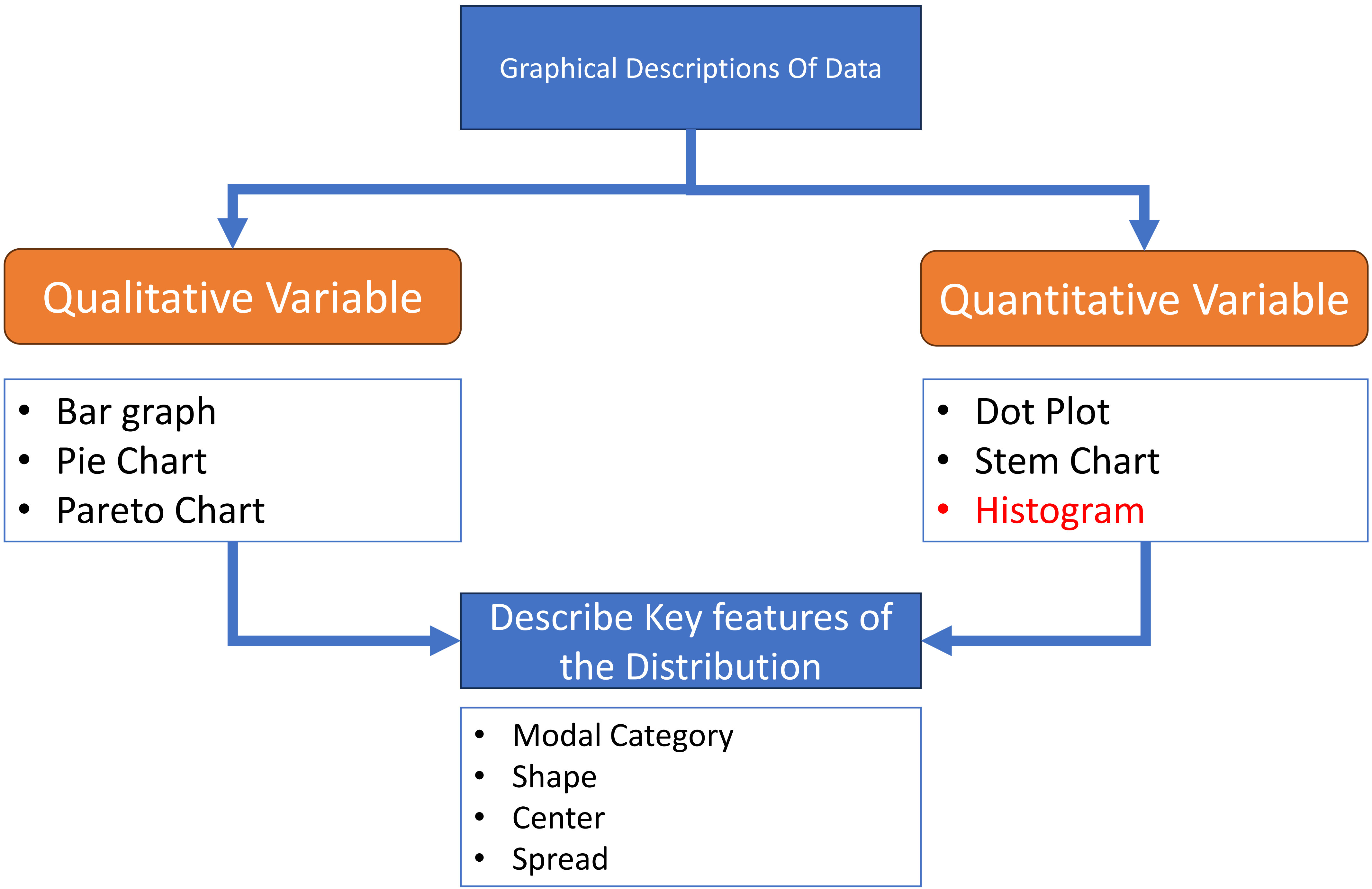
Lecture 3
Describing and Visualizing Distributions Continued

Review

- A natural first step of statistical description is to look graphical summaries of the observations for our variables
- A **distribution** of a variable gives (a) the values that occur and (b) how often each value occurs
- A **frequency table** is a tabular descriptions of the distribution of a variable – it can be applied to either quantitative or qualitative variables


Graphical Descriptions Of Data
Qualitative Variable
Quantitative Variable
Bar graph
Pie Chart
Pareto Chart
Dot Plot
Stem Chart
Histogram
Describe Key features of the Distribution
Modal Category
Shape
Center
Spread

Visualizing Distributions: Quantitative Variables

Stem and leaf plots and **dot plots** are unwieldy for large n

Histogram – uses bars to portray the frequencies or relative frequencies of the possible outcomes for a quantitative variable

Steps to construct a histogram

1. **Divide the range of the data into intervals of equal width**
2. **Compute the frequency of each interval (i.e construct the frequency table)**
3. **Label the x-axis with the values or endpoints of each interval.**
4. **Draw a bar over each value or interval with height equal to its frequency or relative frequency**

How to choose the number of Bins?

- How to choose the best number of bins is not a straightforward question and there is a lot of literature on the subject
- We can construct our histogram using a specific binwidth w or under a set number of bins k
- $w = \dfrac{\max x - \min x}{k}$ or $k = \dfrac{\max x - \min x}{w}$
- Square root method: $k = \mathrm{round}(\sqrt{n})$ (A fairly safe and basic rule of thumb)
- Sturges Rule[1]: $k = \mathrm{round}(\log_2 n) + 1$ (not great for $n < 30$)
- Rices Rule[2]: $k = 2\sqrt[3]{n}$

