Exploiting Structure in Rank-Constrained and Approximated Semidefinite Programs

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Collaborators

Joint work with:

- Yang Zheng[2]
- Mario Sznaier[1]
- Antonis Papachristodoulou[3]
- Biel Roig-Solvas[1]

1. Northeastern University
2. Harvard University
3. Oxford University

SDPs and Decompositions

Semidefinite Programs

Standard problem in convex optimization

$$\begin{aligned} \min_{X} \quad & \langle C, X \rangle \\ \text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\ & X \in \mathbb{S}^n_+ \end{aligned}$$

$C, A_1, \ldots, A_m \in \mathbb{S}^n$ $\qquad$ $\langle M, N \rangle = \mathrm{Tr}(M^\top N)$

Interior Point: ϵ-optimality in polynomial time

Rank-Constrained SDP

SDPs reformulations of combinatorial problems

$$\begin{aligned} \min_{X} \quad & \langle C, X \rangle \\ \text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\ & X \in \mathbb{S}^n_+ \\ & \text{rank}(X) \leq t \end{aligned}$$

NP-Hard in general

Exact recovery if $\text{rank}(X_{SDP}) \leq t$

Decomposition Method Motivation

How algorithms scale in problem size:

- Interior Point (primal-dual)[1]: $O(n^2 m^2 + n^3 m)$
- Rank Constrained: Worse

How to decrease problem size:

- m: Linear Dependence/Pray
- n: Break up large PSD cone into smaller cones

[1] Alizadeh, Haeberly, and Overton 1998.

Chordal Decomposition

Chordal Graphs

All cycles of length 4+ have a one-edge shortcut (chord)

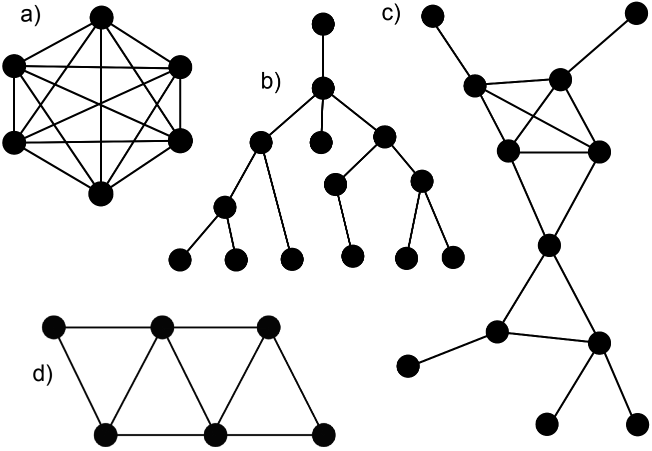


Figure 1: Examples of Chordal Graphs

Aggregate Sparsity

Based on sparsity of (C, A_i)

Stack (C, A_i) together, treat as adjacency matrix

Forms 'aggregate sparsity graph' $\mathcal{G}(\mathcal{V}, \mathcal{E})$

$$
\begin{array}{c|c}
 & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\hline
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} &
\begin{bmatrix}
x_{11} & x_{12} & ? & ? & ? & x_{16} \\
x_{12} & x_{22} & x_{23} & x_{24} & ? & x_{26} \\
? & x_{23} & x_{33} & x_{34} & ? & ? \\
? & x_{24} & x_{34} & x_{44} & x_{45} & x_{46} \\
? & ? & ? & x_{45} & x_{55} & x_{56} \\
x_{16} & x_{26} & ? & x_{46} & x_{56} & x_{66}
\end{bmatrix}
\end{array}
$$

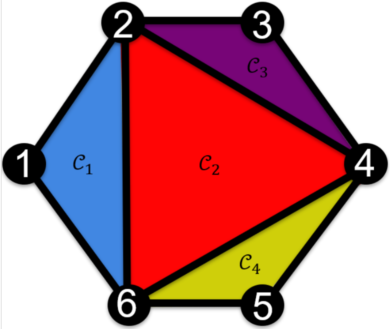


Figure 2: Cone $\mathbb{S}^n_+(\mathcal{E}, ?)$ and its chordal graph

Sparse and Completable Matrices

Sparse symmetric matrices:

$$\mathbb{S}^n(\mathcal{E},0) = \{X \in \mathbb{S}^n \mid \; X_{ij} = 0, \; \forall (i,j) \notin \mathcal{E}^*\},$$
$$\mathbb{S}^n_+(\mathcal{E},0) = \mathbb{S}(\mathcal{E},0) \cap \mathbb{S}^n_+$$

Completable Matrices (dual of sparse matrices)

$$\mathbb{S}^n_+(\mathcal{E},?) = \{X \in \mathbb{S}^n(\mathcal{E},?) \mid \exists M \in \mathbb{S}^n_+ \; X_{ij} = M_{ij} \; \forall (i,j) \in \mathcal{E}^* \}$$

Grone's Theorem

Sparse matrix $X \in \mathbb{S}^n(\mathcal{E}, 0)$ also $\in \mathbb{S}^n_+(\mathcal{E}, ?)$ iff:

$$\forall \text{ maximal cliques } \{\mathcal{C}_k\}_{k=1}^p \text{ in } \mathcal{G}(\mathcal{V}, \mathcal{E}): \quad E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^\top \succeq 0$$

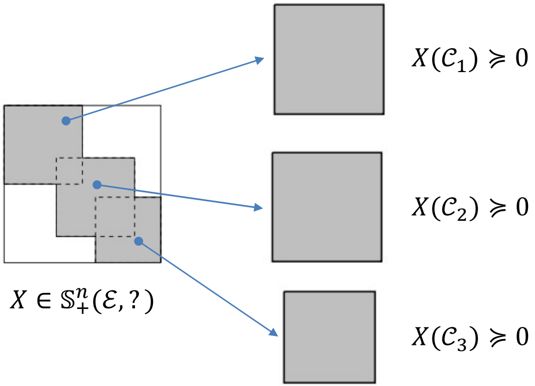


Figure 3: All clique submatrices should be PSD

Choices of Completions

Grone[2]: Exists a unique maximum-determinant completion

Dancis[3]: Exists at least one completion where:

$$\mathrm{rank}(X) = \max_k \mathrm{rank}(E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^{\mathsf{T}})$$

[2] Grone et al. 1984.

