

## Week 1 Questions

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Q1

- a) If each letter appears once then the 1st can be any of the 10, then for the next letter there are only 9 remaining options, then 8, and so on..  $10! = 3628800$
- b) If the letters E and F must be next to each other, then there are  $8!$  combinations for when they are the first two letters  $\rightarrow EF*****$ . The pair can be in 9 different positions, and they can be the other way round (FE), which means  $9 \times 2 \times 8! = 725760$
- c) There are  $6!$  ways of arranging the letters of BANANA. However, they are not all different ie B A N1 A N2 A and B A N2 A N1 A are both counted. Similarly for the three different As, so we divide by  $(3! \times 2!)$   $\rightarrow$  the different combinations of the 3 Ns and 2 As, to get **60**
- d) The first of the three letters drawn can be any of the 5, the next can be one of the 4 remaining letters, and the last can be one of 3. Due to the fact each group of three letters can be arranged in  $3!$  different ways, and the question specifies order doesn't matter, so we divide..  $(5 \times 4 \times 3) / (3 \times 2 \times 1) = 10$

Q2

- a) The die can land on any of the 6 faces every time you throw it, so after 4 throws, there are  $6 \times 6 \times 6 \times 6$  possibilities = **1296**
- b) If you have two 3s and two of any other of the 5 faces, that is  $1 \times 1 \times 5 \times 5 = 25$  possibilities. The threes can appear at different rolls though, so we use permutations:  $4! / (2! \times 2!) = 6$ . The first 2! is the group of threes and the other is group of non-threes. So we get  $25 \times 6 = 150$
- c) There is one possibility where all 4 rolls are a 3. There are 150 where there are 2, and there are  $(1 \times 1 \times 1 \times 5) \times (4! / 3!) = 5 \times 4 = 20$

Q3

- a) There are  $8!$  non-distinct combinations, and there are two copies of each of the four aces, so it is  $8! / (2! \times 2! \times 2! \times 2!) = 2520$
- b) The first card can be any of the four suits, and the second can be any of the remaining three,  $4 \times 3 = 12$ . But that counts HeartsSpades and SpadesHearts etc, which means we divide by two to avoid double counting.  $12 / 2 = 6$
- c) You can have either Hearts or Diamonds for each card dealt  $(2 \times 2)$ , but this counts both HeartsDiamonds and DiamondsHearts, so we subtract one.  $4 - 1 = 3$