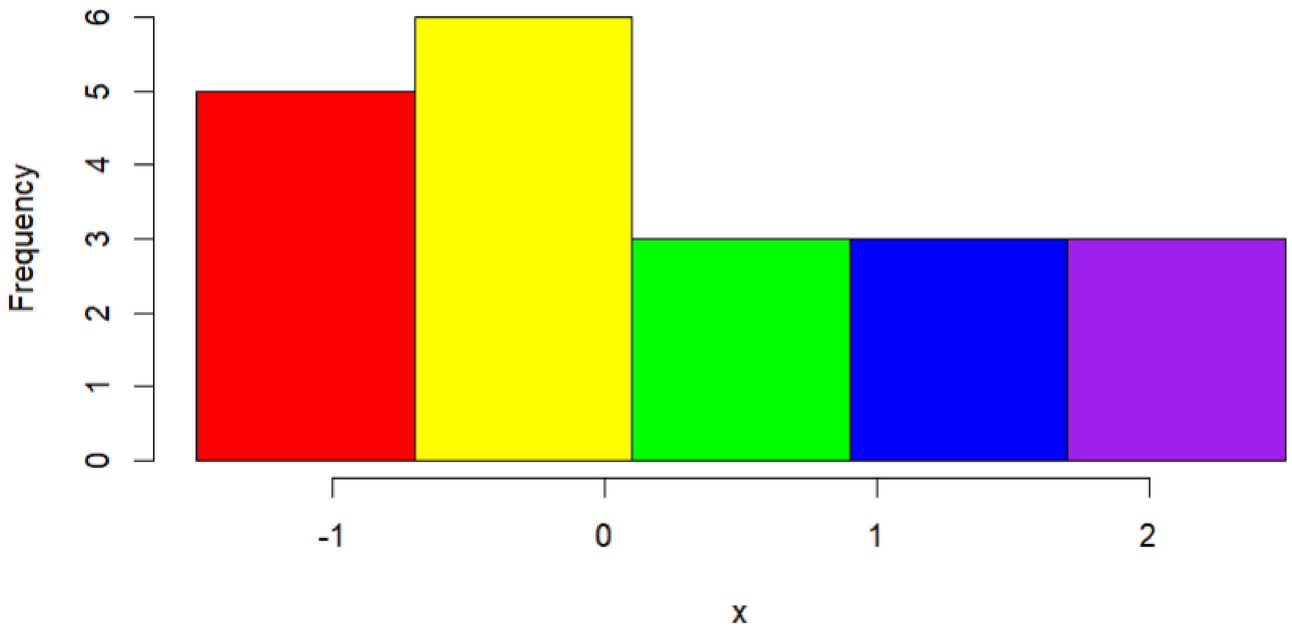
[1] Sturges, Herbert A. "The choice of a class interval." Journal of the american statistical association 21.153 (1926): 65-66.

[2] Lane, David. *Online statistics education: A multimedia course of study*. Association for the Advancement of Computing in Education (AACE), 2003. – Chapter 2 “Graphing Distributions

Try it out: Histogram

Consider the following $n = 20$ observations of a continuous variable

	Bin 1	Bin 2	Bin 3	Bin 4	Bin 5
x =	-1.5, -1.2, -1.0, -0.8, -0.7,	-0.6, -0.1, -0.1, 0.1, 0.1, 0.1,	0.6, 0.6, 0.8,	1.1, 1.2, 1.3,	1.8, 1.9, 2.4



Some tips

- If <u>too few</u> intervals are used, then the graph will be too crude

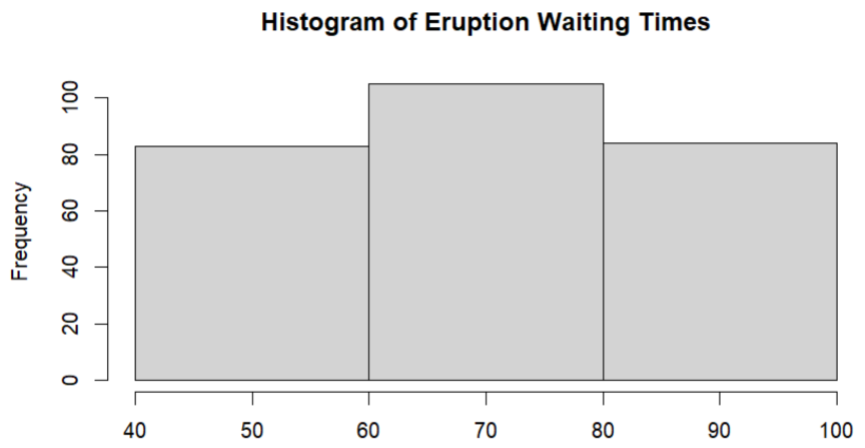
- If <u>too many</u> intervals are used, graph will contain many short bars and gaps.

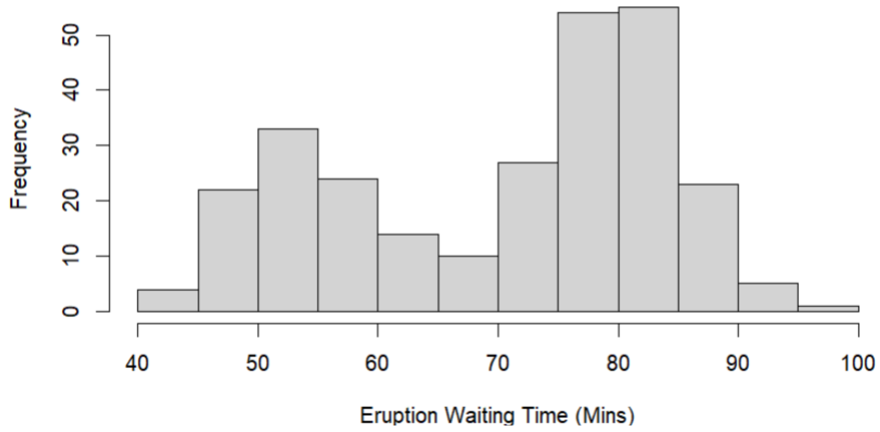
  Usually between 5 - 15 intervals are enough.

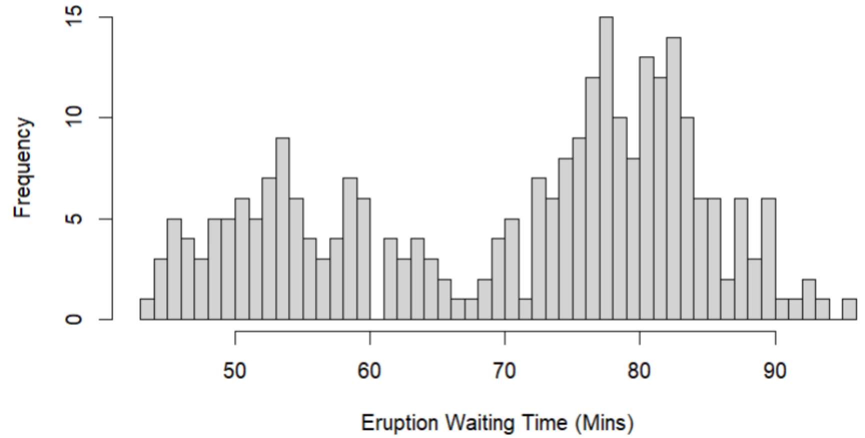
- Most plotting software will automatically choose the number of bins.

