I.1 Antiderivatives and integration basics

a. Explain briefly in your own words why we write $+C$ after evaluating an indefinite integral.

b. Let $f(x) = 8x^7 - \sin x$. If $F(x)$ satisfies $F'(x) = f(x)$ and $F(0) = 0$, what is $F(x)$?

c. Evaluate the indefinite integral $\int \frac{4x^2}{x^2+1}\,dx$. *Hint:* $x^2 = x^2 + 1 + (-1)$.

I.2 Definite integrals and area

a. Evaluate the definite integral

$$\int_1^2 \frac{dx}{4\sqrt{2x-1}}$$

You may leave any radicals in your answer uncalculated.

b. If $f(x)$ is a (mystery) function that satisfies $f(-x) = f(x)$ for all x, and

$$\int_0^1 f(x)\,dx = 2\pi$$

Then what is the value of $\displaystyle\int_{-2}^{2} f(x/2)\,dx$?

I.3 Integration by substitution

a. Which of the following integrals can be solved easily using u-substitution? After circling your answer, write down your choice for u next to the integral. (Do not evaluate the integral.)

   i. $\displaystyle\int x^2 \sin(x^2)\, dx$

   ii. $\displaystyle\int x^3 \sin(x^2)\, dx$

   iii. $\displaystyle\int x^2 \sin(4x^3)\, dx$

   iv. $\displaystyle\int x^2 \sin(3x^2)\, dx$

b. Evaluate the indefinite integral $\displaystyle\int xe^{-x^2}\, dx$.

c. Evaluate the indefinite integral $\displaystyle\int \frac{\cos x}{\sin x \cdot \ln \sin x}\, dx$.

VI.1 Integration by Parts

a. Of the six integrals below, circle those for which *substitution doesn't work* and *integrating by parts with $u = x$ yields a $\int v\,du$ that can be evaluated easily* (i.e. without a second round of IbP or trigonometric substitution)

i. $\displaystyle\int x \sin(x^2)\,dx$ ii. $\displaystyle\int x \cos x\,dx$ iii. $\displaystyle\int x e^x\,dx$

iv. $\displaystyle\int x\sqrt{1-x^2}\,dx$ v. $\displaystyle\int x^3\sqrt{1-x^2}\,dx$ vi. $\displaystyle\int x \tan x\,dx$

b. Evaluate $\displaystyle\int x \sin x\,dx$.

c. Evaluate $\displaystyle\int_0^\pi x \sin x\,dx$ (you may of course use your answer to part b.).