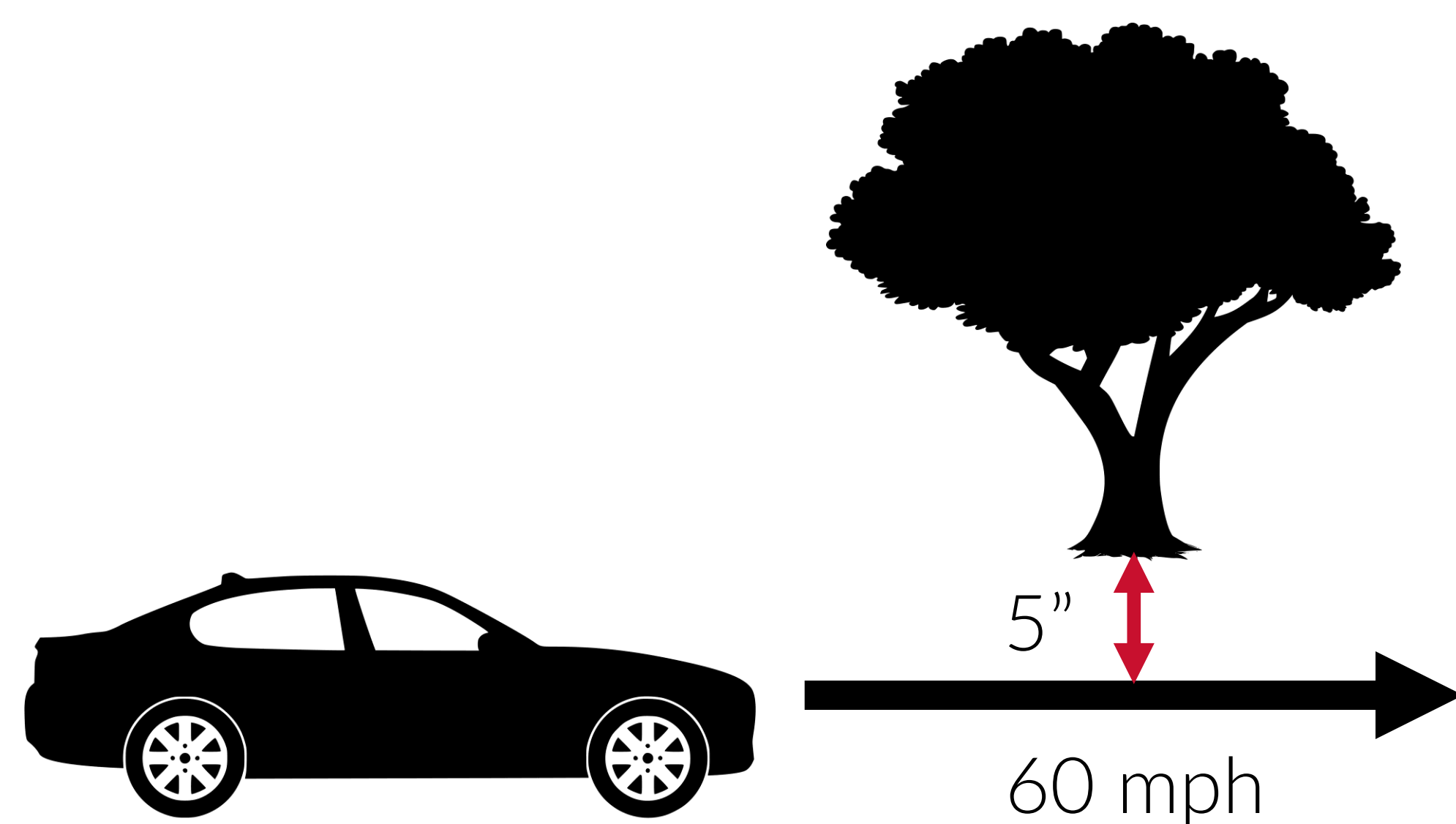


# Safety Analysis and Control using Measures

Jared Miller, Mario Sznaier (Robust Systems Lab)

