) - \log(1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2})]$$

Then each log expression is solved using the definition of a logarithm

$$\begin{aligned}\beta &= 10 \times [1\frac{\mathrm{W}}{\mathrm{m}^2} - (-12)\frac{\mathrm{W}}{\mathrm{m}^2}]\,\mathrm{d}\beta \\ &= 10 \times [1\frac{\mathrm{W}}{\mathrm{m}^2} + 12\frac{\mathrm{W}}{\mathrm{m}^2}]\,\mathrm{d}\beta \\ &= 130\,\mathrm{d}\beta\end{aligned}$$

Both approaches yield the same solution, $130\,\mathrm{d}\beta$.

As with questions about pH, sometimes a problem will require you to work "backwards" to find the intensity of a sound wave for a given sound level.

Example 4

What is the sound intensity of a noise with a sound level measured at 60 decibels?

Begin with the sound intensity equation

$$\beta = 10 \times \log \frac{I}{I_0}$$

Substitute in 60 $\mathrm{d}\beta$ for β and $1 \times 10^{-12}\frac{\mathrm{W}}{\mathrm{m}^2}$ for I_0

$$60\,\mathrm{d}\beta = 10 \times \log \frac{I\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\frac{\mathrm{W}}{\mathrm{m}^2}}$$

Divide each side by 10 to get

$$6\,\mathrm{d}\beta = \log \frac{I\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\frac{\mathrm{W}}{\mathrm{m}^2}}$$

The variable of interest, I, is still stuck in the numerator of the fraction inside a log function. The quotient rule is used to simplify the previous expression to

$$6\,\mathrm{d}\beta = \log(I\frac{\mathrm{W}}{\mathrm{m}^2}) - \log(1 \times 10^{-12}\frac{\mathrm{W}}{\mathrm{m}^2})$$

Using the definition of a logarithm and the power rule, further simplify the equation to

$$6\,\mathrm{d}\beta = \log(I\frac{\mathrm{W}}{\mathrm{m}^2}) + 12$$

Subtract 12 from both sides and rearrange terms

$$\log(I\frac{\mathrm{W}}{\mathrm{m}^2}) = -6$$

Using the definition of a logarithm, the expression is rearranged to be in exponential form $I = 1 \times 10^{-6}\frac{\mathrm{W}}{\mathrm{m}^2}$.

Additional Examples

E1: The K_a of an acid whose buffer has a pH of 5.22 in a solution containing equal M of acid and conjugate base is closest to:

a) 9.23×10^{-8}
b) 6.02×10^{-6}
c) 5.22×10^{-5}
d) 1.49×10^{-3}

Begin by using the Henderson-Hasselbalch equation

$$\mathrm{pH} = \mathrm{pK}_a + \log \frac{[\mathrm{A}^-]}{[\mathrm{HA}]}$$

where $[\mathrm{A}^-]$ is the concentration of the conjugate base and [HA] is the concentration of the acid.

Although the problem doesn't specify what $[\mathrm{A}^-]$ and [HA] are, it does say that that they are the same, and any quantity divided by an equal quantity $= 1$. From the definition of a logarithm, $\log 1 = 0$, which eliminates the portion of the equation dealing with the ratio of the conjugate base to the acid.

$$\begin{aligned} \mathrm{pH} &= \mathrm{pK}_a + \log 1 \\ &= \mathrm{pK}_a + 0 \\ &= \mathrm{pK}_a \end{aligned}$$

Substituting the value of pH given in the problem yields the value of pK_a

$$5.22 = \mathrm{pK}_a$$

The question, however, asks for the K_a, not the pK_a, which requires solving

$$\begin{aligned} \mathrm{pK}_a &= -\log \mathrm{K}_a \\ 5.22 &= -\log \mathrm{K}_a \end{aligned}$$

Approximate the value of the quantity from its logarithm using the extension of the approximation rule

$$\begin{aligned} -\log(n \times 10^{-m}) &\approx m - 0.n \\ -\log(7.8 \times 10^{-6}) &\approx 6 - 0.78 \quad (\text{because } 5.22 = 6 - .78) \end{aligned}$$

Therefore $\mathrm{K}_a \approx 7.8 \times 10^{-6}$, which is closest to the correct answer b) 6.03×10^{-6}.

