Safety Quantification for Nonlinear and Time-Delay Systems using Occupation Measures

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April 3, 2023

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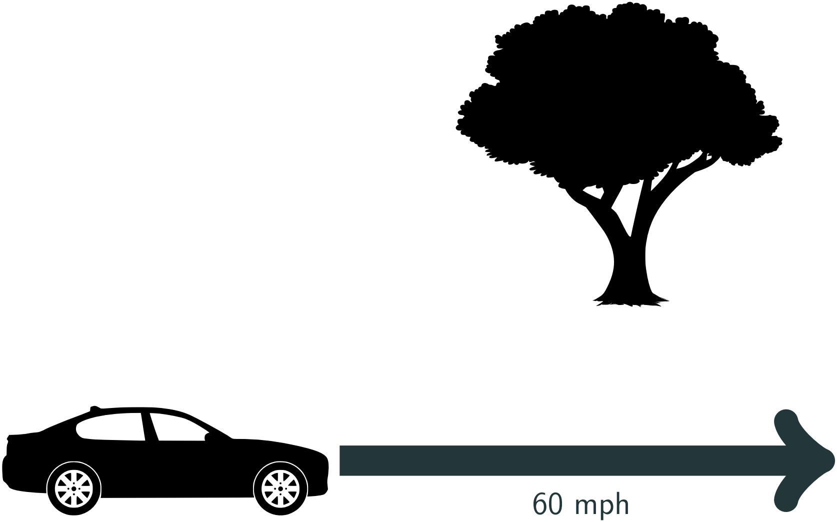
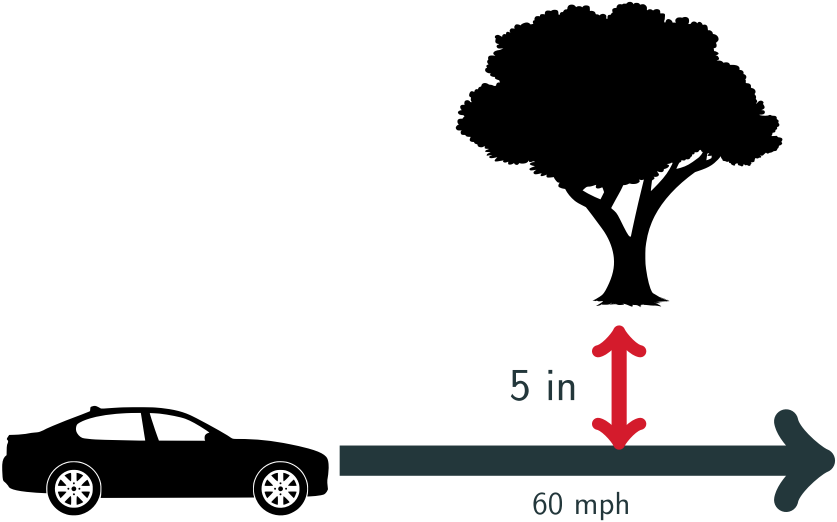
Safety Example



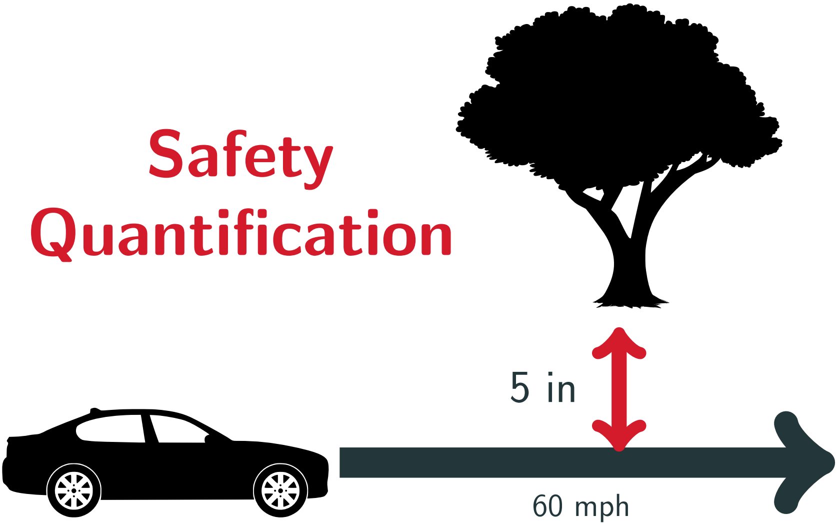
Safety Example (Barrier/Density Function)

Safety Example (Distance Estimate)



Safety Example

Safety Quantification



Motivation: Epidemic

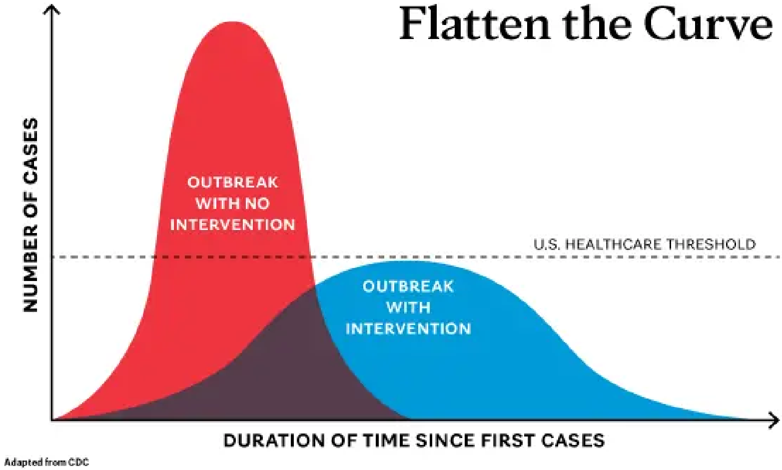
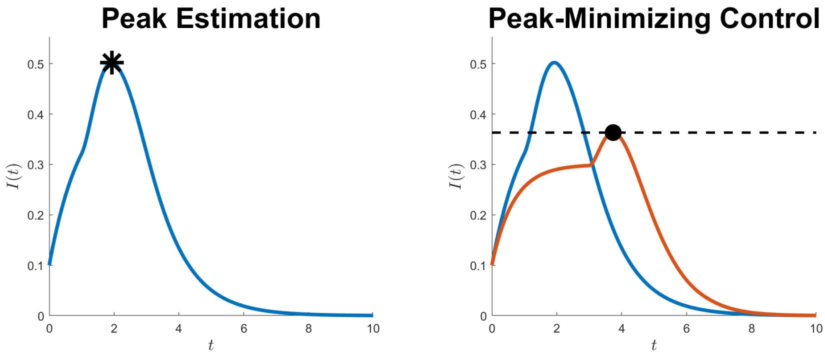


