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MODULE ParReachProofs
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This module contains TLAPS checked proofs of the safety properties asserted in module ParReach—namely, the invariance of Inv and that the parallel algorithm implements the safety part of Misra's algorithm under the refinement mapping defined there.

EXTENDS ParReach, Integers, TLAPS

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LEMMA TypeInvariant \stackrel{\triangle}{=} Spec \Rightarrow \Box Inv
\langle 1 \rangle 1. Init \Rightarrow Inv
      BY RootAssump DEF Init, Inv, ProcSet
\langle 1 \rangle 2. Inv \wedge [Next]_{vars} \Rightarrow Inv'
    BY SuccAssump DEF Inv, Next, Terminating, vars, ProcSet, p, a, b, c
\langle 1 \rangle 3. QED
   BY \langle 1 \rangle 1, \langle 1 \rangle 2, PTL DEF Spec
THEOREM Spec \Rightarrow R!Init \wedge \Box [R!Next]_R!vars
\langle 1 \rangle 1. Init \Rightarrow R!Init
      By ProcsAssump def Init, R! Init, pcBar, vrootBar, ProcSet
\langle 1 \rangle 2. Inv \wedge [Next]_{vars} \Rightarrow [R!Next]_R!vars
   \langle 2 \rangle suffices assume Inv,
                                       [Next]_{vars}
                        PROVE [R!Next]_R!vars
      OBVIOUS
   \langle 2 \rangle USE DEF Inv, Next, Terminating, vars, R! Next, R! vars, vrootBar, pcBar
   \langle 2 \rangle 1. Assume new self \in Procs,
                          a(self)
           PROVE [R!Next]_R!vars
      \langle 3 \rangle 1. Assume vroot \neq \{\}
              PROVE UNCHANGED R!vars
          BY \langle 2 \rangle 1, \langle 3 \rangle 1 DEF a
      \langle 3 \rangle 2. Assume vroot = \{\}
              PROVE [R!Next]_R!vars
         \langle 4 \rangle 1. Assume vrootBar = \{\}
                 PROVE [R!Next]_R!vars
           By \langle 2 \rangle 1, \langle 3 \rangle 2, \langle 4 \rangle 1 Def a, R!a
         \langle 4 \rangle 2. Assume vrootBar \neq \{\}
                 PROVE UNCHANGED R!vars
            \langle 5 \rangle 1. \quad \exists \ q \in Procs \setminus \{self\} : pc[q] \neq "Done"
               BY \langle 4 \rangle 2, \langle 3 \rangle 2, \langle 2 \rangle 1 DEF a
            \langle 5 \rangle 2. pcBar' \neq "Done"
               By \langle 5 \rangle 1, \langle 3 \rangle 2, \langle 2 \rangle 1 def a
            \langle 5 \rangle.QED
              BY \langle 5 \rangle 2, \langle 3 \rangle 2, \langle 2 \rangle 1 DEF a
         \langle 4 \rangle 3. QED
           BY \langle 4 \rangle 1, \langle 4 \rangle 2
      \langle 3 \rangle 3. QED
         BY \langle 3 \rangle 1, \langle 3 \rangle 2 DEF R!Next
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\langle 2 \rangle 2. \text{ Assume new } self \in Procs, \\ b(self) \\ \text{Prove } [R! Next]_R! vars \\ \text{By } \langle 2 \rangle 2 \text{ Def } b, R! a \\ \langle 2 \rangle 3. \text{ Assume new } self \in Procs, \\ c(self) \\ \text{Prove } [R! Next]_R! vars \\ \text{By } \langle 2 \rangle 3 \text{ Def } c \\ \langle 2 \rangle 4. \text{Case unchanged } vars \\ \text{By } \langle 2 \rangle 4 \\ \langle 2 \rangle 5. \text{ Qed} \\ \text{By } \langle 2 \rangle 1, \langle 2 \rangle 2, \langle 2 \rangle 3, \langle 2 \rangle 4 \text{ Def } Next, Terminating, p \\ \langle 1 \rangle 3. \text{ Qed} \\ \text{By } \langle 1 \rangle 1, \langle 1 \rangle 2, TypeInvariant, PTL \text{ Def } Spec
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 $[\]backslash * \ {\it Modification History}$

^{*} Last modified Sun Apr 14 16:55:36 PDT 2019 by lamport

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