E2 A detector with a surface area of 1 square meter is placed 1 meter from a piece of construction equipment and measures the sound as 10^{-2} W. What is the sound intensity of the piece of construction equipment?

a) -12 dβ
b) 10 dβ
c) 12 dβ
d) 100 dβ

Substituting $1 \times 10^{-2} \frac{\text{W}}{\text{m}^2}$ for I and $1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}$ for I_0 into the sound intensity equation yields

$$\beta = 10 \times \log \frac{1 \times 10^{-2} \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}$$

Apply the quotient rule

$$\beta = 10 \times [\log(10^{-2} \frac{\text{W}}{\text{m}^2}) - \log(1 \times 10^{-12} \frac{\text{W}}{\text{m}^2})]$$

And the definition of a logarithm

$$\beta = 10 \times [-2 \frac{\text{W}}{\text{m}^2} - (-12) \frac{\text{W}}{\text{m}^2}]$$

Rearrange terms to get

$$\begin{aligned}\beta &= 10 \times [-2 \frac{\text{W}}{\text{m}^2} + 12 \frac{\text{W}}{\text{m}^2}] \, \text{d}\beta \\ &= 10 * 10 \, \text{d}\beta \\ &= 100 \, \text{d}\beta\end{aligned}$$

Making the correct answer d) $100 \, \text{d}\beta$

E3: Which of the following correctly designates a true statement about pH?

I. pH = $\log(\frac{1}{[H^+]})$
II. pH = $\log(\frac{[OH^-]}{[K_w]})$
III. $\frac{pK_w}{pOH}$

a) I only
b) I and II only
c) I, II and III
d) III only

Although no actual numbers are given in this question, the rules of logarithms are necessary to determine which statements are correct.

Starting with I, by definition

$$pH = -\log[H^+]$$

Use the power rule to move the negative sign from out in front of the log operator

$$pH = \log([H^+]^{-1})$$

Using the rules of exponents

$$pH = \log(\frac{1}{[H^+]})$$

Therefore, I is a correct statement.

To evaluate statement II, begin with the equation that relates K_w, $[H^+]$ and $[OH^-]$

$$K_w = [H^+][OH^-]$$

Rearrange the terms to express the equation in terms of $[H^+]$

$$[H^+] = \frac{K_w}{[OH^-]}$$

To calculate pH, take the $-\log$ of both sides of the expression

$$-\log[H^+] = -\log(\frac{K_w}{[OH^-]})$$

$$pH = -\log(\frac{K_w}{[OH^-]})$$

Then use the power rule to move the negative sign inside the log operator

$$\mathrm{pH} = \log((\frac{\mathrm{K}_w}{[\mathrm{OH}^-]})^{-1})$$

Distribute the negative sign

$$\mathrm{pH} = \log((\frac{\mathrm{K}_w^{-1}}{[\mathrm{OH}^-]^{-1}}))$$

Finally, use the rules of exponents to rearrange the terms inside the log expression

$$\mathrm{pH} = \log(\frac{[\mathrm{OH}^-]}{\mathrm{K}_w})$$

So statement II is also correct.

Finally, statement III. Begin with the same relationship between K_w, $[\mathrm{H}^+]$ and $[\mathrm{OH}^-]$

$$[H+] = \frac{\mathrm{K}_w}{[\mathrm{OH}^-]}$$

Taking the $-\log$ of both sides

$$-\log[\mathrm{H}^+] = -\log(\frac{\mathrm{K}_w}{[\mathrm{OH}^-]})$$

$$\mathrm{pH} = -\log(\frac{\mathrm{K}_w}{[\mathrm{OH}^-]})$$

However,

$$\mathrm{pH} = -\log(\frac{\mathrm{K}_w}{[\mathrm{OH}^-]}) \neq \frac{-\log \mathrm{K}_w}{-\log[\mathrm{OH}^-]} = \frac{\mathrm{pK}_w}{\mathrm{pOH}}$$

The correct equality is

$$\mathrm{pH} = -\log(\mathrm{K}_w) - (-\log[\mathrm{OH}^-])$$

or equivalently

$$\mathrm{pH} = \mathrm{pOH} - \mathrm{pK}_w$$

Therefore, answer b) I and II only is correct.