Image credit to Mayo Clinic News Network

Problems Covered



Main Ideas

Pose safety quantification problems

Want convex, convergent, bisection-free algorithms

Formulate using convex linear programs in measures

Increasing-quality bounds using Semidefinite Programming

Overview of Presentation

Peak estimation background

1. Survey of Thesis Work
2. Peak Value-at-Risk Estimation
3. Time-Delay Systems

Wrap-up

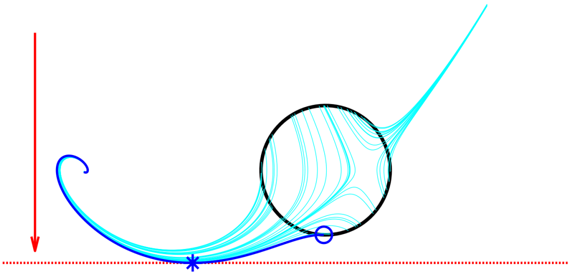
Peak Estimation Background

Peak Estimation Background

Find extreme value of $p(x)$ along trajectories

$$P^* = \sup_{t,\ x_0 \in X_0} p(x(t \mid x_0))$$

$$\dot{x}(t) = f(t, x(t)) \qquad \forall t \in [0, T], \qquad x(0) = x_0.$$



$$p(x) = -x_2, \qquad \dot{x} = [x_2, -x_1 - x_2 + x_1^3/3]$$

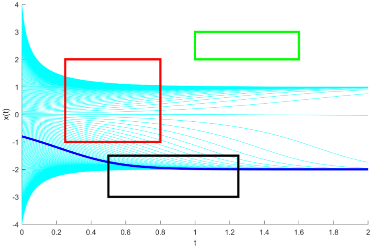
Occupation Measure

Time trajectories spend in set

Test function
$v(t, x) \in C([0, T] \times X)$

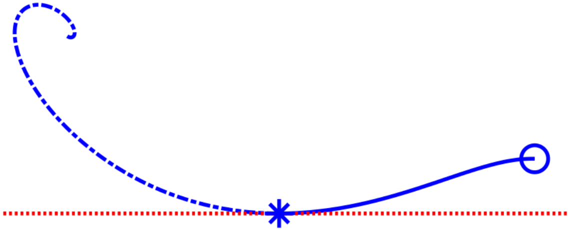
Single trajectory:
$\langle v, \mu \rangle = \int_0^T v(t, x(t \mid x_0))dt$

Averaged trajectory: $\langle v, \mu \rangle =$
$\int_X \left(\int_0^T v(t, x)dt \right) d\mu_0(x)$



$x' = -x(x+2)(x-1)$

Connection to Measures



Measures: Initial μ_0, Peak μ_p, Occupation μ

For all functions $v(t,x) \in C([0,T] \times X)$

$$\begin{aligned}
\mu_0^* : \quad & \langle v(0,x), \mu_0^* \rangle = v(0, x_0^*) \\
\mu_p^* : \quad & \langle v(t,x), \mu_p^* \rangle = v(t_p^*, x_p^*) \\
\mu^* : \quad & \langle v(t,x), \mu^* \rangle = \int_0^{t_p^*} v(t, x^*(t \mid x_0^*)) dt
\end{aligned}$$

Liouville Equation

Lie derivative (instantaneous change along f) $\forall v \in C^1$:

$$\mathcal{L}_f v = \partial_t v(t,x) + f(t,x) \cdot \nabla_x v(t,x) \tag{1a}$$

Conservation law: final = initial + accumulated change

$$\langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle \tag{1b}$$

$$\mu_p = \delta_0 \otimes \mu_0 + \mathcal{L}_f^\dagger \mu \tag{1c}$$

Liouville 'represents' dynamics $\dot{x}(t) = f(t, x(t))$

Measures for Peak Estimation

Infinite-dimensional Linear Program (Cho, Stockbridge, 2002)

$$p^* = \sup \quad \langle p(x), \mu_p \rangle \tag{2a}$$
$$\langle 1, \mu_0 \rangle = 1 \tag{2b}$$
$$\langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle \quad \forall v \tag{2c}$$
$$\mu, \mu_p \in \mathcal{M}_+([0,T] \times X) \tag{2d}$$
$$\mu_0 \in \mathcal{M}_+(X_0) \tag{2e}$$

Instance of Optimal Control Program (Lewis and Vinter, 1980)

$(\mu_0^*, \mu_p^*, \mu^*)$ is feasible with $P^* = \langle p(x), \mu_p^* \rangle \leq p^*$

$P^* = p^*$ if compactness, Lipschitz properties hold

Moments for Peak Estimation

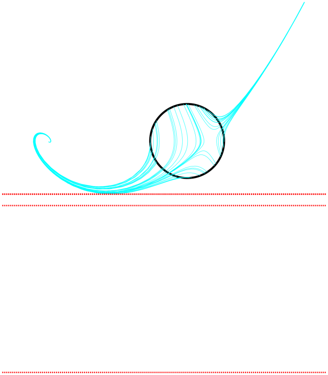
Moment: $y_\alpha = \langle x^\alpha, \nu \rangle \; \forall \alpha \in \mathbb{N}^n$

Moment matrix $\mathbb{M}[y]_{\alpha\beta} = y_{\alpha+\beta}$ is PSD

$$\mathbb{M}_2[y] = \begin{bmatrix} y_{00} & y_{10} & y_{01} & y_{20} & y_{11} & y_{02} \\ y_{10} & y_{20} & y_{11} & y_{30} & y_{21} & y_{12} \\ y_{01} & y_{11} & y_{02} & y_{21} & y_{12} & y_{03} \\ y_{20} & y_{30} & y_{21} & y_{40} & y_{31} & y_{11} \\ y_{11} & y_{21} & y_{12} & y_{31} & y_{22} & y_{13} \\ y_{02} & y_{12} & y_{03} & y_{22} & y_{13} & y_{04} \end{bmatrix} \succeq 0$$

Liouville induces affine relation in $(\mu^0, \mu^p, \mu) \rightarrow (y^0, y^p, y)$

Peak Estimation Example Bounds



Converging bounds to min. $x_2 = -0.5734$ (moment-SOS)

Box region $X = [-2.5, 2.5]$, time $t \in [0, 5]$

Max. PSD size: $\binom{(n+1)+(d+\lfloor \deg f/2 \rfloor)}{n+1}$ (Fantuzzi, Goluskin, 2020)

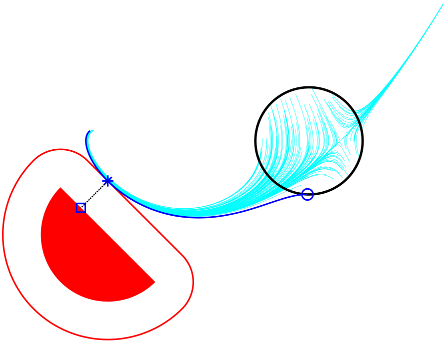
Survey of Thesis Work

Distance Estimation Problem

Unsafe set X_u, point-set distance $c(x; X_u) = \inf_{y \in X_u} c(x, y)$

$$P^* = \inf_{t,\, x_0 \in X_0} c(x(t \mid x_0); X_u)$$

$$\dot{x}(t) = f(t, x(t)) \qquad \forall t \in [0, T], \quad x(0) = x_0.$$



L_2 bound of 0.2831

Distance Program (Measures)

