TASK

Introduction to Computer Science Fundamentals and Big O Notation

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Introduction

Welcome to the Introduction to Computer Science Fundamentals and Big O Notation Task!

In this task, you will be in induced Big O notation, which is used to describe the performance or complexity of an algorithm.

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What is Computer Science?

As the name suggests, computer science is a science that was born through the invention of computers. Computer scientists are mostly concerned with software and software systems. This includes their theory, design, development, and application.

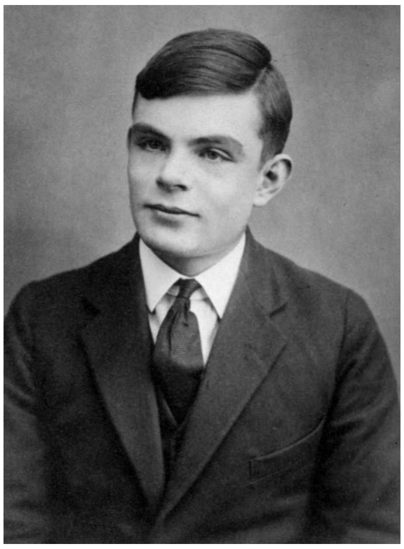
Although it is important to know how to program, programming is only a small element of computer science. A computer scientist should understand the "why" behind computer programs. Computer scientists design and analyse algorithms to invent new ways to solve problems and study the performance of computer hardware and software.

Computer science can be seen on a higher level, as a science of problem solving and computational thinking. Computer scientists must be adept at modeling and analysing problems. They must also be able to design solutions and verify that they are correct.

Computer science can be used in various other disciplines like engineering, healthcare, and business. Finding solutions to problems these disciplines face, requires both computer science expertise and knowledge of that particular industry. Thus, computer scientists often become proficient in many other fields.

The Father of Modern-Day Computing

Alan Turing (1912 – 1954) was a British mathematician, logician, and cryptographer. He is considered by many to be the father of modern computer science. He contributed to the design of some of the earliest electronic, programmable, digital computers.


Alan Turing in 1927, age 16 (turingarchive.org)

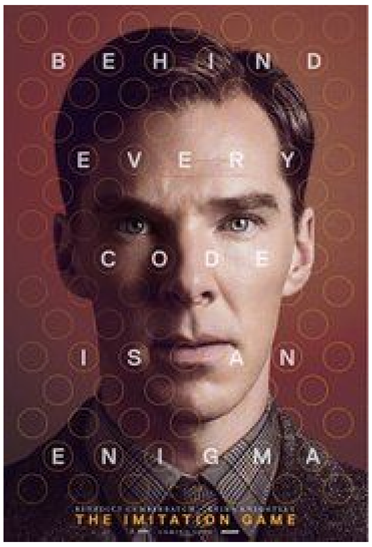
During the Second World War, Turing was recruited by the military to head a classified mission at Bletchley Park. This mission was to crack the Nazi's Enigma machine code which was used to send secret military messages. Many historians believe that breaking the Enigma code was key to bringing an end to the war in Europe.

In 1936, Turing published a paper called "On Computable Numbers" that is now recognised as the foundation of computer science. He proved in this paper that there cannot exist any universal algorithmic method of determining truth in mathematics and that mathematics will always contain undecidable propositions. The "Universal Turing Machine" was also introduced in this paper. This machine is capable of computing anything that is computable. The central concept of the modern computer was based on Turing's paper.

Alan Turing is famous for being a homosexual in a time when it was illegal. This lead to him being convicted of acts of gross indecency in 1952. He was given a choice between prison or a course of hormone therapy. He chose the hormone therapy, which eventually rendered him impotent.

Turing died on June 7, 1954. The postmortem exam concluded that the cause of death was asphyxia due to cyanide poisoning and his death was ruled a suicide. However, many people suggest foul play and that he was actually assassinated due to his sexuality.

British Prime Minister Gordon Brown made an official public apology in 2009 on behalf of the British government. Queen Elizabeth II granted him a posthumous pardon in 2013.



The Imitation Game (imdb.com)

In 2015 Alan Turing's life was brought to the big screen in the movie The Imitation Game, starring Benedict Cumberbatch and Keira Knightley.