## Background

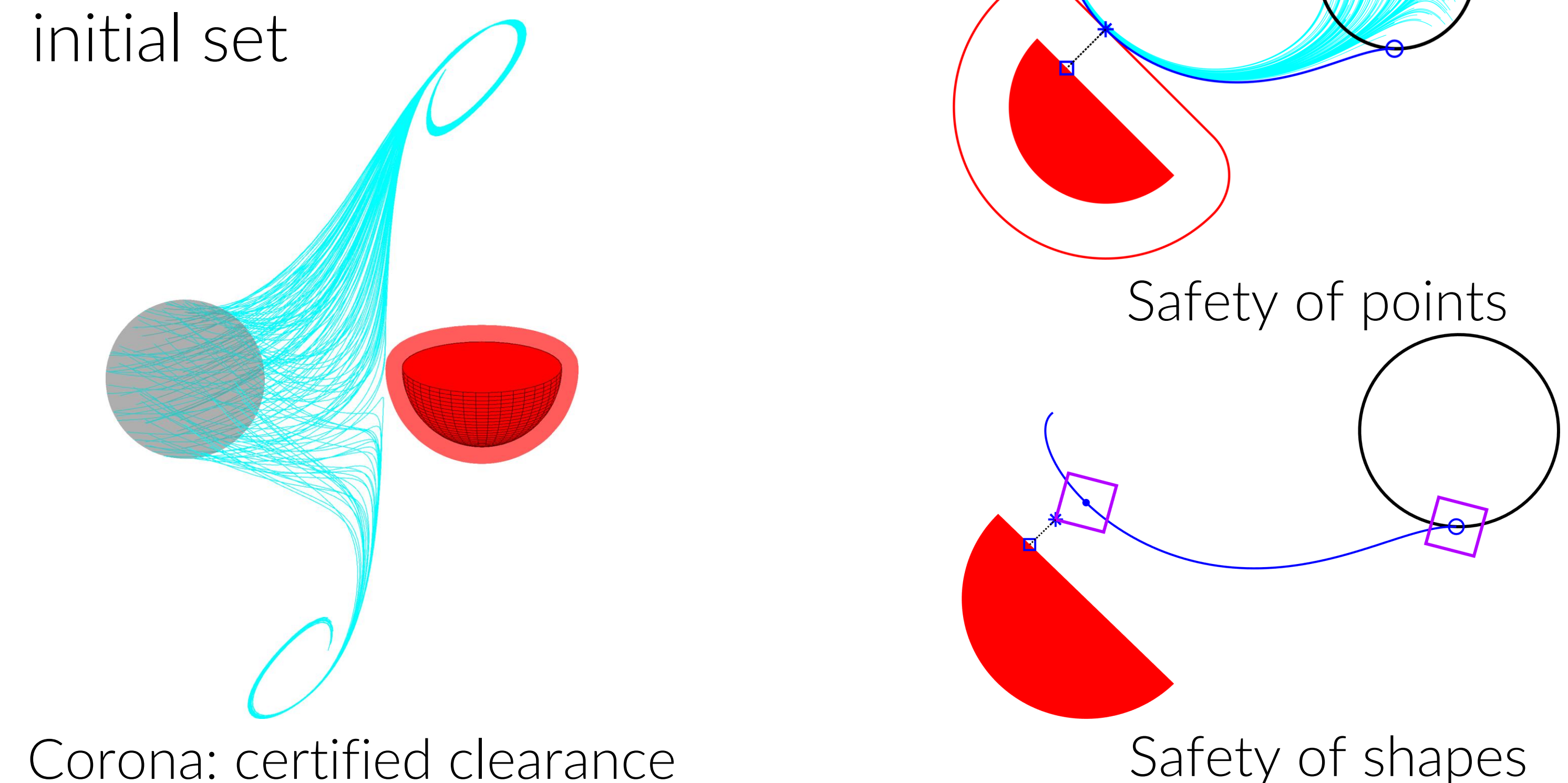
Quantify the safety of trajectories



Use convex optimization to compute convergent distance-lower-bounds

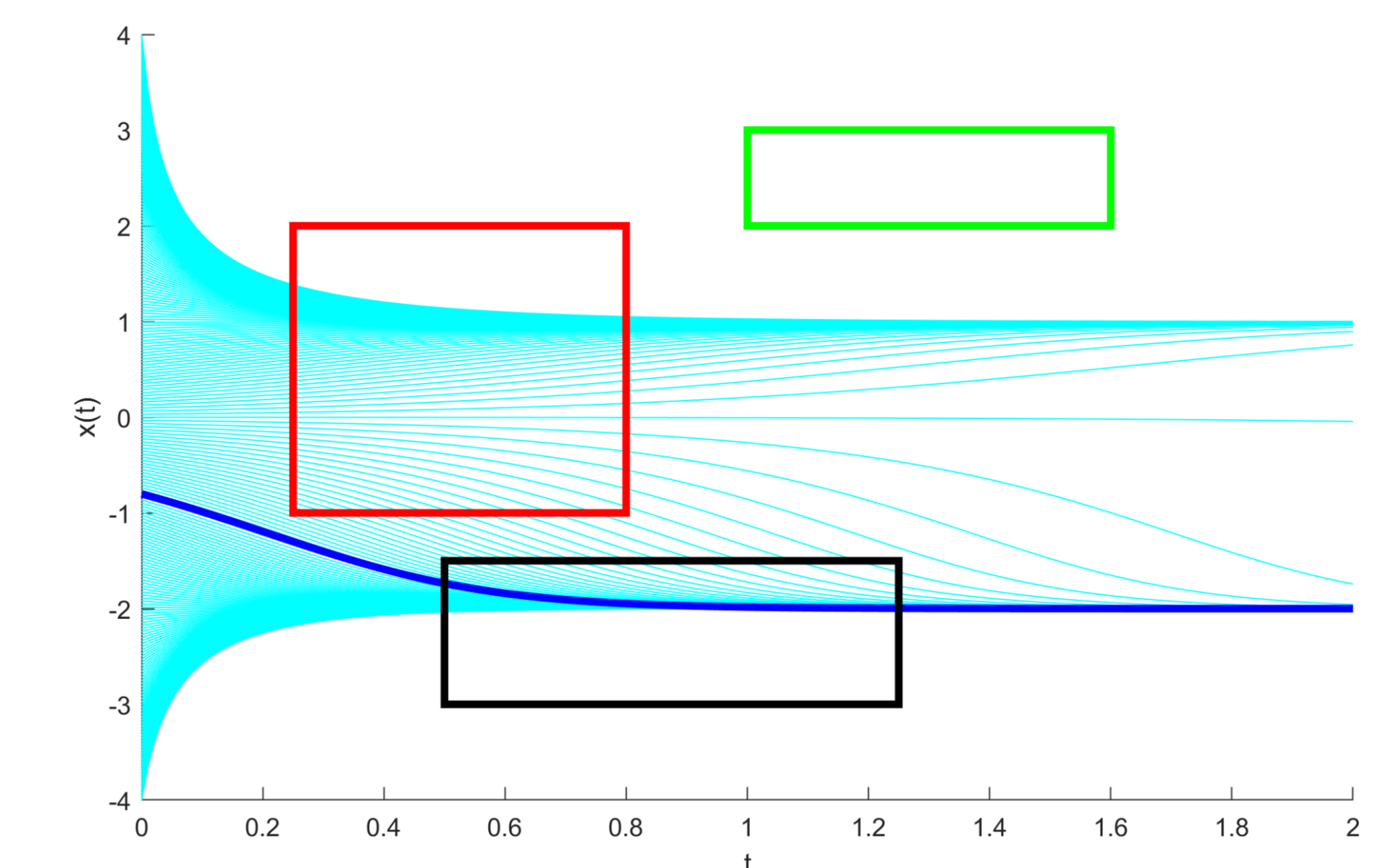
