Benchmarks for subduction zone models

Subduction zone workshop, University of Michigan, July 2003

Introduction

In early October 2002 a group of researchers met at the University of Michigan at Ann Arbor, MI to discuss the modeling of the thermal structure and dynamics of subduction zones (www.geo.lsa.umich.edu/~keken/subduction02.html). At the workshop it became clear that the community could greatly benefit from a set of benchmarks that allow for code testing and comparisons. We can identify two fundamental approaches for subduction zone modeling: a) fully dynamic, where the deformation of the slab is computed using descriptions of rheology and buoyancy; and b) wedge dynamical models, where the geometry and velocity of the slab is imposed kinematically, with a dynamic solution only for the wedge. This draft paper formulates a set of benchmarks of increasing complexity focusing on the second category. The first category requires a fundamentally more difficult approach. For a discussion of a number of potential dynamic benchmarks see www.geobench.org. We hope that this this set of benchmarks will evolve to a standard in subduction modeling, similar to the role of the mantle convection benchmarks formulated in Blankenbach et al., 1989; Busse et al., 1993; and Van Keken et al., 1997.

Description of governing equations and parameters

Conservation of mass for incompressible fluid:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

Heat transport equation for an incompressible medium:

$$\rho c_p (\mathbf{v} \cdot \nabla) T = \nabla \cdot (k \nabla T) + Q + Q_{sh} \tag{2}$$

Conservation of momentum for viscous flow:

$$\nabla \cdot \tau - \nabla P = \mathbf{0} \tag{3}$$

with deviatoric stress tensor τ:

$$\tau = 2\eta\dot{\varepsilon} \tag{4}$$

and components of the strain rate tensor $\dot{\varepsilon}$:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] \tag{5}$$

The effective shear viscosity η follows from (4):

$$\eta = \frac{\tau}{2\dot{\varepsilon}} \tag{6}$$

where the second invariants of the strain-rate and stress tensors are defined by $\dot{\varepsilon} = [\frac{1}{2}\sum_{ij}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}]^{\frac{1}{2}}$ and $\tau = [\frac{1}{2}\sum_{ij}\tau_{ij}\tau_{ij}]^{\frac{1}{2}}$.

A general equation for the viscosity of olivine deformation by diffusion creep is

$$\eta(T) = A_{diff}\, e^{(E_{diff}+pV_{diff})/RT} \tag{7}$$

and for deformation by dislocation creep

$$\eta(T, \dot\varepsilon) = A_{disl}\, e^{(E_{disl}+pV_{disl})/nRT}\ \dot\varepsilon^{(1-n)/n} \tag{8}$$

See, for example, Karato and Wu (1993).

Definition of symbols

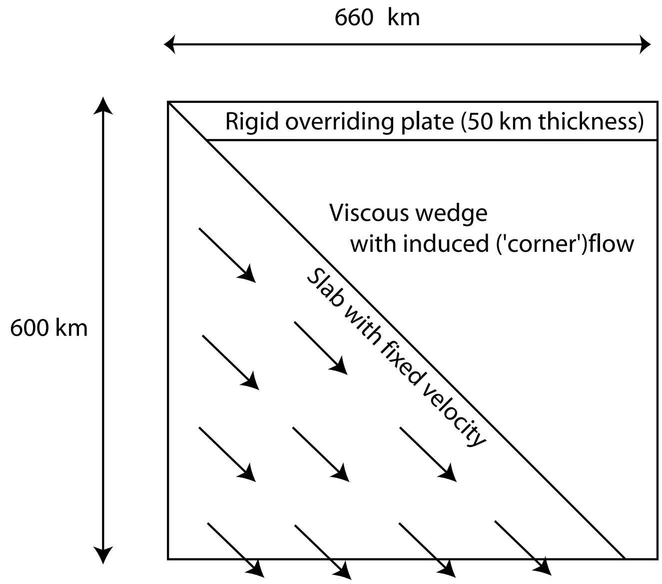
Quantity	Symbol	Reference value and/or SI units
Velocity	$\mathbf{v}$	m/s
Dynamic viscosity	η	$\eta_0 = 10^{21}$ Pa $\cdot$ s
Stress tensor	τ	Pa
Strain rate tensor	$\dot\varepsilon$	1/s
Dynamic pressure	P	Pa
Dynamic pressure gradient	∇P	Pa/m
Density	ρ	$\rho_0 = 3300$ kg/m^3
Temperature	T	$T_0 = 1573$ K = 1300 °C
Thermal conductivity	k	$k = 3$ W/mK
Heat capacity	c_p	1250 J/kgK
Radiogenic heating	Q	W/m^3
Shear heating	Q_{sh}	W/m^3
Thermal diffusivity	$\kappa = k/\rho c_p$	0.7272×10^{-6} m^2/s
Activation energy for diffusion creep	E_{diff}	335 kJ/mol
Activation energy for dislocation creep	E_{disl}	540 kJ/mol
Powerlaw exponent for dislocation creep	n	3.5
Pre-exponential constant for diffusion and dislocation creep	A_{diff}, A_{disl}	Pa $\cdot$ s$^{1/n}$
Hydrostatic pressure	p	Pa
Activation volume for diffusion and dislocation creep	V_{diff}, V_{disl}	0 m^3/mol
Gas constant	R	8.3145 J/molK

