

Math 19: Fall 2014  
Midterm 1  
Blue version

**NAME:**

**LECTURE:**

Time: **75 minutes**

For each problem, you should write down all of your work carefully and legibly to receive full credit. When asked to justify your answer, you should use theorems and/or mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the Stanford Honor Code:

- You may not use a calculator or any notes or book during the exam.
- You may not access your cell phone during the exam for any reason.
- You are required to sit in your assigned seat.
- You are bound by the Stanford Honor Code, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: \_\_\_\_\_

Problem	Value	Score
1	10	
2	10	
3	5	
4	5	
5	28	
6	8	
7	14	
8	11	
9	9	
TOTAL	100	

**Problem 1 : (10 points)**

a) (4 points) Suppose that  $f$  is a function that is one-to-one. What is the definition of the inverse of  $f$ ?

b) (4 points) Consider the function  $f: [-1, 1] \rightarrow \mathbb{R}$ ,  $f(x) = \arcsin x$ . What **function** is the inverse of  $f$ ?

c) (2 points) Find the value:

$$\arcsin \left( \sin \left( \frac{7\pi}{6} \right) \right).$$

**Problem 2 : (10 points)**

a) Simplify the following number completely:

$$\sin\left(\frac{5\pi}{12}\right)$$

Hint:  $\frac{5}{12} = \frac{1}{4} + \frac{1}{6}$

b) Simplify the following number completely:

$$\log_2 8 - \log_2 \frac{1}{4}$$

**Problem 3 : (5 points)** Give the form of the partial fraction decomposition. You do not need to solve for the constants.

$$\frac{2x^2 + x + 1}{x^3 - x^2 + x - 1}$$

**Problem 4 : (5 points)** Let  $\theta$  be angle such that  $\sin \theta = \frac{x}{4}$ . Write down an expression for  $\tan \theta$  in terms of  $x$ .

**Problem 5 : (28 points)** Compute the following limits, and answer the follow-up question.

a)     i.  $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

- ii. Let  $f$  be a function given by the rule  $f(x) = \frac{\sqrt{x}-1}{x-1}$ . What is the behavior of  $f$  near  $x = 1$ ? Is  $f$  continuous? If not, does  $f$  have a removable discontinuity, a jump discontinuity, or an infinite discontinuity? In the case where  $f$  has a jump discontinuity or an infinite discontinuity, compute the one-sided limits of  $f$  on both sides of the discontinuity.

b)    i.  $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{|x + 2|}$

- ii. Let  $f$  be a function given by the rule  $f(x) = \frac{x^2+3x+2}{|x+2|}$ . What is the behavior of  $f$  near  $x = -1$ ? Is  $f$  continuous? If not, does  $f$  have a removable discontinuity, a jump discontinuity, or an infinite discontinuity? In the case where  $f$  has a jump discontinuity or an infinite discontinuity, compute the one-sided limits of  $f$  on both sides of the discontinuity.

c)    i.  $\lim_{x \rightarrow -2} \frac{x^2 + 3x + 2}{|x + 2|}$

- ii. Let  $f$  be a function given by the rule  $f(x) = \frac{x^2+3x+2}{|x+2|}$ . What is the behavior of  $f$  near  $x = -2$ ? Is  $f$  continuous? If not, does  $f$  have a removable discontinuity, a jump discontinuity, or an infinite discontinuity? In the case where  $f$  has a jump discontinuity or an infinite discontinuity, compute the one-sided limits of  $f$  on both sides of the discontinuity.



d)    i.  $\lim_{x \rightarrow 2} \frac{x^2 + 3x + 2}{x^2 - 4x + 4}$

- ii. Let  $f$  be a function given by the rule  $f(x) = \frac{x^2+3x+2}{x^2-4x+4}$ . What is the behavior of  $f$  near  $x = 2$ ? Is  $f$  continuous? If not, does  $f$  have a removable discontinuity, a jump discontinuity, or an infinite discontinuity? In the case where  $f$  has a jump discontinuity or an infinite discontinuity compute the one-sided limits of  $f$  on both sides of the discontinuity.

**Problem 6 : (8 points)** Let  $f$  be a function given by the rule

$$f(x) = \frac{x+2}{x+5}.$$

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a) Give the rule for the function  $f^{-1}$ .

b) What is the domain of  $f^{-1}$ ? Give your answer in interval notation.

c) What is the range of  $f$ ? Give your answer in interval notation.

**Problem 7 : (14 points)** Consider the function  $f$  given by the rule

$$f(x) = \frac{3x^2 + 9x + 6}{x^2 + 3x + 2}.$$

a) (3 points) Find all the  $x$ - and  $y$ -intercepts of  $f$ .

b) (4 points) Use limits to find the equation(s) for all horizontal asymptote(s) of  $f$ .

c) (3 points) Find the equation(s) for all vertical asymptote(s) of  $f$ .

- d) (4 points) For each vertical asymptote, find out the behavior of the function on either side of the asymptote.

**Problem 8 : (11 points)** Consider the function  $f: [-8, \infty) \rightarrow \mathbb{R}$  given by the rule

$$f(x) = \begin{cases} x + 5 & \text{if } -8 \leq x \leq -4, \\ (x + 4)^2 + 1 & \text{if } -4 < x < 0, \\ 3 - x & \text{if } 0 \leq x. \end{cases}$$

a) (5 points) Sketch the graph of  $y = f(x)$ .

b) (2 points) Is  $f$  one-to-one? Justify.

Recall the function we are considering in this problem:  $f: [-8, \infty) \rightarrow \mathbb{R}$ ,

$$f(x) = \begin{cases} x + 5 & \text{if } -8 \leq x \leq -4, \\ (x + 4)^2 + 1 & \text{if } -4 < x < 0, \\ 3 - x & \text{if } 0 \leq x. \end{cases}$$

c) (4 points) Is  $f$  continuous at  $x = -4$ ? Justify.

**Problem 9 : (9 points)**

- a) (5 points) Prove that the polynomial  $f(x) = x^5 - 4x + 2$  has a root in the interval  $[-1, 1]$ .

- b) (4 points) Can the same be said of the rational expression  $g(x) = \frac{x-2}{x}$  ?