

Assignment 1 - OLS Regression

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Introduction

Complex real estate markets influence the prices of homes, and determining what features are most impactful to the value goes beyond physical characteristics. In this analysis, we use ordinary least squares regression to investigate the variation in median home sales prices for Census block groups in the city of Philadelphia, PA. We use a multivariate linear regression model to determine the correlation between median home value for owner occupied housing units in block groups and four predictive variables:

- proportion of residents in a block group with at least a bachelor's degree;
- proportion of housing units that are vacant;
- percent of housing units that are detached single family houses; and
- number of households living in poverty

We include the proportion of residents in a block group with at least a bachelor's degree because previous research has indicated that there is a correlation between income and educational attainment¹. This correlation results in households that lack a higher education degree having less wealth and are thus likely to only be able to purchase homes in areas with lower median home values.

We include the percentage of lots which are vacant as a predictor because previous research indicates that there is a correlation between vacant lots and median home sales prices in Philadelphia. Gravin et al notes that there are 40,000 vacant parcels in Philadelphia and most of these are concentrated in low income areas².

The proportion of housing units that are detached single family homes is included as a predictor, because detached single family homes tend to be larger and there is typically a correlation between home size and property value.

We include the number of households with income below 100% poverty levels because low income households are less likely to be able to afford homes in expensive areas.

Methods

Data Cleaning

The data set used in our analysis contains information from the 2000 US Census for Philadelphia, with neighborhood characteristic variables included for 1,720 block groups. Our analysis incorporates the following variables:

- POLY_ID: Census Block Group ID
- MEDHVAL: Median value of all owner occupied housing units
- PCBACHMORE: Proportion of residents in a block group with at least a bachelor's degree
- PCTVACANT: Proportion of housing units that are vacant
- PCTSINGLES: Percent of housing units that are detached single family houses
- NBELPOV100: Number of households with incomes below 100% poverty level (i.e., number of households living in poverty)
- MEDHHINC: Median household income

The original data set included 1,816 block groups and was cleaned based on the following criteria, which reduced to the total number of observations to 1,720:

- Block groups where population is less than 40 people
- Block groups without housing units
- Block groups where the median house value is lower than \$10,000
- One North Philadelphia block group which had a very high median house value (over \$800,000) and a very low median household income (less than \$8,000)

Exploratory Data Analysis

We will examine the summary statistics and distributions of the data set's variables, including the mean and standard deviation of our dependent variable and predictor variables.

As part of our exploratory data analysis, we will examine the Pearson correlations between the predictors. A Pearson correlation, denoted in this analysis by "r", is a standardized measurement of the strength and direction of the linear relationship between two variables. The correlation between two variables is calculated using the following equation:

¹ Sean Reardon, The Widening Academic Achievement Gap Between the Rich and the Poor: New Evidence and Possible Explanations, Stanford University, <https://cepa.stanford.edu/sites/default/files/reardon%20whither%20opportunity%20-%20chapter%205.pdf>

² Eugenia Garvin et. al, More Than Just An Eyesore: Local Insights And Solutions on Vacant Land And Urban Health, Journal of Urban Health, <https://link.springer.com/article/10.1007/s11524-012-9782-7>

$$r = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

A Pearson correlation value ranges between -1 to 1, with no units of measurement attached, and the observed variables are interchangeable between the x axis and y axis. A value of -1 represents a perfect negative linear relationship and a value of 1 represents a perfect positive linear relationship - in either case, points on a graph would appear in a straight line with either a negative or positive slope, respectively. A value of 0 indicates that there is no linear relationship between two variables. However, a different type of relationship can exist, such as an exponential or quadratic relationship, that the Pearson correlation does not measure - in those cases, a Spearman correlation would be more appropriate to calculate.

Multiple Regression Analysis

Ordinary least square (OLS) regression is a statistical method used to examine the linear relationship between a variable of interest (dependent variable) and one or more explanatory variables (predictors or independent variables). This type of regression tests the strength of the relationship, the direction of the relationship (positive, negative, or no relationship) and goodness of model fit - how well a model will predict a future set of observations. Regressions can also calculate the amount that the dependent variable changes when a predictor variable changes by one unit (holding all other predictors constant). However, if an explanatory variable is a significant predictor of the dependent variable, it does not imply causation.

When more than one predictor is present, multiple regression is used. In a multiple regression analysis, there are $K > 1$ predictors, so rather than getting a line in 2 dimensions from a simple linear regression, there is instead a surface in $K + 1$ dimensions (+1 accounts for the dependent variable). Here, each independent variable will have its own slope coefficient which indicates the relationship of that particular predictor with the dependent variable, when all other independent variables are held constant.