- **<u>ALWAYS</u>** plot the histogram to get an idea about the shape of the distribution of a quantitative variable

- Is the number of observations is small (say n < 50) then it’s a good idea to supplement a histogram with a dot plot or stem plot

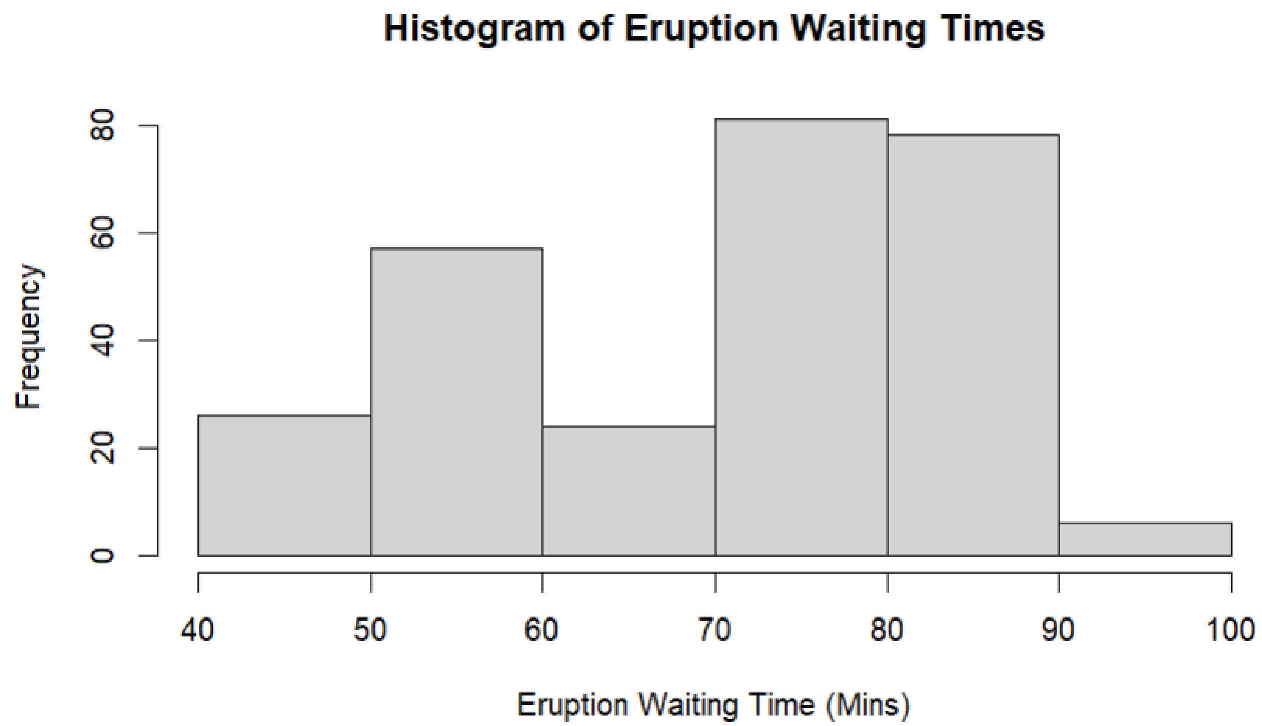