All models are 2D Cartesian (assuming no variation in the third dimension). For simplicity we will not make a distinction between potential and ’real’ temperature.

1. Constant viscosity wedge

Model description

Computational domain: box of 660x600 km (see Figure 1). Straight slab at 45 degree angle. The top of slab starts in the upper left hand corner at ($x = 0$ km, $z = 0$ km). The mantle below the slab moves with the same velocity. The speed of the slab is 5 cm/yr. The overriding plate above the wedge is assumed rigid and extends to a depth of 50 km. Reference values for k, ρ, c_p, and constant viscosity η. Note that this yields a value for κ which is not equal to the often used standard value of 10^{-6}. No radiogenic or shear heating. Age of the incoming lithosphere at x=0 is 50 Myr.



Benchmark cases

a) Temperature field using the analytical Batchelor solution.
Use the analytical expression for cornerflow (e.g., Batchelor, 1967) to describe (u,v). This model requires only the solution of the temperature equation with the following boundary conditions: $T_s = 273\ K$ at the surface. $T_m = 1573$ K at the inflow portion of the wedge (below the overriding plate). This results in a linear temperature gradient in the overriding plate at the left hand boundary. The temperature at the slab inflow boundary is described by the standard error function consistent with the 50 Myr age of the lithosphere

$$T(z) = T_s + (T_m - T_s) erf\left(\frac{z}{2\sqrt{\kappa T_{50}}}\right)$$

where T_{50} is the age of the lithosphere in seconds. At the slab and wedge outflow boundaries: $\nabla T \cdot \mathbf{v} = 0$. Specify $P = 0$ at the inflow boundary at the interface between overriding plate and wedge ($x = 660$ km, $z = 50$ km).

b) Dynamically computed wedge flow - 1.
As in a), but now with a dynamical solution for the velocity in the wedge only. The flow is kinematically driven by the slab (i.e., no buoyancy due to thermal expansion). In order to compare results with a) we will describe the same boundary conditions for velocity: velocity is 0 at the top of the wedge. The velocity is equal to the slab velocity at the slab-wedge interface. The velocity of the inflow and outflow boundary are imposed by the Batchelor solution. See for example the results shown in Appendix A of Van Keken et al. (2003).

c) Dynamically computed wedge flow - 2.
As in b), but now with natural boundary conditions for stress at the inflow and outflow boundary. Both the normal and the tangential component of the total stress $\tau - P\mathbf{I}$, where $\mathbf{I}$ is the identity tensor should be set to zero.

