* WAVE-PARTICLE DUALITY *

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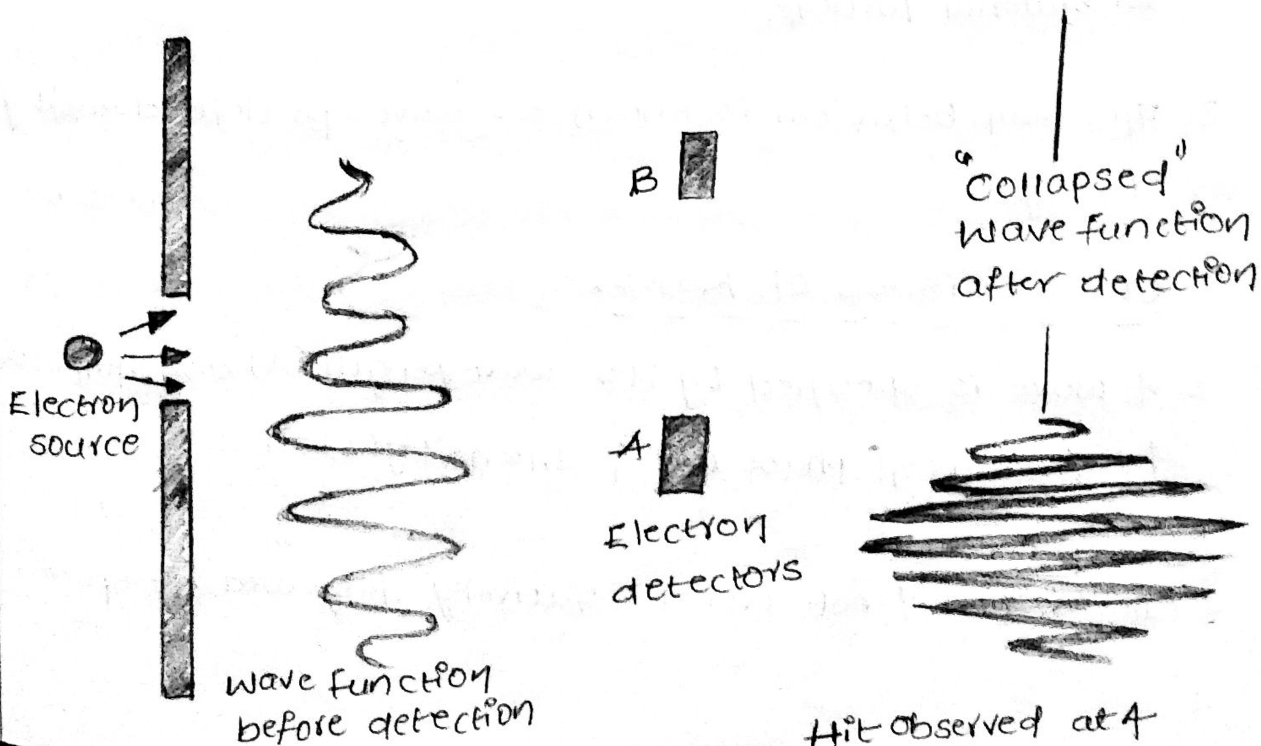
Name of the project :- "Wave-particle Duality".

Introduction :-

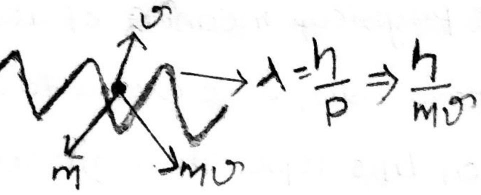
A well-known mystery in Quantum mechanics is Wave-particle duality. Is an electron a point mass or a physical wave?. What is the physical meaning of its wave function? About a hundred years ago, there was a famous debte between Bohr and Einstein on this topic. Their questions is still open today. This paper reviews a new theoretical frame work to address this problem. Here, it is hypothesized that both photons and electrons are quantized excitation waves of the vacuum, the physical properties of which can be modeled based on the Maxwell theory. Using the method of Helmholtz decomposition, one can show the wave function of the particle is associated with an electric vector potential called 'Z', which plays the role of basic field for the excitation wave.