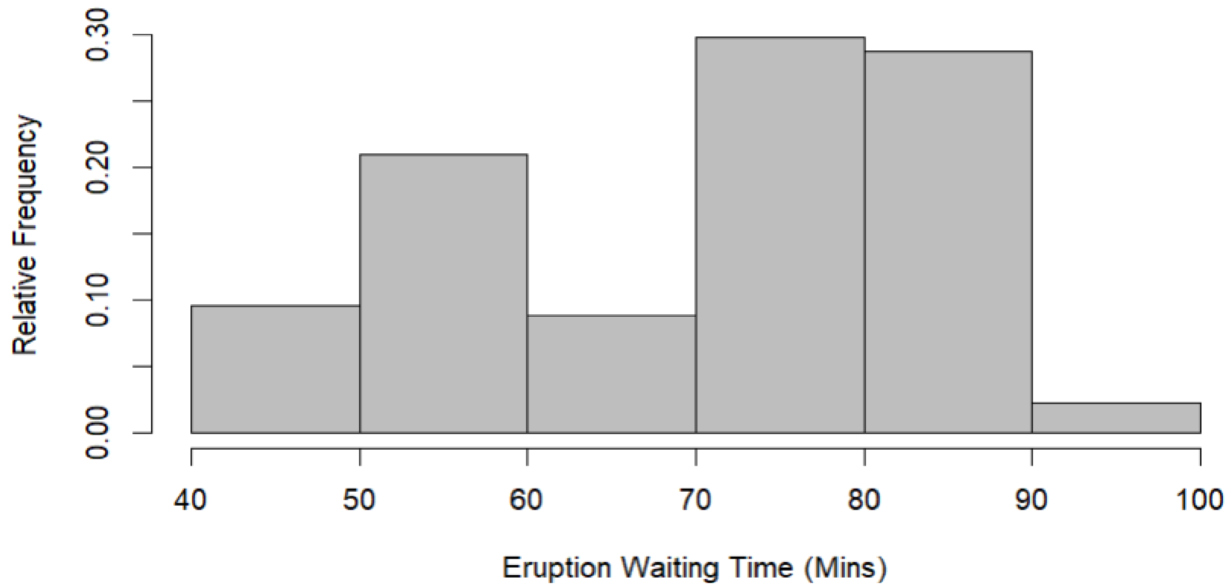




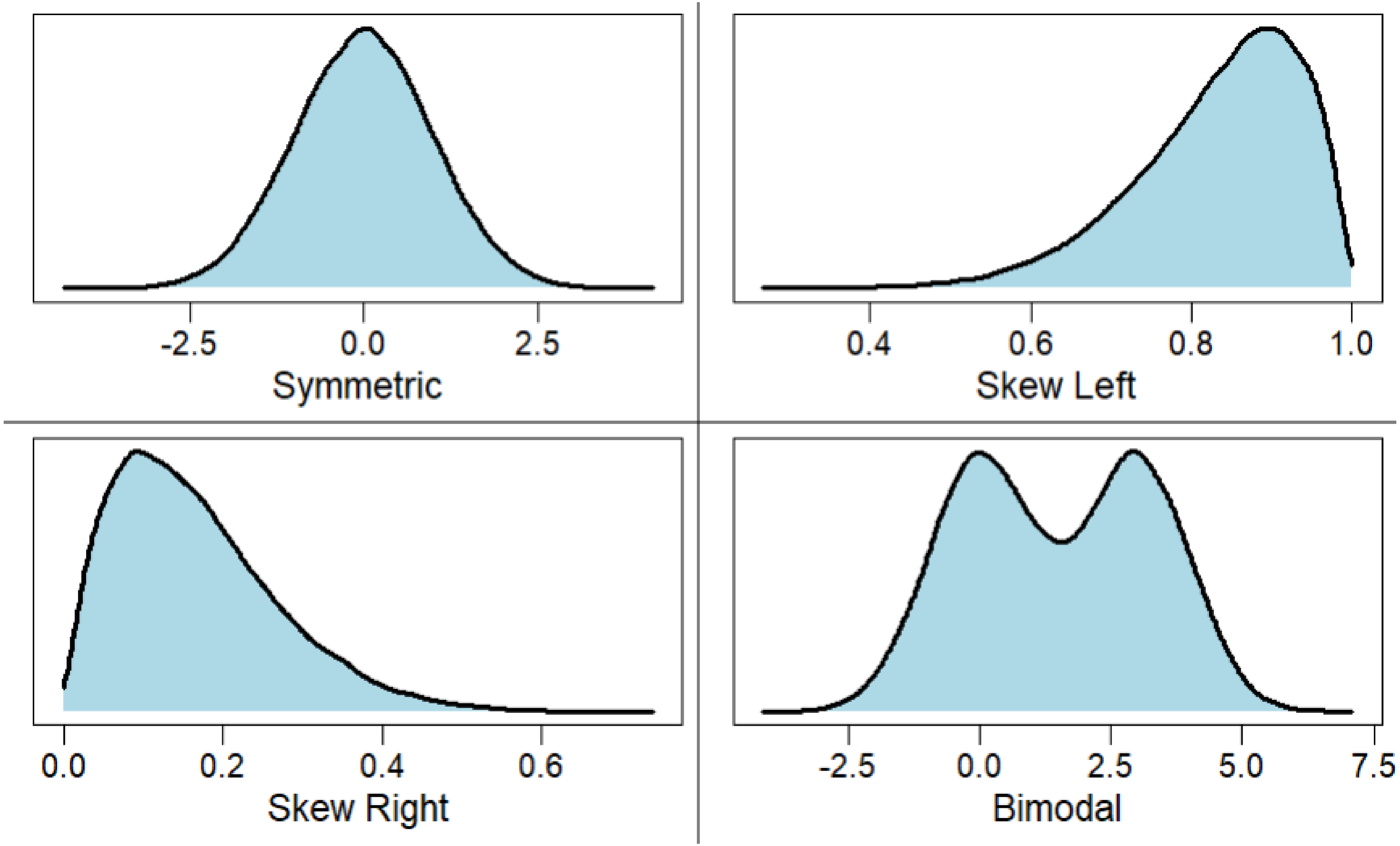
Example: Old Faithful Eruption Times

Waiting Time (Min)	Frequency	Relative Frequency	Cumulative Relative Frequency
< 50	21	0.077	0.077
50 - 60	56	0.206	0.283
60 - 70	26	0.096	0.379
70 - 80	77	0.283	0.662
80 - 90	80	0.294	0.956
> 90	12	0.044	1