[3] Dancis 1992.

Example of Completion

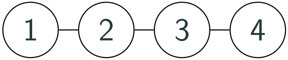
Max Det Completion: 1 — 2 — 3 — 4

$$\begin{bmatrix} 5 & 0.25 & ? & ? \\ 0.25 & 3 & 0.5 & ? \\ ? & 0.5 & 1 & 0.75 \\ ? & ? & 0.75 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0.25 & 0.042 & 0.031 \\ 0.25 & 3 & 0.5 & -0.72 \\ 0.042 & 0.5 & 1 & 0.375 \\ 0.031 & 0.375 & 0.75 & 1 \end{bmatrix}$$

$$\lambda = \{5.04, 3.22, 1.50, 0.25\}$$

$$|X| = 5.99$$

Example of Completion

Rank 2 Completion: 

$$\begin{bmatrix} 5 & 0.25 & ? & ? \\ 0.25 & 3 & 0.5 & ? \\ ? & 0.5 & 1 & 0.75 \\ ? & ? & 0.75 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0.25 & -2.09 & -2.09 \\ 0.25 & 3 & 0.5 & -0.72 \\ -2.09 & 0.5 & 1 & 0.75 \\ -2.09 & -0.72 & 0.75 & 1 \end{bmatrix}$$

$$\lambda = \{6.77, 3.23, 0, 0\}$$

$$|X| = 0$$

Low Rank Chordal SDPs

Grone in Optimization

SDP has equivalent optimum

$$\begin{aligned} \min_{X, X_k} \quad & \langle C, X \rangle \\ \text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\ & X_k = E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^\mathsf{T}, \; k = 1, \ldots, p, \\ & X_k \in \mathbb{S}_+^{|\mathcal{C}_k|}, \quad k = 1, \ldots, p \end{aligned}$$

Rank Constrained Chordal SDP

Use minimal-rank completion on Grone

$$
\begin{aligned}
\min_{X, X_k} \quad & \langle C, X \rangle \\
\text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\
& X_k = E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^\mathsf{T}, \; k = 1, \ldots, p, \\
& X_k \in \mathbb{S}_+^{|\mathcal{C}_k|}, \quad k = 1, \ldots, p \\
& \mathrm{rank}(X_k) \leq t
\end{aligned}
$$

Rank Soft-Constrained Chordal SDP

Rank Regularization

$$
\begin{aligned}
\min_{X, X_k, t} \quad & \langle C, X \rangle + \lambda t \\
\text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\
& X_k = E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^{\mathsf{T}},\ k = 1, \ldots, p, \\
& X_k \in \mathbb{S}_+^{|\mathcal{C}_k|}, \quad k = 1, \ldots, p \\
& \operatorname{rank}(X_k) \leq t
\end{aligned}
$$

Log-Det Heuristic

Replace Rank with Log-Det

$$
\begin{aligned}
\min_{X, X_k} \quad & \langle C, X \rangle \\
\text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \dots, m, \\
& X_k = E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^{\mathsf{T}},\ k = 1, \dots, p, \\
& X_k \in \mathbb{S}_+^{|\mathcal{C}_k|}, \quad k = 1, \dots, p \\
& \log |X_k| \leq M_t
\end{aligned}
$$

Reweighted Trace Heuristic

Local Linearization of Log-det

Nuclear norm $\|X\|_* = \sum_{i=1}^{n} \sigma_i(X) = Tr(X) = \langle I, X \rangle$

$\|X\|_*$ penalizes all σ_i equally

Use weighting $\langle W, X \rangle$ to add penalty to higher σ_i

Repeat process[4]:

- Start with $W = 0$ or $W = I$
- Solve SDP with cost $C + W$ for to find X^*
- Set $W = (X^* + \delta I)^{-1}$

[4] Mohan and Fazel 2010.

Chordal Trace penalty

Sparsity preserved between reweighting iterations (after chordal extension)

$$
\begin{aligned}
\min_{X, X_k} \quad & \langle C, X \rangle + \sum_k \langle W_k, X_k \rangle \\
\text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\
& X_k = E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^{\mathsf{T}}, \ k = 1, \ldots, p, \\
& X_k \in \mathbb{S}_+^{|\mathcal{C}_k|}, \quad k = 1, \ldots, p
\end{aligned}
$$

Cost function is $\langle C + W_{\mathcal{C}}, X \rangle$ where $W_{\mathcal{C}} = \sum_{k=1}^{p} E_{\mathcal{C}_k}^{\mathsf{T}} W_k E_{\mathcal{C}_k}$

Maxcut Reweighting (10 iterations)