On the basis of dual nature of light in 1924 Louis de-Broglie suggested that the dual nature is not only of light, but "each moving material particle has the dual nature.

He assumed "a wave to be associated with each moving material particle which is called the matter wave, and wave length of this matter wave is given by "$\lambda = \frac{h}{p}$"

Now if mass of the particle is 'm' & velocity is v then momentum $P = mv$ so, $\lambda = \frac{h}{mv}$



Wave-particle duality :- [Theory]

1. This wave particle duality was discovered by "Louis de-broglie" in 1924

2. According to him the matter also has dual charateristics i.e, Matter must also exbits both particle & wave nature, what is true for an energy packet (quanta/photon) is also true for material particle.

3. This dual behaviour is known as wave-particle duality of light.

Charateristics of Waves :-

1. A wave is specified by its wave length (λ) and Amplitude (A) frequency (η); phase (ϕ) & Intensity (I) etc....

2. It is spread onto over a relatively large area of space.

Charateristics of particle:-

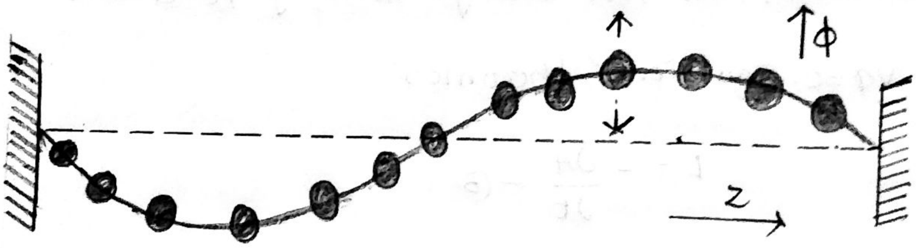
1. A particle is specified by its mass, velocity, momentum (P) and Energy (E) etc...

2. It occupies a definate position at an instant of time.

3. phenomenon of Photo-electric effect and compte effect are the evidences for the particle nature of light.

Wave - Generating Mechanism in the Vacuum:-

In quantum mechanics, one usually associates the wave function of a particle with the variation of a "basic field". This basic-field (or) quantum field. can be considered as a measure of Movement of the wave medium. This field is not the same as the "classical field" (such as the electric field or Magnetic field



We know the photon is an electromagnetic wave. Thus, one way think that the basic field for a photon is either the electric or magnetic field. However, this thinking is not true., Because there are certain mathematical requirements for the basic field. For example, one-dimensional (1D) string, the Lagrangian density is known to be.

$$\boxed{\mathcal{L} = \frac{1}{2}\rho\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}F_i\left(\frac{\partial\phi}{\partial z}\right)^2}$$

Here ϕ is the basic field which represents the vertical displacement of the string (For details of other symbols. Fig.1)
Since the Lagrangian density of the electromagnetic field in the vacuum is

$$\mathcal{L} = \frac{1}{2}(\varepsilon_0 E^2 - \mu_0 H^2),$$

E or H does not appear to be suitable for playing the role of a basic field.. Then, what else can play the role of a basic field for the photon? In the Maxwell theory E & H can be derived from the scalar potential Φ and the vector potential $\vec{A}$.

$$\begin{cases} B = \nabla \times A & \text{—①} \\ E = -\nabla\phi - \dfrac{\partial A}{\partial t} & \text{—②} \end{cases}$$

In the vacuum, the free charge density : $\rho_e = 0$ and thus one can set $\nabla\phi = 0$. Equation (2) becomes.

$$E = -\frac{\partial A}{\partial t} \quad \text{—③}$$

substituting Equations.

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t} = 0 \quad \text{—④}$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$ is the speed of light. This suggest that Vector potential A could play the role of basic field for the photon. Indeed, the Lagrangian density composed of A does appear to have the right form.

$$\mathcal{L} = \frac{1}{2}\left[\varepsilon_0 \left|\frac{\partial A_x}{\partial t}\right|^2 - \frac{1}{\mu_0}\left|\frac{\partial A_x}{\partial z}\right|^2\right] \quad \text{—⑤}$$