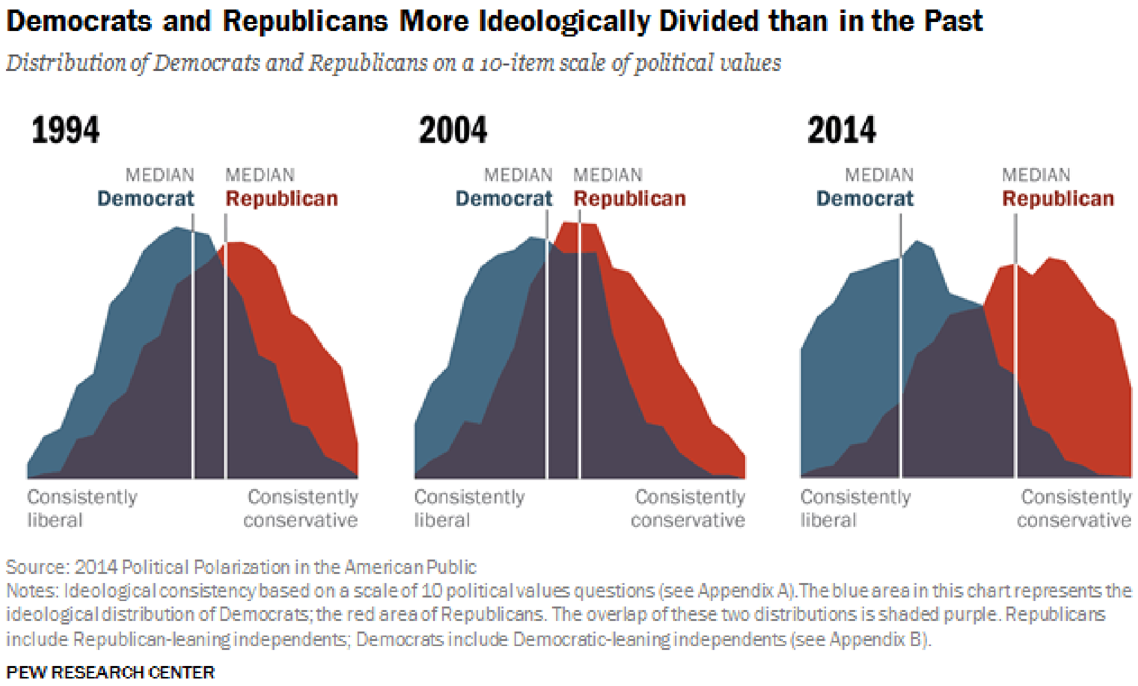


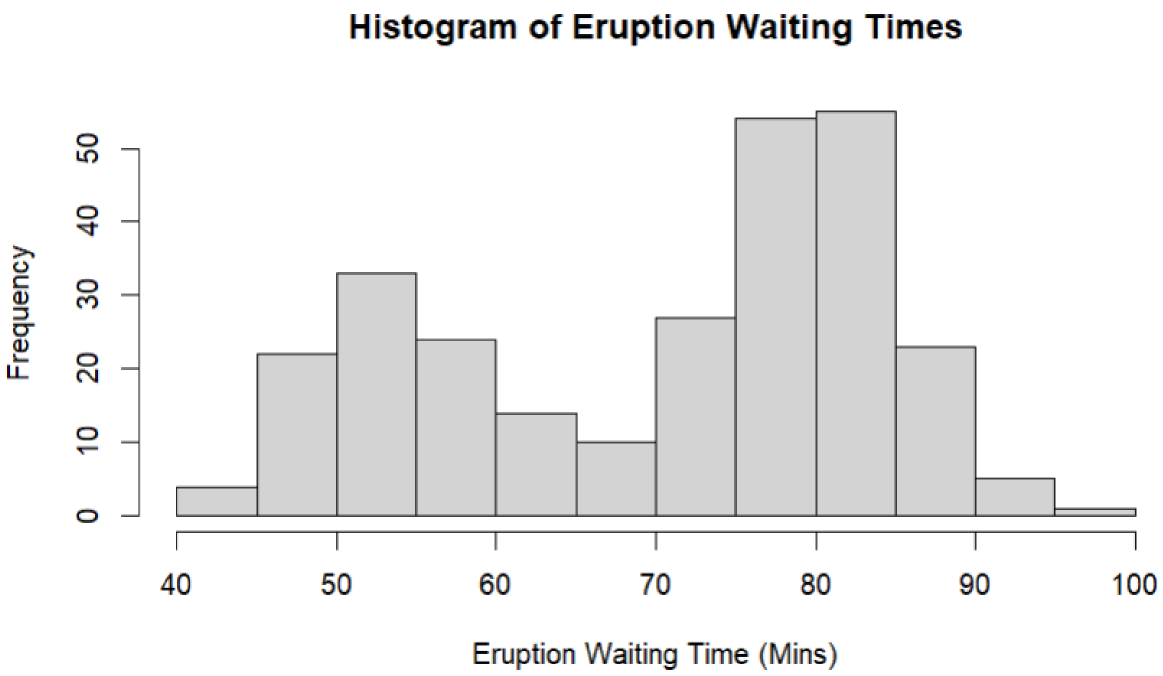


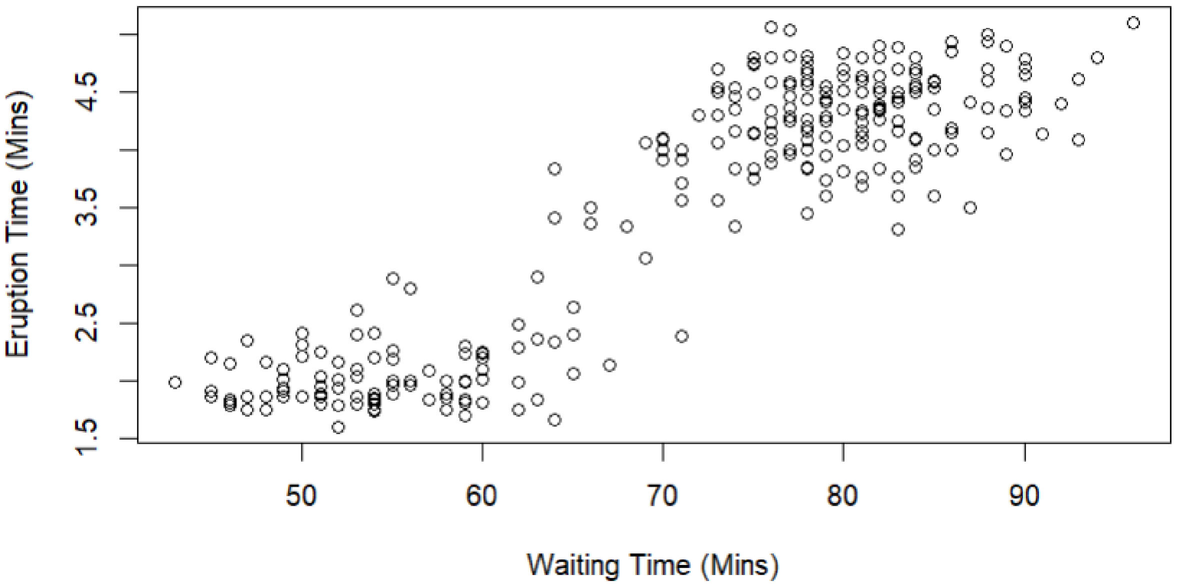
Shape of a distribution



- Bimodal distributions can arise when
  - A population is polarized on a controversial issue
  - When observations come from two different populations







Measures of Central Tendency

- The (arithmetic) **mean** is the average of a set of observations
  it measures the center of mass of a distribution (the balancing point)

$$\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n} = \sum_{i=1}^{n} \frac{x_i}{n}$$

- We can also express the mean in terms of the frequency F or the relative frequency RF

$$\bar{x} = \frac{1}{n}\sum_{x} x\, F(x) \quad \text{or} \quad \bar{x} = \sum_{x} x\, RF(x)$$

Where the sum is over all distinct values of the variable x

- ➢ the mean is usually not equal to any of the values observed in the sample
- ➢ The mean is highly influenced by **outliers** - observations that take on extreme values relative to the distribution

Measures of Central Tendency

- The **median** is the middle value of a set of observations

How to compute the median:

1. Compute the median by first ordering the observations from smallest value to largest value and choose the number in the middle

2. If the n is odd the median is the middle number

   - If n is even the median is the sum of the two middle values divided by 2

Ex.)