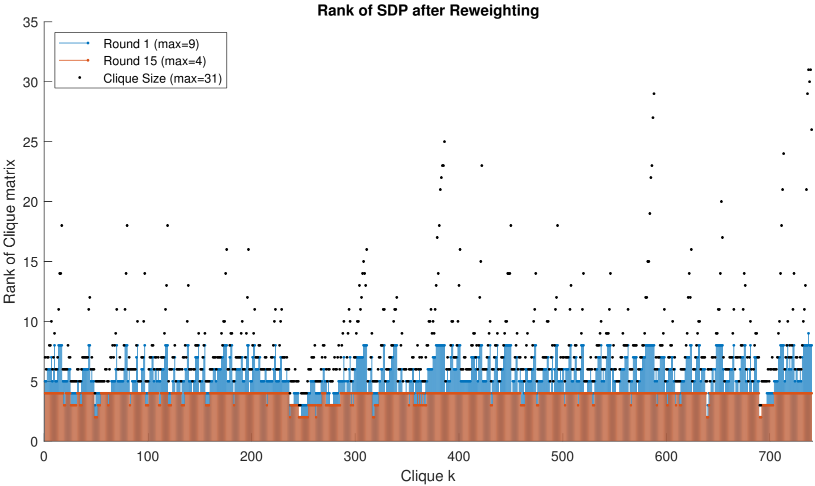


Figure 4: 1000 vertex Maxcut reweighting: Rank 9 → 4

Preservation of Sparsity

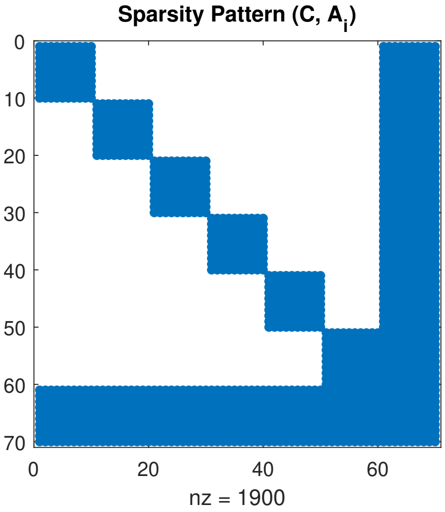


Figure 5: Random Block Arrow SDP

Preservation of Sparsity

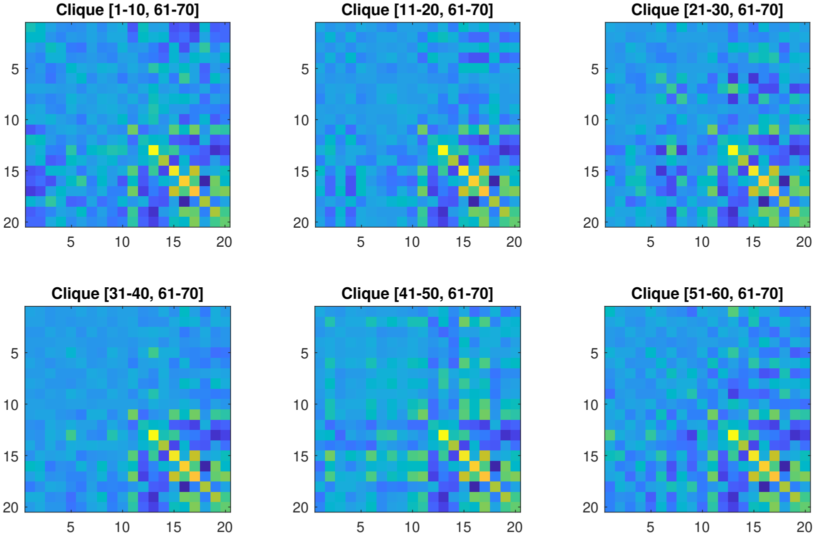


Figure 6: Cliques X_k^* at optimality

Preservation of Sparsity

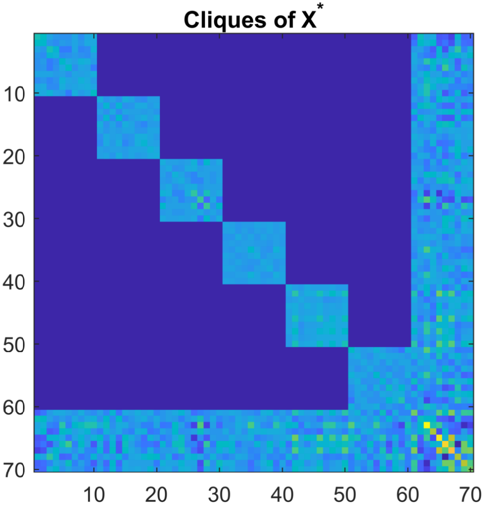


Figure 7: Collecting cliques X_k^* into matrix X^*

Preservation of Sparsity

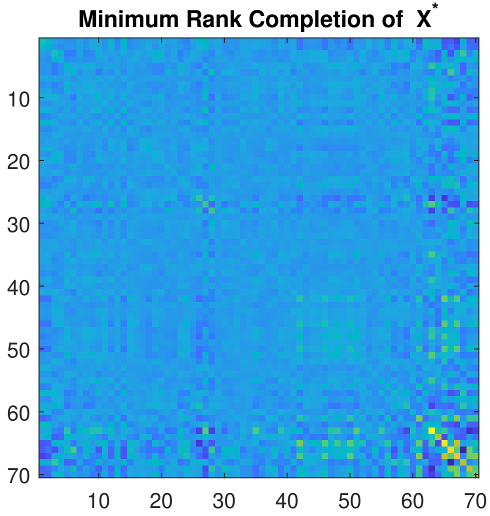


Figure 8: Rank-5 Completion of X^*

Preservation of Sparsity

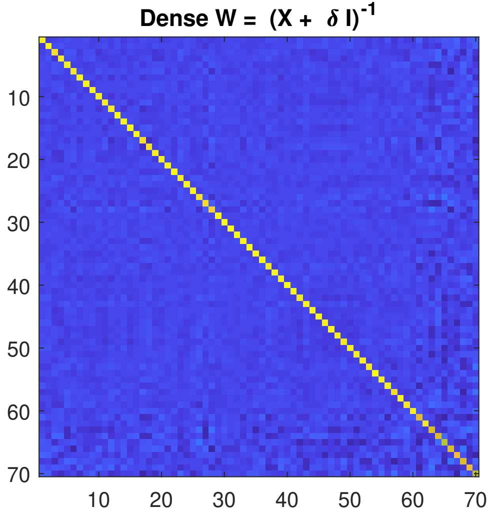


Figure 9: Dense Reweighted Heuristic W

Preservation of Sparsity

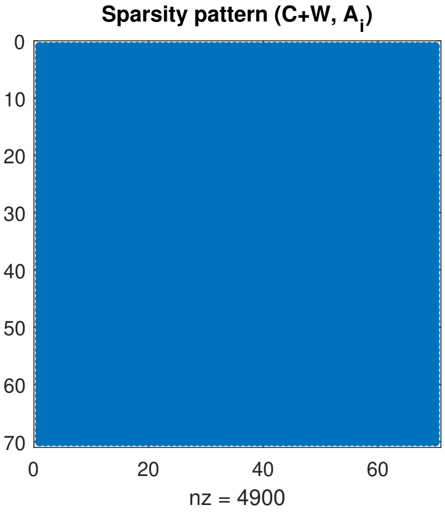


Figure 10: Sparsity pattern destroyed for next iteration

Preservation of Sparsity

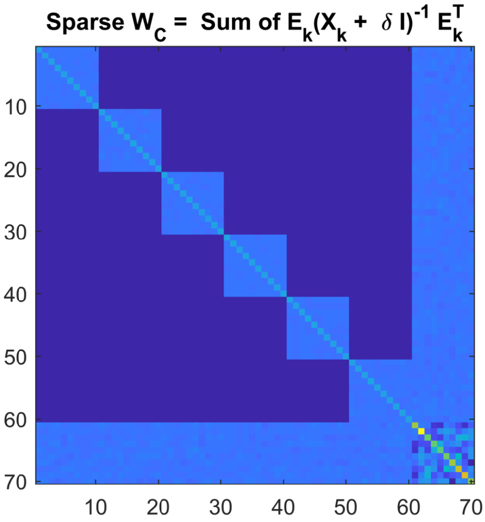


Figure 11: Sparse Reweighted Heuristic W

Preservation of Sparsity

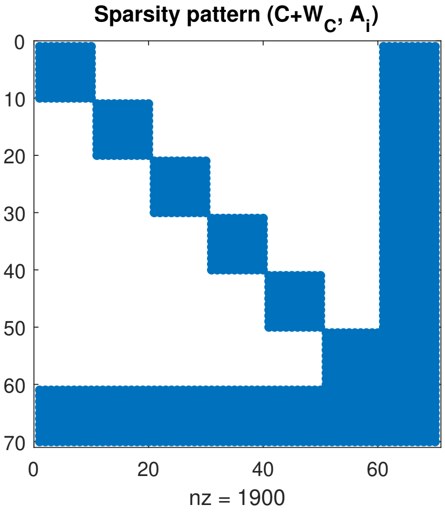


Figure 12: Sparsity pattern retained for next iteration

Subspace Clustering (SSC)

Problem brief

Determine which points came from which subspaces

Applications to Switched System Identification

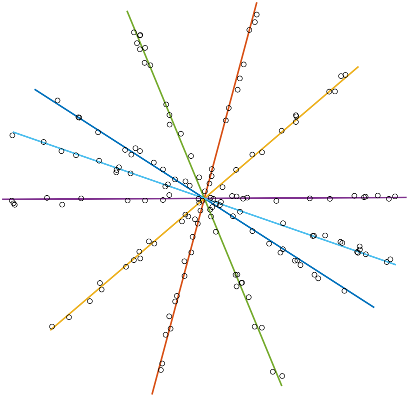


Figure 13: SSC under bounded noise

Problem Formulation

N_p points $x_j \in \mathbb{R}^D$, N_s subspace normals $r_i \in \mathbb{R}^D$

Binary classification $s_{ij} = 1$ if $|r_i^\mathsf{T} x_j| \leq \epsilon$

Nonconvex quadratic feasibility for (r_i, s_{ij})