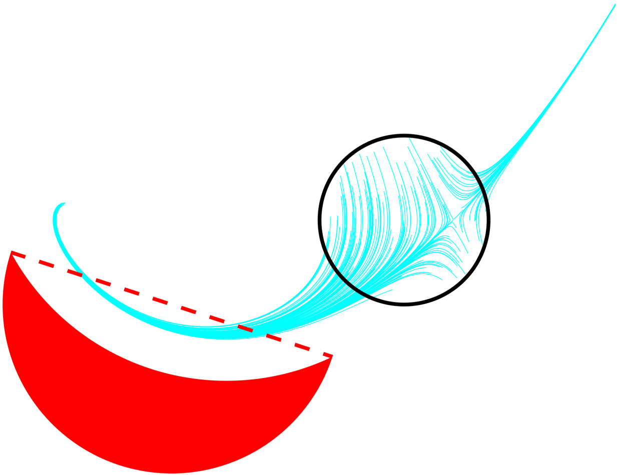
Infinite Dimensional Linear Program (Convergent)

$$
\begin{aligned}
p^* = \ \inf \quad & \langle c(x,y), \eta(x,y) \rangle && (3a)\\
& \langle 1, \mu_0 \rangle = 1 && (3b)\\
& \langle v(t,x), \mu_p \rangle = \langle v(0,x), \mu_0 \rangle + \langle \mathcal{L}_f v(t,x), \mu \rangle \quad \forall v && (3c)\\
& \langle w(x), \eta(x,y) \rangle = \langle w(x), \mu_p(t,x) \rangle \quad \forall w && (3d)\\
& \eta \in \mathcal{M}_+(X \times X_u) && (3e)\\
& \mu_p, \ \mu \in \mathcal{M}_+([0,T] \times X) && (3f)\\
& \mu_0 \in \mathcal{M}_+(X_0) && (3g)
\end{aligned}
$$

Probability measures: (μ_0, μ_p, η)

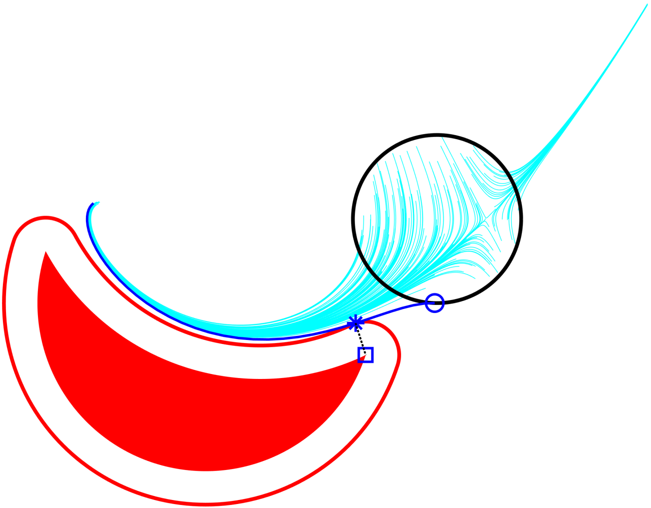
Near-optimal trajectories if moment-matrix $\approx$ rank-1

Distance Example (Flow Moon)



Collision if X_u was a half-circle

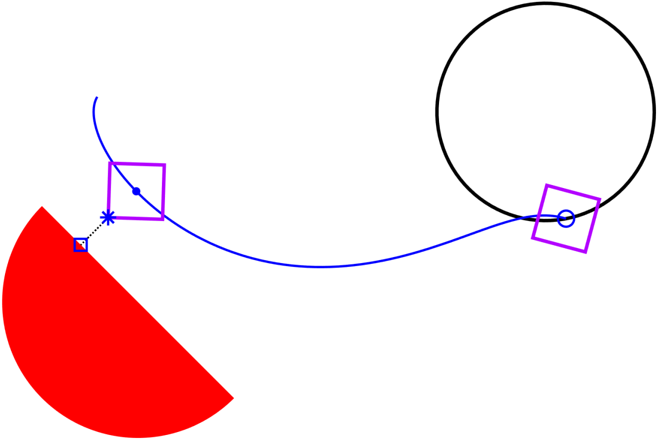
Distance Example (Flow Moon)



L_2 bound of 0.1592

Safety of Shapes

Points on shape S with orientation ω (e.g., rigid body motion)

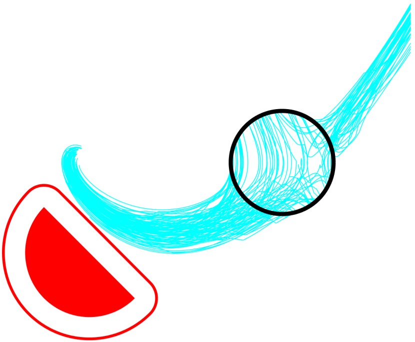


L_2 bound of 0.1465, rotating square

Distance with Bounded Uncertainty

Dynamics $\dot{x}(t) = f(t, x(t), w(t))$ with $w(t) \in W$

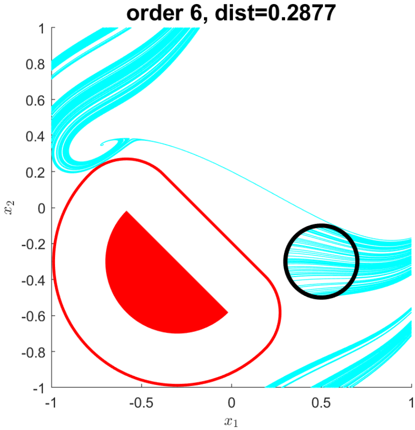
Young measure $\mu(t, x, w)$, Liouville term $\langle \mathcal{L}_f v(t, x, w), \mu \rangle$



L_2 bound of 0.1691, $w(t) \in [-1, 1]$

Hybrid Systems

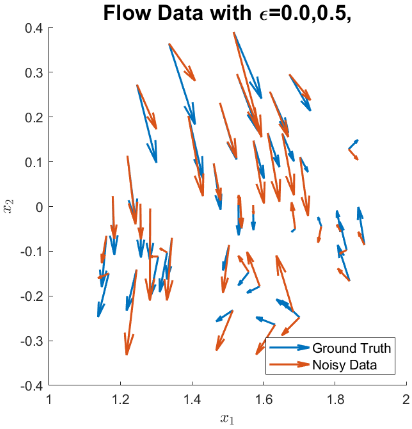
Continuous dynamics with discrete jumps/transitions



$$R_{\text{left}\to\text{bottom}} = [1 - x_2; x_1], \qquad R_{\text{right}\to\text{top}} = [x_2; x_1]$$

Sampling: Flow System

Data $\mathcal{D} = \{(t_j, x_j, \dot{x}_j)\}_j$ under mixed L_∞-bounded noise



$$\dot{x} = [x_2, -x_1 - x_2 + x_1^3/3]$$

Dynamics Model

Given data $\mathcal{D}$, budget ϵ, system model $\{f_0, f_\ell\}$

Parameterize ground truth F by functions in dictionary

$$\dot{x}(t) = f(t, x, w) = f_0(t, x) + \sum_{\ell=1}^{L} w_\ell f_\ell(t, x)$$

Ground truth satisfies corruption $J(w^*) \leq \epsilon$

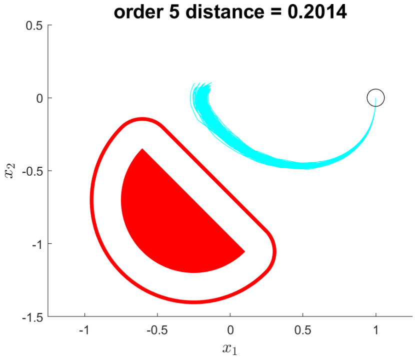
L_∞ example: $J(w) = \max_j \| f(t_j, x_j, w) - \dot{x}_j \|_\infty$

Distance Estimation Example (Flow)

Input-affine + Semidefinite Representable uncertainty

$$\mathcal{L}_f v(t, x, w) \leq 0 \qquad \forall (t, x, w) \in [0, T] \times X \times W$$