To examine the relationship between median house values and several neighborhood characteristics, we run a multiple regression. Using Philadelphia data at the Census block group level, we regressed the natural log of the median value of all owner occupied housing units (LNMEDHVAL) on the following variables for each block group:

- The proportion of housing units that are vacant (PCTVACANT)
- The percent of housing units that are detached single family homes (PCTSINGLES),
- The proportion of residents in a block group with at least a bachelor's degree (PCTBACHMOR), and
- The natural log of the number of households with incomes below 100% poverty level (LNNBELPOV100).

The equation is stated as:

$$LNMEDHHINC = \beta_0 + \beta_1 PCTVACANT + \beta_2 PCTSINGLES + \beta_3 PCTBACHMOR + \beta_4 LNNBELPOV100 + \epsilon$$

Performing a logarithmic transformation on the dependent variable MEDHVAL and one the independent variables NBELPOV100 helps us to achieve better normality of residuals, which is a core assumption when running an OLS regression that we discuss in greater detail in the regressions assumptions section of the study.

The Beta coefficient β_i of each predictor is interpreted as the amount by which the dependent variable changes as the independent variable increases by one unit, holding all other independent variables constant. The sign indicates whether the relationship between the dependent variable and the independent variables is positive (direct) or negative (inverse). It is important to look at the sign and value of the β_i when the coefficient is statistically significant and different from zero. The β_i is considered statistically significant and different from zero when the p-value falls below our alpha threshold of 0.05. The variable ϵ is commonly referred to as the residual term or random error term in the model. The residual term ϵ allows the regression line to fall above ($\epsilon > 0$) or below ($\epsilon < 0$) the actual data points. ϵ is the difference between observed values of y and the values of y predicted by the regression model (denoted by \hat{y}).

Regression Assumptions

Prior to making conclusions about the model estimates or using the model for predictions, certain assumptions must be met. These assumptions include linearity, independence of observations, normality of residuals, homoscedasticity, and no multicollinearity.

Linearity assumes that there is a linear relationship between the dependent variable y and each of the predictors x . Linearity can be checked by creating scatterplots between y and each of the predictors.

Independence of observations assumes that there should be no spatial, temporal, or other forms of dependence in the data. This means that each observation in the data must be independent of the others. In order to test for independence of observations, one can examine the Global Moran's I value for the residuals or the values of y to examine whether regression residuals, or the dependent variable itself, are spatially autocorrelated.

Normality of residuals is violated when either the dependent variable or independent variables themselves are distributed non-normally, or the linearity assumption is violated. This assumption is not as essential to follow as the three previously stated assumptions, especially when analyzing a data set with a large sample size. In this context, a large sample size is generally defined as having 30+ observations with 10 additional observations for every additional predictor after the first one. One can test for this assumption by looking at the histogram of residuals to see if they are normal.

No homoscedasticity refers to the variance of the residuals ε being constant regardless of the values of each x (or the values of \hat{y} , i.e., values of y predicted by the model). Heteroscedasticity is present when this assumption is violated, which occurs when the residuals ε differ across all values of the independent variables and means that there is systematic under-or over-predictions happening in the model. This assumption can be checked by looking at scatterplots of standardized residuals against each predictor to see if variance of residuals remains the same for different values of each predictor.

No multicollinearity only applies to multiple regression - no multicollinearity occurs when predictor variables are not strongly correlated with each other. Multicollinearity is when two or more predictors are very strongly correlated with each other: $r > 0.8$ or $r < -0.8$. If multicollinearity is present in a model, it will become difficult for the model to estimate the relationship between each predictor and the dependent variable independently, making it difficult to identify significant predictors. One can check for this assumption by reviewing a correlation matrix of the predictors to check if $r > 0.8$ or $r < -0.8$ for two predictors. If two or more predictors are strongly correlated, only one of these correlated predictors should be included in the regression model.

Multiple Regression Parameters & Estimation

Performing multiple regression requires one to estimate the values for a critical set of parameters. These parameters include σ^2 , which determines the amount of variability inherent in a regression model; a regression constant β_0 ; and one regression coefficient β_i for each independent variable in the model. The regression constant (i.e., the intercept) β_0 represents the mean value of the dependent variable when all independent variables are equal to 0. The regression coefficient, as stated previously, is interpreted as the amount by which the dependent variable changes as the independent variable increases by one unit, holding all other independent variables constant. In multiple regression we estimate these parameters by finding the values β_0 and β_i that minimize the Sum of Squared Errors (SSE) of prediction, the amount of variability in y that is not explained when accounting for the predictors in the model. SSE will produce the Least Square estimates $\hat{\beta}_0$ & $\hat{\beta}_k$. The equation for SSE is stated as:

$$SSE = \sum (y_i - \hat{y}_i)^2 = \sum [y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \hat{\beta}_2 x_{2i} + \dots + \hat{\beta}_k x_{ki})]^2$$

For multiple regression, the equation for σ^2 is stated as $\sigma^2 = \frac{SSE}{n-(k+1)} = MSE$, where k = # predictors and n = # observations. Here, MSE stands for mean squared error.

