Data Simulation Algorithm For Hierarchical Networks

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Simulating The Network

- 1. Generate the top layer Graph:
 - 1.1 initiate the graph of the top layer (i.e ℓ^{th} layer) consisting of n_{ℓ} nodes.

If the top layer is fully connected:

1.1.1 Generate a Wattz Strogratz graph with n_{ℓ} nodes. Each node is connected to its n_{ℓ} nearest neighbors and with probability p_{ℓ} of rewiring. This yields a graph $\mathcal{G}_{\ell} = \{E, V\}$ with edges $E = \{e_{ij} \ \forall \ i \neq j\}$ and nodes $V = \{n_i\}_{i=1}^{n_{\ell}}$

If the top layer is disconnected:

- 1.1.2 Generate a graph of n_{ℓ} nodes with no edges. This yields a graph $\mathcal{G}_{\ell} = \{E, V\}$ where $E = \emptyset$ and $V = \{n_i\}_{i=1}^{n_{\ell}}$
- **2.** For the remaining $\ell 1$ layers in the hierarchy:

For each node in \mathcal{G}_{ℓ} :

2.1 Generate a small world, scale free, or random subgraph $\mathcal{G}_{\ell-1}$ with $m \sim \text{Uniform}(a,b)$ nodes:

If the subgraph is a small world graph:

 $\mathcal{G}_{\ell-1}$ is generated as a Watts Strogatz graph with m nodes where each node is connected to its k nearest neighbors and with probability of rewiring $p_{\ell-1} = p_{\ell}/(\frac{1}{2}a + b)$

If the subgraph is a random graph:

 $\mathcal{G}_{\ell-1}$ is generated as a Erdős-Rényi graph with m nodes and $E\sim$ uniform(a,b) edges

If the subgraph is a scale free graph:

 $\mathcal{G}_{\ell-1}$ is generated as a Barabási-Albert graph with m nodes and $g \sim \text{uniform}(2, m-1)$ possible edges for each node

