Cpt S 450 Homework 1

1. $\{\langle n\rangle\#\langle p\rangle\#\langle q\rangle : n = p\cdot q$, where p, q are primes $\}$, where $\langle n\rangle$ is the binary representation of number n, and $\#$ is the delimiter.

$\{\langle n\rangle : n = p\cdot q$, for some p, q being primes $\}$

$\{\langle A\rangle\#w : A$ accepts $w\}$, where $\langle A\rangle$ is the string encoding of an NFA A (NFA is a program, and a program is a string, right?)

$\{\langle A\rangle : A$ accepts w for some $w\}$

2. (1). Notice that $2n^3 - 18n \leq cn^3$ with c=2 for almost all n. Hence, it is $O(n^3)$. One can also argue that $2n^3 - 18n \leq cn^4$ with $c = 1$ for almost all n and hence, it is also $O(n^4)$. To show that it is not $O(n^2 \log n)$, we argue that one can not find a $c > 0$ such that $2n^3 - 18n \leq cn^2 \log n$ for almost all n. Otherwise, $\frac{2n^3-18n}{n^2\log n} \leq c$ for infinitely many n. this is not possible, since, as $n \to \infty$, the LHS goes to ∞.
2. (2). We need only establish $3n^2 2^{2n} \leq 2cn$ for some $c > 0$ and for almost all n. Take $c = 3$, the result follows since

$$\lim_{n\to\infty} \frac{3n^2 2^{2n}}{2^{3n}} = \lim_{n\to\infty} \frac{3n^2}{2^n} = 0.$$

(The last limit is obtained using l'Hôpital's rule twice)