Solow Simulations

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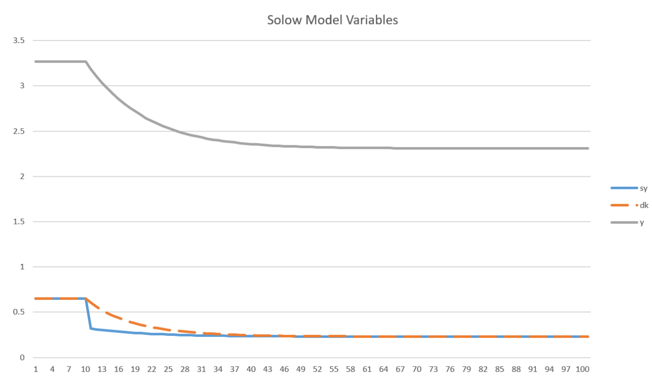
Introduction

This analysis shows how changing different economic variables affect each other. The analysis will change different variables with the Solow model to demonstrate these changes. It will also provide insight into how these variables relate. For our first experiment, imagine a new tax law has discouraged investing, leading to decreases in household savings. What other changes will that create? Economies tend to drift toward a steady state where economic variables like depreciation, capital, output and investing are in balance with each other. The new tax law has caused a change in savings. This leads to changes in economic conditions until that steady state is reached again.

1 Experiment 1: Tax Law Changing Savings Rate

The tax laws discouraged investing, decreasing the savings rate of U.S. households to 0.1. Because individuals are investing less, they have less capital and have less purchasing power. This means there will be an overall depreciation of capital in the economy.

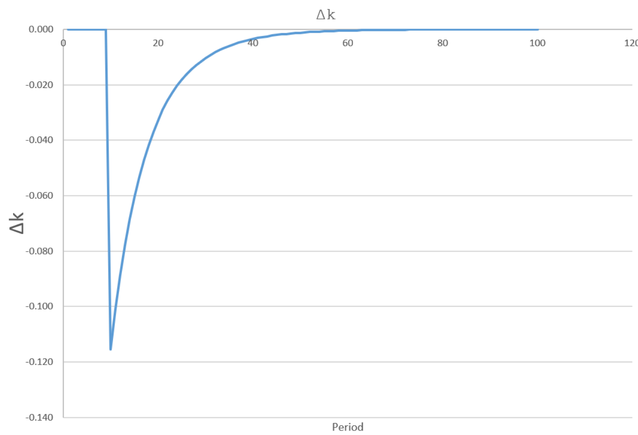
1.1 Graphical Representation of Economic Variables



Above is a graphical representation of this decrease. First, total economic output decreases (y) with the change in the savings rate. The housing market is an example of this. There are not as many homes being built because people don't have money saved to invest in building homes. This is visualized by a dip in the gray line.

When the tax laws take effect, there is a sharp drop in investing (sy). this is represented by the dip in the blue line. Depreciation (δk) also drops, but at a slower rate. People will react immediately to the change in tax laws. This causes investing to drop faster than depreciation. Depreciation takes more time to drop because its moving from older goods to new goods. People can't afford to purchase new homes with the drop in savings. Eventually the newer homes depreciate and become affordable.

The rate at which capital changes is also affected. Δk represents the change in capital.



The change in capital sharply decreases in sync with the new savings rate. This is the beginning of capital adjusting back to the steady state. Initially, the adjustment is quick, but then slows exponentially back to 0 change in capital. In the housing example, there is a sudden drop in people's ability to invest in a home. Over time and with the help of depreciation, homes become more affordable. People then invest in the cheaper homes or cheaper capital.