$$\begin{aligned} \operatorname*{find}_{r,s} \quad & s_{ij}|r_i^\mathsf{T} x_j| \leq s_{ij}\epsilon & \qquad s_{ij} = s_{ij}^2 \\ & \sum_{i=1}^{N_s} s_{ij} = 1 & \qquad r_i^\mathsf{T} r_i = 1 \end{aligned}$$

Coordinates of r_i are $\{r_i^{(k)}\}_{k=1}^D$

SDP Reformulation

Define variable $X \in \mathbb{S}^{1+N_s(D+N_p)}$:

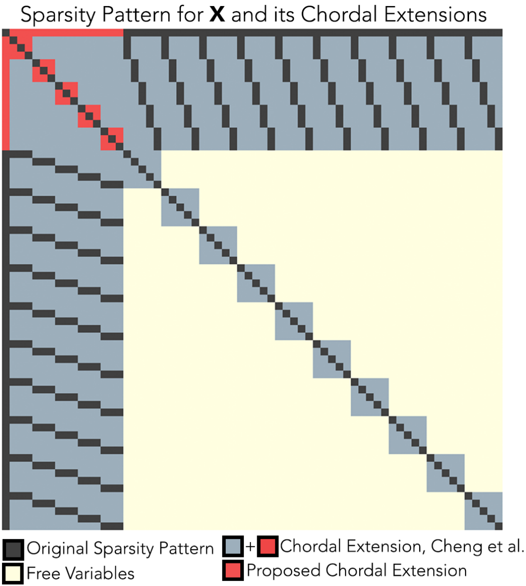
$$X = [1\ r_i\ s_{ij}][1\ r_i\ s_{ij}]^{\mathsf{T}} = \begin{bmatrix} 1 & r_i^{\mathsf{T}} & s_{ij}^{\mathsf{T}} \\ r_i & r_i r_i^{\mathsf{T}} & r_i s_{ij}^{\mathsf{T}} \\ s_{ij} & s_{ij} r_i^{\mathsf{T}} & s_{ij} s_{ij}^{\mathsf{T}} \end{bmatrix}$$

Translate constraints: $s_{ij} = s_{ij}^2 \rightarrow X_{1,s_{ij}} = X_{s_{ij},s_{ij}}$

Table 1: Sparsity in SSC problem

Monomial	Index in problem
$s_{ij}s_{i'j'}$	$i = i',\ j = j'$
$s_{ij}r_i^{(k)}$	$i = i'$
$r_i^{(k)}r_{i'}^{(k')}$	$i = i',\ k = k'$

Sparsity and Experiments



Chordal Extensions

Previous work by Cheng et. al.[5]

SSC algorithm with new chordal decomposition is:

- Linear in number of points, subspaces
- NP-hard in dimension of space

	Rank 1 PSD		Other PSD		
Problem	# Cliques	Size Cliques	# Cliques	Size Cliques	% edges
Full X	1	$1 + N_s(D + N_p)$	$\varnothing$	$\varnothing$	1637%
Cheng	1	$1 + N_s D$	N_p	$1 + N_s(D + 1)$	350%
Ours	N_s	$D + 1$	$N_p N_s$	$D + 2$	13%

Table 2: Chordal Extensions for SSC ($D = 3$, $N_p = 10$, $N_s = 5$)

[5] Cheng et al. 2016.