Coefficient of Multiple Determination R^2

R^2 is the proportion of observed variance in the dependent variable y that is explained by the model by all k predictors - it is often referred to as the coefficient of multiple determination. Higher values of R^2 are indicative of a better model. R^2 is calculated as $R^2 = 1 - \frac{SSE}{SST}$, where SSE is the sum of squared residuals and SST is the total variability in the dependent variable. Adjusted R^2 is the R^2 value adjusted for the number of predictors in the model and is used to account for a high number of predictor variables in the model, which can increase R^2 based on presence alone. Larger values for Adjusted R^2 , like R^2 , are also indicative of a better model. The equation for Adjusted R^2 is stated as:

$$R_{adj}^2 = \frac{(n-1)R^2 - k}{n - (k+1)}$$

Hypothesis Testing

For hypothesis testing, we examine a baseline - the null hypothesis - compared to an alternative hypothesis. To examine the overall significance of our regression model, we first use the F-Ratio to test the null hypothesis $H_0 : \beta_i = 0$ that none of the independent variables in the model are a significant predictor of the dependent variable (LNMEDHVAL) against the alternative hypothesis $H_a : \beta_i \neq 0$ that at least one of the independent variables is a significant predictor of the dependent variable (LNMEDHVAL).

Second, we use T-test hypothesis testing to determine the significance of each of the four predictors on our dependent variable LNMEDHVAL.

PCTVACANT:

- $H_0 : \beta_i = 0$: Implies that the variable PCTVACANT is not a significant predictor of the dependent variable (LNMEDHVAL).
- $H_a : \beta_i \neq 0$: Implies that the variable PCTVACANT is a significant predictor of the dependent variable (LNMEDHVAL) for $1 \leq i \leq k$

PCTSINGLES:

- $H_0 : \beta_i = 0$: Implies that the variable PCTSINGLES is not a significant predictor of the dependent variable (LNMEDHVAL).
- $H_a : \beta_i \neq 0$: Implies that the variable PCTSINGLES is a significant predictor of the dependent variable (LNMEDHVAL).

PCTBACHMOR:

- $H_0 : \beta_i = 0$: Implies that the variable PCTBACHMOR is not a significant predictor of the dependent variable (LNMEDHVAL).
- $H_a : \beta_i \neq 0$: Implies that the variable PCTBACHMOR is a significant predictor of the dependent variable (LNMEDHVAL).

LNNBELPOV100:

- $H_0 : \beta_i = 0$: Implies that the variable LNNBELPOV100 is not a significant predictor of the dependent variable (LNMEDHVAL).
- $H_a : \beta_i \neq 0$: Implies that the variable LNNBELPOV100 is a significant predictor of the dependent variable (LNMEDHVAL).

Additional Analyses

Stepwise Regression

Stepwise regression is a data mining method which selects predictors based on the following criteria:

- P-values are below a certain threshold (variables where the P-value < 0.1)
- The smallest value of the Akaike Information Criterion (AIC), which measures the relative quality of statistical models

There are a number of limitations when using stepwise regression, which include:

- The final model is not guaranteed to be optimal in any specified sense
- The stepwise procedure produces a single final model, although there are often several equally good models
- It does not consider a researcher's knowledge of the predictors
- While the order in which the variables are removed or added can provide valuable information, it is important not to over-interpret the order
- One should not conclude that all the important variables for predicting y have been identified or that all unimportant variables have been eliminated

K-Fold Cross-Validation

The k-Fold Cross-Validation approach involves randomly dividing the set of observations into k groups (or folds) of approximately equal size. In practice, typical folds to use are $k = 5$ or $k = 10$. The first fold is treated as a validation set and the model is fitted on the remaining $k - 1$ folds (training data set). The MSE is then computed on the observations in the held-out fold. The procedure is repeated k times with a different fold treated as the validation set. This process results in k estimates of MSE.

The k-fold MSE estimate is computed by averaging the MSEs across the k folds. The k-fold root mean squared error (RMSE) is computed as the square root of the MSE: $RMSE = \sqrt{\frac{\sum_{i=1}^n \epsilon_i^2}{n}}$. RMSE values can be compared across different models, with the model that has the smallest RMSE considered the best model.

Software

This report uses the open source software R to conduct statistical analyses.

