a) $\int x\sin(2x)\,dx$ $\quad u = x \quad dv = \sin(2x)\,dx$

$du = 1dx \quad v = -\frac{1}{2}\cos(2x)$

$-\frac{1}{2}x\cos(2x) - \int -\frac{1}{2}\cos(2x)\,dx$

$\boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

b) $\int xe^{x^2}dx \quad u = x^2 \quad du = 2x \quad x = \frac{1}{2}du$

$\int \frac{1}{2}e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{x^2} + C}$

c) $\int xe^x dx \quad dv = e^x dx \quad v = e^x$

$u = x \quad du = 1dx$

$xe^x - \int e^x dx = \boxed{xe^x - e^x + C}$

e) $\int x\sqrt{1+x}\,dx \quad u = 1+x \quad du = dx$

$\int(u-1)(\sqrt{u}) = \int u^{3/2} - u^{1/2} = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \boxed{\frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C}$

f) $\int \sec(\theta)\,d\theta \Rightarrow \sec\theta\,\frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}\,d\theta$

$\frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} \quad u = \sec\theta + \tan\theta$

$du = \sec\theta\tan\theta + \sec^2\theta\,d\theta$

$\int \frac{du}{u} = \ln(u) + C$

$= \boxed{\ln(\sec\theta + \tan\theta) + C}$

g) $\int \sec^2(\theta)\,d\theta \quad t = \tan(x) \quad dt = \sec^2(x)$

$\int dt = t + C = \boxed{\tan(x) + C}$

h) $\int \operatorname{sech}^2(\theta)\,d\theta \quad t = \tanh x \ldots ?$

i) $\int \frac{x^2+2}{7-x^2} \Rightarrow -\int \frac{-2-x^2}{7-x^2} \Rightarrow -\int \frac{7-x^2}{7-x^2} - \frac{9}{7-x^2}$

$-x + \int \frac{9}{7-x^2} = (x+\sqrt{7})(x-\sqrt{7}) \quad \frac{A}{x+\sqrt{7}} + \frac{B}{x-\sqrt{7}} \quad 9 = Ax - \sqrt{7}A + Bx + \sqrt{7}B$

$A + B = 0 \quad +\sqrt{7}B + \sqrt{7}B = 9 \quad 2\sqrt{7}B = 9 \quad B = \frac{9}{2\sqrt{7}}$

$-x + \int \frac{-\frac{9}{2\sqrt{7}}}{x+\sqrt{7}} + \frac{\frac{9}{2\sqrt{7}}}{x-\sqrt{7}} = -x + \frac{9}{2\sqrt{7}}\int \frac{-1}{x+\sqrt{7}} + \frac{1}{x-\sqrt{7}}$

$-x + \frac{9}{2\sqrt{7}}\left[-\ln(x+\sqrt{7}) + \ln(x-\sqrt{7})\right] \Rightarrow \boxed{\frac{9}{2\sqrt{7}}\ln\left(\frac{x-\sqrt{7}}{x+\sqrt{7}}\right) - x} + C$

j. $\int \frac{1}{ap-bp^2}\,dp \qquad \frac{A}{p} + \frac{B}{a-bp}$

ODE's

a) $\frac{dx}{dt} = ex \qquad x(0)=1.0$

$\int \frac{1}{ex}\,dx = \int dt \qquad t = \frac{1}{e}\ln(x) \qquad \boxed{x = e^{te} + 0}$

b) $\frac{dx}{dt} = 3tx \qquad x(0)=1.0$

$\int \frac{1}{x}\,dx = 3t\,dt + C \qquad \ln(x) = \frac{3}{2}t^2 + C$

$\boxed{x = e^{3/2t^2+0}}$

c) $\frac{dx}{dt} = 0.1x - 0.003x^2 \qquad x(0)=4$

$\int \frac{1}{0.1x-0.003x^2}\,dx = \int dt \qquad t=$

$-1000\int \frac{1}{3x^2-100x}\,dx \qquad -1000\int \frac{1}{x^2}\left(\frac{1}{3-100/x}\right)dx$

$u = 3 - \frac{100}{x} \qquad du = \frac{100}{x^2}dx \Rightarrow dx = \frac{x^2}{100}du$

