

Assignment 2 - Q4

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1 Definitions

```
or :: [Bool] -> Bool
or []           = False                (or.1)
or (x:xs)       = x || (or xs)        (or.2)

match :: Integer -> [Integer] -> [Bool]
match x []       = []                  (match.1)
match x (y:ys)    = (x == y):(match x ys) (match.2)

elem :: Integer -> [Integer] -> [Bool]
elem x []         = False              (elem.1)
elem x (y:ys)     = (x == y) || (elem x ys) (elem.2)
```

2 Proof Goals

We want to prove that for all finite lists:

$$\text{or}(\text{match } x \text{ } xs) = \text{elem } x \text{ } xs \quad (\text{EQ})$$

First we are going to prove the base case

$$\text{or}(\text{match } x \text{ } []) = \text{elem } x \text{ } [] \quad (\text{BASE})$$

Then we are going to prove the induction step:

$$\text{or}(\text{match } x \text{ } (x:xs)) = \text{elem } x \text{ } (x:xs) \quad (\text{IND})$$

Assuming the hypothesis:

$$\text{or}(\text{match } x \text{ } xs) = \text{elem } x \text{ } xs \quad (\text{HYP})$$

3 Proving the Base Case

$\text{or}(\text{match } x \text{ } []) = \text{elem } x \text{ } []$ (BASE)

Left Hand Side (LHS):

$\text{or}(\text{match } x \text{ } [])$
 $= \text{or}([])$ by (match.1)
 $= \text{False}$ by (or.1)

Right Hand Side (RHS):

$\text{elem } x \text{ } []$
 $= \text{False}$ by (elem.1)

LHS = RHS.

4 Induction Step

$\text{or}(\text{match } x \text{ } (x:xs)) = \text{elem } x \text{ } (x:xs)$ (IND)

Left Hand Side (LHS):

$\text{or}(\text{match } x \text{ } (x:xs))$
 $= \text{or}((x == x):(\text{match } x \text{ } xs))$ (match.2)
 $= (x == x) || (\text{or } xs)$ (or.2)

Right Hand Side (RHS):

$\text{elem } x \text{ } (x:xs)$
 $= (x == x) || (\text{elem } x \text{ } xs)$ (elem.2)
 $= (x == x) || \text{or}(\text{match } x \text{ } xs)$ (hyp)
 $= (x == x) || \text{or}((x == x):(\text{match } x \text{ } xs))$ (match.2)
 $= (x == x) || (x == x) || (\text{or } xs)$ (or.2)
 $= (x == x) || (\text{or } xs)$ (||)

LHS = RHS.

End of Proof.