Week 5 Questions

Q1

a) The chance that the second marble drawn is the same as the first is 4/9.

$$E[X] = x_1 P(X = x_1) + x_2 P(X = x_2)$$

$$1.1 * \frac{4}{9} + (-1) * \frac{5}{9} = 0.4889 - 0.5556 = -0.0667$$

b) Using the formula for variance:

$$Var(X) = \sum_{i=1}^{n}(x_i - \mu)^2 p(x_i)$$

$$(1.1 - (-0.0667))^2 * \frac{4}{9} + (-1 - (-0.0667))^2 * \frac{5}{9}$$

$$0.60497284 + 0.48391605 = 1.0889$$

Q2

a)

$$E[X_i] = 1 * 0.6 + 0 * 0.4 = 0.6$$

b)

$$Var(X_i) = (1 - 0.6)^2 * 0.6 + (0 - 0.6)^2 * 0.4$$

$$Var(X_i) = 0.096 + 0.144 = 0.24$$

c) $E[Y]$ is the overall expected number of people who voted, out of the n who took part in the poll, $n * E[X] = 0.6n$.
$E[X] = E[X_i]$ for all i, because people were sampled independently, hence
$E[Y] \neq E[X], 0.6n \neq 0.6$ ie they are not the same.

d) $E[\frac{1}{n}Y] = E[\frac{1}{n}\sum_{i=1}^{n} X_i] = E[X] = 0.6$

e)

$$Var(\frac{1}{n}Y) = \frac{1}{n^2}Var(Y) = \frac{1}{n^2}Var(\sum_{i=1}^{n} X_i)$$

$$Var(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n}Var(X_i) = nVar(X)$$

$$\therefore Var(\frac{1}{n}Y) = \frac{1}{n}Var(X)$$

Q3

a) 5 + 8 = 13 balls total (for X_1, 12 for X_2).

JPMF	x=0	x=1	P(X2=y)
y=0	14/39	10/39	24/39
y=1	10/39	5/39	15/39
P(X1=x)	8/13	5/13	1

b) No, they are not independent.

$$P(X_1 \cap X_2) \neq P(X_1) * P(X_2)$$

$$\frac{5}{13} * \frac{15}{39} = 0.1479 \neq 0.1282 = \frac{5}{39}$$

c)

$$E[X_2] = 1 * \frac{15}{39} + 0 * \frac{24}{39} = \frac{15}{39}$$

d)

$$P(Y = y|X = x) = \frac{P(Y = y \text{ '' and '' } X = x)}{P(X = x)}$$

$$E[Y|X = x] = \sum_y yP(Y = y|X = x)$$

$$\frac{5}{39} \div \frac{5}{13} = \frac{1}{3}$$

$$\frac{10}{39} \div \frac{5}{13} = \frac{2}{3}$$

$$(\frac{1}{3} * 1) + (\frac{2}{3} * 0) = E[X_2|X_1 = 1] = \frac{1}{3}$$