Growth Accounting Report

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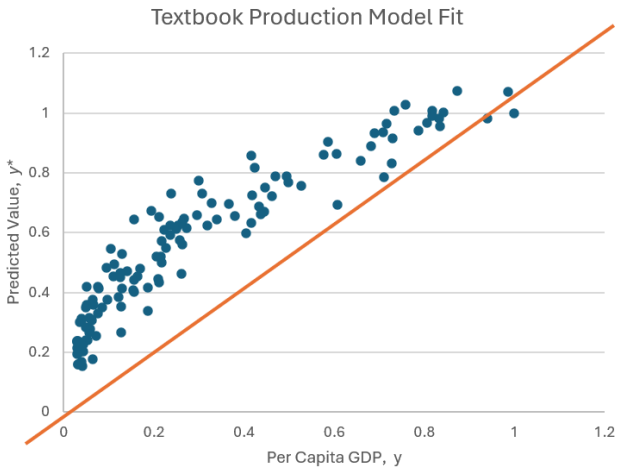
1 The Cobb-Douglas Model

To anticipate anything, it is helpful to have a system of beliefs. This system is called a model. An important model for policy makers is the Cobb-Douglas function. Its purpose is to help us understand production within an economy. To do this, it gives an estimate of what can be produced with the labor and capital within the economy. It is denoted as:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where Y is output, $\bar{A}$ is total factor productivity (TFP), K is capital, L is labor, and α represents the output elasticity of capital.

However, the model does not stand up to empirical scrutiny. When graphed, countries are significantly poorer than the model suggests.



The orange line represents the Cobb-Douglas function. The vertical axis shows the model's predictions for per capita GDP compared to the U.S. According to the graph, every country should be further along the X-axis enjoying the fruits of more GDP per capita, but that's not what is shown. Nearly every country is above the line.

Depending on how much capital per person is available the model estimates a country's potential output, but it goes wrong in two ways. First, the magnitude of the model is off due to diminishing returns to capital (MDK). The model assumes that MDK is constant across all countries, leading to an overestimation of capital's impact. This makes it seem like poorer countries should get more out of their capital than they actually do.

The second part of the problem is that the model inaccurately assumes that some countries, like Switzerland, Japan, and Norway, should be as rich as the U.S. This occurs because it holds $\bar{A}$ (TFP) constant across all countries. However, some countries are better at turning inputs into outputs. $\bar{A}$ is not

the same for everyone, which can be due to factors such as culture, government policies, and inefficient utilization of TFP.

2 The Hall-Jones Model

The Hall-Jones model predicts GDP per capita levels that are closer to reality for some countries. The Hall-Jones model refines TFP by incorporating human capital (h). Unlike the Cobb-Douglas model, it does not assume that labor is constant across all countries.

Human capital is the economic value of an individual's skill, knowledge, and behaviors. For example, take a society that has a capital input of laptops. A programmer will be able to produce more with a laptop than a non-programmer, as their programming skill augments their human capital. Human capital differs from labor, which is defined as the exertion or effort put into achieving an outcome.

The Hall-Jones model makes the distinction between labor and human capital clearer:

$$Y = \bar{A}K^{\alpha}(hL)^{1-\alpha} \tag{2}$$

The Hall-Jones model adjusts $hL^{1-\alpha}$ to account for human capital in the model. The Hall and Jones approximate human capital as a function of years in school, assuming diminishing returns after excessively extended schooling. However, this does not explain all the variation in the data.

While this is a sufficient way to incorporate human capital into the model, it is imperfect. Time in school does not directly translate to higher human capital. Factors such as educational quality, student engagement, and external conditions (e.g., access to resources) matter. For example a student can get an education in Taiwan and have an accelerated learning experience in regards to math, while someone taking U.S. curriculum may be on track to learn math over a longer period of time. This causes some of the variation in the model.

Another reason for the added variation this model produces is the inability of a country to use its human capital effectively. For example a person can get an engineering and robotics degree in Ecuador, and due to the lack of jobs in that field, end up selling tractors. If they were in a different country they could be producing more with the added opportunities to use their education.