See the Common Errors section at the end of this guide for other common arithmetic mistakes using the log function.

E4: The K_a for H_3PO_4 is 7.6×10^{-3} at 25°C. What is the approximate pK_b for $H_2PO_4^-$ at 25°C?

a) 2.25
b) 7.51
c) 11.88
d) 14.00

At 25° C, for an acid and its conjugate base

$$pK_w = pK_a + pK_b = 14$$

Substitute in the K_a value from the problem

$$\begin{aligned} pK_a &= -\log(K_a) \\ &= -\log(7.6 \times 10^{-3}) \end{aligned}$$

Use the product rule and power rule and rearrange terms

$$\begin{aligned} pK_a &= -\log(7.6) + (-\log 10^{-3}) \\ &= 3 - \log(7.6) \end{aligned}$$

Finally, approximate the value of the logarithm using $\log(7.6) \approx 0.76$

$$\begin{aligned} pK_a &= 3 - .76 \\ &= 2.24 \end{aligned}$$

Using the first identity, substitute in the value of pK_a and solve for pK_b

$$2.24 + pK_b = 14$$

$$pK_b = 11.76$$

making the answer b) 11.88.

E5: A musician owns a keyboard with a quietest setting of 2 dβ and a loudest setting of 32 dβ. How many keyboards playing at their softest setting would it take to equal the sound intensity of one keyboard playing as loud as possible?

a) 10
b) 30
c) 100
d) 1000

Use the sound level equation to find the sound intensity for both the lowest and highest settings on the keyboard from their respective sound level.

Solving for the sound intensity at the softest setting yields

$$2\,\mathrm{d}\beta = 10 \times \log \frac{I\,\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2}}$$
$$0.2\,\mathrm{d}\beta = \log \frac{I\,\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2}}$$

Use the quotient rule

$$0.2\,\mathrm{d}\beta = \log I + 12\frac{\mathrm{W}}{\mathrm{m}^2}$$

Then subtract $12\frac{\mathrm{W}}{\mathrm{m}^2}$ from both sides to isolate $\log I$ on one side of the equation

$$-11.8\,\frac{\mathrm{W}}{\mathrm{m}^2} = \log I$$

Use the definition of a logarithm to express the equation in exponential form as

$$I = 1 \times 10^{-11.8}\,\frac{\mathrm{W}}{\mathrm{m}^2}$$

Solving for the sound intensity at the loudest setting

$$32\,\mathrm{d}\beta = 10 \times \log \frac{I\,\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2}}$$
$$3.2\,\mathrm{d}\beta = \log \frac{I\,\frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12}\,\frac{\mathrm{W}}{\mathrm{m}^2}}$$

Use the quotient rule and then subtract $12\frac{\text{W}}{\text{m}^2}$ from both sides to isolate $\log I$ on one side of the equation

$$
\begin{aligned}
3.2\,\mathrm{d}\beta &= \log I + 12\frac{\text{W}}{\text{m}^2} \\
-8.8\frac{\text{W}}{\text{m}^2} &= \log I
\end{aligned}
$$

Use the definition of a logarithm to express the equation in exponential form as

$$I = 1 \times 10^{-8.8}\frac{\text{W}}{\text{m}^2}$$

To determine how many keyboards playing at the softest setting it would take to equal the sound intensity of one playing at its loudest setting, take the ratio of the two sound intensities and use the rules of exponents to calculate

$$
\begin{aligned}
\frac{1 \times 10^{-8.8}\frac{\text{W}}{\text{m}^2}}{1 \times 10^{-11.8}\frac{\text{W}}{\text{m}^2}} &= 10^{(11.8-8.8)} \\
&= 10^3 \\
&= 1000
\end{aligned}
$$

which is answer d).