Results

Exploratory Results

Summary Statistics

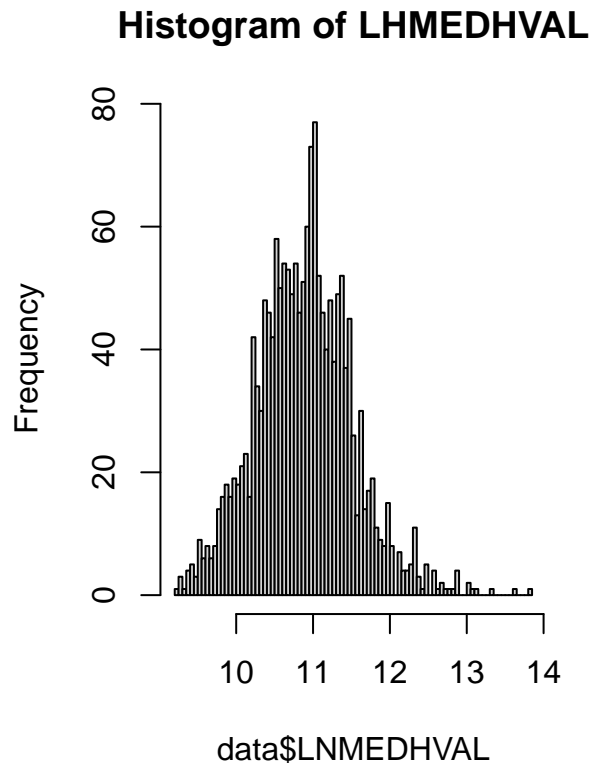
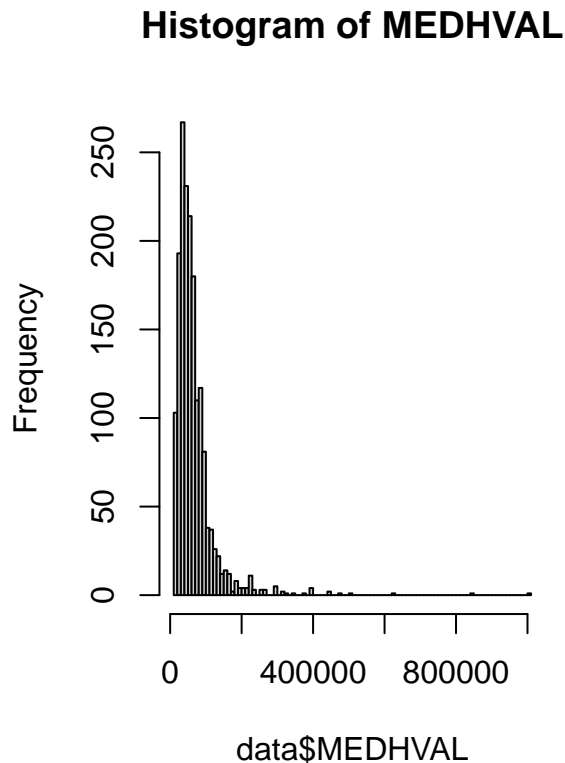
The following table provides the mean and standard deviation for our dependent variable (median house value by block group) and our four independent variables. The average median home value for census block groups in Philadelphia is \$66,287.73 and the standard deviation is \$60,006 USD. The standard deviation is nearly equal to the mean, which indicates a large amount of variability in average home sale prices across block groups in the city.

Variable	Mean	Standard Deviation
Median Home Value of all occupied housing units	66287.733139	60006.075990
% of Individuals with Bachelor Degrees or Higher	16.081372	17.769558
# Households Living in Poverty	189.770930	164.318480
% of Vacant Houses	11.288529	9.628472
% of Single House Units	9.226473	13.249250

Histograms

Median Home Value of Owner-Occupied Housing Units: The two histograms below show the distribution of our dependent variable, the median home values of owner occupied housing units by block group before and after applying a natural log transformation. The histogram without the natural log transformation peaks near \$75,000 and is right-skewed - it does not have a normal distribution. After applying a natural log transformation, the mean home value has a near normal distribution with a peak around 11.

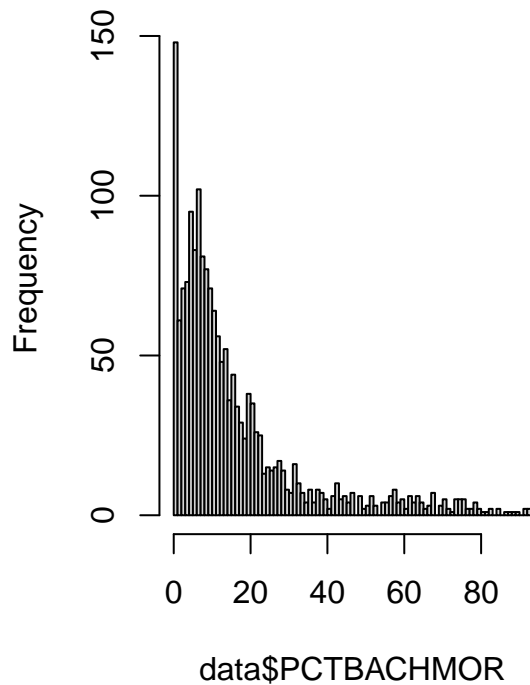
Our analysis will use a natural log transformation of median home sales value as the independent variable because a core assumption of linear regression is that variables are normally distributed. Using a normally distributed variable can help mitigate issues resulting from a non-linear relationship between our independent and dependent variables. The natural log transformation is most widely used with positive-skewed (i.e., right-skewed) data such as our dependent variable.