1.2 Formulas for Change in Steady State

Eventually, all the variables stabilize at a much lower level than before the new tax policy. To show this mathematically, we will use the steady state formula. It describes the point at which economic variables are in balance with each other, or in other words, where savings and depreciation are equal to one another.

$$k^* = (\frac{s\bar{A}}{\delta})^{\frac{1}{1-\alpha}}$$

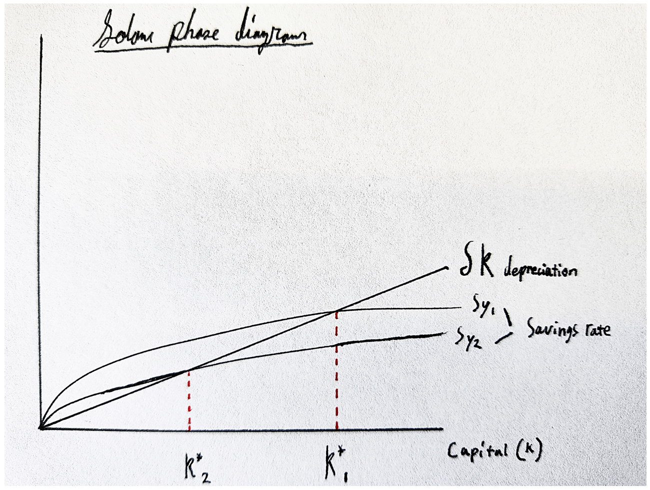
Here is k^* or steady state capital before the tax laws:

$$k_1^*(\text{before}) = (\frac{0.2 * 1}{0.15})^{\frac{1}{1-(1/3)}} \approx 1.53$$

Here is k^* after the tax laws take effect:

$$k_1^*(\text{before}) = (\frac{0.1 * 1}{0.15})^{\frac{1}{1-(1/3)}} \approx 0.54$$

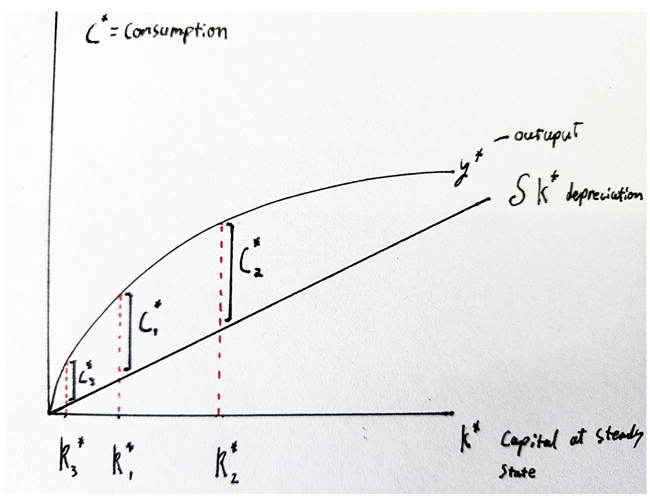
1.3 Solow Model Phase Diagram



This is a representation of what is happening in the Solow model. The experimental tax laws cause the U.S. to move from k_1^* to k_2^*. k^* represents two different steady states where economic variables are balanced. Investing immediately moves while output and deprecation move over time toward k_2^*.

1.4 Policy Implications

These tax laws will decrease U.S. production and, consequentially, consumption. There is a better solution. Instead of reducing the savings rate, find the savings rate that allows the U.S. to have more product after savings. Consumption is the amount of product left over after removing savings. This is not the common definition of consumption as defined by consumerism culture.



This is a graphical representation of consumption in the Solow Model with the change in tax laws, or savings rate. Choosing an optimal savings rate will allow for more consumption. k_1^* represents the original savings rate at 0.2. After the change in tax laws consumption shifts to k_3^* at 0.1.

The savings rate that maximizes U.S. consumption is at the k_2^* steady state. This happens at the rate where savings is equal to the capital share of income. In other terms, it is the k^* where: $\alpha = s$ is true. This is known as the golden rule or the golden level of savings. In our experiment, policies should encourage a savings rate equal to roughly 1/3. This is why consumption at k_2^* is the largest, represented by C_2^*.

$$c^* = y^*(1-s)$$

Consumption with decreased savings rate:

$$c^* = 0.82(1-0.1) \approx 0.73$$

Consumption with golden savings rate

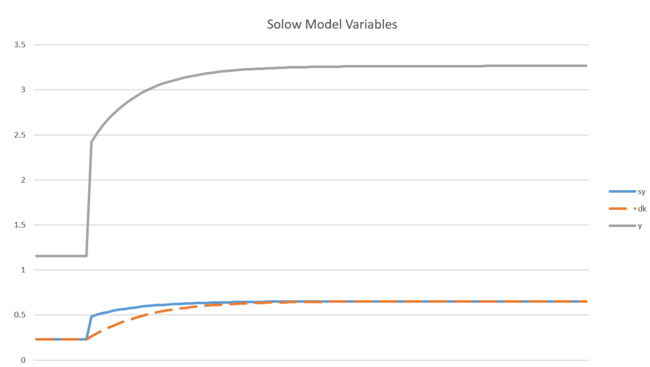
$$c^* = 1.49(1-\frac{1}{3}) \approx 0.99$$

Consumption is about 30% larger at a savings rate of $\frac{1}{3}$. Setting laws that would encourage a savings rate of $\frac{1}{3}$ would be the most beneficial to the country.