To improve the measurement of human capital, additional factors such as grades, work experience, health, multilingual abilities, and many others could be incorporated. This added detail would enhance the accuracy of human capital measurements.

3 Statistical Relationship Between Human Capital and TFP

The Hall Jones model operates under the assumption that human capital and total factor production are correlated. At first glance, this makes sense. Historically, college degrees have nearly guaranteed superior employment opportunities. A linear regression model can reflect this reality.

The model uses two variables, the Human Capital Per Capita index and the Total Factor Production variable (TFP or $\bar{A}$). The Human Capital Per Capita Index (HCI) is an index developed to measure how well a country is using its human capital. It ranks each country on a scale between 0 and 1 with 1 being the maximum potential reached. TFP is measured by the total output of an economy compared to the United States' production. The null hypothesis for the linear regression model is that HIC does not correlate with TFP. The alternative hypothesis is that HIC and TFP correlate.

$$H_0 : \beta_1 = 0 \quad \text{(HIC does not correlate with TFP)}$$
$$H_A : \beta_1 \neq 0 \quad \text{(HIC and TFP correlate)}$$

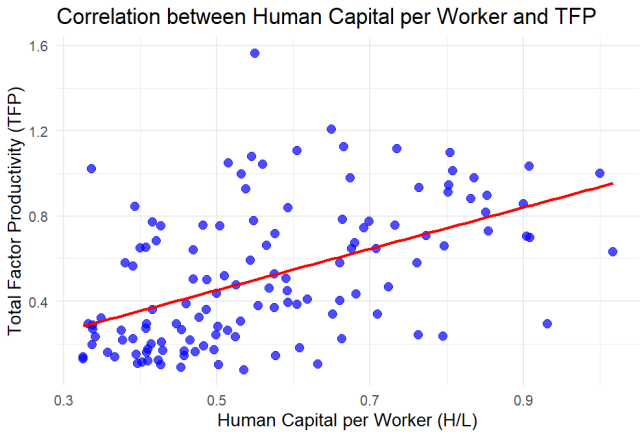
Here is the formula for the regression model:

$$A(k,h) = \beta_0 + \beta_1 \frac{H}{L} + \varepsilon \tag{3}$$

where:

- $A(k,h)$ represents TFP in the Hall-Jones model
- H/L is the Human Capital per capita index
- ε is the error term

Here are the regression results:



	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.03128	0.08851	-0.3534	0.7244
`H/L`	0.9687	0.1501	6.453	2.189e-09

Fitting linear model: `A(k,h)` ~ `H/L`

Observations	Residual Std. Error	R^2	Adjusted R^2
127	0.2829	0.2499	0.2439

The regression has a positive correlation between Human Capital Per Capita (H/L) and TFP. The intercept is not statistically significant, meaning the starting TFP to HCI varies. However, the slope is statistically significant, indicating that there is a relationship between TFP and HCI. The slope estimate is significant with a p-value next to 0. This indicates that a one-unit increase in HCI leads to roughly a 0.99 increase in TFP. Almost a one to one ratio.

Although human capital raises TFP, its impact varies from country to country. Some countries with high human capital are unable to transfer it to TFP. As theorized, this is due to inefficient institutions and practices. Infrastructure, innovation, and use of government power also influence the transfer of HCI to TFP.

This suggests that an improvement in human capital does not necessarily improve a country's TFP. A brilliant mind put into an environment that limits its ability to produce will have difficulty making meaningful contributions to TFP.

4 Conclusion

The Cobb-Douglas and Hall-Jones models are both steps toward better defining what an economy is and where it is going. They both fall short in perfectly predicting what will happen, but do provide valuable insight. They show how labor and capital can, in theory, explain a country's production. The Hall-Jones model builds off of the Cobb-Douglas model. It was a huge step in better understanding a country's production. It added to the system of beliefs that the Cobb-Douglas model described. This advancement was accomplished by narrowing the definition of labor. Adding a deeper truth to a general system of beliefs creates progress.

Sources

References

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