Data = 1,1,4,5,6

Median = 4

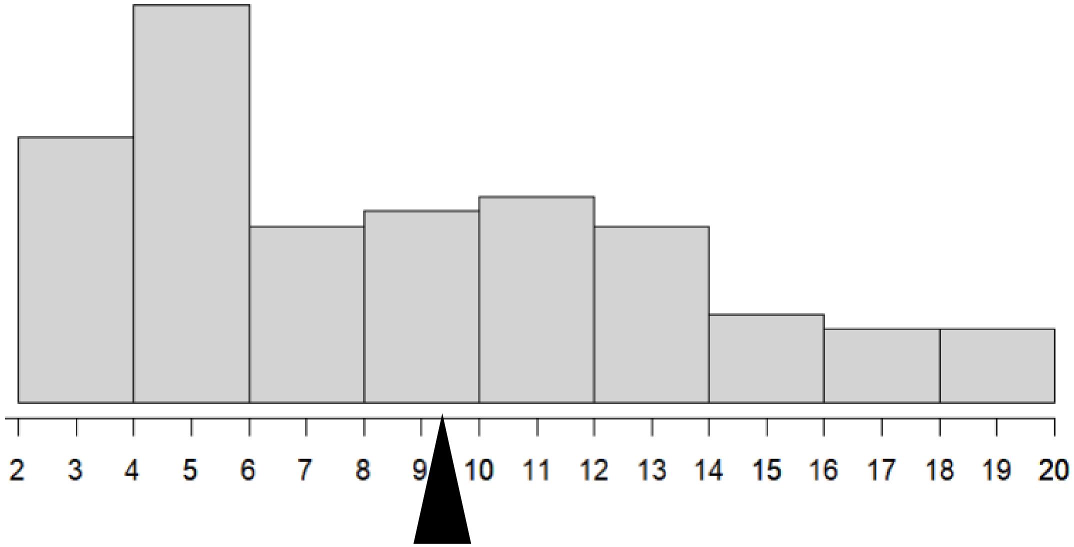
Mode = 1

Data = 1,1,4,5,6,6

Median $= \frac{4+5}{2} = 4.5$

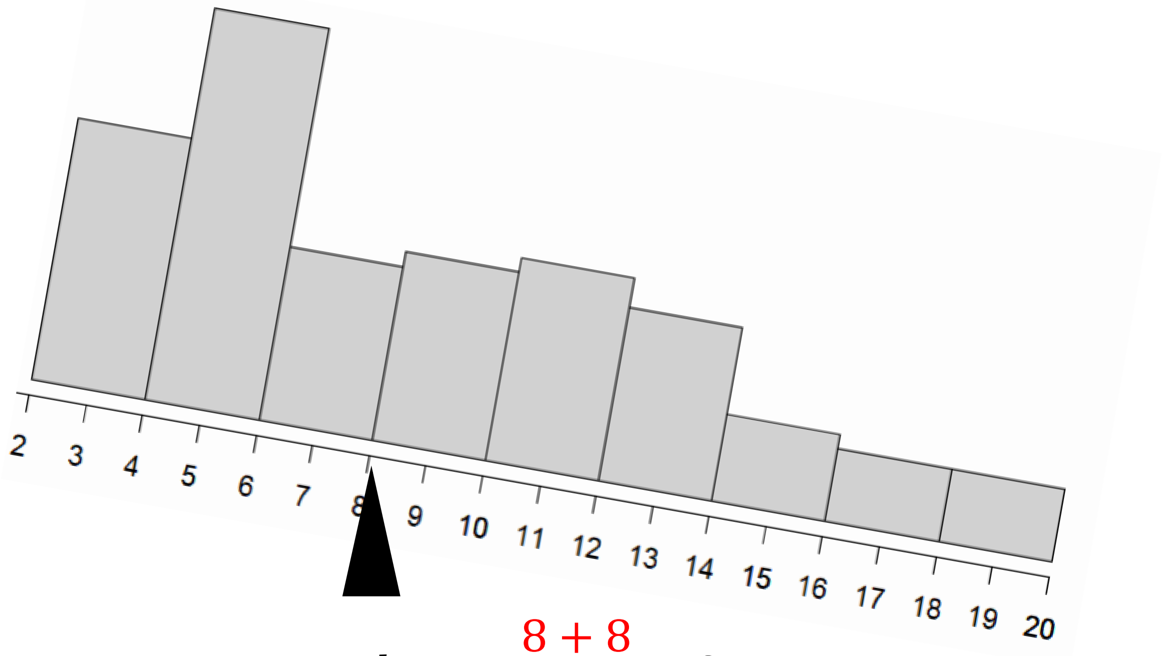
Mode 1, 6

Data: 3 3 3 3 3 3 3 4 4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 5
5 5 5 5 5 5 5 5 5 6 6 6 6 6 6 6 6 6 6 7 7 7 7 7 7 7 7
7 8 8 8 8 8 8 9 9 9 9 9 9 9 10 10 10 10 10 10 10 11 11 11 11 11 12
12 12 12 12 12 12 12 13 13 13 14 14 14 14 14 14 15 15 15 16 16 16 16 17 17 17 18
18 20 20 20 20 20



$\bar{x} = 9.2$

The mean is the center of gravity



$$median = \frac{8+8}{2} = 8$$

The median is the middle value

Measure of Central Tendency

- The **mode** is the value with the largest relative frequency (i.e the value that occurs most often)

  - Can be used with categorical data (mean and median cannot)
    - e.g the most frequent category

  - It may not be unique if two or more values have the same frequency

  - <u>**Caution**</u> for quantitative data, the mode <u>may not</u> anywhere near the center of the distribution.

Ex.)

Data = 1,1,4,5,6

Mode = 1

Data = 1,1,4,5,6,6

Mode   1, 6

Practice:

- Roll a six-sided die $n = 10$ times and record the number rolled each time
- Data = 1,2,3,3,4,4,4,5,6,6

x	$f(x)$	$rf(x)$
1	1	0.1
2	1	0.1
3	2	0.2
4	3	0.3
5	1	0.1
6	2	0.2

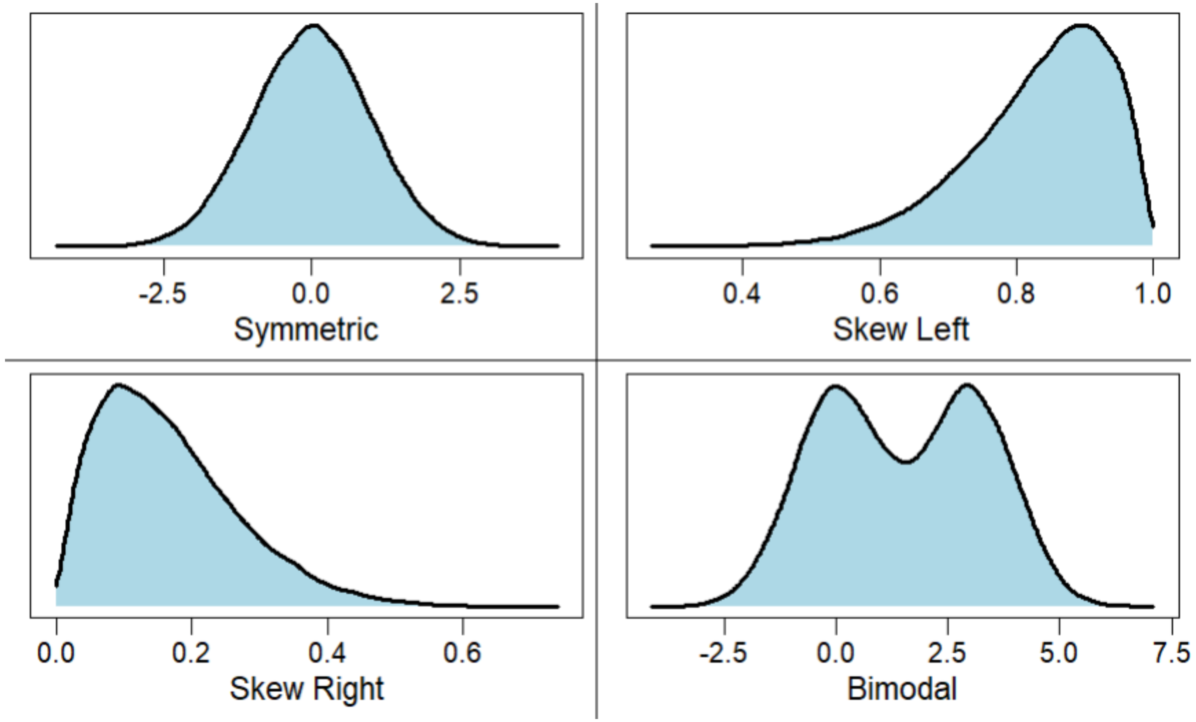
Compute the **mean** using all 3 equations:

Compute the **median**

Compute the **mode**

Comparing the Mean, Median, and Mode

- The shape of a distribution influences whether the mean is larger or smaller.
- Skew left = mean < median
- Skew right = mean > median
- When a distribution is symmetric the mean will equal the median



Comparing the Mean, Median, and Mode

- The median is a robust estimate of the mean
- The median is not usually affected by the presence of outliers
- The median is usually preferred for highly skewed distributions

- Ex.) take using the following 9 data points: 0.3, 0.4, 0.8, 1.4, 1.8, 2.1, 5.9, 11.6, 16.9

