

Table 5.1 Common EDMs, showing their variance function $V(\mu)$, cumulant function $\kappa(\theta)$, canonical parameter θ , dispersion parameter ϕ , unit deviance $d(y, \mu)$, support S (the permissible values of y), domain Ω for μ and domain Θ for θ . For the Tweedie distributions, the case $\xi = 2$ is the gamma distribution, and $\xi = 1$ with $\phi = 1$ is the Poisson distribution. \mathbb{R} refers to the real line; \mathbb{N} refers to the natural numbers $1, 2, \dots$; superscript $+$ means positive values only; superscript $-$ means negative values only; subscript 0 means zero is included in the space (Sect. 5.3.5)

EDM	$V(\mu)$	$\kappa(\theta)$	θ	ϕ	$d(y, \mu)$	S	Ω	Θ	Reference
Normal	1	$\theta^2/2$	μ	σ^2	$(y - \mu)^2$	\mathbb{R}	\mathbb{R}	\mathbb{R}	Chaps. 2 and 3
Binomial	$\mu(1 - \mu)$	$\frac{\exp \theta}{1 + \exp \theta}$	$\log \frac{\mu}{1 - \mu}$	$\frac{1}{m}$	$2 \left\{ y \log \frac{y}{\mu} + (1 - y) \log \frac{1 - y}{1 - \mu} \right\}$	$0, 1, \dots, m$ m	$(0, 1)$	\mathbb{R}	Chap. 9
Negative binomial	$\mu + \frac{\mu^2}{k}$	$-\log(1 - \exp \theta)$	$\log \frac{\mu}{\mu + k}$	$\frac{1}{k}$	$2 \left\{ y \log \frac{y}{\mu} - (y + k) \log \frac{y + k}{\mu + k} \right\}$	\mathbb{N}_0	\mathbb{R}^+	\mathbb{R}^-	Chap. 10
Poisson	μ	$\exp \theta$	$\log \mu$	1	$2 \left\{ y \log \frac{y}{\mu} - (y - \mu) \right\}$	\mathbb{N}_0	\mathbb{R}^+	\mathbb{R}	Chap. 10
Gamma	μ^2	$-\log(-\theta)$	$-\frac{1}{\mu}$	ϕ	$2 \left\{ -\log \frac{y}{\mu} + \frac{y - \mu}{\mu} \right\}$	\mathbb{R}^+	\mathbb{R}^+	\mathbb{R}	Chap. 11
Inverse Gaussian	μ^3	$-\sqrt{-2\theta}$	$-\frac{1}{2\mu^2}$	ϕ	$\frac{(y - \mu)^2}{\mu^2 y}$	\mathbb{R}^+	\mathbb{R}^+	\mathbb{R}_0^-	Chap. 11
Tweedie ($\xi \leq 0$ or $\xi \geq 1$)	μ^ξ	$\frac{\{(1 - \xi)\theta\}^{(2 - \xi)/(1 - \xi)}}{2 - \xi}$	$\frac{\mu^{1 - \xi}}{1 - \xi}$	ϕ	$2 \left\{ \frac{\max(y, 0)^{2 - \xi}}{(1 - \xi)(2 - \xi)} - \frac{y\mu^{1 - \xi}}{1 - \xi} + \frac{\mu^{2 - \xi}}{2 - \xi} \right\}$	$\xi < 0: \mathbb{R}$ $1 < \xi < 2: \mathbb{R}_0^+$ $\xi > 2: \mathbb{R}^+$	\mathbb{R}^+	\mathbb{R}_0^+ \mathbb{R}^+ \mathbb{R}_0^-	Chap. 12
		for $\xi \neq 2$	for $\xi \neq 1$		for $\xi \neq 1, 2$				