2 Experiment 2: Technology Transfers

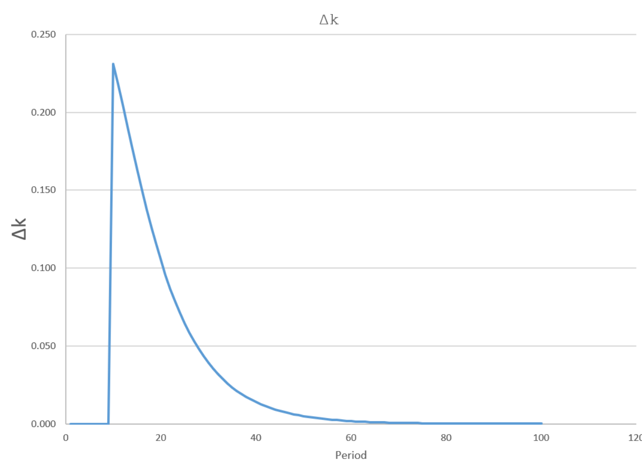
China develops new policy that allows them increased access to foreign technology. This results in a permanent increase in Total Factor Production (TFP or $\bar{A}$) from 1 to 2.

2.1 Graphical Representation of Economic Variables



An improvement of TFP will cause output (y) and savings(sy) to spike. Savings increases because people react to the change in technology. An example of this is AI's recent public availability. People invested much more than usual because of the positive impact they assumed AI would have on the economy. Depreciation adjusts more slowly. As new goods are produced, depreciation will increase with the increase in production. Older goods will still work until they depreciate fully, leaving only the more expensive goods. An example of this is the auto industry and AI.

In the example, AI's initial introduction to the auto industry will boost productivity. The new AI technology will improve the quality of cars. This will increase the auto industry's profits, allowing them to invest more in production. Savings spikes as people try and take advantage of these improvements. Depreciation won't spike. Older cars are unaffected by AI's introduction into the market and their depreciation speed remains the same. It takes time for older cars to become completely obsolete. Older cars become redundant as consumer and producer preference shifts to cars produced with AI. Depreciation then eventually catches up with the AI cars, reaching a new steady state equal to savings.



Capital change spikes drastically with the change in TFP. New technology instantly opens opportunities for the creation of more capital. This would be analogous to Prometheus's gift of fire. As soon as that technology is given, it allows humans much more freedom of movement now that warmth and shelter is easier to obtain. It opens up hunting in colder regions as well as the ability to cook and store food.

Eventually the change in capital exponentially slows back to 0 as the new technology is fully integrated. As the people more thoroughly incorporate fire into daily living, there is less and less change in capital. Every need related to fire is met and it is no longer changing the people's life since the changes resulting from the new technology have already occurred.

2.2 Formulas for Change in Steady State

As the new technology China has introduced into its economy becomes more incorporated into daily living, it causes economic variables to once more reach a steady state. Here it is mathematically:

$$k^* = (\frac{s\bar{A}}{\delta})^{\frac{1}{1-\alpha}}$$

Here is k^* or steady state capital before technology changes:

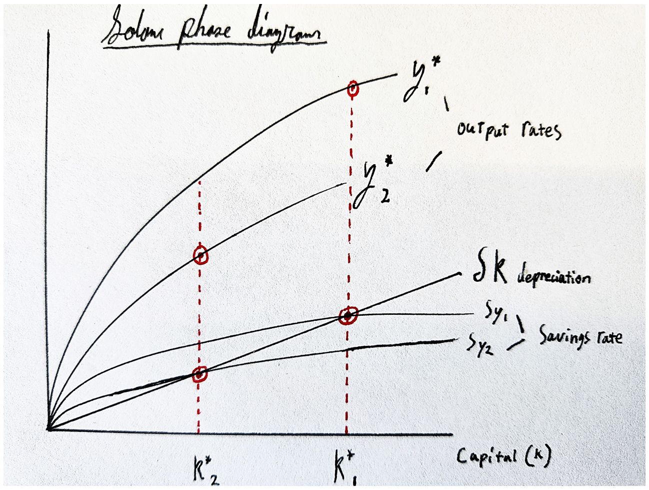
$$k_1^*(\text{before}) = (\frac{0.2 * 1}{0.15})^{\frac{1}{1-(1/3)}} \approx 1.54$$

Here is k^* after the technology changes take effect:

$$k_1^*(\text{before}) = (\frac{0.2 * 2}{0.15})^{\frac{1}{1-(1/3)}} \approx 4.35$$

A change in technology leads to an increases k^* by almost 3 times as much as the original k^*

2.3 Solow Model Phase Diagram

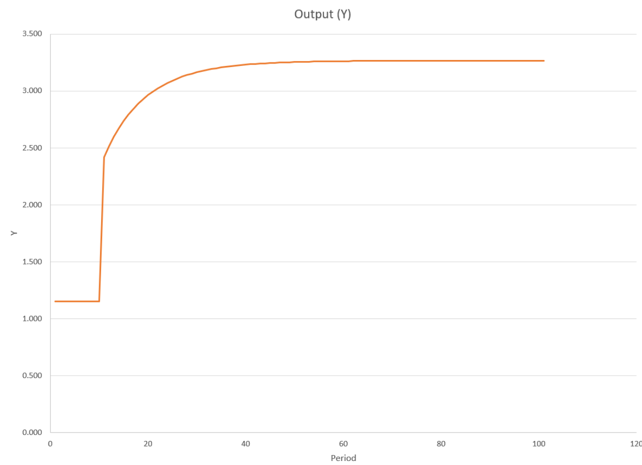


A change in technology causes more than a shift in the savings line. It also causes a shift in output. Depreciation remains constant. Initially China is at k_2^* and y_2^*. With the improvements China shifts to k_1^* and y_1^*.

2.4 Policy implications

There is a U.S. policy that suggests a one-time technology transfer to developing nations. The aim of the policy is to reduce poverty in developing nations.

If this policy were to be enacted, the output of developing nations would increase.



In the beginning output improves almost instantly, causing short term economic growth. However, as seen in the graph, the growth caused by this technology transfer slows and then stabilizes. This suggests an improvement in technology would only help temporarily. This is demonstrated in the graph. There is a sharp increase then output levels plateau.

This type of policy would fail in its initiative. A few short years after technology is introduced into these developing countries their respective economies would stabilize once more. A better way to facilitate economic growth in developing countries would be to intact policy that encourages technology transfers over time, the development of institutes, incentivize domestic innovation, etc.

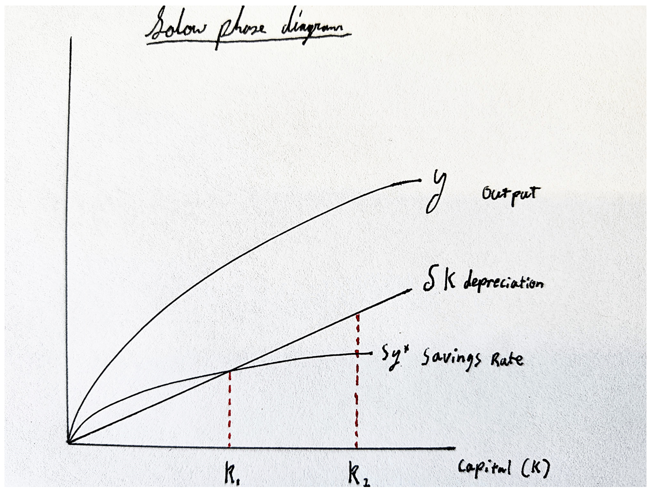
Returning to the caveman example, it would be like a group of caveman introducing fire to another group who had never seen it. The ignorant group gets handed several torches. This leads to disaster. Wildfires are started, some accidentally burn themselves on the handheld sun they were just given. Others might even worship the flame. Eventually fire would be understood and incorporated, but it wouldn't change how the group operates. Other than the new output produced by the understanding of flame there are no systems in place to facilitate more change. They go stagnant.