## Distance Estimation

Find minimum distance from **unsafe set** starting from initial set



## Occupation Measures

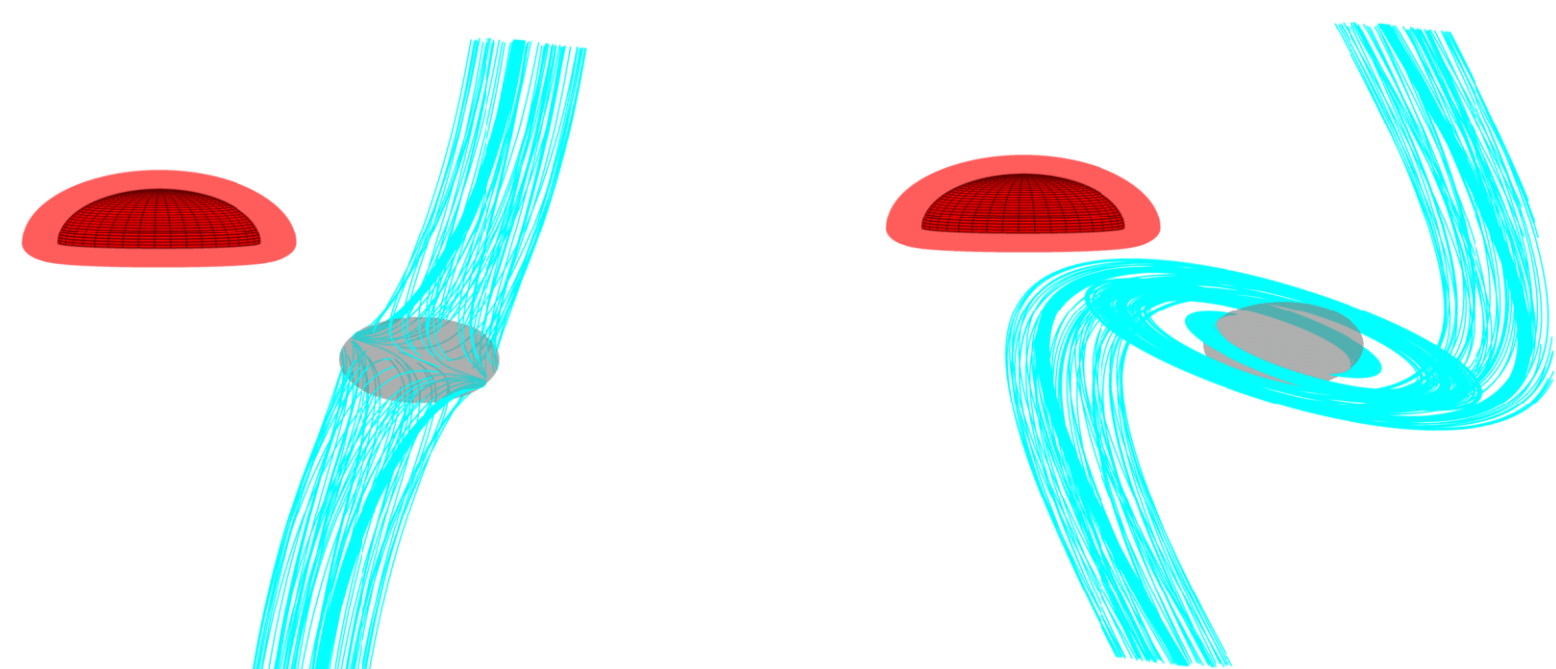
Returns the time trajectories spend in each set (given initial distribution)



Infinite-dimensional linear program

## Hybrid Systems

Continuous dynamics with jumps  
Guards and reset maps (e.g., contact)