History of Computer Science

Before 1900

- People have been using mechanical devices to aid calculation for thousands of years. For example, the abacus existed in Babylonia (present-day Iraq) about 3000 B.C.E.
- The ancient Greeks developed a device now called the Antikythera mechanism, used for predicting the motions of the stars and planets.

1940's

- John von Neumann proposed that programs and data should be stored in memory. He began work on EDVAC, the first computer based on this idea.

1950's

- Jack Kilby and Robert Noyce invented the integrated circuit in 1959. The invention of integrated circuits (transistors, wiring and other components on a single chip) reduced the cost and size of computers even further.

1960's

- New programming languages were invented, such as BASIC
- Ted Hoff and Federico Faggin from Intel designed the first microprocessor

1970's

- Unix, a very influential operating system, was developed at Bell Laboratories

- Other new programming languages, such as C, C++, Pascal, and Ada were developed.

1980's

- This decade also saw the rise of the personal computer, thanks to Steve Wozniak and Steve Jobs, founders of Apple Computer.
- In 1981, the first truly successful portable computer was marketed, the Osborne I.
- In 1984, Apple first marketed the Macintosh computer.

1990's and Beyond

- The age of modern day computers.
- We witnessed the appearance of laptops, iPads, improvements in secondary storage media and the use of multimedia, and the phenomenon of virtual reality.

Algorithms

An algorithm is a precise and unambiguous specification of a sequence of steps that can be carried out mechanically. One of the characteristics of algorithms is that they do not require any intelligence to carry out; they are mechanical processes in which each step follows from the last according to a simple set of rules. In other words, an algorithm is like a recipe. It is a step by step method for solving a problem. You have probably already encountered many non-technical algorithms in your everyday life, such as a recipe to bake a cake or instructions on how to assemble a piece of furniture.

Algorithms can be written in any commonly understood language, but are most often expressed as programs in a programming language or as pseudocode in computer science. Ideally, they should be easy to understand and easy to turn into a working program.

Big O Notation

In order to start taking advantage of algorithms, you need to know how fast they are so you can choose the most efficient one. In computer science, Big O notation is used to describe the performance or complexity of an algorithm. Specifically, Big O describes the worst-case scenario and can be used to describe the execution time required or the space used by an algorithm.

Big O doesn't use measurements of time, such as seconds or minutes, to determine how fast an algorithm is. It instead uses the number of operations, and how these operations grow over time.

In order to use Big O notation, you will need to know some mathematics. Most notably the concept of logarithms. There is no need to panic if you are uncomfortable with mathematics however. You don't need to know that much about it. We will explain the basics of logarithms next.

Logarithms

A logarithm is another way of thinking about exponents. For example, 2 raised to the 3rd power (23) is 8. We can express this as 23 = 8. Now, suppose I asked, "2 raised to which power equals 8?" The answer would obviously be 3. We can express this by the logometric equation $\log_2(8) = 3$ which is read as "log base two of eight is three".

The general definition of a logarithm is as follows:

$$\mathrm{Log}_n(x) = y$$

Where:

- n is the base
- y is the exponent
- x is the argument

For example, let's look at the following Logarithm:

$$\mathrm{Log}_5(100)$$

This expression is asking "how many times do I need to multiply 5 by itself in order to reach the number 100?"
Well, we know that 5*5*5 = 125 (53=125) so we need to raise 5 to the power of 3 in order get to a number that is equal or greater than 100. The result of this logarithm will therefore be 3.

$$\mathrm{Log}_5(100) = 3$$

As you probably noticed, the calculation was not exactly equal; we went past 100 and ended up at 125. We think of 3 as worst case number of steps involved when calculating that particular item. In this case, the number 100 represents the number of items we have to execute our algorithm against. Therefore, if you were using an algorithm that runs over a collection of items, the length of the items would be 100.