A better idea would be to slowly introduce fire. Maybe instead of handing them a torch show them flint and steel. Introduce them to sparks in a contained fashion maybe even small bits of fire. Help them incorporate it while demonstrating the dangers of fire. Introduce the new technology over time. Teach them systems of maintaining safe fires. Have fire masters watch over others to reduce risks. Have some cavemen experiment with the fire and discover what it is. This approach would lead to long term growth and new ideas. It wouldn't stagnate. This approach needs to be taken in regards to foreign aid in developing countries.

3 Experiment 3.1: Foreign Aid in Ethiopia

After COVID-19, Ethiopia received a foreign aid package that doubled the countries capital. There are no other changes. This experiment will look at how the sudden increase in capital affects Ethiopia's economy.

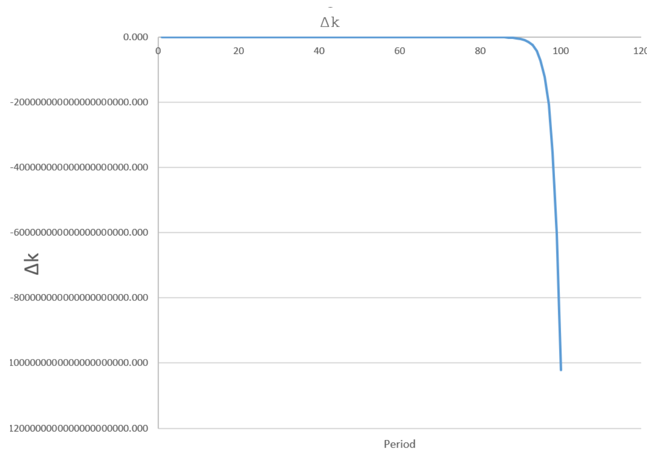
3.1 Solow Model Phase Diagram



As seen in the graph above, doubling capital does not change a countries steady state. It will drift back to an equilibrium. The change in capital will increase output and depreciation. Savings decreases, leaving Ethiopia with less consumption. Charles Jones, the author of the textbook Macro Economics, summed up the effect of supplemental capital on this economy when he said "Capital accumulation cannot serve as the engine of long-run economic growth."(Jones 116).

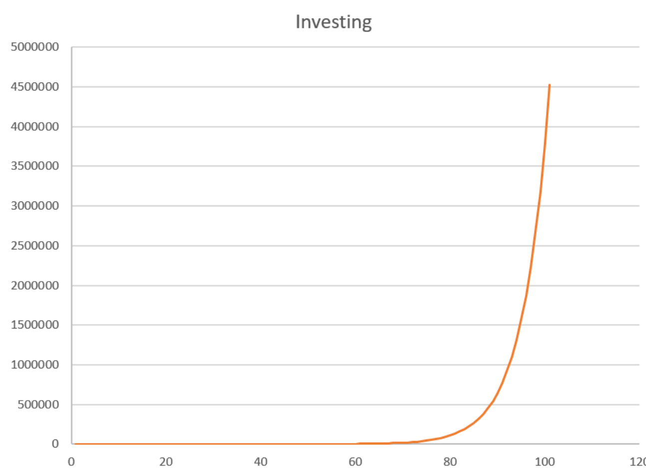
Although these affects are temporary, they have a huge impact on the different economic variables. Deprecation immediately jumps from k_1^* to k_2. Output also leaps up to meet the new k_2. This is shown in the graph above.

The change in capital sharply decreases.



This happens partially because capital is tied directly to depreciation. Capital doubles, meaning depreciation also doubles. Capital then loses value at twice its previous rate.