Practice Problems

Q1: The pH of a solution with $[HClO_4]$ = 0.008 M is approximately equal to:

a) 2
b) 3
c) 4
d) 8

Q2: The sound of a door slamming is measured at 80 dβ. What is the intensity of the sound of the door slam?

a) $1 \times 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m}^2}$
b) $1 \times 10^{-4}\,\frac{\mathrm{W}}{\mathrm{m}^2}$
c) $1 \times 10^{4}\,\frac{\mathrm{W}}{\mathrm{m}^2}$
d) $1 \times 10^{8}\,\frac{\mathrm{W}}{\mathrm{m}^2}$

Q3: A scientist wishes to set up a buffer at a pH of 9.4. Of the following acids, the scientist should use:

a) hydrofluoric acid ($K_a = 7.2 \times 10^{-4}$)
b) benzoic acid ($K_a = 6.3 \times 10^{-5}$)
c) acetic acid ($K_a = 1.8 \times 10^{-5}$)
d) hydrocyanic acid ($K_a = 6.2 \times 10^{-10}$)

Q4: The pH of blood is about 7.4. What is the $[OH^-]$?

a) 1.63×10^{-10}
b) 9.01×10^{-9}
c) 2.51×10^{-7}
d) 7.33×10^{-5}

Q5: What is the sound level in decibels of a sound with intensity $1 \times 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m}^2}$?

a) $-20\,\mathrm{d}\beta$
b) $-4\,\mathrm{d}\beta$
c) $4\,\mathrm{d}\beta$
d) $40\,\mathrm{d}\beta$

Practice Problem Solutions

Q1: The pH of a solution with [$HClO_4$] = 0.008 M is approximately equal to:

a) 2
b) 3
c) 4
d) 8

A1: Percholoric acid is a strong acid that will fully dissociate in solution. Therefore

$$\mathrm{pH} = -\log[\mathrm{H}^+]$$

Before doing any calculations or approximation, it is useful to express [$HClO_4$] in scientific notation

$$0.008 = 8 \times 10^{-3}$$

Beginning with the pH equation, and substituting in the [H^+] given in the problem

$$\begin{aligned}\mathrm{pH} &= -\log[\mathrm{H}^+] \\ &= -\log(8 \times 10^{-3})\end{aligned}$$

Use the product rule

$$\mathrm{pH} = -\log 8 + (-\log 10^{-3})$$

Then the power rule and definition of a logarithm and rearrange terms

$$\mathrm{pH} = 3 - \log 8$$

Approximate the value of $\log 8$ with 0.8

$$\mathrm{pH} \approx 3 - 0.8 \approx 2.2$$

So the correct answer is a) 2.

Q2: The sound of a door slamming is measured at 80 dβ. What is the intensity of the sound of the door slam?

a) $1 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2}$
b) $1 \times 10^{-4} \frac{\mathrm{W}}{\mathrm{m}^2}$
c) $1 \times 10^{4} \frac{\mathrm{W}}{\mathrm{m}^2}$
d) $1 \times 10^{8} \frac{\mathrm{W}}{\mathrm{m}^2}$

A2: Begin with the sound intensity equation and substitute in 80 dβ for β and $1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}$ for I_0

$$\beta = 10 \times \log \frac{I}{I_0}$$

$$80\,\mathrm{d}\beta = 10 \times \log \frac{I \frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}}$$

Divide each side by 10

$$8\,\mathrm{d}\beta = \log \frac{I \frac{\mathrm{W}}{\mathrm{m}^2}}{1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2}}$$

Use the quotient rule

$$8\,\mathrm{d}\beta = \log(I \frac{\mathrm{W}}{\mathrm{m}^2}) - \log(1 \times 10^{-12} \frac{\mathrm{W}}{\mathrm{m}^2})$$

Use the definition of a logarithm and the power rule

$$8\,\mathrm{d}\beta = \log(I \frac{\mathrm{W}}{\mathrm{m}^2}) + 12$$

Subtract 12 from both sides and rearrange terms

$$\log(I \frac{\mathrm{W}}{\mathrm{m}^2}) = -4$$

Use the definition of a logarithm to express the equation in exponential form as $I = 1 \times 10^{-4} \frac{\mathrm{W}}{\mathrm{m}^2}$, which is answer choice b).