Requested output:

For each of these models provide
Required:
2D arrays of T, P, dT/dx, dT/dz, dP/dx and dP/dz. Provide these quantities at x_i, z_j, where $x_i = (i-1)\Delta x$ and $z_j = (j-1)\Delta z$ with $\Delta x = \Delta z = 6$ km. This allows us to difference various solutions as function of type of code, assumptions about boundary conditions and resolution of the computational grid. Also provide heat flow in this output grid points at the surface $(x_i, 0)$.
Format of the output files. Please provide these quantities in **dimensional** units (T in °C, P in Pa, heat flow in W/m^2, and the pressure and temperature derivatives in Pa/m and °C/m). The 2D arrays should be in ASCII (text) files with 101 rows, each containing 111 numbers. The top row corresponds to the surface of the model.
Optional:
Integrated quantities, e.g., rms-velocity, average temperature, average pressure, and average dissipation in the wedge.

Possible extensions:
1) Inclusion of shear heating along the seismogenic zone; 2) Inclusion of radiogenic heating in the overriding plate; 3) Variations of subduction speed and age of the incoming lithophere; 4) Variations of the subduction angle.

2. Dynamic wedge flow with variable viscosity

Model description

As in 1c, but now with variable viscosity with general form for olivine with constant grainsize (eqns 7+8).

Benchmark cases

a) Diffusion creep
Use viscosity formulation (7) with $E_{diff} = 335$ kJ/mol, $V_{diff} = 0$, potential temperature T, and pre-exponential factor $A_{diff} = 1.32043 \times 10^9$ Pa $\cdot$ s. This prefactor follows from the arbitrary assumption that $\eta = \eta_0$ at 1473 K.

b) Dislocation creep
Use the viscosity formulation (8) with $E_{disl} = 540$ kJ/mol, $V_{disl} = 0$, $n = 3.5$, potential temperature T and pre-exponential factor $A_{disl} = 28968.6$ Pa $\cdot$ $s^{1/n}$ (corresponding to the dislocation creep parameters of Karato and Wu, 1993).

Requested output

As in 1. For the non-Newtonian case also provide an output fi le containing viscosity.

3. Test for convection dominated flows

Model description

a) Transport of smooth function
As in 1a (isoviscous wedge using analytical Batchelor solution) but with low diffusivity and variable temperature at inflow boundary, e.g.,

$$T(0,z) = \cos(n\pi(z - z_0)/\lambda)$$

where z_0 is the thickness of the overriding plate (50 km) and λ is the distance between the base of the overriding plate and the transition between in- and outflow of the Batchelor solution. Use $n = 1, n = 3, n = 5 \cdots$.

b) Transport of discontinuous function
As in 3a), but now with a discontinuous function of the form

$$T(0,z) = T_0 \quad \text{if } \cos(n\pi(z - z_0)/\lambda) \le 0$$

$$T(0,z) = T_0 + \Delta T \quad \text{if } \cos(n\pi(z - z_0)/\lambda) > 0$$

Requested output:

Show the temperature at the outflow boundary of the wedge.

References

Batchelor, G.K., An introduction to fluid dynamics, 615 pp., Cambridge University Press, New York, 1967.

Blankenbach, B., F. Busse, U. Christensen, L. Cserepes, D. Gunkel, U. Hansen, H. Harder, G. Jarvis, M. Koch, G. Marquart, D. Moore, P. Olson, H. Schmeling, T. Schnaubelt, A benchmark comparison for mantle convection codes, *Geophys. J. Int.*, *98*, 23-38, 1989.

Busse, F.A., Christensen, U., Clever, R., Cserepes, L., Giannandrea, E., Guillou, L., Nataf, H.-C., Ogawa, M., Parmentier, M., Sotin, C., and Travis, B, Convection at infinite Prandtl number in Cartesian geometry - a benchmark comparison. *Geophys. Astrophys. Fluid Dyn.*, *75*, 35-59, 1993.

Karato, S., and P. Wu, Rheology of the upper mantle: a synthesis, *Science, 260*, 771-778, 1993.

Van Keken, P.E., King, S.D., Schmeling, H., Christensen, U.R., Neumeister, D., Doin, M.P., A comparison of methods for the modeling of thermochemical convection, *J. Geophys. Res.*, *102*, 22,477-22,495, 1997.

Van Keken, P.E., B. Kiefer, and S. Peacock, High resolution models of subduction zones: implications for mineral dehydration reactions and the transport of water into the deep mantle, *Geochemistry, Geophysics, and Geosystems, 3*, 2002. doi: 10.1029/2001GC000256.