Simulation

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1 Quantum Mechanics

1.1 PHYS 4P51, Jared Wogan, November 19, 2021

Note: My code is not very well documented or commented, I hope that is okay. Most of the functions should be self explanatory though.

1.1.1 Quantum States, Gates, and Measurements

[ ]:
```
from Quantum import State, zero, one, minus, plus, plusi, minusi
from Quantum import B00, B01, B10, B11, random_state, bits_to_send,␣
↪receive_state
from Quantum import H, CH, X, CX, NOT, CNOT, Y, CY, Z, CZ, P, CP, SWAP
from Quantum import state_n_zeros, H_transform, DJU, DJU_result

import sympy as sym
```

[ ]:
```
zero.show()
one.show()
minus.show()
plus.show()
minusi.show()
plusi.show()
```

$|0\rangle$

$|1\rangle$

$\frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}}{2}|1\rangle$

$\frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|1\rangle$

$\frac{\sqrt{2}}{2}|0\rangle - \frac{\sqrt{2}i}{2}|1\rangle$

$\frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}i}{2}|1\rangle$

[ ]:
```
B00.show()
B00.entangled()
B01.show()
```

```
B01.entangled()
B10.show()
B10.entangled()
B11.show()
B11.entangled()
```

$\frac{\sqrt{2}}{2}\left|00\right\rangle + \frac{\sqrt{2}}{2}\left|11\right\rangle$

Entanglement Status: Entangled

$\frac{\sqrt{2}}{2}\left|01\right\rangle + \frac{\sqrt{2}}{2}\left|10\right\rangle$

Entanglement Status: Entangled

$\frac{\sqrt{2}}{2}\left|00\right\rangle - \frac{\sqrt{2}}{2}\left|11\right\rangle$

Entanglement Status: Entangled

$\frac{\sqrt{2}}{2}\left|01\right\rangle - \frac{\sqrt{2}}{2}\left|10\right\rangle$

Entanglement Status: Entangled

[ ]:
```
state1 = State(
    [sym.Rational(1, 2), sym.Rational(1, 2),
     sym.Rational(1, 2), sym.Rational(1, 2)],
    ["00", "01", "10", "11"]
)
state1.show()
state1.entangled()
```

$\frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{1}{2}\left|10\right\rangle + \frac{1}{2}\left|11\right\rangle$

Entanglement Status: Not Entangled

[ ]:
```
state1.measure(0).show()
```

$\frac{\sqrt{2}}{2}\left|10\right\rangle + \frac{\sqrt{2}}{2}\left|11\right\rangle$

[ ]:
```
state_long = State(
    [
        1/sym.sqrt(10), sym.sqrt(3)/sym.sqrt(10), sym.sqrt(2) /
        sym.sqrt(10), 1/sym.sqrt(10), sym.sqrt(3)/sym.sqrt(10)
    ],
    ["000", "010", "101", "111", "110"]
)
state_long.show()
state_long.measure(0).show()
```

$\frac{\sqrt{10}}{10}\left|000\right\rangle + \frac{\sqrt{30}}{10}\left|010\right\rangle + \frac{\sqrt{5}}{5}\left|101\right\rangle + \frac{\sqrt{10}}{10}\left|111\right\rangle + \frac{\sqrt{30}}{10}\left|110\right\rangle$

$\frac{\sqrt{3}}{3}\left|101\right\rangle + \frac{\sqrt{6}}{6}\left|111\right\rangle + \frac{\sqrt{2}}{2}\left|110\right\rangle$

[ ]:
```
state2 = State(
    [1],
    ["11"]
)
state2.show()
state2.entangled()
```

$|11\rangle$

Entanglement Status: Not Entangled

[ ]:
```
state3 = State(
    [sym.Rational(1, 2), sym.Rational(1, 2),
     sym.Rational(1, 2), sym.Rational(1, 2)],
    ["00", "01", "10", "11"]
)
state3.show()
state3.entangled()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$

Entanglement Status: Not Entangled

[ ]:
```
X(state1, 0).show()
NOT(state1, 0).show()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$

[ ]:
```
CX(state1, 0, 1).show()
CNOT(state1, 0, 1).show()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|11\rangle + \frac{1}{2}|10\rangle$

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|11\rangle + \frac{1}{2}|10\rangle$

[ ]:
```
Y(state1, 0).show()
```

$-\frac{i}{2}|00\rangle - \frac{i}{2}|01\rangle + \frac{i}{2}|10\rangle + \frac{i}{2}|11\rangle$

[ ]:
```
CY(state1, 0, 1).show()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{i}{2}|11\rangle - \frac{i}{2}|10\rangle$