Optimality Interpretation

Only rank-1 solution to SDP yields valid SSC

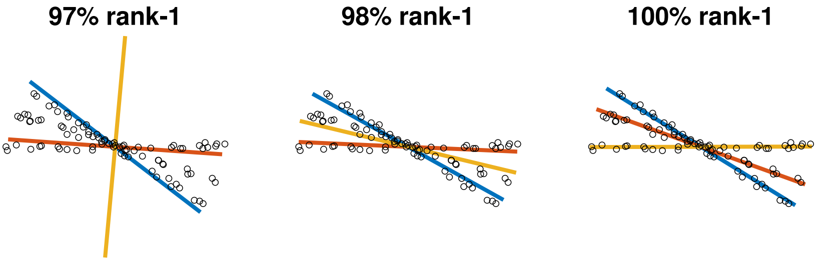


Figure 14: SSC with $N_s = 3, N_p = 90, D = 2, \epsilon = 0.15$

rank-1-ness: $\lambda_1 / (\sum_i \lambda_i)$

Merging Cliques

Chordal Decomposition adds clique overlap equality constraints:

$$E_k^{\mathsf{T}} X E_k = X_k$$

Interior Point Methods scale poorly under new Equality Constraints

Min rank completion (orignal $\mathcal{E}$)

SparseCoLO[6]: Merge cliques for optimization ($\mathcal{E}_C \supseteq \mathcal{E}$)

[6] Fujisawa et al. 2009.

Numerical Experiments

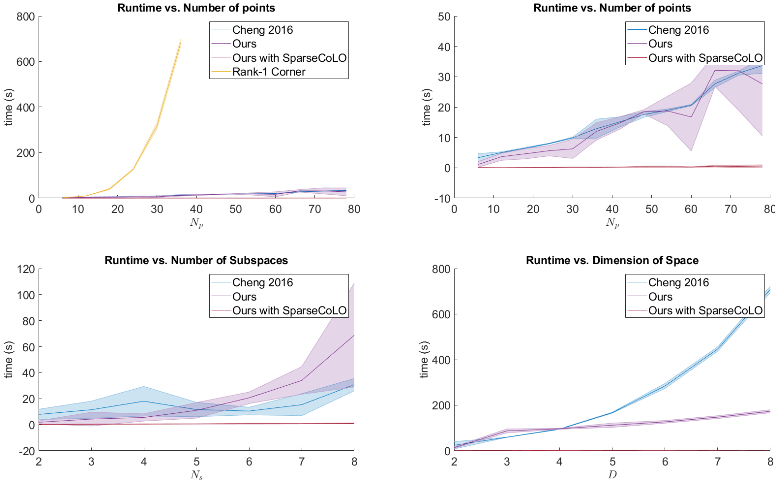


Figure 15: SSC Experiments, changing individually N_p (top), N_s (bottom left), and D (bottom right).

Structured Subsets in Decompositions

Structured Subset Motivation

Large PSD constraints are intractable

Use inner approximation $K \subset \mathbb{S}_+$ (lose low-rank ability)

Examples[7],

$$\mathcal{D}^n = \{A \in \mathbb{S}^n : A = \mathrm{diag}(a_1, \ldots, a_n), a_i \geq 0\},$$
$$\mathcal{DD}^n = \{A \in \mathbb{S}^n : \; a_{ii} \geq \sum_{j \neq i} |a_{ij}|, i = 1, 2, \ldots, n\},$$
$$\mathcal{SDD}^n = \{A \in \mathbb{S}^n : \; \exists D \in \mathcal{D}^n \mid DAD \in \mathcal{DD}^n\}.$$

$$\mathcal{D}^n \subset \mathcal{DD}^n \subset \mathcal{SDD}^n \subset \mathbb{S}^n_+.$$

[7] Ahmadi and Majumdar 2017.

Change of Basis

Membership in K is basis dependent

Generic $M \in \mathbb{S}^n$ is diagonal in eigenvector basis

After solving $X_0 = L_0 L_0^\mathsf{T}$:

$$\begin{aligned} X_1 = \underset{X}{\operatorname{argmin}} \quad & \langle C, X \rangle \\ \text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\ & L_0 X L_0^\mathsf{T} \in K. \end{aligned}$$

Strict decrease in $\langle C, X \rangle$, not guaranteed to solve SDP[8].

[8] Ahmadi and Hall 2014.

Decomposed Structured Subsets

Even with decompositions, SDP may still be intractable

Graph $\mathcal{E}$ with maximal cliques $\{\mathcal{C}_k\}_{k=1}^{p}$

$\mathcal{K} = \{K_s\}_{k=1}^{p}$ list of cones per clique

Define new cones

$$\mathcal{K}(\mathcal{E},0) := \Big\{ Z \in \mathbb{S}^n \mid Z = \sum_{k=1}^{p} E_{\mathcal{C}_k}^{\mathsf{T}} Z_k E_{\mathcal{C}_k}, \\ Z_{\mathcal{C}_k} \in K_k,\ k = 1, \ldots, p. \Big\},$$

$$\mathcal{K}(\mathcal{E},?) := \Big\{ X \in \mathbb{S}^n \mid E_{\mathcal{C}_k} X E_{\mathcal{C}_k}^{\mathsf{T}} \in K_k,\ k = 1, \ldots, p. \Big\}.$$

Properties of Decomposed Structured Subsets

Partial order: $\mathcal{K} \subseteq \tilde{\mathcal{K}}$ iff $K_k \subseteq \tilde{K}_k \ \forall k = 1 \dots p$

If $\mathcal{K} \subseteq \tilde{\mathcal{K}}$, then:

$$\mathcal{K}(\mathcal{E}, 0) \subseteq \tilde{\mathcal{K}}(\mathcal{E}, 0) \quad \text{and} \quad \mathcal{K}(\mathcal{E}, ?) \subseteq \tilde{\mathcal{K}}(\mathcal{E}, ?)$$

Dual Space ($\mathcal{K}^* = \{K_k^*\}$):

$$\mathcal{K}(\mathcal{E}, ?)^* = \mathcal{K}^*(\mathcal{E}, 0) \quad \text{and} \quad \mathcal{K}(\mathcal{E}, 0)^* = \mathcal{K}^*(\mathcal{E}, ?).$$