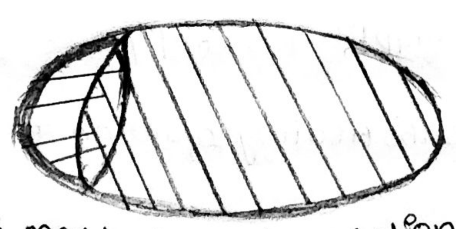
Origin of the concept of vector potential :- Theorem of Helmholtz decomposition :-

In order to determine whether A is a proper measure of the vacuum displacement, let us first review the original meaning of A. In the Maxwell theory, the vector potential A was defined from the magnetic flux B based on the relation.

$$B = \nabla \times A \quad \text{-(6)}$$

According to the literature, Maxwell proposed Eq(6). because he wanted to interpret Faraday's concept of "electro-tonic state." Using the concept of vector potential introduced by Thomson. However, from a mathematical point of view, one way recognize that Equation (6) could be originated from the theorem of Helmholtz decomposition. According to this theorem, any sufficiently smooth vector field in a three-dimensional (3D) space can be resolved into the sum of an irrotational vector field & a solenoidal (divergence-free) vector field.

Flow vector Component Model



Any flow F may have both rotations & divergence.

The divergence distribution calculating $\nabla \cdot F$

The vorticity distribution by calculating $\nabla \times F$

Helmholtz Decomposition Theorem

$\nabla \cdot F \Rightarrow \nabla \cdot (\nabla . x)$

irrotational flow ∇x

from $\nabla \times F \Rightarrow \nabla \times (\nabla \times A)$

non divergent flow $\nabla \times A$

Basic Wave Equation of the Vacuum

In this model, we propose that both the radiation wave and matter wave are excitation waves of the vacuum. Then not only z can represent the wave function of light, it can also represent the wave function of the matter wave. One can connect the vector potential z with the quantum wave function using following relation.

$$Z(x,t) = \epsilon_K \psi(x,t) \quad -(7)$$

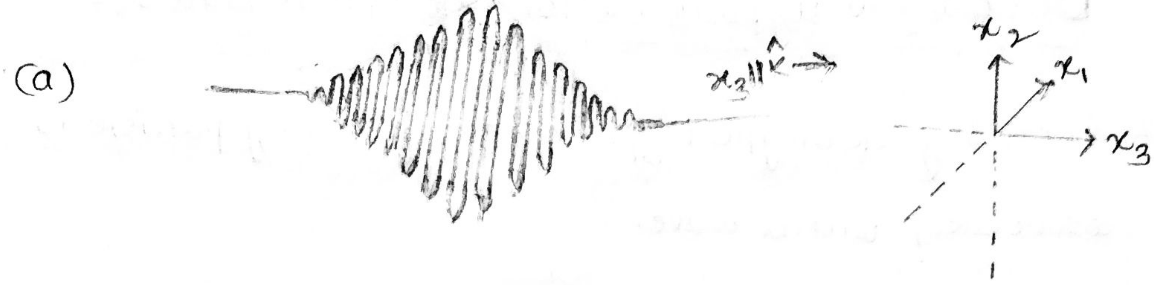
Where ϵ_K is a polarization factor which specifies the orientation of z. Let us denote the poisition vector as $x = (x_1, x_2, x_3)$ and choose the axis x_3 as parallel to the motion of the particle, i.e. $x_3 \parallel K$. Since z is a transverse wave; ϵ_K can be written as

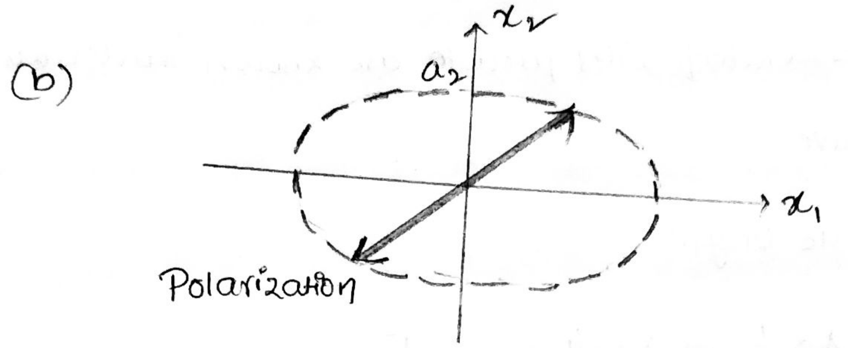
$$\epsilon_K = a_1 e^{i\theta_1}\hat{x}_1 + a_2 e^{i\theta_2}\hat{x}_2 \quad -(8)$$

Where a_1, a_2 & θ_1, θ_2 are amplitudes and phase angles, they are fitting constants. By substituting equation 7, 8. we have.

