a) $\int x\sin(2x)\,dx$ $\quad u = x \quad dv = \sin(2x)\,dx$

$\quad du = 1dx \quad v = -\frac{1}{2}\cos(2x)$

$-\frac{1}{2}x\cos(2x) - \int -\frac{1}{2}\cos(2x)\,dx$

$-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$

b) $\int x e^{x^2}\,dx \quad u = x^2 \quad du = 2x \quad x = \frac{1}{2}du$

$\int \frac{1}{2}e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$

c) $\int x e^x\,dx \quad dv = e^x dx \quad v = e^x$

$\quad u = x \quad du = 1\,dx$

$xe^x - \int e^x\,dx = xe^x - e^x + C$

e) $\int x\sqrt{1+x}\,dx \quad u = 1+x \quad du = dx$

$(u-1)(\sqrt{u}) = \int u^{3/2} - u^{1/2} = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C$

f) $\int \sec(\theta)\,d\theta \Rightarrow \sec\theta\,\frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}\,d\theta$

$\frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} \quad u = \sec\theta + \tan\theta$

$\quad du = \sec\theta\tan\theta + \sec^2\theta\,d\theta$

$\int \frac{du}{u} = \ln(u) + C$

$= \ln(\sec\theta + \tan\theta) + C$

g) $\int \sec^2(\theta)\,d\theta \quad t = \tan(x) \quad dt = \sec^2(x)$

$\int dt = t + C = \tan(x) + C$

h) $\int \operatorname{sech}^2(\theta)\,d\theta \quad t = \tanh x \ldots ?$

i) $\int \frac{x^2+2}{7-x^2} \Rightarrow -\int \frac{-2-x^2}{7-x^2} \Rightarrow -\int \frac{7-x^2}{7-x^2} - \frac{9}{7-x^2}$

$-x + \int \frac{9}{7-x^2} = (x+\sqrt{7})(x-\sqrt{7}) \quad \frac{A}{x+\sqrt{7}} + \frac{B}{x-\sqrt{7}} \quad 9 = Ax - \sqrt{7}A + Bx + \sqrt{7}B$

$A + B = 0 \quad +\sqrt{7}B + \sqrt{7}B = 9 \quad 2\sqrt{7}B = 9 \quad B = \frac{9}{2\sqrt{7}}$

$-x + \int \frac{-\frac{9}{2\sqrt{7}}}{x+\sqrt{7}} + \frac{\frac{9}{2\sqrt{7}}}{x-\sqrt{7}} = -x + \frac{9}{2\sqrt{7}}\int\left[\frac{-1}{x+\sqrt{7}} + \frac{1}{x-\sqrt{7}}\right]$

$-x + \frac{9}{2\sqrt{7}}\left[-\ln(x+\sqrt{7}) + \ln(x-\sqrt{7})\right] \Rightarrow \boxed{\frac{9}{2\sqrt{7}}\ln\left(\frac{x-\sqrt{7}}{x+\sqrt{7}}\right) - x} + C$

j. $\int \frac{1}{ap-bp^2}\,dp \qquad \frac{A}{p} + \frac{B}{a-bp}$

ODE's

a) $\frac{dx}{dt} = ex \qquad x(0) = 1.0$

$\int \frac{1}{ex}\,dx = \int dt \qquad t = \frac{1}{e}\ln(x) \qquad x = e^{te} + 0$

b) $\frac{dx}{dt} = 3tx \qquad x(0) = 1.0$

$\int \frac{1}{x}\,dx = 3t\,dt + C \qquad \ln(x) = \frac{3}{2}t^2 + C$

$x = e^{3/2 t^2 + 0}$

c) $\frac{dx}{dt} = 0.1x - 0.003x^2 \qquad x(0) = 4$

$\int \frac{1}{0.1x - 0.003x^2}\,dx = \int dt \qquad t =$

$-1000 \int \frac{1}{3x^2 - 100x}\,dx \qquad -1000 \int \frac{1}{x^2}\left(\frac{1}{3 - 100/x}\right) dx$

$u = 3 - \frac{100}{x} \qquad du = \frac{100}{x^2}\,dx \Rightarrow dx = \frac{x^2}{100}\,du$

$-1000 \int \left(\frac{1}{3 - \frac{100}{x}}\right) \frac{du}{100} \qquad -10 \int \ln(u) + C = -10 \ln\left(3 - \frac{100}{x}\right) + C$

$e^{\left(\frac{t}{-10} - C\right)} = 3 - \frac{100}{x} \qquad -(100) / \left(e^{t/-10 - C} - 3\right) = x \qquad t = \frac{-100}{e^{-C} - 3} = 4 \qquad e^{-C} = 28$

d) $\frac{dx}{dt} = 0.1x - 0.003x^2 \;, \quad x(0) = 400 \qquad e^{-C} - 3 = 25 \quad -C = \ln$

$(C = -3.33)$

$\frac{-100}{\quad} = 400$

$e^{-C} - 3 = 0.25$

$e^{-C} = 3.25$

$C = -1.1786 \qquad \ln 3.25 = -C$

c) $x = \frac{-100}{e^{((t/-10) + \ln(28))} - 3}$

d $x = \frac{-100}{e^{((t/-10) + \ln(3.25))} - 3}$