[ ]:
```
Z(state1, 0).show()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle - \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

[ ]:
```
CZ(state1, 0, 1).show()
```

$\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle - \frac{1}{2}|11\rangle$

[ ]:
```
H(state1, 0).show()
```

$\frac{\sqrt{2}}{2}\left|00\right\rangle + \frac{\sqrt{2}}{2}\left|01\right\rangle$

[ ]:
```
CH(state1, 0, 1).show()
```

$\frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{\sqrt{2}}{4}\left|10\right\rangle - \frac{\sqrt{2}}{4}\left|11\right\rangle + \frac{\sqrt{2}}{4}\left|10\right\rangle + \frac{\sqrt{2}}{4}\left|11\right\rangle$

[ ]:
```
SWAP(state1, 0, 1).show()
```

$\frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|10\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{1}{2}\left|11\right\rangle$

[ ]:
```
P(state1, 0, phase=sym.pi/4).show()
```

$\frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{\sqrt{2}(1+i)}{4}\left|10\right\rangle + \frac{\sqrt{2}(1+i)}{4}\left|11\right\rangle$

[ ]:
```
CP(state1, 0, 1, phase=sym.pi/4).show()
```

$\frac{1}{2}\left|00\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{1}{2}\left|10\right\rangle + \frac{\sqrt{2}(1+i)}{4}\left|11\right\rangle$

2 Quantum Teleportation

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[ ]:
```
alice_some_state = random_state()
alice_some_state.show()
```

$\frac{\sqrt{161114}(387+622i)}{483342}\left|0\right\rangle + \frac{\sqrt{161114}(562+773i)}{483342}\left|1\right\rangle$

[ ]:
```
gamma = alice_some_state * B00
CNOT12_gamma = CNOT(gamma, 0, 1)
H1_CNOT12_gamma = H(CNOT12_gamma, 0)
result = H1_CNOT12_gamma.measure(0, 1)
result.show()
```

$\frac{\sqrt{161114}(387+622i)}{483342}\left|000\right\rangle + \frac{\sqrt{161114}(562+773i)}{483342}\left|001\right\rangle$

[ ]:
```
alice, bob = bits_to_send(result)
alice.show()
bob.show()
```

$\left|00\right\rangle$

$\frac{\sqrt{161114}(387+622i)}{483342}\left|0\right\rangle + \frac{\sqrt{161114}(562+773i)}{483342}\left|1\right\rangle$

[ ]:
```
bob_some_state = receive_state(alice, bob)
bob_some_state.show()
alice_some_state.show()
```

$\frac{\sqrt{161114}(387+622i)}{483342}\left|0\right\rangle + \frac{\sqrt{161114}(562+773i)}{483342}\left|1\right\rangle$

$\frac{\sqrt{161114}(387+622i)}{483342}|0\rangle + \frac{\sqrt{161114}(562+773i)}{483342}|1\rangle$

3 Deutsch-Jozsa Algorithm

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[ ]:
```
n = 3
zeros = state_n_zeros(n)
psi = zeros * one
psi.show()
```

$|0001\rangle$

[ ]:
```
H_1ton_psi = H_transform(psi, n)
H_1ton_psi.show()
H_last_H_1ton_psi = H(H_1ton_psi, n)
H_last_H_1ton_psi.show()
```

$\frac{\sqrt{2}}{4}|0001\rangle + \frac{\sqrt{2}}{4}|0011\rangle + \frac{\sqrt{2}}{4}|0101\rangle + \frac{\sqrt{2}}{4}|0111\rangle + \frac{\sqrt{2}}{4}|1001\rangle + \frac{\sqrt{2}}{4}|1011\rangle + \frac{\sqrt{2}}{4}|1101\rangle + \frac{\sqrt{2}}{4}|1111\rangle$

$\frac{1}{4}|0000\rangle - \frac{1}{4}|0001\rangle + \frac{1}{4}|0010\rangle - \frac{1}{4}|0011\rangle + \frac{1}{4}|0100\rangle - \frac{1}{4}|0101\rangle + \frac{1}{4}|0110\rangle - \frac{1}{4}|0111\rangle + \frac{1}{4}|1000\rangle - \frac{1}{4}|1001\rangle + \frac{1}{4}|1010\rangle - \frac{1}{4}|1011\rangle + \frac{1}{4}|1100\rangle - \frac{1}{4}|1101\rangle + \frac{1}{4}|1110\rangle - \frac{1}{4}|1111\rangle$

[ ]:
```
U_H_last_H_1ton_psi = DJU(H_last_H_1ton_psi, "random")
U_H_last_H_1ton_psi.show()

H_1ton_U_H_last_H_1ton_psi = H_transform(U_H_last_H_1ton_psi, n)
result = H_1ton_U_H_last_H_1ton_psi.measure(*[i for i in range(n)])
result.show()
DJU_result(result)
```

$-\frac{1}{4}|0000\rangle + \frac{1}{4}|0001\rangle - \frac{1}{4}|0010\rangle + \frac{1}{4}|0011\rangle - \frac{1}{4}|0100\rangle + \frac{1}{4}|0101\rangle - \frac{1}{4}|0110\rangle + \frac{1}{4}|0111\rangle - \frac{1}{4}|1000\rangle + \frac{1}{4}|1001\rangle - \frac{1}{4}|1010\rangle + \frac{1}{4}|1011\rangle - \frac{1}{4}|1100\rangle + \frac{1}{4}|1101\rangle - \frac{1}{4}|1110\rangle + \frac{1}{4}|1111\rangle$

$-\frac{\sqrt{2}}{2}|0000\rangle + \frac{\sqrt{2}}{2}|0001\rangle$

```
DJU Algorithm Result: f(x) is Constant
```

4 Classical Computer vs. Quantum Computer

Clearly, this is being run on a classical computer, so the speedup that can be observed by performing the Deutsch-Jorsza algortihm on a quantum computer cannot be seen. However, the fact that the algorithm can be run on a classical computer is of no issue. We are simulating the algorithm in this notebook, and in order to do so, we are indeed actually calculating all 2^n values of $f(x)$ behind the scenes.

This is a problem, as once n grows large enough, the simulation will take an extremely long time to finish. For example:

- For $n = 7$, the simulation takes approximately 10 seconds to run in total.
- For $n = 8$, the simulation takes approximately 40 seconds to run in total.

- For $n = 9$, the simulation takes upwards of 160 seconds to run in total.

This will only continue to take longer and longer as we increase n. So while we may be able to simulate the algorithm on a classical computer for small n, if we had a system of a large number of qubits, say $n = 64$, we would have a much more difficult time simulating the algorithm (unless you are willing to wate hundreds of thousands of years for the result).