PSD Size 8568 $\to$ 56 ($L = 10$) using robust counterparts



$$\dot{x} = [x_2,\ \text{cubic}(x_1, x_2)]$$

Crash-Safety

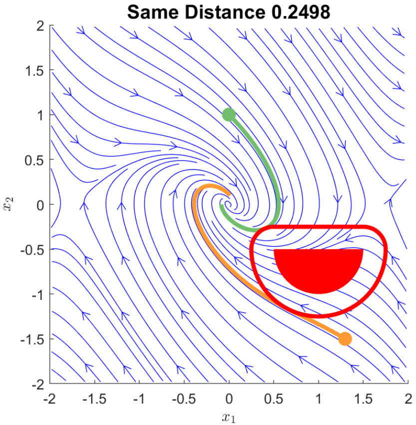
How much data corruption is needed to crash?

$$
\begin{aligned}
Q^* = \inf_{t^*,\ x_0,\ w} & \left[\sup_{t \in [0,t]} J(w(t)) \right] \\
& \dot{x}(t) = f(t, x(t), w(t)) & \forall t \in [0, t^*] \\
& x(t \mid x_0, w(\cdot)) \in X_u \\
& w(\cdot) \in W,\ \ t^* \in [0, T],\ \ x_0 \in X_0
\end{aligned}
$$

Model safe if $Q^* > \epsilon$

Example Crash-Bounds

Two trajectories have same distance, different crash-bounds



Green-Top $Q^* = 0.316$, Yellow-Bottom $Q^* = 0.622$

Peak-Minimizing Control

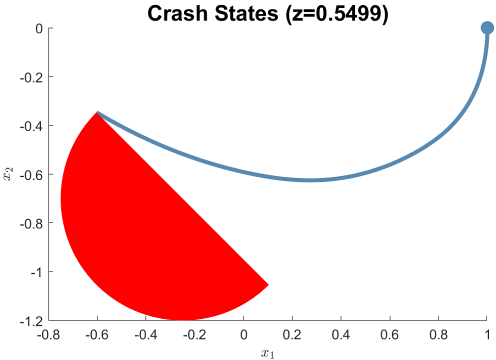
Add state $\dot{z} = 0$ (Molina, Rapaport, Ramírez 2022)

$$
\begin{aligned}
Q_z^* = \inf_{t^*,\ x_0,\ z,\ w} \quad & z && && \text{(4a)}\\
& \dot{x}(t) = f(t, x(t), w(t)) && \forall t \in [0, t^*] && \text{(4b)}\\
& \dot{z}(t) = 0 && \forall t \in [0, t^*] && \text{(4c)}\\
& J(w(t)) \leq z && \forall t \in [0, t^*] && \text{(4d)}\\
& x(t^* \mid x_0, w(\cdot)) \in X_u && && \text{(4e)}\\
& w(\cdot) \in W,\ \ t^* \in [0, T] && && \text{(4f)}\\
& x_0 \in X_0, z \in [0, J_{\max}] && && \text{(4g)}
\end{aligned}
$$

Equivalent formulation, $Q^* = Q_z^*$

Data-Driven Flow Crash-Bound

CasADi matches degree-4 moment-SOS crash bound



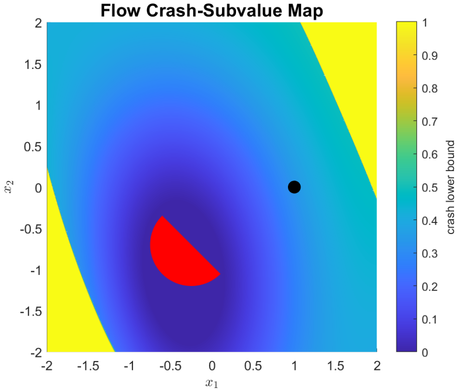
Terminal measure $\mu_p \in \mathcal{M}_+([0, T] \times X_u)$

True $\epsilon = 0.5 < 0.5499$, distance ≈ 0.2014

Flow Crash-Subvalue

Piecewise-polynomial subvalue for crash-safety

Based on Joint+Marginal optimization (Lasserre, 2010)



Bound of $0.3399 \leq 0.5499$, but valid everywhere in X

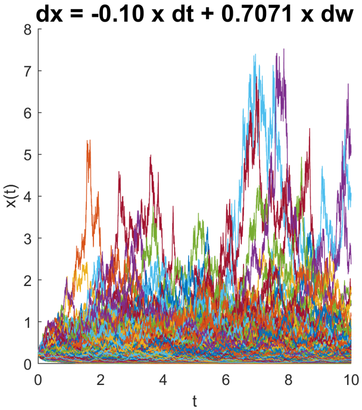
Peak Value-at-Risk Estimation

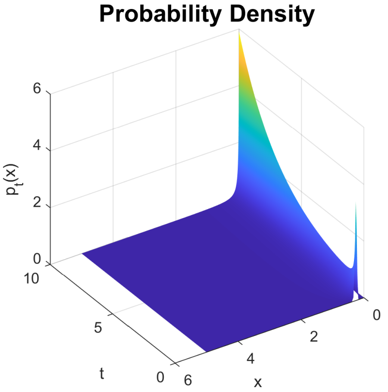
with M. Tacchi, M. Sznaier, A. Jasour

Stochastic Differential Equation

Multivariate SDE $dx = f(t,x)dt + g(t,x)dw$ (Itô)

Drift f and Diffusion g



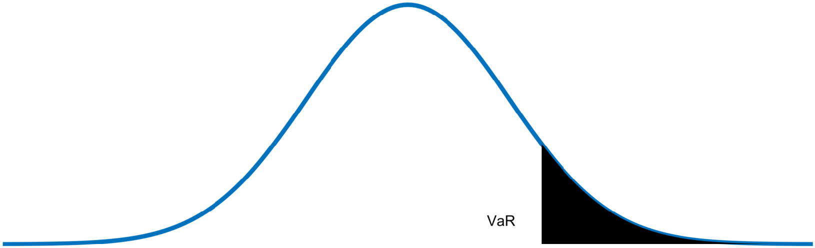


Geometric Brownian Motion

Value-at-Risk (Quantile)

ϵ-VaR of univariate measure $\omega(q)$ is unique number with

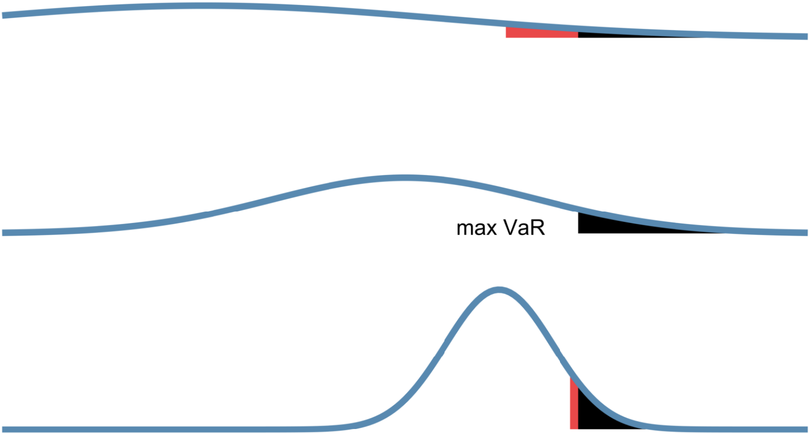
$$\text{Prob}_{\omega}(q \geq VaR_{\epsilon}(\omega)) = \epsilon$$