  The **mean** is about 4.58

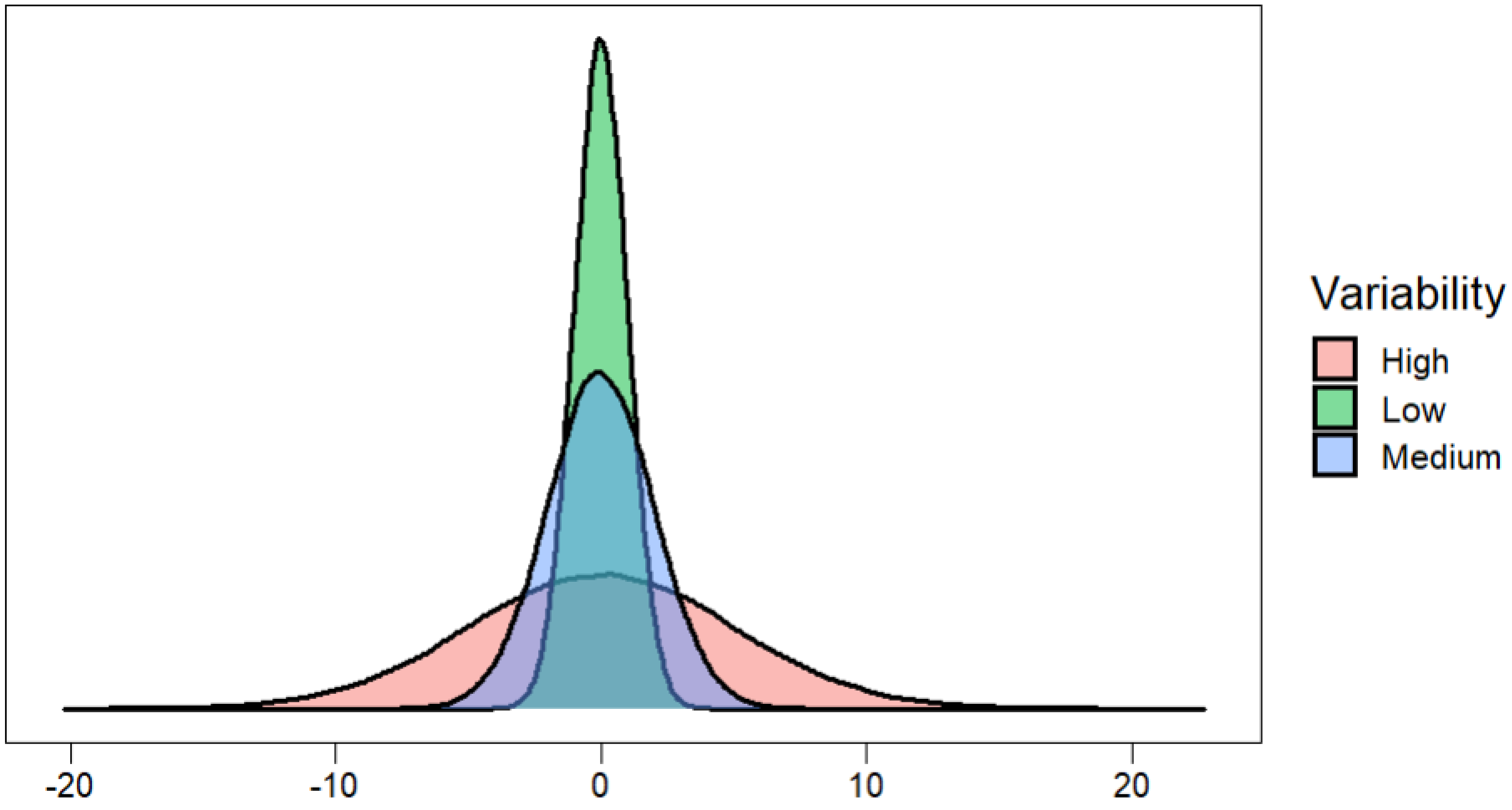
  The **median** is 1.8

- Change one of the data points to be an outlier, for example, we change **16.9** to **90**

  The **mean** becomes 12.7

  While the **median** is still 1.8

Variability of A Distribution: Measures of Spread



Measures of Spread: Range

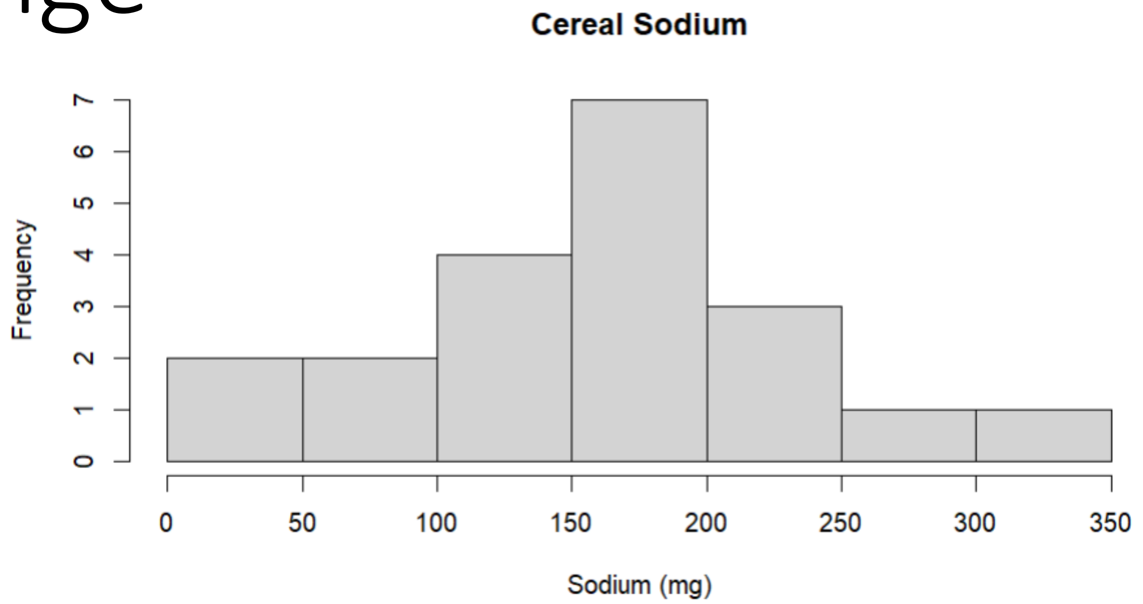
- The **range** is a measure of the distance between the smallest and largest values in the data

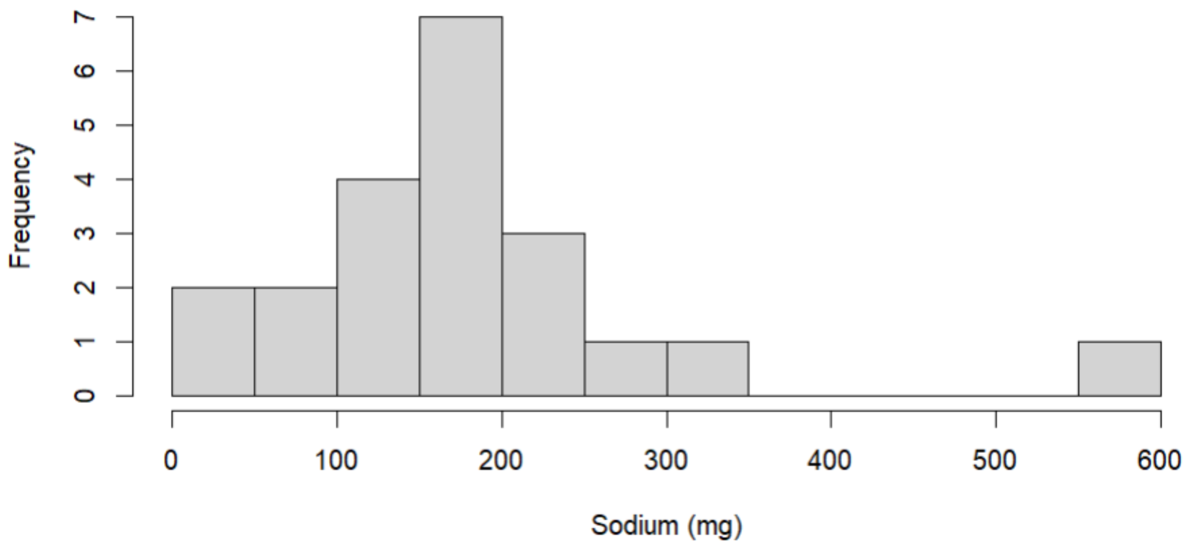
  The range can be computed with only two data points the minimum value and maximum value