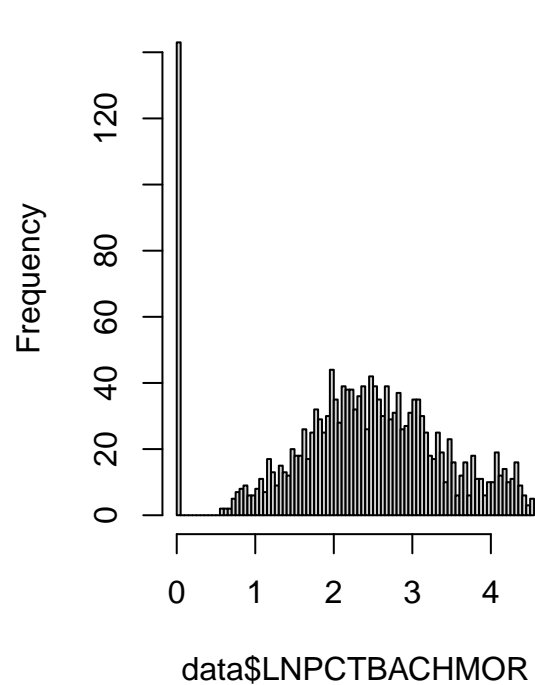
Percent of Population with a Bachelor Degree: The following histograms show the distribution of the percent of the population with a bachelor's degree by block group before and after applying a natural log transformation. The histogram without the natural log transformation is right-skewed, and there are 143 block groups where 0% of the population has a bachelor's degree.

After applying a natural log transformation, the 143 census blocks which had a value of 0% continue to have a value of 0. The distribution with the natural log transformation applied is not normal, indicating that the data has a zero-inflated distribution. Because both the variable with and without the natural log transformation applied are both not normal, we will use the variable without the natural log transformation (PCTBACHMORE) for our model.

Histogram of PCTBACHMOR

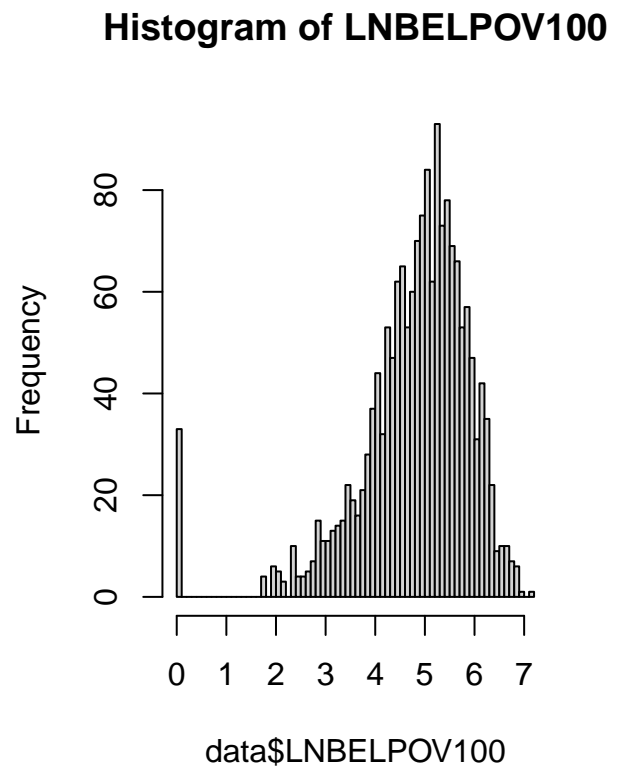
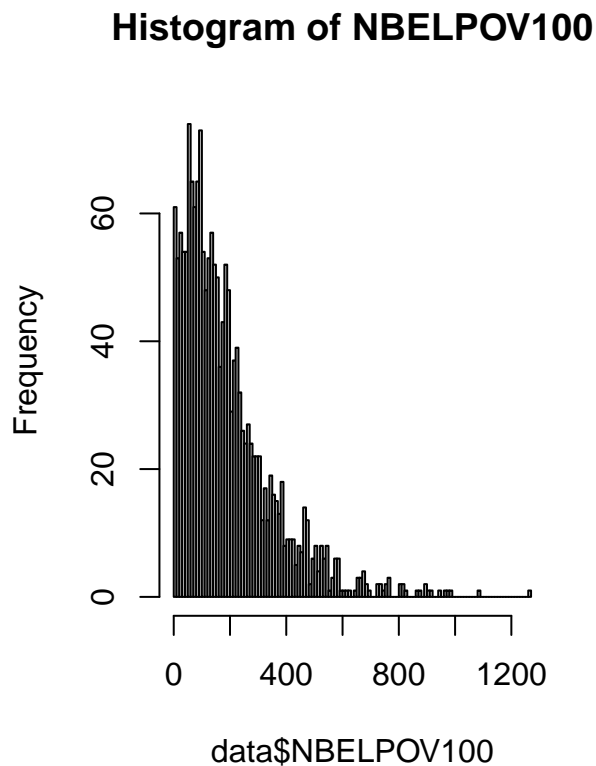