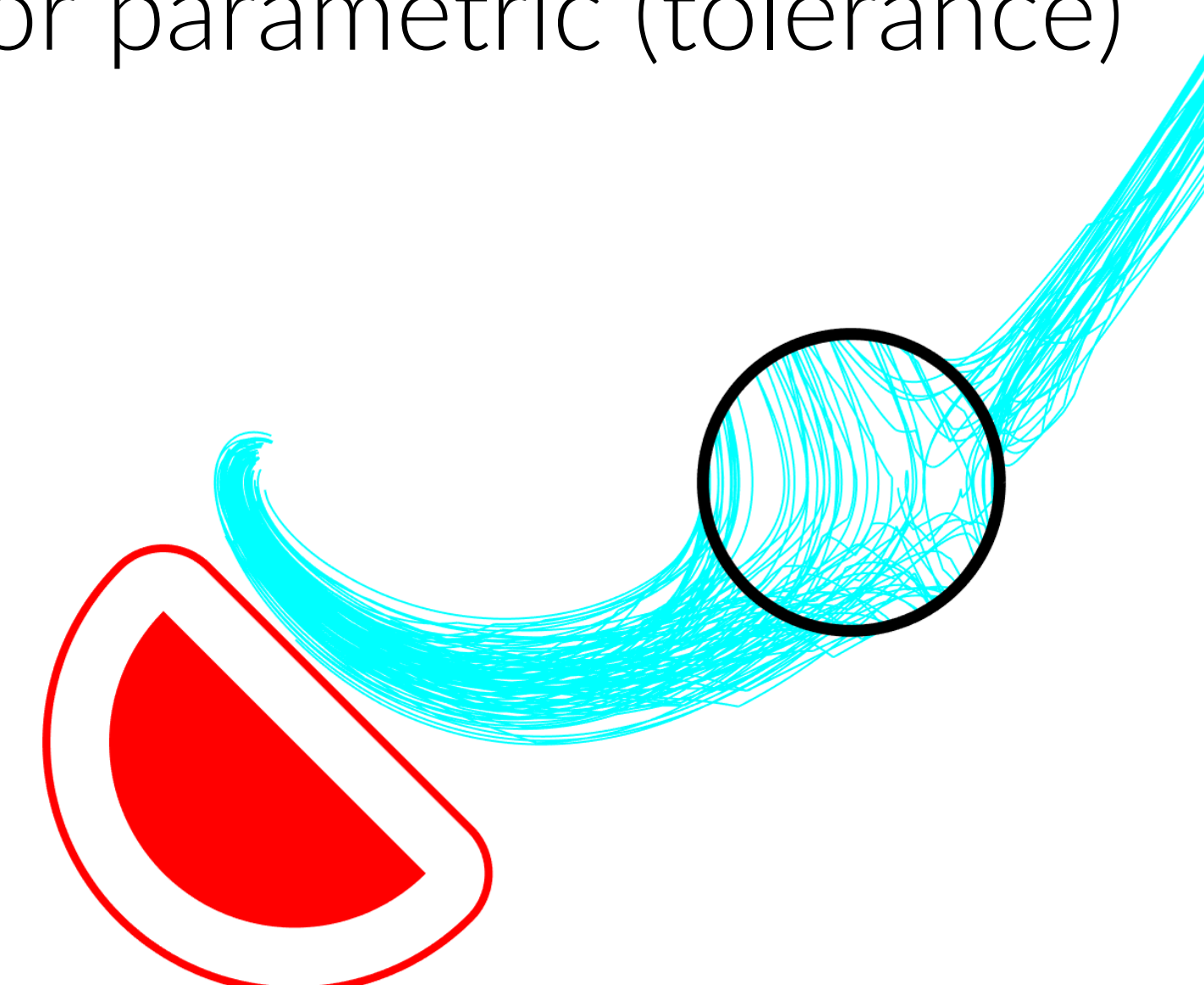
Diverge to boundary      Controlled to gray ball