$VaR = 1.282$ for unit normal distribution at $\epsilon = 10\%$

Maximal Value at Risk

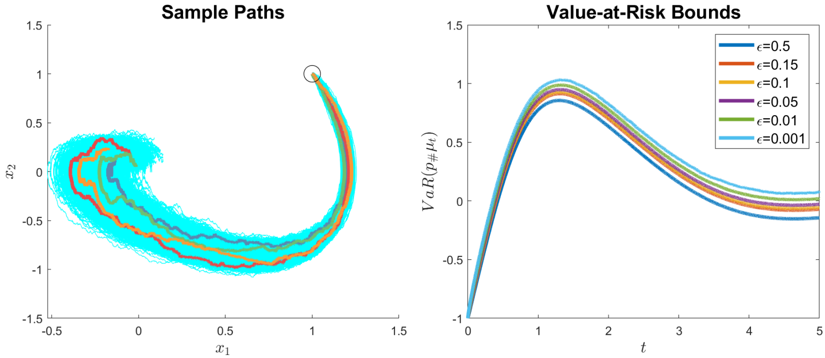
Maximize ϵ-VaR among multiple distributions



Red + Black areas = 10% probability

Value-at-Risk Example (Monte Carlo)

50,000 samples with $T = 5,\ \Delta t = 10^{-3}$



VaR of $p = -x_2$ along $dx = \begin{bmatrix} x_2 \\ -x_1 - x_2 - \frac{1}{2}x_1^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} dw$

Chance-Peak Problem

Maximize VaR of $p(x)$ along SDE trajectories

$p_{\#}\mu_{t^*}$: distribution of $p(x(t))$ at time t^*

$$
\begin{aligned}
P^* = \sup_{t^* \in [0, T]} \quad & VaR_\epsilon(p_{\#}\mu_{t^*}) && \text{(5a)} \\
dx &= f(t, x)dt + g(t, x)dw && \text{(5b)} \\
& \text{stopping time of } \min(t^*, \text{exit from } X) && \text{(5c)} \\
x(0) &\sim \mu_0. && \text{(5d)}
\end{aligned}
$$

Value-at-Risk Bounds

Concentration inequalities can upper-bound VaR

$$\mathit{VaR}_{\epsilon}(\omega) \leq \mathrm{stdev}(\omega) r + \mathrm{mean}(\omega)$$

Name	r	Condition
Cantelli	$\sqrt{1/(\epsilon) - 1}$	ω probability distribution
VP	$\sqrt{4/(9\epsilon) - 1}$	ω unimodal, $\epsilon < 1/6$

Coherent Risk Measures (e.g., CVaR) can also bound VaR

Concentration-Bounded Chance-Peak

Apply concentration inequalities to get upper bound $P_r^* \geq P^*$

Objective upper-bounds VaR w.r.t. time-t^* distribution μ_{t^*}

$$
\begin{aligned}
P_r^* = \sup_{t^* \in [0,T]} \quad & r\sqrt{\langle p^2, \mu_{t^*}\rangle - \langle p, \mu_{t^*}\rangle^2} + \langle p, \mu_{t^*}\rangle && \text{(6a)} \\
& dx = f(t,x)dt + g(t,x)dw && \text{(6b)} \\
& \quad \text{stopping time of } \min(t^*, \text{exit from } X) && \text{(6c)} \\
& x(0) \sim \mu_0. && \text{(6d)}
\end{aligned}
$$

Max-Mean: $\epsilon = 0.5,\ r = 0$ (Cho, Stockbridge, 2002)

Occupation Measure Formulation

Occupation measure μ, terminal measure μ_τ

Second-Order Cone Program in measures (3d SOC)

$$p_r^* = \sup r\sqrt{\langle p^2, \mu_\tau\rangle - \langle p, \mu_\tau\rangle^2} + \langle p, \mu_\tau\rangle \tag{7a}$$

$$\mu_\tau = \delta_0 \otimes \mu_0 + \mathcal{L}^\dagger \mu \tag{7b}$$

$$\mu_\tau,\ \mu \in \mathcal{M}_+([0, T] \times X) \tag{7c}$$

Generator $\mathcal{L}v = \partial_t v + f \cdot \nabla_x v + g^T(\nabla^2_{xx} v)g/2$ (Dynkin's)

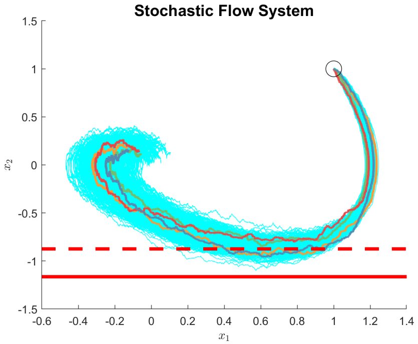
Results in upper-bound $p_r^* \geq P_r^* \geq P^*$, use moments

Chance-Peak Examples

Two-State

Stochastic Flow system from Prajna, Rantzer with $T = 5$

$$dx = \begin{bmatrix} x_2 \\ -x_1 - x_2 - \frac{1}{2}x_1^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} dw.$$

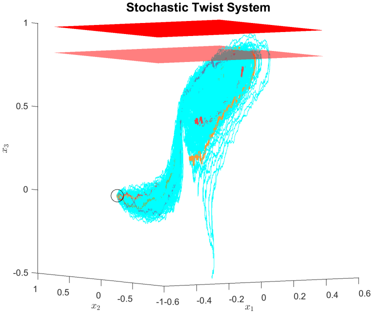


Maximize $-x_2$ with $d = 6$ (dashed=50%, solid=85% [ours])

Three-State

Stochasic Twist system with $T = 5$

$$dx = \begin{bmatrix} -2.5x_1 + x_2 - 0.5x_3 + 2x_1^3 + 2x_3^3 \\ -x_1 + 1.5x_2 + 0.5x_3 - 2x_2^3 - 2x_3^3 \\ 1.5x_1 + 2.5x_2 - 2x_3 - 2x_1^3 - 2x_2^3 \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix} dw.$$

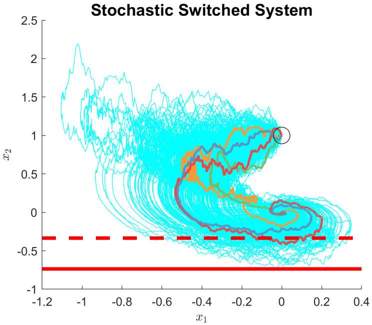


Maximize x_3 with $d = 6$ (translucent=50%, solid=85%)

Two-State Switching

Switching subsystems at $T = 5$

$$dx = \left\{ \begin{bmatrix} -2.5x_1 - 2x_2 \\ -0.5x_1 - x_2 \end{bmatrix}, \begin{bmatrix} -x_1 - 2x_2 \\ 2.5x_1 - x_2 \end{bmatrix} \right\} dt + \begin{bmatrix} 0 \\ 0.25x_2 \end{bmatrix} dw$$

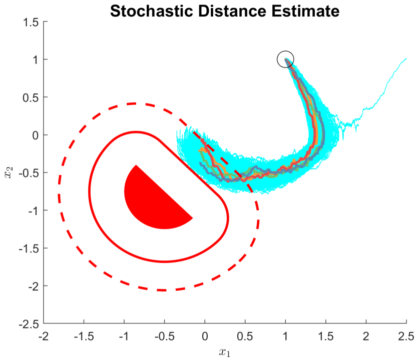


Maximize $-x_2$ with $d = 6$ (dashed=50%, solid=85%)

Two-State Distance

Half-circle unsafe set X_u

Based on distance estimation program



Minimize L_2 distance to X_u with $d = 6$ (dashed=50%, solid=85%)