Histogram of LNPCTBACHMOR



Population Below the Poverty Line: The following histograms show the distribution of the population living below the poverty line in each block group with and without a natural log transformation applied. The histogram without the natural log transformation is again right skewed and peaks around 100 households. There is a very long tail to the right, and multiple outliers are present. Notably, the maximum value is 1,267 households below the poverty line in one census tract - which is more than six times larger than the mean value.

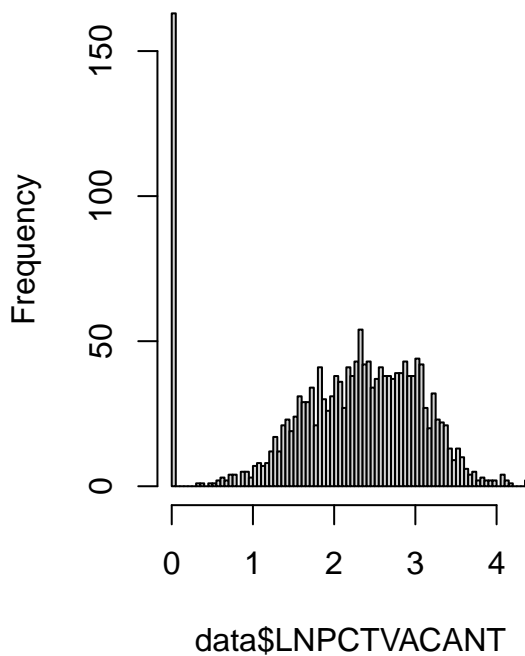
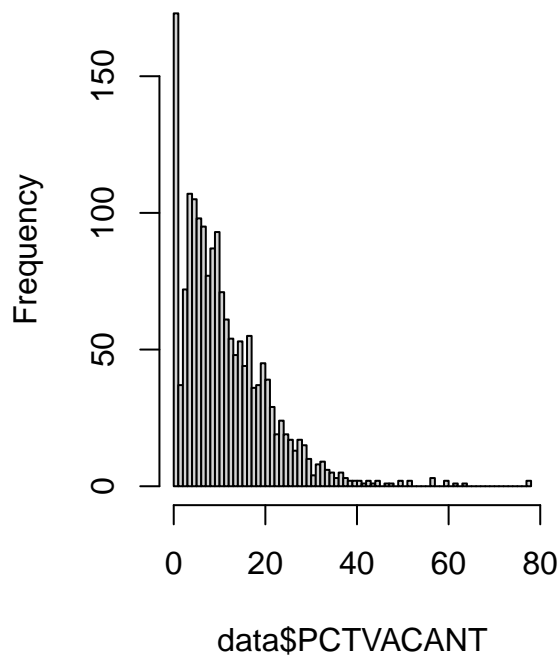
After applying a natural log transformation, the variable displays a distribution which is closer to a normal distribution. There is a clear peak around 5.5, but the data is slightly skewed to the left and is zero-inflated, but is closer to a normal distribution than the distribution prior to transformation. Because the natural log transformed variable is closer to a normal distribution we use the natural log transformation of the Population Below the Poverty Line (LNBELPOV100) in our regression. Additionally, the large positive skew in the non natural log transformed data supports the usage of the natural log transformed variable.



Percent of Vacant Housing Units: The following histograms show the distribution for the percent of housing units in a block group which are vacant with and without a natural log transformation. The histogram without the natural log transformation is right skewed and has a long tail, with multiple outliers present. Additionally, there are 163 block groups where 0% of the housing units are vacant.

After applying the natural log transformation, the 163 block groups which have a value of 0% still have a value of 0. The presence of the large number of block groups with a value of zero prevents the distribution from being considered normal and results in a zero-inflated distribution. Because neither distribution is normal, we use the variable without the natural log transformation in our regression analysis (PCTVACANT).