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

We will call this the "Basic Wave Equation" (BWE) of the quantum vacuum.





Figures : Excitation wave in the vacuum. (a) The wave function represents the electric vector potential z; which is a transverse wave moving in the direction of k. The amplitude of z oscillates along the particle pathway and is described by the quantum wave function $\varphi(x,t)$ (b) z is a polarized vector. The orientation of its polarization factor ϵ_K is shown here by the red arrow.

The Wave function of a photon

The simplest solution of this basic wave equation in a plane wave.

$$\varphi_{\hat{k}}(x,t) \propto e^{i(\hat{k}x-\omega t)}$$

Where k and ω are the wave vector and frequency, respectively. This is identical to the known dispersion relation of radiation wave. Thus, we may identify the plane was solution of BWE as wave function of a photon. One may note that the plane wave solution is only is the simplest solution of BWE. The general solutions of the BWE represent the matter wave of certain massive particle. while the quantum wave equation for the electron is Dirac equation.

De-broglie Hypothesis for the Matter Waves:-

1. According de broglie Hypothesis, any moving particle is associated with a wave.

2. The wave associated with particle are known 'matter wave' (or) de-broglie wave.

3. According to de broglie.

$$\lambda \propto \frac{1}{p} \Rightarrow \lambda = \frac{h}{mv}$$

$$\lambda \propto \frac{1}{mv} \quad \boxed{\lambda = \frac{h}{p}}$$

h → planks constant

$h = 6.62 \times 10^{-34}$ J/sec

$\therefore \lambda = \frac{h}{p}$ is de-broglie wavelength

Derivation for De-broglie wavelength

1. According to Einstien mass - Energy equivalent

$$E = mc^2 \quad \text{—} ①$$

$\therefore$ m = mass of the particle, c = velocity of light

2. Acording planks the energy of photon is $E = h\nu$ — ②

comparing eq^n ① & ②

$$mc^2 = h\nu$$

$$mc^{\cancel{2}} = \frac{hc}{\lambda} \qquad \therefore \nu \propto \frac{1}{\lambda}$$

$$mc = \frac{h}{\lambda} \qquad \therefore \nu = \frac{c}{\lambda}$$

$$\lambda = \frac{h}{mc}$$

$$\boxed{\lambda = \frac{h}{p}} \qquad \therefore mc = p$$

$$mv = p$$

De-broglie wavelength associates with an accelerated charged particle:-

if $e^{\ominus}$ is accelerated by internal diff(V)

$$e \rightarrow V$$

$$KE = eV$$

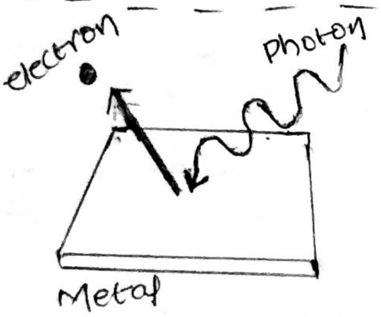
$$\frac{1}{2}mv^2 = eV$$

$$mv^2 = 2eV$$

$$v^2 = \frac{2eV}{m}$$

$$\boxed{v = \sqrt{\frac{2eV}{m}}}$$

Dual nature of radiation - (photo electric Effect)