Motivation for Decomposed Structured Subsets

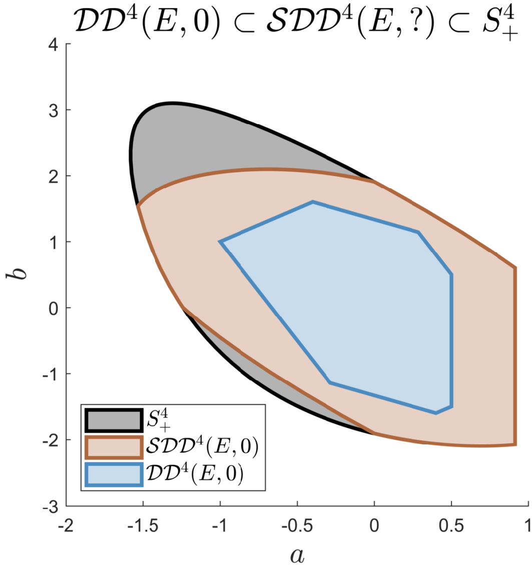
Consider $M(a, b)$

$$M(a,b) = \begin{bmatrix} 1 & 1+a & ? & ? \\ 1+a & 2-b & -2a & a+b \\ ? & -2a & 5 & b/2 \\ ? & a+b & b/2 & 2 \end{bmatrix}$$

Cliques are $\mathcal{C}_1 = (2, 3, 4) \qquad \mathcal{C}_2 = (1, 2)$

Plot feasibility sets of $M(a, b) \in \mathcal{K}(\mathcal{E}, ?)$

Decomposed DD, SDD, PSD



Mixing DD and SDD

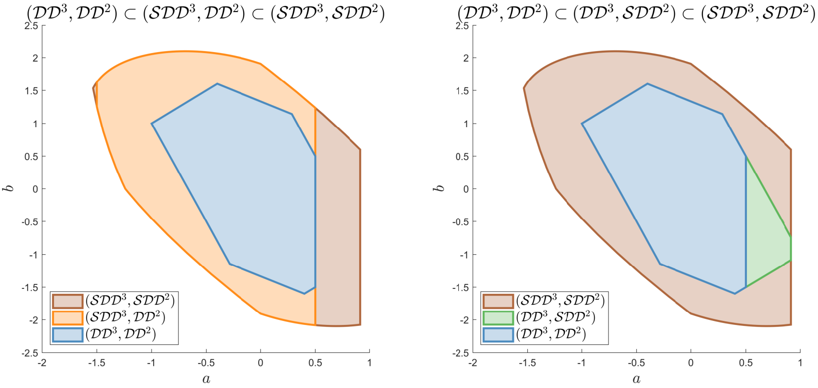


Figure 16: Mixing cones broadens feasibility regions

Cone Completions

Constraint on x_{11}:

$X \in \mathcal{DD}$	$x_{11} \geq \lvert x_{12} \rvert + \lvert x_{13} \rvert + \lvert x_{14} \rvert + \lvert x_{15} \rvert + \lvert x_{16} \rvert$
$X \in \mathcal{DD}(\mathcal{E}, ?)$	$x_{11} \geq \lvert x_{12} \rvert + \lvert x_{16} \rvert$

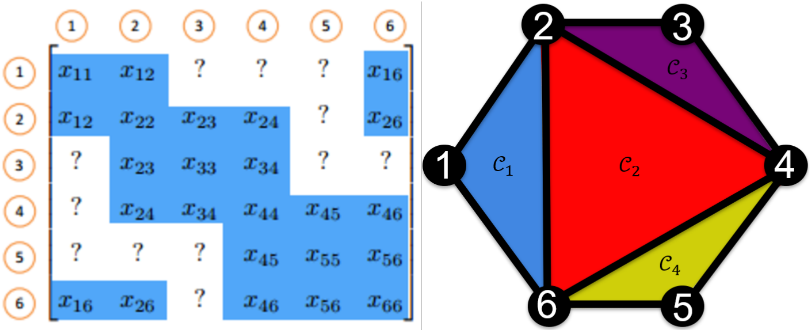


Figure 17: Cone $\mathbb{S}_+^n(\mathcal{E}, ?)$ and its chordal graph

Cone Completions

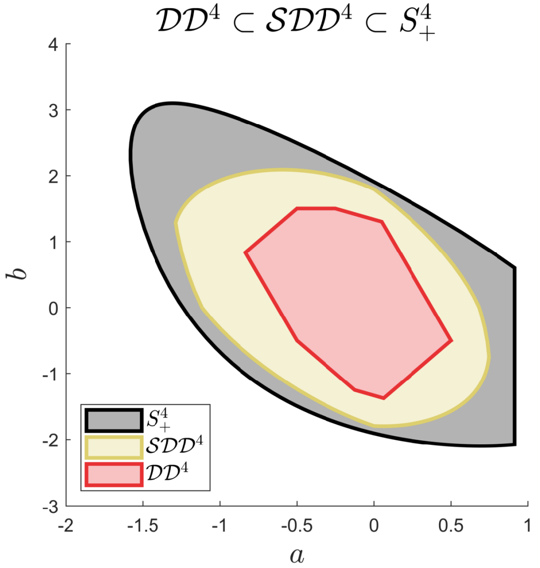
Matrix $M \in \mathbb{S}^n(\mathcal{E}, 0)$ may have K-completion $M^c \in K^n$

Equivalent for $K = \mathbb{S}_+$ (Grone).

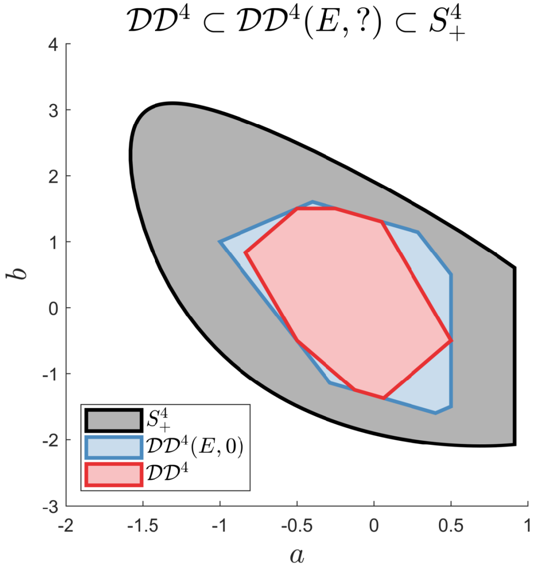
Distinct for Decomposed Structured Subsets:

$$\begin{array}{ccc} K^n & \subset & K^n(\mathcal{E}, ?) \\ \cap & & \cap \\ \tilde{K}^n & \subset & \tilde{K}^n(\mathcal{E}, ?) \end{array}$$