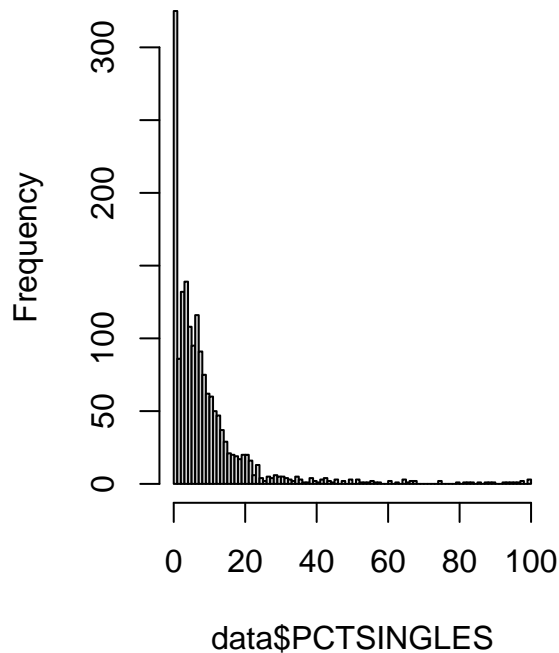
Histogram of data\$PCTVACANT Histogram of data\$LN PCTVACANT



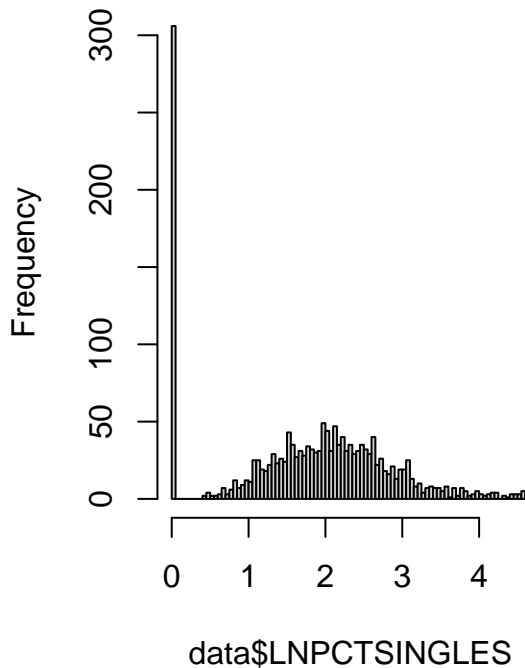
Percent of Detached Single Family Houses Units The following histograms show the distribution for the percent of housing units that are detached single family homes by block group with and without a natural log transformation. The histogram without the natural log transformation is right-skewed, and the vast majority of block groups (i.e: 1,548) have a percentage less than 20% - 306 block groups have a percentage of 0%. There are 172 census block groups which have percentages above 20%, including three homes with values of 100%. The extreme outliers are likely a result of the inclusion of suburban block groups in Northeast and Northwest Philadelphia, where most homes are detached single family homes.

After applying a log transformation, the 306 block groups where the percentage of detached single family homes is 0% continue to have a value of 0 resulting in a zero-inflated distribution. Because both the natural log transformed and non-natural log transformed variable do not have a normal distribution, we use the non-natural log transformed variable in our regression analysis.