Q3: A scientist wishes to set up a buffer at a pH of 9.4. Of the following acids, the scientist should use:

a) hydrofluoric acid ($K_a = 7.2 \times 10^{-4}$)
b) benzoic acid ($K_a = 6.3 \times 10^{-5}$)
c) acetic acid ($K_a = 1.8 \times 10^{-5}$)
d) hydrocyanic acid ($K_a = 6.2 \times 10^{-10}$)

A3: When producing a buffer, the acid chosen should have a pK_a that is as close as possible to the desired pH. Use the product rule and power rule as well as the approximation for small logarithms to find the pK_a for each acid to determine which one is closest to the desired pH.

For hydrofluoric acid,

$$\begin{aligned} K_a &= 7.2 \times 10^{-4} \\ pK_a &= -\log(7.2 \times 10^{-4}) \\ &= -\log(7.2) - \log(10^{-4}) \\ &= -\log(7.2) - (-4) \\ &= 4 - \log(7.2) \\ &= 4 - 0.72 \\ &= 3.28 \end{aligned}$$

For benzoic acid,

$$\begin{aligned} K_a &= 6.3 \times 10^{-5} \\ pK_a &= -\log(6.3 \times 10^{-5}) \\ &= -\log(6.3) - \log(10^{-5}) \\ &= -\log(6.3) - (-5) \\ &= 5 - \log(6.3) \\ &= 5 - 0.63 \\ &= 4.37 \end{aligned}$$

For acetic acid,

$$\begin{aligned} K_a &= 1.8 \times 10^{-5} \\ pK_a &= -\log(1.8 \times 10^{-5}) \\ &= -\log(1.8) - \log(10^{-5}) \\ &= -\log(1.8) - (-5) \\ &= 5 - \log(1.8) \\ &= 5 - 0.18 \\ &= 4.82 \end{aligned}$$

And finally, for hydrocyanic acid,

$$\begin{aligned}
K_a &= 6.2 \times 10^{-10} \\
pK_a &= -\log(6.2 \times 10^{-10}) \\
&= -\log(6.2) - \log(10^{-10}) \\
&= -\log(6.2) - (-10) \\
&= 10 - \log(6.2) \\
&= 10 - 0.62 \\
&= 9.38
\end{aligned}$$

The acid with a pK_a closest to the desired pH is d) hydrocyanic acid ($K_a = 6.2 \times 10^{-10}$).

Q4: The pH of blood is about 7.4. What is the $[OH^-]$?

a) 1.63×10^{-10}
b) 9.01×10^{-9}
c) 2.51×10^{-7}
d) 7.33×10^{-5}

A4: Beginning with the relationship between pH and pOH

$$pH + pOH = 14$$

Substitute in 7.4 for the pH and solve for pOH

$$\begin{gathered}
7.4 + pOH = 14 \\
pOH = 6.6
\end{gathered}$$

Use the extension of the approximation for small logarithms—$m - 0.n \approx -\log(n \times 10^{-m})$—to find the $[OH^-]$.

$$\begin{gathered}
pOH = -\log[OH^-] \\
6.6 = -\log[OH^-] \\
7 - 0.4 \approx -\log(4 \times 10^{-7})
\end{gathered}$$

Therefore $[OH^-] \approx 4 \times 10^{-7}$, which is closest to answer c) 2.51×10^{-7}.