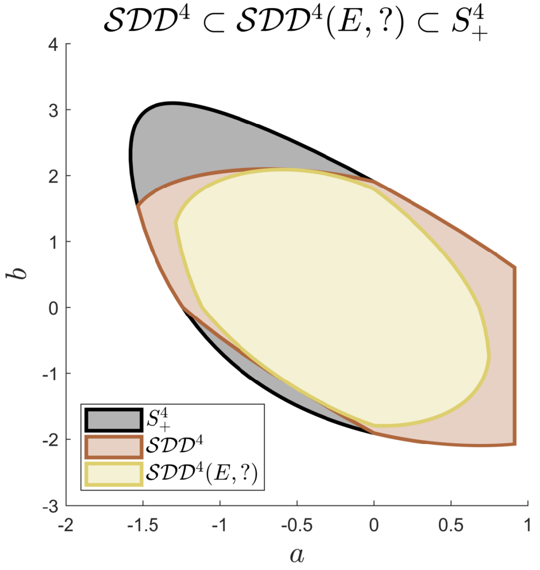
Standard DD, SDD



Standard vs. Decomposed DD



Standard vs. Decomposed SDD



Block Arrow Example

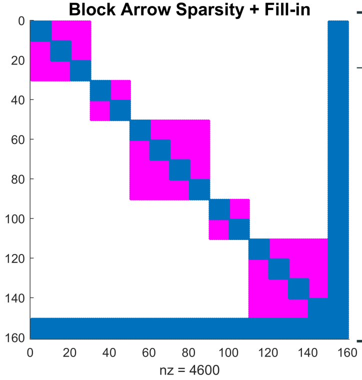


Figure 18: Sparsity Pattern

	K	$K(\mathcal{E}_F, ?)$	$K(\mathcal{E}, ?)$
$\mathcal{DD}$	Inf.	Inf.	Inf.
$\mathcal{SDD}$	64.5	34.7	19.4
B_2	51.4	27.1	13.9
B_5	32.1	15.0	5.34
B_{10}	20.8	7.10	-1.23
$\mathbb{S}_+$	-1.23	-1.23	-1.23

Table 3: Cost vs. Cone

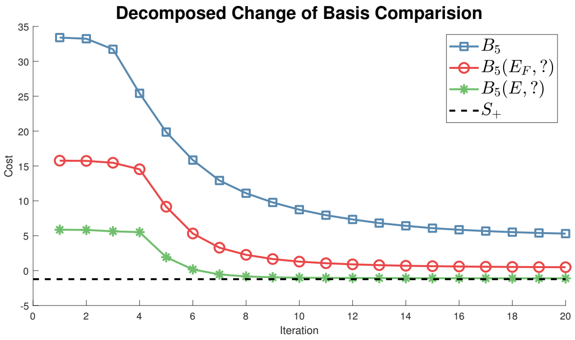
B_n: Block Factor-Width 2 matrix[9] with block-size n

[9] Zheng, Sootla, and Papachristodoulou 2019.

Decomposed Change of Basis

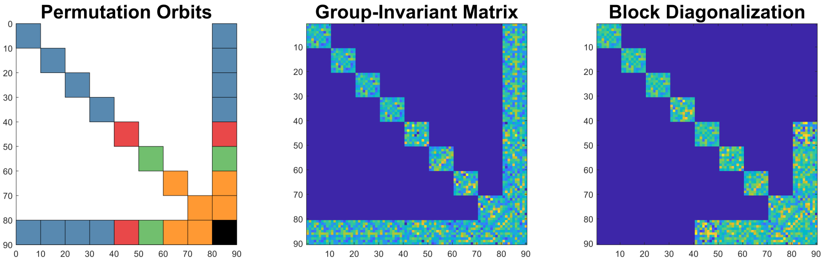
Different basis L_k on each clique $\mathcal{C}_k$

$$\begin{aligned} X^* = \underset{X}{\operatorname{argmin}} \quad & \langle C, X \rangle \\ \text{subject to} \quad & \langle A_i, X \rangle = b_i, i = 1, \ldots, m, \\ & L_k X L_k^{\mathsf{T}} \in K_k. \end{aligned}$$



Combining Decompositions

Use Symmetry and Sparsity together:



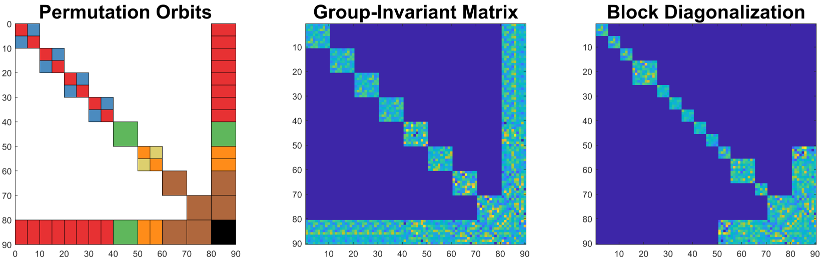
Cost	Full	Sym.
Full	10.82	10.37
Sparse	7.40	7.40

Time (s)	Full	Sym.
Full	55.07	12.39
Sparse	13.39	7.11

Table 4: $\mathcal{SDD}$ Block Arrow with Symmetry

Combining Decompositions (cont.)