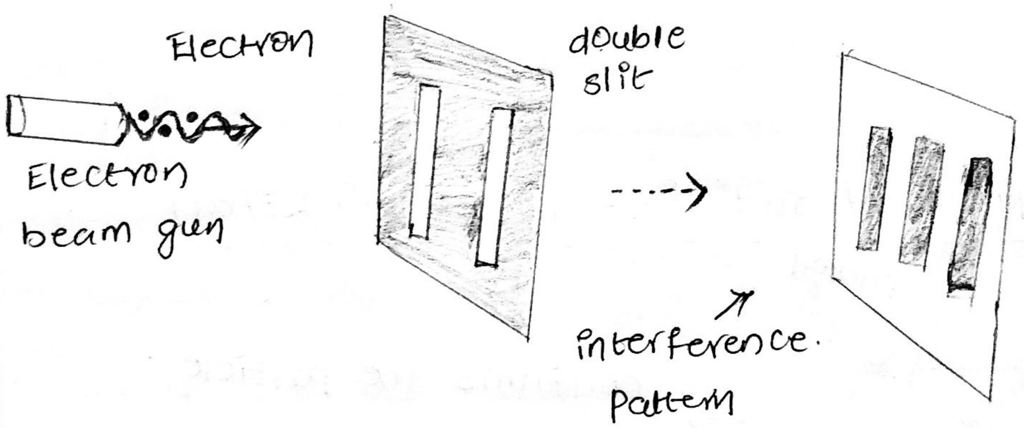
→ confirms the particle nature of light

light behaves both as a wave and as a particle which is referred to as dual nature (or) wave-particle duality of radiation. We say that the nature of light depends on the nature of our observation. If you are observing phenomenon like the interference, diffraction or reflection, you will find that light is a wave.

Related Experiment

① YOUNG'S DOUBLE SLIT EXPERIMENT

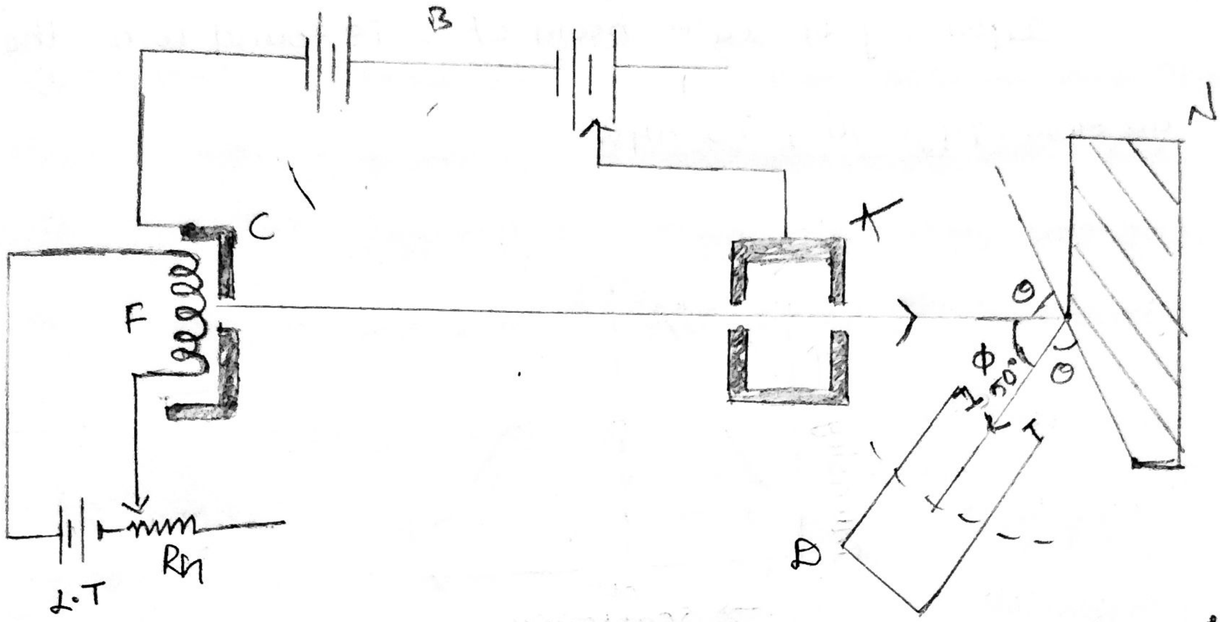
S is a narrow slit illuminated by Monochromatic source of Light. At a suitable distance (10cm) from S.; There are two fine slits A & B placed symmetrically parallel to S. When a screen is placed at a long distance (2m) from the slits A & B then alternate bright & dark Fringes appears on the screen.



Explanation :- The appearance of bright & dark fringes can be explained on the basis of interference of light. According to Huygen's principle, monochromatic source of light illuminating the slit's sends spherical wave front. In these wave fronts solid arc reprents crest & dotted arc reprents trough; Thus the two wave of same amplitude and same of frequences. ⊙ reprents constructive interference ⊗ reprents distructive interference.