Q5: Given the threshold of human hearing is $1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}$, what is the sound level in decibels of a sound with intensity $1 \times 10^{-8} \frac{\text{W}}{\text{m}^2}$?

a) $-20\,\text{d}\beta$
b) $-4\,\text{d}\beta$
c) $4\,\text{d}\beta$
d) $40\,\text{d}\beta$

A5: Use the sound intensity equation

$$\text{d}\beta = 10 \times \log \frac{I \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}$$

And substitute in for $I = 1 \times 10^{-8} \frac{\text{W}}{\text{m}^2}$

$$\text{d}\beta = 10 \times \log \frac{1 \times 10^{-8} \frac{\text{W}}{\text{m}^2}}{1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}}$$

Use the quotient rule for exponents. Note that $-8 - (-12) = 4$

$$\text{d}\beta = 10 \times \log(1 \times 10^4 \frac{\text{W}}{\text{m}^2})$$

Apply the definition of a logarithm

$$\text{d}\beta = 10 \times 4 = 40\,\text{d}\beta$$

Which is answer d).

Proofs of Selected Properties of Logarithms

Proof of the Product Rule

$$\log MN = \log M + \log N$$

Proof:

Let $x = \log M$ and $y = \log N$. Converting to exponential equations, $10^x = M$ and $10^y = N$, yields

$$MN = 10^x \times 10^y = 10^{(x+y)}$$

Take the log of both sides

$$\begin{aligned} \log MN &= \log 10^{(x+y)} \\ &= x + y \end{aligned}$$

Recalling what x and y represent

$$\begin{aligned} \log MN &= x + y \\ &= \log M + \log N \end{aligned}$$

Proof of the Power Rule

$$\log M^p = p \times \log M$$

Proof:

Let $x = \log M$. The equivalent exponential expression is $10^x = M$. Raise both sides to the power p

$$\begin{aligned} (10^x)^p &= M^p \\ 10^{xp} &= M^p \end{aligned}$$

Take the log of both sides

$$xp = \log M^p$$

Using the original identity $x = \log M$

$$\log M \times p = \log M^p$$

And rearranging terms

$$p \times \log M = \log M^p$$

Proof of the Quotient Rule

$$\log \frac{M}{N} = \log M - \log N$$

Proof:

Let $x = \log M$ and $y = \log N$. Expressed in exponential form, $10^x = M$ and $10^y = N$. Therefore,

$$MN = 10^x \times 10^y = 10^{(x+y)}$$

Take the log of both sides and applying the product and power rules

$$\begin{aligned} \log \frac{M}{N} &= \log(M \times N^{-1}) \\ &= \log M + \log N^{-1} \text{ (by the Product Rule)} \\ &= \log M + (-1) \times \log N \text{ (by the Power Rule)} \\ &= \log M - \log N \end{aligned}$$

Common Errors

$\log MN \neq (\log M) \times (\log N)$ The logarithm of a product is *not* the product of the logarithms.

$\log(M + N) \neq (\log M) + (\log N)$ The logarithm of a sum is *not* the sum of the logarithms.

$\log(\frac{M}{N}) \neq \frac{(\log M)}{(\log N)}$ The logarithm of a quotient is *not* the quotient of the logarithms.

$(\log M)^p \neq p \times (\log M)$ The power of a logarithm is *not* the exponent times the logarithm.

Approximation for Small Logarithms

To approximate the logarithm of a small number ($1 < n < 10$), $\log n \approx 0.n$.

Values of log(n) calculated using the approximation and using a calculator for $1 < n < 10$ are shown in the table below. Remember that the authors of the MCAT expect students to use this approximation, and the results it provides will be good enough to select the correct answer from among the choices.

	Calculation of log(n)	
n	$\approx 0.n$	Using a calculator
1	0.1	0.100
2	0.2	0.301
3	0.3	0.477
4	0.4	0.602
5	0.5	0.699
6	0.6	0.778
7	0.7	0.845
8	0.8	0.903
9	0.9	0.954
10	1.0	1.000

Best of luck on your upcoming MCAT!

About the Author

Chelsea Myers has enjoyed a decade-long career as a research biostatistician in the Department of Ophthalmology and Visual Sciences at the University of Wisconsin - Madison. In addition, she has taught mathematics, statistics and biostatistics to numerous aspiring doctors, nurses, physical and occupational therapists—including many non-traditional students—-at Rollins College and Valencia College in Florida. She lives with her husband and three sons in the Orlando area.

Final Reminder