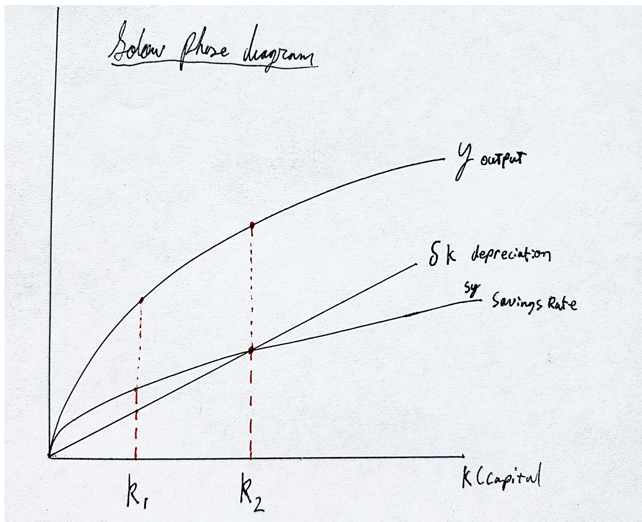
An example of this is potato farmers. If suddenly six potato farmers were given 600,000 acres of land they would be hard pressed to keep it all farmed. Most of the 600,000 acres would rapidly lose value due to the farmers being unable to use the land to its full potential.



Investing is a different story. It sharply spikes as more people are able to invest more capital. This would be analogous to someone gifting the farmers several billion potatoes. They have more potatoes then they ever wanted, so they get as much potato seed as they can from the gifted potatoes. In Ethiopia's case, they are given so much it ends up hurting more then helping. The economy will just return to its steady state in spite of the gift of foreign aid or potatoes. All the gifts will do is create an economic upheaval.

4 Experiment 3.2: Foreign Aid in Ethiopia

Foreign aid in Ethiopia under different circumstances could be beneficial. The last experiment assumed that Ethiopia was at steady state. Now our assumption is that Ethiopia's capital is at exactly half of its steady state. This means foreign aid would bring capital to a steady state.



Ethiopia's foreign aid is shown graphically by capital moving from k_1 to k_2^*. Output and depreciation increase with the change is shown by y and δk respectively.

The variables in this experiment behave similarly to the variables in the previous experiment 3.1. The graphs are almost identical. The key difference is that in experiment 3.2 the foreign aid is retained in their economy. This is because Ethiopia's economy reached a steady state in experiment 3.2

Imagine some potato farmers have a hard year. They produce less because winter was longer then expected. They can only afford to produce half as many potatoes as they normally do. The farmers can eventually get back to their previous rate of production, it just takes time. However, they have a neighbor willing to give them half of their normal crop, bring them back to normal levels of potato production. The farmers don't have to take that time working to get back to their normal production rates. Instead they can keep producing at the rate they had been.

4.1 Implications for foreign aid

Giving a country capital can have an extremely positive or negative effect. It is important to carefully calculate how much aid is needed to bring an economy back to a steady state, if it is needed at all.

A positive example of foreign aid is the Marshall Plan. In 1948 the U.S., "...approved funding that would eventually rise to over $12 billion for the rebuilding of Western Europe" (Marshall). The Marshall plan boosted Germany to unprecedented economic growth and prosperity in the country. The result was trade was reopened with Germany more quickly benefiting both parties.

An example of foreign "aid" going awry is U.S. aid in Afghanistan. The U.S. sent in military forces to uphold the Afghan government to keep power out of the Taliban's hands. The military aid was a form of capital foreign aid. Eventually officials in the U.S. government viewed the task as too costly, leading to a disastrous withdraw in 2021. As a result of this drastic cut in humanitarian aid "More than a third of Afghans are enduring dangerous levels of food deprivation" (Saito et al.). Foreign aid hurt the Afghan population.

5 Concluding Thoughts

Providing strategic aid is a concept that can be applied to more than foreign aid policy. There must be understanding for growth in any field to occur. More capitol is rarely the answer and oftentimes detrimental. Sometimes its better to provide mentalities and systems to create improvement.

Works Cited

References

[1] Jones, Charles I. *Macroeconomics*. 4th ed., W.W. Norton, 2018.

[2] Marshall Plan, 1948. *Office of the Historian, U.S. Department of State*, https://history.state.gov/milestones/1945-1952/marshall-plan. Accessed 15 Feb. 2025.

[3] Saito, Mari, et al. "Famine, NGO, Afghanistan." *Reuters*, 10 Dec. 2024, 11 a.m. GMT. https://www.reuters.com/investigates/special-report/famine-ngo-afghanistan/.