Back to Big O Notation

As previously stated, Big O notation tells you how fast an algorithm is. It expresses the runtime of the algorithm, not in terms of a measurement of time, but how quickly it grows relative to the input, as the input gets larger.
To break that down:

- **how quickly the runtime grows** - The runtime depends on many things, such as the speed of the processor and other programs the computer is running, so we use big O notation to talk about how quickly the runtime grows and do not measure the actual runtime.
- **relative to the input** - We represent our speed in terms of the size of the input, which we call "n”. We can therefore say that the runtime grows "depending on the size of the input" (O(n)) or "depending on the square of the size of the input" ($O(n^2)$).
- **as the input gets larger** - Our algorithm may have steps that seem expensive when n is small but as n gets larger these steps become insignificant. For big O analysis, we care mostly about the steps that grow fastest as the input grows, because everything else quickly becomes insignificant as n gets very large

As the name suggests, Big O notation is written with a big “O”. The “O” is then followed by some function of n between brackets. Let's now try to understand how big-O notation works using some examples. (Remember that n represents the size of the input.)

O(1)

O(1) describes an algorithm that will always execute in the same time, regardless how large or small the input is. For example:

```java
 public static void printItem(int[] arrayOfItems) {
    System.out.println(arrayOfItems[0]);
}
```

In the example above we can see that it doesn’t matter whether the input array has 1 item or 1000 items, the method would still just require one step.

O(n)

An algorithm whose performance grows linearly and is directly proportional to the size of the input is described by O(n). For example:

```
public static void printItems(int[] arrayOfItems) {
    for (int item : arrayOfItems) {
        System.out.println(item);
    }
}
```

In the example above, if the array has 10 items, we have to print 10 times whereas if it has 1,000 items, we have to print 1,000 times.

O(n^2)

An algorithm whose performance is directly proportional to the square of the size of the input is represented by O(n^2). This is common with algorithms that involve nested loops. For example:

```
public static void printOrderedPairs(int[] arrayOfItems) {
    for (int firstItem : arrayOfItems) {
        for (int secondItem : arrayOfItems) {
            System.out.println(firstItem + ", " + secondItem);
        }
    }
}
```

In the example above we have two nested loops. If the array contains n items, the outer loop runs n times and the inner loop runs n times for each iteration of the outer loop. This gives us n^2 total print statements. If the array has 10 items, for example, we have to print 100 times.

Note: You might not recognise the syntax of the for loop in the code above. This for loop is known as an enhanced for loop. We will be using this notation for the rest of this task so you should become familiar with it. You can read more about them **here**.

O(log n)

Binary search is a technique used to search a sorted set of data. With a binary search, the middle element, or median, of the data set is selected and compared against the value you are searching for. If the value you are searching for is the

same as the middle element the search will return a success. If the value you are searching for is higher than the middle element it will take the upper half of the data set and perform the same operation against it. If the value you are searching for is lower than the middle element it will take the lower half of the data set. The binary search will continue to halve the data set until the value has been found or until data set can no longer be halved.
This algorithm is described by O(log n). The iterative halving of data sets in a binary search produces a growth curve that peaks at the beginning and slowly flattens out as the size of the data sets increases. Doubling the size of the input does not have a large effect on its growth as after a single iteration of the algorithm the data set will be halved and therefore equal to an input data set half the size. This algorithm is therefore very efficient when dealing with large inputs.

n Can be the Actual Input or the Size of the Input

Look at the methods below:

```
 public static void sayHi(int n) {
    for (int j = 0; j < n; j++) {
        System.out.println("hi");
    }
}

public static void printItemsInArray(int[] theArray) {
    for (int item : theArray) {
        System.out.println(item);
    }
}
```

The runtime for both of these methods is O(n), even though one takes an integer as its input and the other takes an array. Sometimes, n is an actual number that's an input to our method, and other times other is the number of items in an input object.

Dropping Constants

When you're calculating the Big O of something, you can just drop the constants. For example:

```
public static void printAllItemsTwice(int[] theArray) {
    for (int item : theArray) {
        System.out.println(item);
    }
```

```java
    for (int item : theArray) {
        System.out.println(item);
    }
}
```

The above example is O(n + n) which simplifies to O(2n), which we can in turn just call O(n).

Another example:

```java
// This method prints the first item in the array, then prints out hi 100 times
public static void printHi(int[] myArray) {

    System.out.println(myArray[0]);

    int middleIndex = myArray.length / 2;
    int index = 0;

    while (index < middleIndex) {
        System.out.println(myArray[index]);
        index++;
    }

    for (int i = 0; i < 100; i++) {
        System.out.println("hi");
    }
}
```

The method above is O(1+n/2+100), which we can further simplify to O(n).