- If the range of a set of data is large, then the data vary more

- The range is <u>severely</u> affected by the presence of outliers

- We typically <u>do not</u> use the range to measure variability





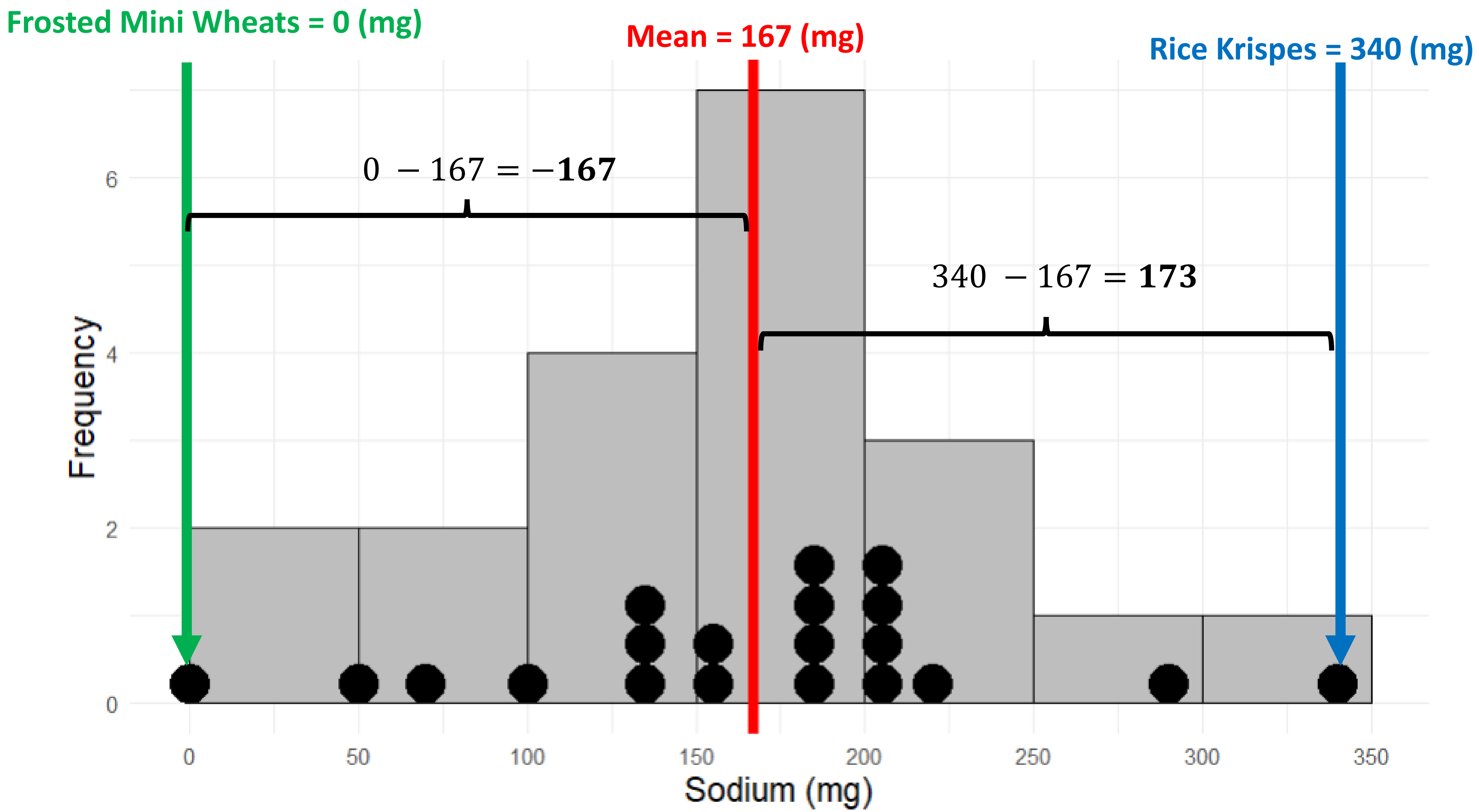
Measures of Spread: Deviation

- A better measure of variability that uses *all* the data is based on **deviations**

- **deviations** are the distances of each value from the mean of the data:

  Deviation of an observation $x_i = (x_i - \bar{x})$

- Every observation will have a deviation from the mean



Frosted Mini Wheats = 0 (mg)
Mean = 167 (mg)
Rice Krispes = 340 (mg)
0 − 167 = −167
340 − 167 = 173
Frequency
0
2
4
6
0
50
100
150
200
250
300
350
Sodium (mg)

Measures of Spread: Variance

- The sum of all deviations is zero. $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

- We typically use either the **squared deviations** or their **absolute value**

  Squared deviation of an observation $x_i = (x_i - \bar{x})^2$

- The **Variance** of a distribution is the average squared deviation from the mean

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

- The sum $\sum_{i=1}^{n}(x_i - \bar{x})^2$ is called the sum of squares

Measures of Spread: Standard Deviation

- Since the variance uses the squared deviation, we usually take its square root called the **standard deviation**

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

- The standard deviation represents (roughly) the average distance of an observation from the mean

- The greater s is the greater the variability in the data is

- We denote the population parameter for the variance and standard deviation using σ for s and σ^2 for s^2

Why divide by $n-1$?

- We divide by $n-1$ because we have only $n-1$ pieces of independent information for s^2

- Since the sum of the deviations must add to zero, then if we know the first $n-1$ deviations we can always figure out the last one

- Ex.) suppose we have two data points and deviation of the first data point is $x \ -\bar{x}=-5$
    - Then the deviation of the second data point <u>has</u> to be 5 for the sum of deviations to be zero.

Try it out: Computing s and s^2

- Roll a six-sided die $n = 10$ times and record the number rolled each time
- Data = 1,2,3,3,4,4,4,5,6,6
- Mean = 3.8