② Wave Theory of Electron (or) Davisson and Germer Experiment

The wave Nature of electron was verified by davisson & Germer Experiment.

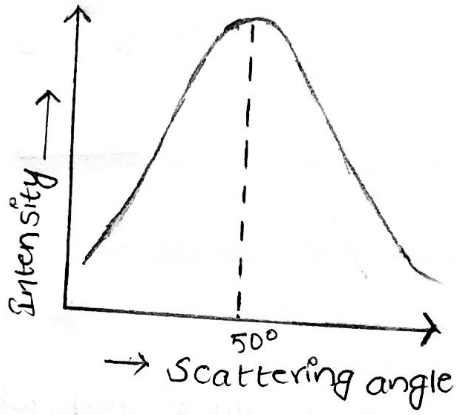


Construction:- It consist of an e^- gun which comprises of a tungsten Filament (F) coated with barium oxide and heated by two voltage power supply. Electrons Emitted by the Filament are accelerated to a desired velocity by applying suitable potential from a High voltage power supply.

They are made to pass through a cylinder with fine holes along its axis producing a fine collimated beam. The beam is made to fall on the surface nickle crystal the e's are scattered in all the direction by atoms of crystal.

Working:- A beam of e^-s emitted by the e^- gun is made to fall on nickel crystal cut along cubical axis at a particular angle. The scattered beam of e^- is recieved by the detector which can be rotated at an angle ϕ.

Intensity of scatter beam of e^- is found to be the accelerated potential is 54V.



The appearence of peak in a particular direction is due to constructive Interference i.e. the diffraction of take place which proof the wave nature of e^-.

Conclusion of Davisson-Germer:- The conclusion that "electrons exhibit wave nature" also, thus, supporting the Hypothesis given by de-broglie regarding "Wave-particle duality" of matter. This experiment involved bombardment of the nickle crystal in vaccum by accelerated electrons coming out of an Electron gun.

Importance of wave-particle duality:-

Wave-particle duality is deeply embedded into the foundations of quantum mechanics. In the formalism of the theory, all the information about a particle is encoded in its wave function, a complex-valued function roughly analogous to the amplitude of a wave at each point in space. This function evolves according to "schrodinger equation". For particles with mass this equation has solutions that follow the form of the wave equation. propagation of such waves leads to wave-like phenomena such as interference and diffraction.. particle without function that localizes mass in space, a photon wave function can be constructed from Einstein kinematics to localize energy in spatial coordinates.

The particle-like behaviour is most evident due to phenomena associated with measurement in quantum mechanics. Upon measuring the location of the particle, the particle will be forced into a more localized state as given by the uncertainty principle. When viewed through this formalism, the measurement of the wave function will randomly lead to wave function collapse to a sharply peaked function at some location. For particles with mass, the likelihood of detecting the particles at any particular location is equal to the squared amplitude of the wave function there.

Applications & uses

Although it is difficult to draw a line separating wave-particle duality from the rest of quantum mechanics it is nevertheless possible to list some applications of this basic idea.

* Wave-particle duality is exploited in electron microscopy, where the small wavelengths associated with the electron can be used to view objects much smaller than what is visible using visible light.

* photos are now able to show this dual nature, which may lead to new ways of examining and recording this behaviour.

* Similarly, neutron diffraction uses neutrons with a wavelength of about 0.1nm, the typical spacing of atoms in a solid, to determining the structure of solids.

* Because electrons behave as waves, they can be used to "illuminate" objects in a manner similar to light. An electron microscope is an instrument that takes advantage of this situation. Electrons are given energy by accelerating them in a manner similar to the way a TV tube works.

CONCLUSION :-

It was a wonderful learning experience for me while working on this project. This project has developed my thinking skills related to the topics. This project gave me real insight into the "Wave-particle duality" world. I enjoyed each and every bit work I had put into this project.

The only logical conclusion is that light is both a particle and a wave. From this, they concluded that all matter is simultaneously a particle and a wave, but when it comes to relatively heavy things like electrons, the particle behavior is a lot easier to observe than the wave behavior.

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