You might be asking, “How can we just drop these constants?”. However, remember that we are looking at what happens as n gets arbitrarily large. As n gets infinitely big, adding 100 or dividing by 2 has little overall effect.

Dropping Less Significant Terms

You can also drop the less significant terms. For example:

```java
 // This method prints out all numbers and then prints out all the sums of pairs

public static void printNumbersAndPairSums(int[] numArray) {

    System.out.println("these are all the numbers:");
    for (int number : numArray) {
        System.out.println(number);
    }

    System.out.println("and these are all their sums:");
    for (int firstNumber : numArray) {
        for (int secondNumber : numArray) {
            System.out.println(firstNumber + secondNumber);
        }
    }
}
```

The runtime for the above example is O(n + n^2), however, we can just call it O(n2). You can simply drop all terms with a lower exponent and only keep the term with the highest exponent.

Other examples are:

- O(n^2/2 + 100n) is O(n^2)
- O(n^3 + 76n2 + 7282) is O(n^3)
- O((n + 100) ∗ (n + 10)) is O(n^2)

Remember that we can do this because the less significant terms have an insignificant effect as n gets larger.

Worst-Case Scenario

Big O describes the worst-case scenario and sometimes the worst case runtime is much worse than the best case runtime. For example:

```
 public static boolean contains(int[] haystack, int needle) {

    // determines whether the haystack contain the needle
    for (int n : haystack) {
        if (n == needle) {
            return true;
        }
    }

    return false;
}
```

Just say that there are 100 items in the haystack. The needle might be the first item in the haystack in which case the method should return true in just a single iteration. However, the runtime for the example above is O(n).

Formal Definition of Big O

Now, let us develop a formal notation of Big O

Consider the following functions:

- $g(n) = n^2$
- f(n) = n2 + 4n + 20

As you can see the function g does not contain a linear term. In other words, it doesn't contain a term with n. Now look at the table below:

n	$g(n) = n^2$	$f(n) = n^2 + 4n + 20$
10	100	160
50	2500	2720
100	10 000	10 420
1000	1 000 000	1 004 020
10000	100 000 000	100 040 020

As you can see, as n becomes larger, the term 4n + 20 in f(n) becomes insignificant, while the term n2 becomes the dominant term. Therefore, we can predict the behaviour of f(n) for large values of n by looking at the behaviour of g(n). If the complexity of a function can be described by the complexity of a quadratic function without the linear term, we say that the function is of $O(n^2)$.

Now, let f and g be real-valued functions. Assume that f and g are nonnegative for all real numbers n, f(n) >= 0 and g(n) >= 0.

Definition: We say that f(n) is Big O of g(n), written f(n) = O(g(n)), if there exists positive constants c and n0 such that f(n) <= cg(n) for all n >= n0.

In general the following theorem exists:

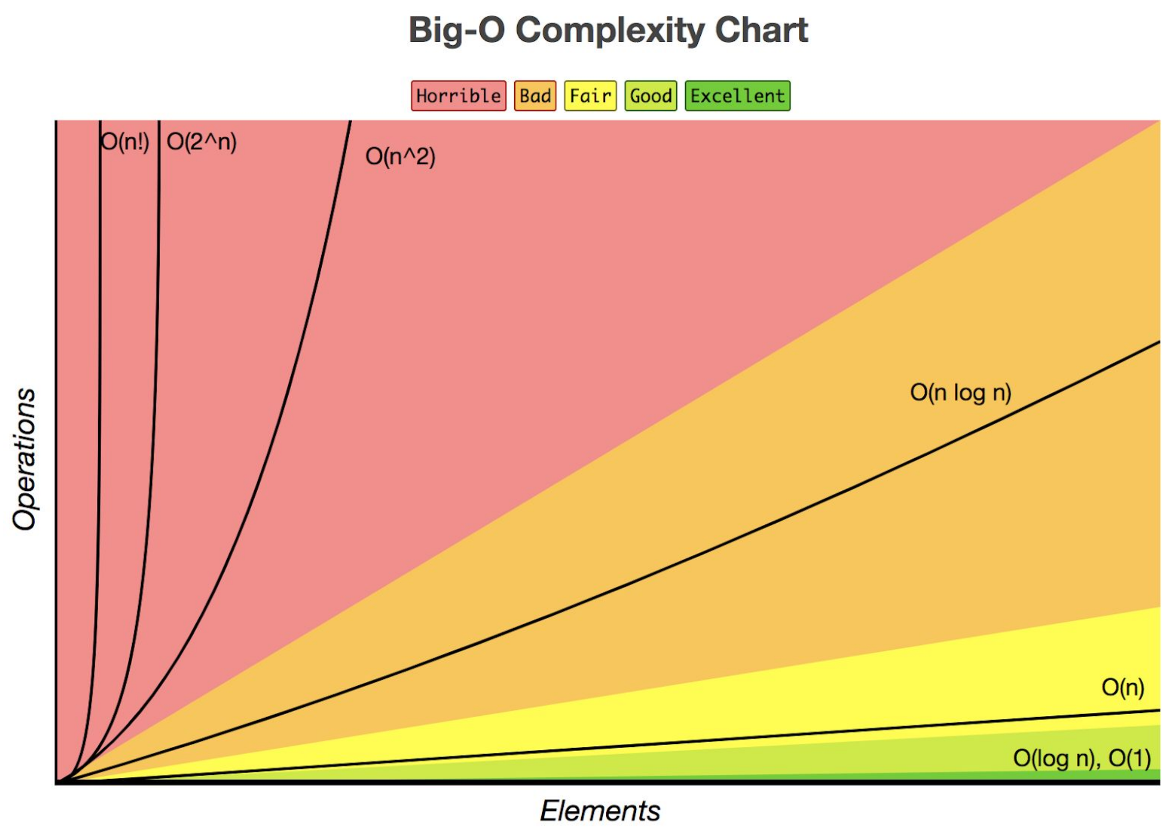
Theorem: Let f(n) be a nonnegative real-valued function such that
$f(n) = a_m n^m + a_{m-1} n^{m-1} + \dots + a_1 n + a_0$,
where a_i's are real numbers, $a_m \neq 0$, n >= 0, and m is a nonnegative integer. Then f(n) =
$O(n^m)$.

In the examples below we use the theorem to establish the Big-O of certain functions:

Function	**Big O**
$f(n) = 2n + 1$	$f(n) = O(n)$
$f(n) = n^2 + 5n + 1$	$f(n) = O(n^2)$
$f(n) = 23n^8 + 30$	$f(n) = O(n^8)$
$f(n) = 89n^{123}$	$f(n) = O(n^{123})$

Summary of Some Common Big O Runtimes

Big O	**Growth Rate**
O(1)	The growth rate is constant and so does not depend on n.
O(log n)	The growth rate is a function of log n. Because a logarithm function grows slowly, the growth rate is slow.
O(n)	The growth rate is linear. The growth rate is directly proportional to the size of the input.
O(nlog n)	The growth rate is faster than the a linear algorithm
$O(n^2)$	The growth rate increases rapidly with the size of the input. The growth rate is quadrupled when the input size is doubled.



Big O complexity chart (bigocheatsheet.com)

Compulsory Task

Answer the following questions:

- What is the Big-O notation of the following algorithm (assume that all variables have been declared):

```
for (int i=1; i<= n; i++)
        sum = sum + i * (i + 1);
```

- What is the Big-O notation of the following algorithm (assume that all variables have been declared):

```
for (int i = 5; i <= 2 * n; i++)
        System.out.println(2 * n + i - 1);
```

- What is the Big-O notation of the following algorithm:

```
for (int i = 1; i <= 2 * n; i++)
        for (int j = 1; j <= n; j++)
                System.out.println(2 * i + j);
System.out.println(" ");
```

- What is the Big-O notation of the following algorithm:

```
for (int i = 1; i <= n; i++)
      for (int j = 1; j <= n; j++)
            for (int k = 1; k <= n; k++)
                  System.out.println(i + j + k);
```

- Each of the following expressions represents the number of inputs for certain algorithms. Write them in Big O notation (e.g, n + 2 -> O(n)):

    - $n^2 + 6n + 4$
    - $5n^3 + 2n + 8$
    - $(n^2 + 1)(3n + 5)$
    - $5(6n + 4)$
    - $n + 2\log_2 n - 6$
    - $4n\log_2 n + 3n + 8$