Further details  
and papers

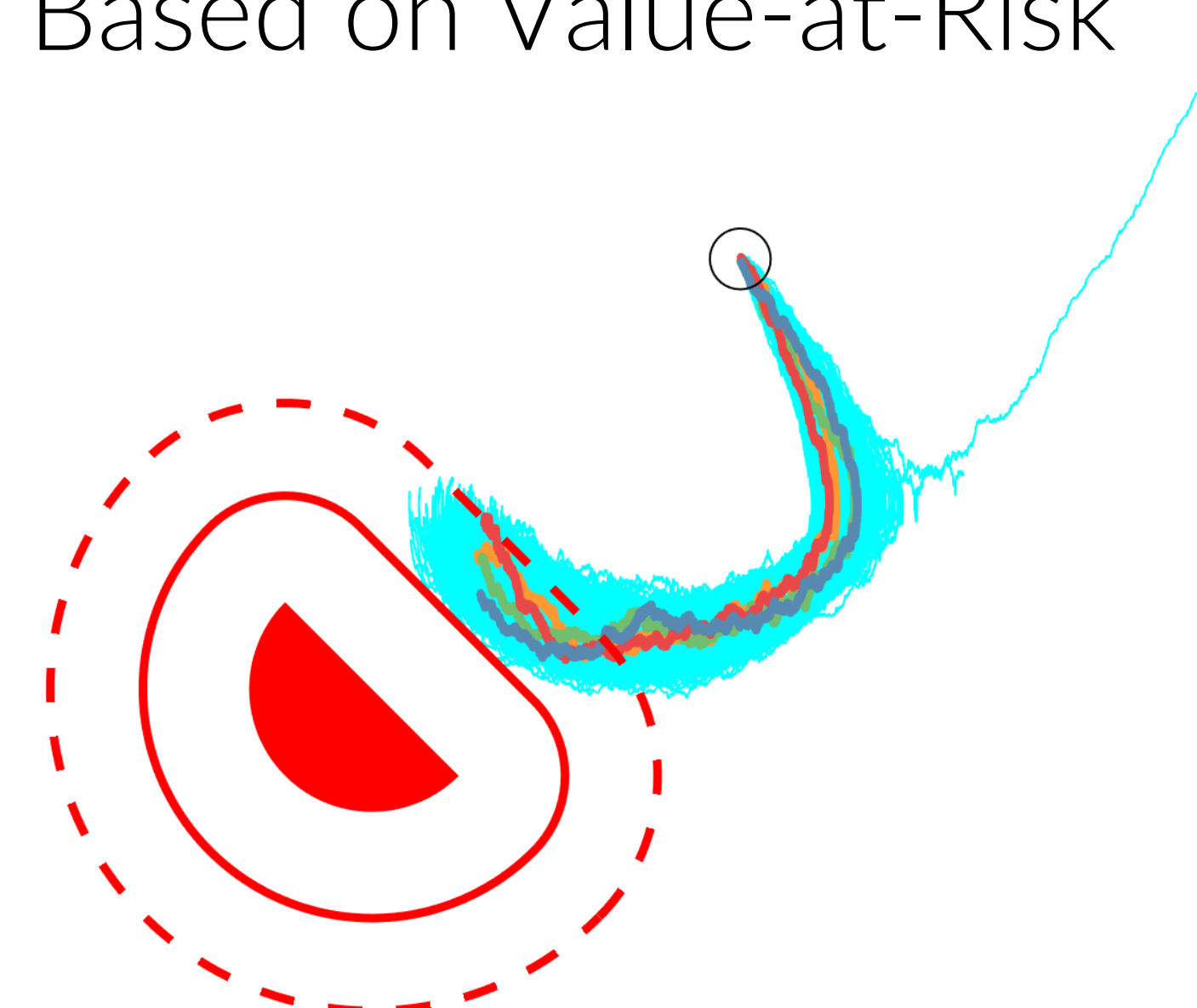
## Uncertain Systems

Bounded uncertainty  
Time dependent (wind)  
or parametric (tolerance)