Time-Delay Peak Estimation

with M. Korda, V. Magron, M. Sznaier

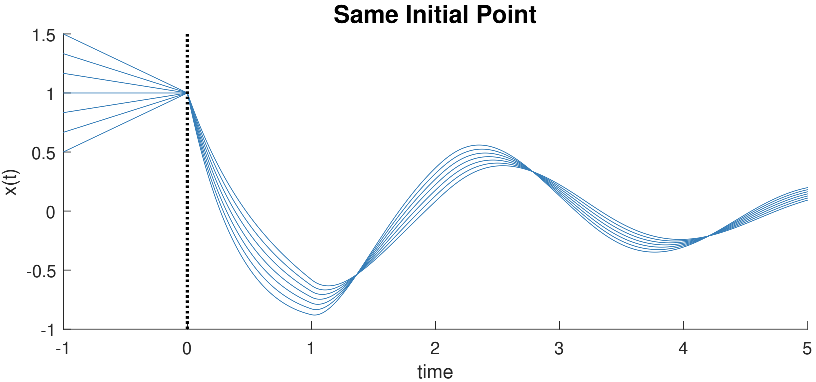
Time-Delay Examples

Delay between state change and its effect on system

$$\dot{x}(t) = f(t, x(t), x(t-\tau)) \qquad \forall t \in [0, T]$$
$$x(s) = x_h(s) \qquad \forall s \in [-\tau, 0]$$

System	**Delay**
Epidemic	Incubation Period
Population	Gestation Time
Traffic	Reaction Time
Congestion	Queue Time
Fluid Flow	Moving in Pipe

Dependence on History

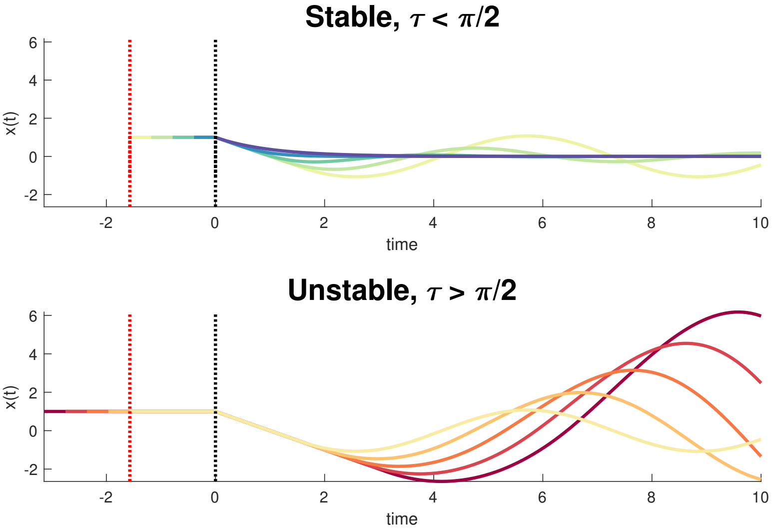


$$x'(t) = -2x(t) - 2x(t-1)$$

All trajectories pass through $(t, x) = (0, 1)$

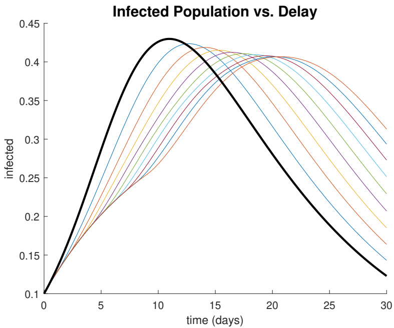
Initial history determines behavior, not just initial point

Delay Bifurcation Example

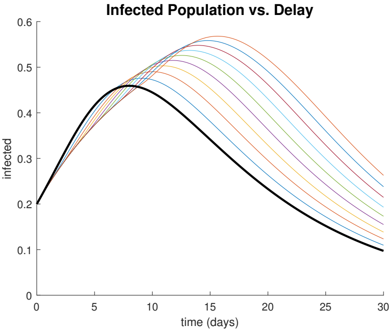


$$x'(t) = -x(t-\tau) \qquad \text{(Fridman 2014)}$$

Peak Value vs. Delay



(a) $I_h = 0.1$, peak decreases



(b) $I_h = 0.2$, peak increases

$$\begin{bmatrix} S'(t) \\ I'(t) \end{bmatrix} = \begin{bmatrix} -0.4S(t)I(t) \\ 0.4S(t-\tau)I(t-\tau) - 0.1I(t) \end{bmatrix}$$

Peak Estimation of Time-Delay Systems

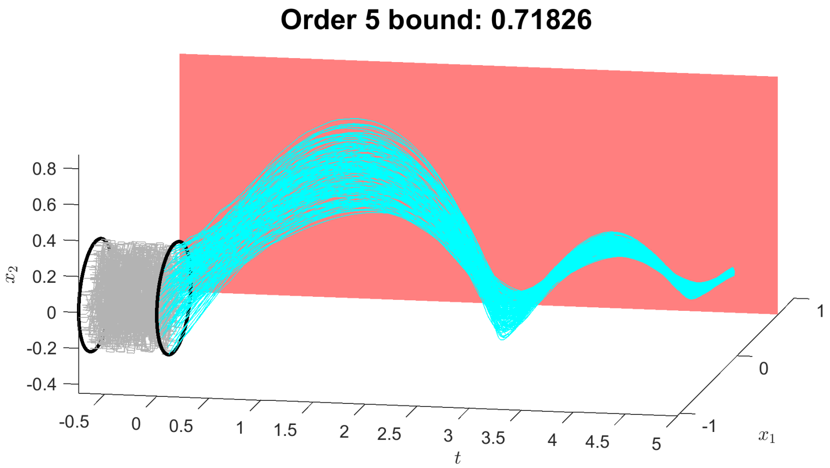
History $x_h(t)$ resides in a class of functions $\mathcal{H}$

Graph-constrained $\mathcal{H}$: $(t, x_h(t))$ contained in $H_0 \subset [-\tau, 0] \times X$

$$
\begin{aligned}
P^* = \sup_{t^*,\, x_h} \ & p(x(t^*)) \\
& \dot{x} = f(t, x(t), x(t-\tau)) && t \in [0, t^*] \\
& x(t) = x_h(t) && t \in [-\tau, 0] \\
& x_h(\cdot) \in \mathcal{H}
\end{aligned}
$$

Represent $x(t \mid x_h) : t \in [-\tau, t^*]$ as occupation measure

Time-Varying Preview



Maximize x_1 on $\dot{x}(t) = \begin{bmatrix} x_2(t)t - 0.1x_1(t) - x_1(t-\tau)x_2(t-\tau) \\ -x_1(t)t - x_2(t) + x_1(t)x_1(t-\tau) \end{bmatrix}$

Existing Methods (very brief)

Certificates of Stability

- Lyapunov-Krasovskii
- Razumikhin
- LMI, Wirtinger
- ODE-Transport PDE

Relaxed control (Warga 1974, Vinter and Rosenblueth 1991-2)

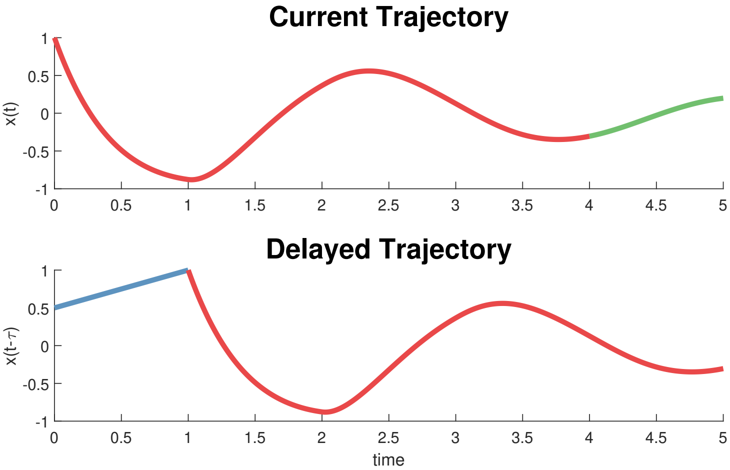
Fixed-terminal-time OCP with gridding (Barati 2012)