$-1000\int\left(\frac{1}{3-\frac{100}{x}}\right)\frac{du}{100} \qquad -10\int \ln(u) + C = -10\ln\left(3-\frac{100}{x}\right) + C$

$e^{\left(\frac{t}{-10}-C\right)} = 3 - \frac{100}{x} \qquad -(100)/\left(e^{t/-10-C} - 3\right) = x \qquad t = \frac{-100}{e^{-C}-3} = 400$

d) $\frac{dx}{dt} = 0.1x - 0.003x^2, \quad x(0) = 400 \qquad e^{-C}-3=25$

$\frac{-100}{} = 400$

$e^{-C} - 3 = 0.25$

$e^{-C} \quad 3.25$

$C \approx -1.1786 \qquad \ln 3.25 = -C$

c) $\boxed{x = \frac{-100}{e^{(t/-10)+\ln(28)}-3}}$

d $\boxed{x = \frac{-100}{e^{((t/-10)+\ln(3.25))}-3}}$

$L = \lim_{n \to \infty} \left(\frac{a_{n+1}}{a_n} \right)$ $L < 1 \therefore$ converges

$\frac{f^{(n)}(a)}{n!} (x-a)^n$

expand functions

radius = ∞

a) $f(x) = \sin(2x)$, $x_0 = 0$

$\sin(0) + \frac{2\cos(0)}{1}(x) - \frac{4\sin(0)}{2}(x^2) - \frac{8\cos(0)}{6}(x^3)$

$= \boxed{0 + 2x - \frac{8}{6}x^3 + \ldots}$

b) $f(x) = \ln(2x)$, $x_0 = 1$

$\ln(2) + (1)(x-1)^1 - \frac{1}{2}(x-1)^2 + \frac{4}{}\,\frac{1}{6}(x-1)^3$

$\boxed{\ln(2) + (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots}$

$\frac{\frac{1}{n+1}(x-1)^{n+1}}{} \cdot \frac{n}{(x-1)^n}$

$\frac{(x-1)n}{n+1}$ $\boxed{ROC = 1}$

$x - 1 < 1$

c) $f(x) = e^{2x}$, $x_0 = 1$

$\boxed{e^2 + 2e^2(x-1)^1 + 4e^2\frac{1}{2}(x-1)^2 + 8e^2\frac{1}{6}(x-1)^3 + \ldots}$

$\frac{(x-1)^{n+1}(2^{n+1})}{(n+1)!} \cdot \frac{n!}{(x-1)^n 2^n} \Rightarrow \frac{2(x-1)}{(n+1)}$

$\boxed{ROC = \infty}$

? d) $f(x) = 3x^2 - 2x + 5$, $x_0 = 0$

$\boxed{5 - 2(x) + 3(x^2)}$?

e) $f(x) = 3x^2 - 2x + 5$ $x_0 = 1$

$6 + 4(x-1) + 3(x-1)^2 = 6 + 4x - 4 + 3x^2 - 6x + 3$

$= \boxed{5 - 2x + 3x^2}$

f) $f(x) = (3x^2 - 2x + 5)^{-1}$ $x_0 = 1$

$-(3x^2-2x+5)^{-2}(6x-2)$

$2(3x^2-2x+5)^{-3}(6x-2) - (3x^2-2x+5)^{-2}(6)$

$\frac{2(4)}{216} - \frac{1}{3}$

$\boxed{\frac{1}{6} - \frac{1}{9}(x-1) + \left(\frac{8}{216} - \frac{1}{3}\right)(x-1)^2\left(\frac{1}{2}\right) + \ldots}$

g) $f(x) = \cosh(x-3)$, $x_0 = 1$

$\boxed{\cosh(-2) + \sinh(-2)(x-1) + \cosh(-2)(x-1)^2\frac{1}{2} + \sinh(-2)(x-1)^3\frac{1}{6}}$

h) $f(x)$ $x_0 = a$

$\boxed{f(a) + f'(a)(x-a) + f''(a)\frac{1}{2}(x-a)^2 + f'''(a)\frac{1}{6}(x-a)^3 + \ldots}$

? i) $\boxed{f(x) + f'(x)(a-x) + f''(x)\frac{1}{2}(a-x)^2 + f'''(x)\frac{1}{6}(a-x)^3 + \ldots}$

j) ?