Exploit switching structure

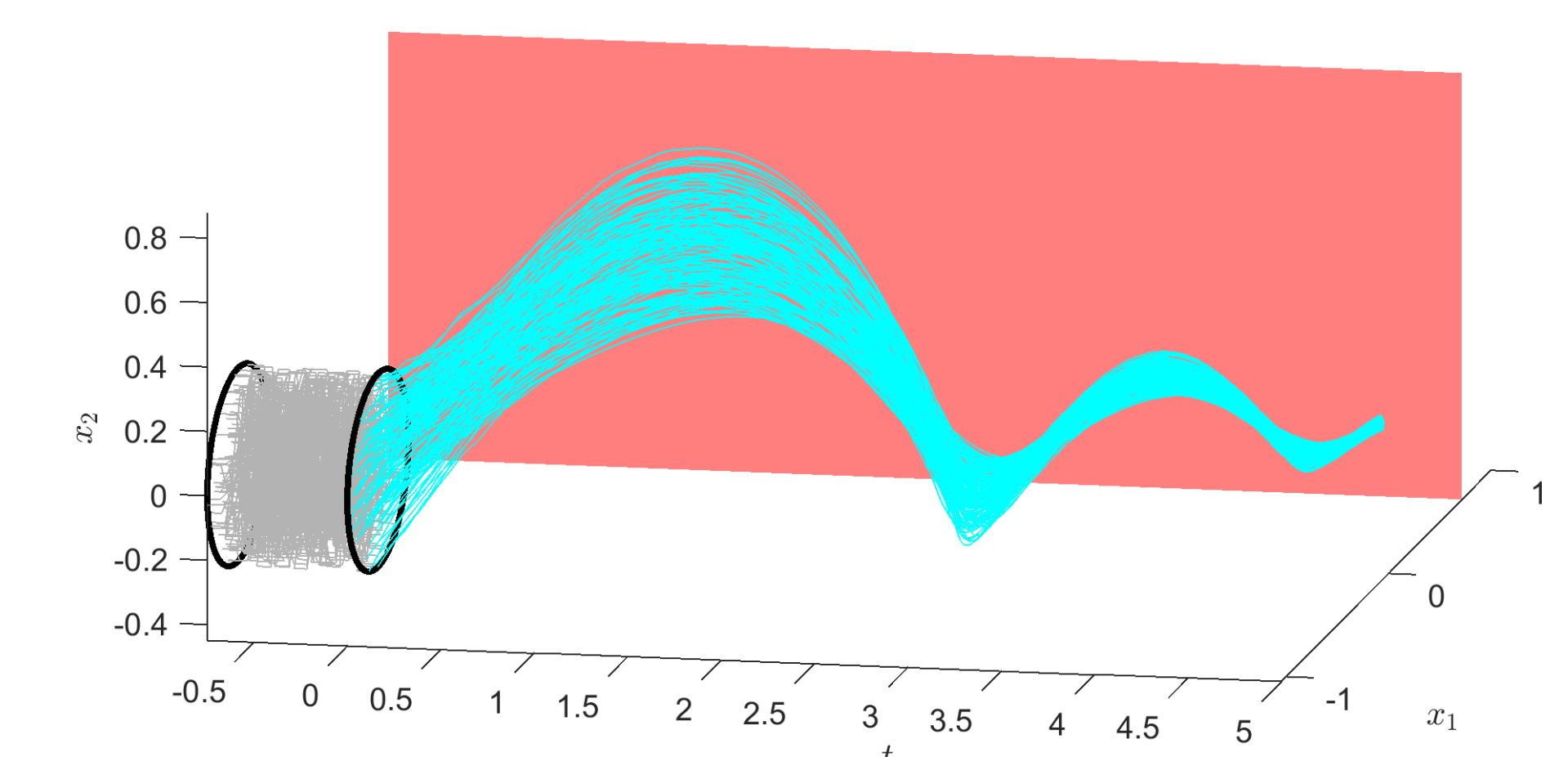
Unbounded (white noise)  
Probabilistic peak value  
Based on Value-at-Risk



Dotted: 50% bound, Solid: 85%

## Time-Delay Systems

Discrete time delay  $\tau$ , history  $x_h$   
Examples: epidemics, population  
 $\dot{x}(t) = f(t, x(t), x(t - \tau)) \quad \forall t \in [0, T]$   
 $x(s) = x_h(s) \quad \forall s \in [-\tau, 0]$



Gray: history, Cyan: trajectory