SOS Barrier (Papachristodoulou and Peet, 2010)

Riesz Operators (Magron and Prieur, 2020)

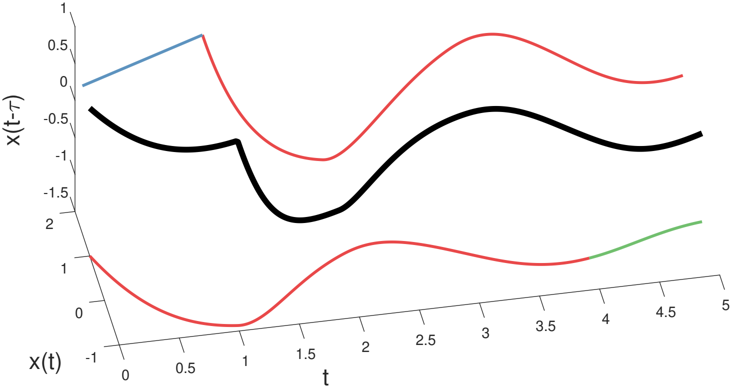
Time-Delay Measure Program

Time-Delay Visualization



$$x(t) = -2x(t) - 2x(t-1), \qquad x_h(t) = 1 - t/2$$

Delay Embedding



Black curve: $(t, x(t), x(t-\tau))$

Measure-Valued Solution

Tuple of measures for the delayed case

Peak	$\mu_p \in \mathcal{M}_+([0, T] \times X)$
Initial	$\mu_0 \in \mathcal{M}_+(X_0)$
History	$\mu_h \in \mathcal{M}_+(H_0)$
Occupation Start	$\bar{\mu}_0 \in \mathcal{M}_+([0, T - \tau] \times X^2)$
Occupation End	$\bar{\mu}_1 \in \mathcal{M}_+([T - \tau, T] \times X^2)$
Time-Slack	$\nu \in \mathcal{M}_+([0, T] \times X)$

Types of Constraints

Initial Conditions

Liouville: Dynamics

Consistency: Time-delay overlaps

Initial Conditions

Point evaluation $\langle 1, \mu_0 \rangle = 1$ at time $t = 0^+$

History $(t, x_h(t))$ defines a curve $[-\tau, 0]$, point at $x_h(0)$

t-marginal of μ_h should be the Lebesgue measure in $[-\tau, 0]$

Liouville

Treat $x(t-\tau) = x_1$ as an external input $\dot{x}_0 = f(t, x_0, x_1)$

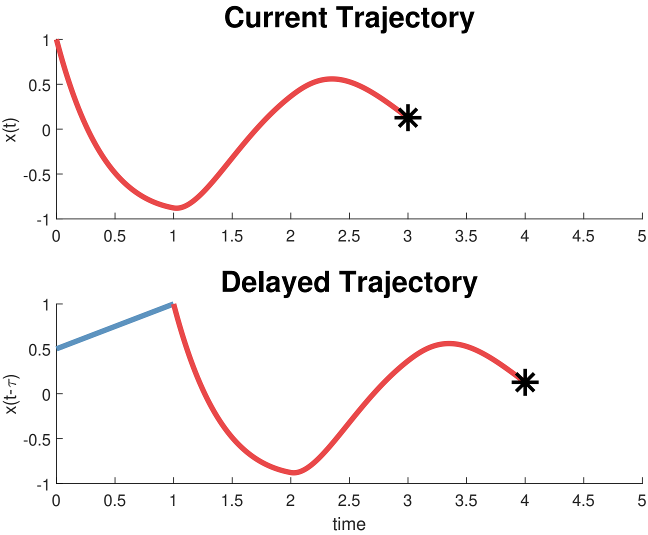
Sum $\bar{\mu} = \bar{\mu}_0 + \bar{\mu}_1$ in times $[0, T-\tau] \cap [T-\tau, T] = [0, T]$

Based on the delay embedding $(t, x(t), x(t-\tau))$

For all test functions $v \in C^1([0, T] \times X)$:

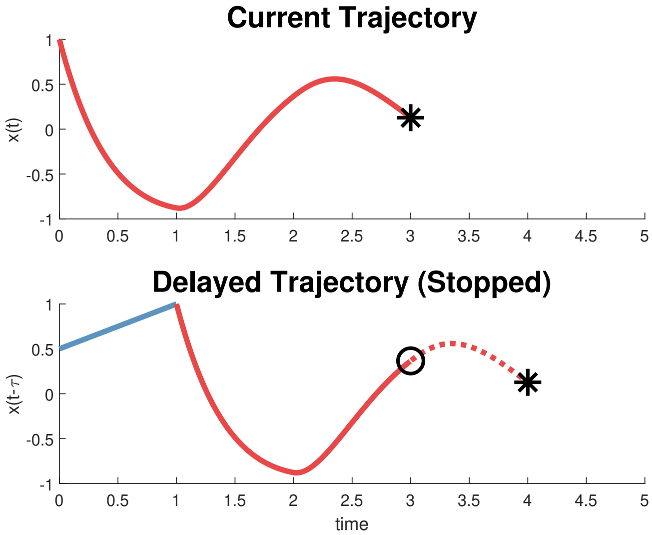
$$\langle v, \mu_p \rangle = \langle v(0, x), \mu_0(x) \rangle + \langle \mathcal{L}_{f(t, x_0, x_1)} v(t, x_0), \bar{\mu}(t, x_0, x_1) \rangle$$

Consistency Issue



Inconsistent elapsed times

Consistency Fix



Early stopping in delayed time

Consistency Constraint

Inspired by changing limits of integrals $t' \leftarrow t - \tau$

$$\left(\int_0^{t^*} + \int_{t^*}^{\min(T, t^*+\tau)}\right) \phi(t, x(t-\tau))dt$$
$$= \left(\int_{-\tau}^{0} + \int_0^{\min(t^*, T-\tau)}\right) \phi(t'+\tau, x(t'))dt'.$$

Shift-push $S^\tau_\#$ with $\langle \phi, S^\tau_\# \mu\rangle = \langle S^\tau \phi, \mu\rangle = \langle \phi(t+\tau, x), \mu\rangle$

Consistency constraint with time-slack ν

$$\pi^{tx_1}_\#(\bar{\mu}_0 + \bar{\mu}_1) + \nu = S^\tau_\#(\mu_h + \pi^{tx_0}_\# \bar{\mu}_0).$$

