Math 112
Exam 2
March, 2013
Professor Hopkins

Name: ______________________________

e-mail: ______________________________

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2.		5.	
3.		6.	
		Total	

Make sure your name is written on every page of this exam.

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1. (20 pts) Suppose that X is a metric space, and that $p_1, p_2, \ldots$ is a sequence of points in X converging to a point p. Show that the set

$$P = \{p\} \cup \{p_1, p_2, \ldots\}$$

is compact.

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2. (5 pts each) True or False.

(a) A continuous function is differentiable.

(b) If X and Y are metric spaces $f : X \to Y$ is continuous, and $K \subset Y$ is compact, then $f^{-1}K$ is compact.

(c) If $f : X \to Y$ is a function, then $f(U \cap V) = f(U) \cap f(V)$.

(d) If $f : X \to Y$ is a function, then $f(U \cup V) = f(U) \cup f(V)$.

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3. (20 pts) Suppose that $f : [a, b] \to \mathbb{R}$ is differentiable at x, and that $f'(x) > 0$. Prove that there exists $\epsilon > 0$ such that if t satisfies $0 < t - x < \epsilon$ then

$$f(t) > f(x).$$

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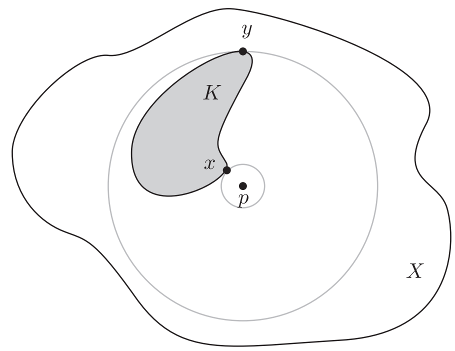
4. (20 pts) Suppose that X a metric space and $p \in X$ is a point. Show that the function

$$f : X \to \mathbb{R}$$

defined by $f(x) = d(p, x)$ is continuous. Show that if $K \subset X$ is compact and $p \notin K$ then there are points $x, y \in K$ such that for all $z \in K$ one has $d(p, x) \leq d(p, z)$ and $d(p, y) \geq d(p, z)$.



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5. **(20 pts)** Let X be a metric space and $E \subset X$ a non-empty subset. For $x \in X$ define the *distance from x to E* by

$$\rho_E(x) = \inf_{z \in E} d(x, z).$$

Show that ρ_E is uniformly continuous.

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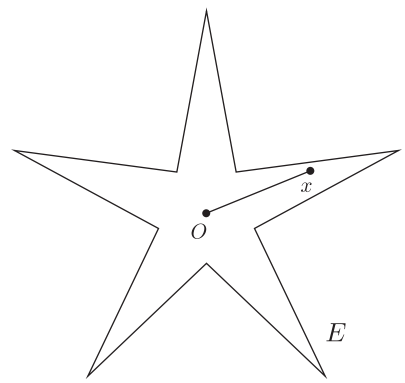
6. **(20 pts)** A subset $E \subset \mathbb{R}^n$ is said to be *starlike* if there is a point $O \in E$ with the property that for each $x \in E$ and each $0 \le t \le 1$, the point $t\,O + (1-t)\,x$ is in E. Geometrically this says that E contains the straight line from O to x, as illustrated below. Let $E \subset \mathbb{R}^n$ be starlike, and $x, y \in E$ two points. Let $f : E \to \mathbb{R}$ be a continuous function. Write $f(x) = a$, $f(y) = b$ and suppose $a < b$. Show that if c is between a and b there is a point $z \in E$ with $f(z) = c$.



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