Additional Symmetry within blocks



Cost	Full	Sym.
Full	12.96	10.86
Sparse	9.49	8.44

Time (s)	Full	Sym.
Full	124.5	19.3
Sparse	38.2	12.0

Table 5: $\mathcal{SDD}$ Block Arrow with in-block Symmetry

Network Robustness Estimation

Linear System and H_∞ norm

State x, input u, output y

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

Transfer function (gain) $G(s) = C(sI - A)^{-1}B + D$

Small Gain Theorem: Robust stability when $\|G\|_\infty < 1$

Following statements are equivalent (Bounded Real Lemma):

1. $\|G\|_\infty < \gamma$
2. There exists a $P \succ 0$ such that
$$\begin{bmatrix} PA + A^\mathsf{T}P + C^\mathsf{T}C & P^\mathsf{T}B + C^\mathsf{T}D \\ B^\mathsf{T}P + D^\mathsf{T}C & -\gamma^2 I \end{bmatrix} \prec 0.$$

Sea Star

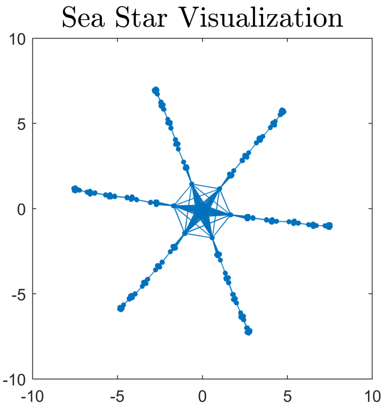
Figure 19: Inspiration for Sea Star Network

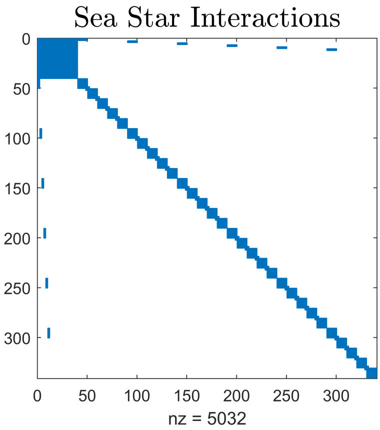
Sea Star Network

40 agents in head, random linear system per agent.

6 arms, 5 knuckles per arm, 10 agents per knuckle.

2 agents communicate between head/adjacent knuckles.

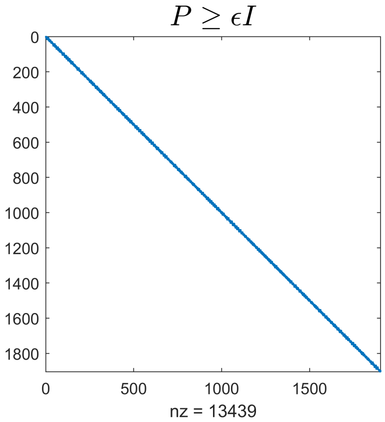


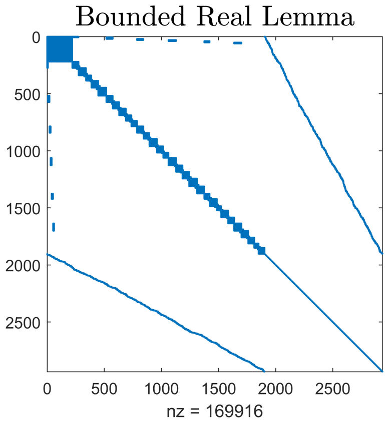


Sea Star SDP

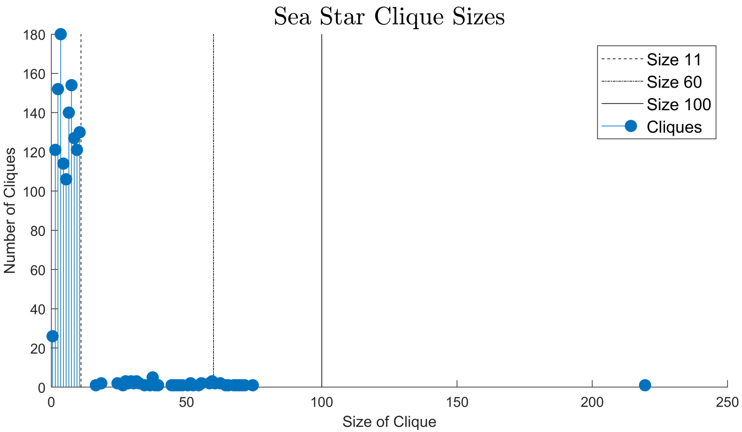
Block Diagonal P: approximation to exploit sparsity.

PSD blocks of size 1903 and 2934.





Sea Star Cliques



$$\mathcal{K}_N : \qquad K_k = \begin{cases} K & |\mathcal{C}_k| > N \\ \mathbb{S}_+ & |\mathcal{C}_k| \leq N \end{cases}$$

Sea Star Costs

	Structured Subset			
Cone	$K(\mathcal{E}, 0)$	$\mathcal{K}_{11}(\mathcal{E}, 0)$	$\mathcal{K}_{60}(\mathcal{E}, 0)$	$\mathcal{K}_{100}(\mathcal{E}, 0)$
$\mathcal{DD}$	Inf.	11.71	4.6	1.282
$\mathcal{SDD}$	1.224	1.224	1.224	1.224
B_3	1.224	1.224	1.224	1.224
B_5	1.224	1.224	1.224	1.224
B_8	1.224	**1.224**	1.224	1.224
B_{15}	1.224	1.224	1.224	1.224
B_{30}	1.224	1.224	1.224	1.224
B_{55}	1.224	1.224	1.224	1.224
B_{70}	1.224	1.224	1.224	1.224
$\mathbb{S}_+$	1.224			

Sea Star Time

	Structured Subset			
Cone	$K(\mathcal{E},0)$	$\mathcal{K}_{11}(\mathcal{E},0)$	$\mathcal{K}_{60}(\mathcal{E},0)$	$\mathcal{K}_{100}(\mathcal{E},0)$
$\mathcal{DD}$	30.5	230.5	387.3	355.4
$\mathcal{SDD}$	282.2	214.9	198.4	213.7
B_3	106.0	132.8	166.7	183.2
B_5	108.5	107.8	125.9	165.4
B_8	92.93	**90.12**	115.2	149.0
B_{15}	106.7	106.4	114.2	164.5
B_{30}	302.0	318.9	322.6	321.8
B_{55}	675.6	632.5	619.1	611.2
B_{70}	1133.0	1139.0	1124.0	1124.0
$\mathbb{S}_+$	1798.0			

Conclusions

Structure in SDPs are powerful

Decompositions allow for tractable programs

Preserve structure if at all possible

Use all possible decompositions before approximating

Questions?

Thank you for your attention

Chordal Rank Minimization:

https://arxiv.org/abs/1904.10041

Decomposed Structured Subsets:

https://arxiv.org/abs/1911.12859

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