Measure Linear Program

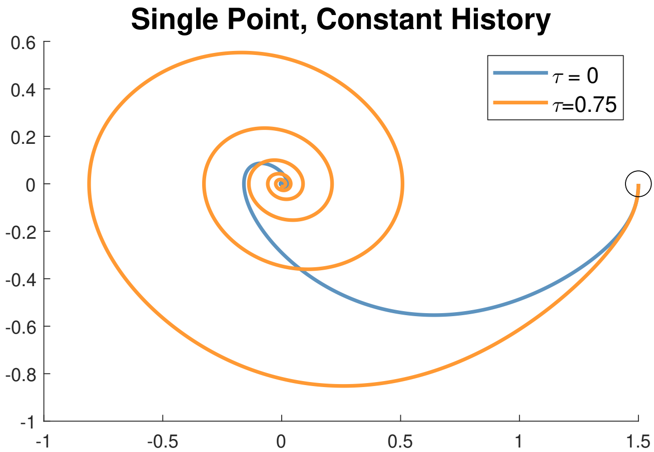
Linear program for time-delay peak estimation

$$
\begin{aligned}
p^* = \ \sup \quad & \langle p, \mu_p \rangle && \text{(8a)} \\
& \text{History-Validity}(\mu_0, \mu_h) && \text{(8b)} \\
& \text{Liouville}(\mu_0, \mu_p, \bar{\mu}_0, \bar{\mu}_1) && \text{(8c)} \\
& \text{Consistency}(\mu_h, \bar{\mu}_0, \bar{\mu}_1, \nu) && \text{(8d)} \\
& \text{Measure Definitions for } (\mu_h, \mu_0, \mu_p, \bar{\mu}_0, \bar{\mu}_1, \nu) && \text{(8e)}
\end{aligned}
$$

Largest measures $\bar{\mu}_0, \bar{\mu}_1$ have $2n + 1$ variables

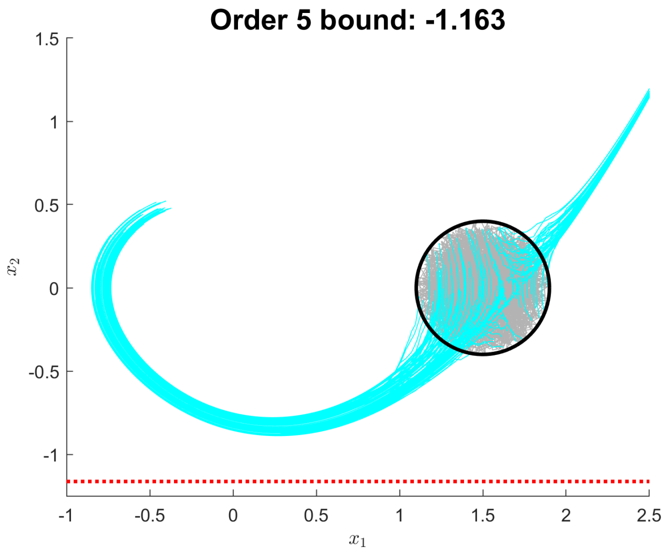
Time-Delay Examples

Delay Comparision



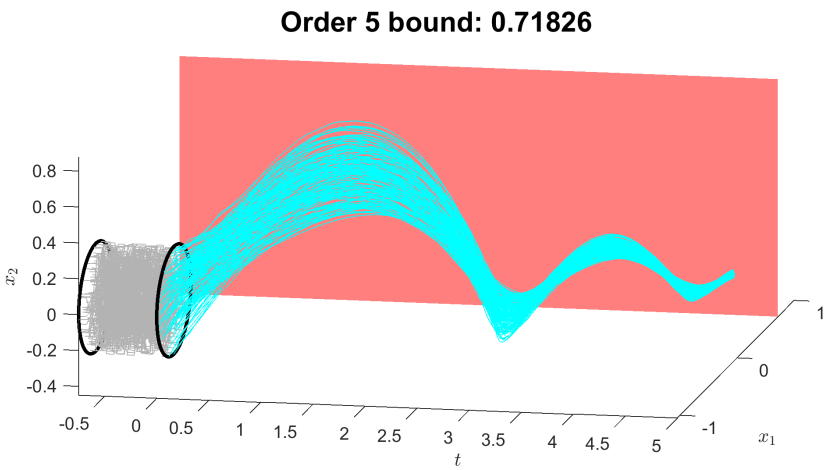
$$\dot{x}(t) = \begin{bmatrix} x_2 \\ -x_1(t-\tau) - x_2(t) + x_1(t)^3/3 \end{bmatrix}$$

Delayed Flow System



Minimize x_2 on the delayed Flow system

Time-Varying System (Reprise)



Maximize x_1 on $\dot{x}(t) = \begin{bmatrix} x_2(t)t - 0.1x_1(t) - x_1(t-\tau)x_2(t-\tau) \\ -x_1(t)t - x_2(t) + x_1(t)x_1(t-\tau) \end{bmatrix}$

Take-aways

Summary

Noted importance of safety quantification

Extended occupation measure methods for peak estimation

Performed data-driven analysis using robust counterparts

Adapted to non-ODE systems (Hybrid, SDE, Time-Delay)

Future Work

- No-relaxation-gap for chance-peak and time-delay system
- High-order concentration inequalities
- Other time-delay models
- Lévy processes, Poisson jumps
- Distance-maximizing control
- Increased scalability, robotic systems
- Real-time computation

Safety is Important

Quantify using Peak Estimation

Journal Papers

Published:

1. J. Miller, D. Henrion, and M. Sznaier, "Peak Estimation Recovery and Safety Analysis," *IEEE Control Systems Letters*, vol. 5, no. 6, pp. 1982–1987, 2021 [link]

Conditionally Accepted:

1. J. Miller and M. Sznaier, "Bounding the Distance to Unsafe Sets with Convex Optimization," (Conditionally accepted by IEEE Transactions on Automatic Control in 2022) [link]

Conference Proceedings

1. J. Miller and M. Sznaier, "Bounding the Distance of Closest Approach to Unsafe Sets with Occupation Measures," in *2022 61st IEEE Conference on Decision and Control (CDC)*, pp. 5008–5013, 2022. [link]
2. J. Miller and M. Sznaier, "Facial Input Decompositions for Robust Peak Estimation under Polyhedral Uncertainty," *IFACPapersOnLine*, vol. 55, no. 25, pp. 55–60, 2022. [link]. IFAC Young Author Award (ROCOND)
3. J. Miller, D. Henrion, M. Sznaier, and M. Korda, "Peak Estimation for Uncertain and Switched Systems," in *2021 60th IEEE Conference on Decision and Control (CDC)*, pp. 3222–3228, 2021. [link]. Outstanding Student Paper Award (CDC 2021)

Preprints

1. J. Miller, M. Korda, V. Magron, and M. Sznaier “Peak Estimation of Time Delay Systems using Occupation Measures, ” 2023. [link]
2. J. Miller, M. Tacchi, M. Sznaier, and A. Jasour, “Peak Value-at-Risk Estimation for Stochastic Differential Equations using Occupation Measures,” 2023. [link]
3. J. Miller and M. Sznaier, “Peak Estimation of Hybrid Systems with Convex Optimization, ” 2023. [link]
4. J. Miller and M. Sznaier “Quantifying the Safety of Trajectories using Peak-Minimizing Control, ” 2023. [link]
5. J. Miller and M. Sznaier, “Analysis and Control of Input-Affine Dynamical Systems using Infinite-Dimensional Robust Counterparts,” 2023. [link]

Acknowledgements

Parents (Wayne and Debbie) and Family

Mario Sznaier, Octavia Camps, RSL

Didier Henrion, POP and MAC groups at LAAS-CNRS

Roy Smith, IfA at ETH Zurich

Jesús A. De Loera, ICERM

Fred Leve, Air Force Office of Scientific Research

Chateaubriand Fellowship of the Office for Science Technology of the Embassy of France in the United States.

National Science Foundation, Office of Naval Research

Last but not least

The Warden

Thank you again for your attention

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Cookies in Dana 429 (RSL)