Histogram of PCTSINGLES



Histogram of LNPCTSINGLES

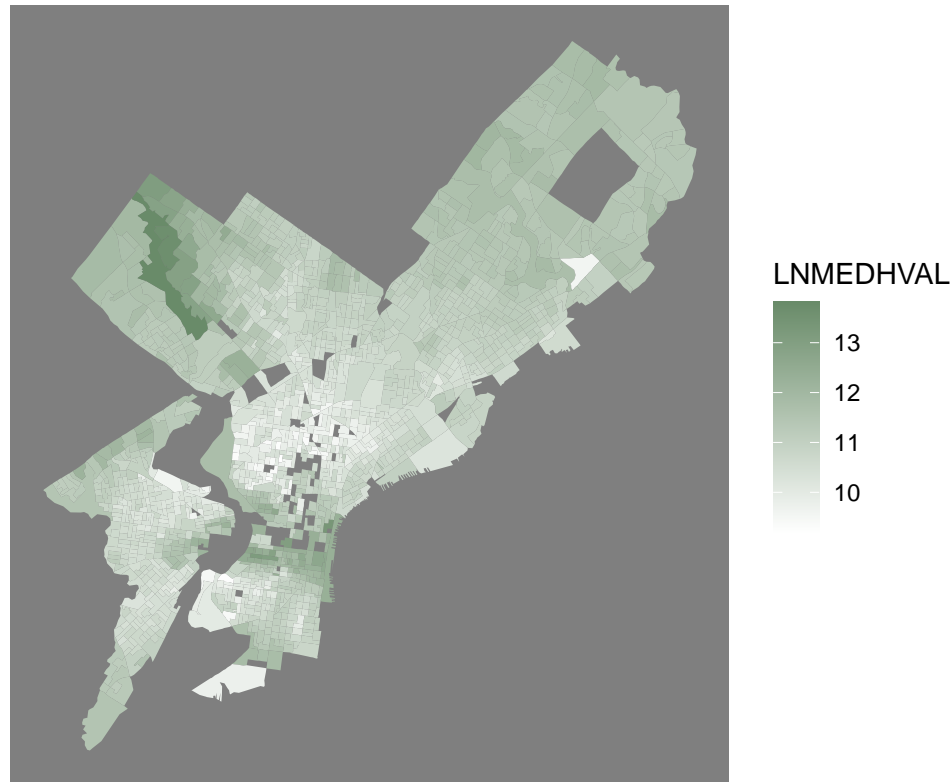


Maps

This section includes choropleth maps of our dependent and four independent variables.

Map of Dependent Variable: The following map shows our dependent variable, which is the median house value of owner occupied units by block group with a natural log transformation. We observe that the census tracts with the highest median home values are primarily clustered in Center City and northwest Philadelphia. The block groups with the lowest median home values are located north of Center City and west of University City.

Natural Log Median Home Value



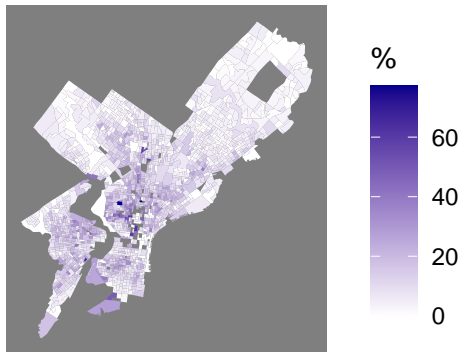
Maps of Independent Variables: The following maps show the spatial patterns on our four independent variables: PCTBACHMOR, PCTVACANT, PCTSINGLES, and LNBELPOV100.

Based on a review of the maps, the PCTBACHMOR variable appears to be the independent variable with the strongest correlation with our dependent variable. Like our dependent variable, the percentage of residents with a bachelor's degree is highest in Center City and northwest Philadelphia. The areas with the lowest percentage of residents with a bachelor degree are located west of University City and north of Center City - these are the same areas where the natural log transformed median home values are lowest.

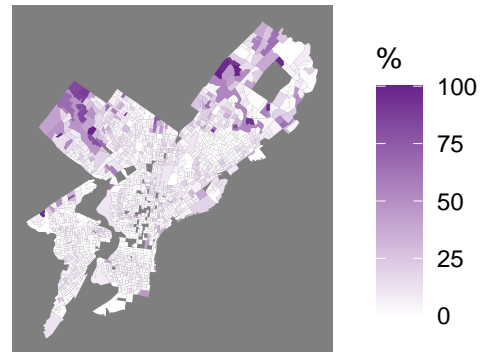
Conversely, the PCTSINGLES variable appears to be less correlated with our dependent variable. This is because the percent of housing units that are detached single family homes tends to be low in Center City and high in northwest Philadelphia, which are both neighborhoods with high median home prices values.

The independent variables PCTBACHMOR and PCTVACANT appear to have strong negative correlations, as areas with a high PCTVACANT rate also have a low PCTBACHMOR rate. Conversely, areas with a high PCTBACHMOR rate have a low PCTVACANT rate. This negative correlation indicates that there may be multicollinearity between these two variables. We will check the strength of this correlation using the Pearson correlation to determine if this multicollinearity could be an issue in our regression.

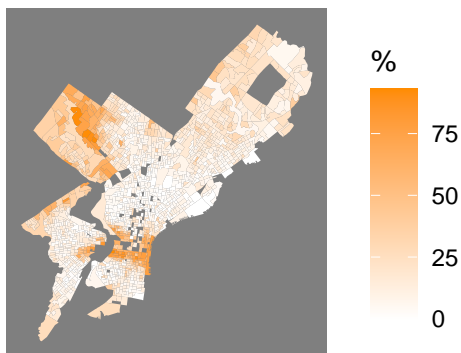
Vacant Lots



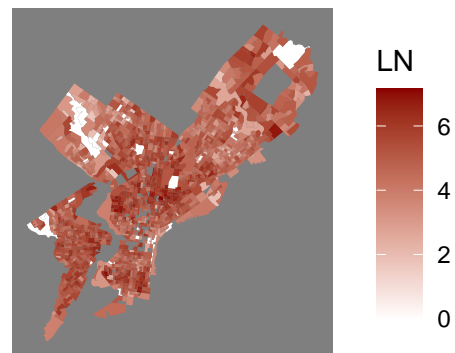
Single Houses



Bachelor's or More



LN Below Poverty

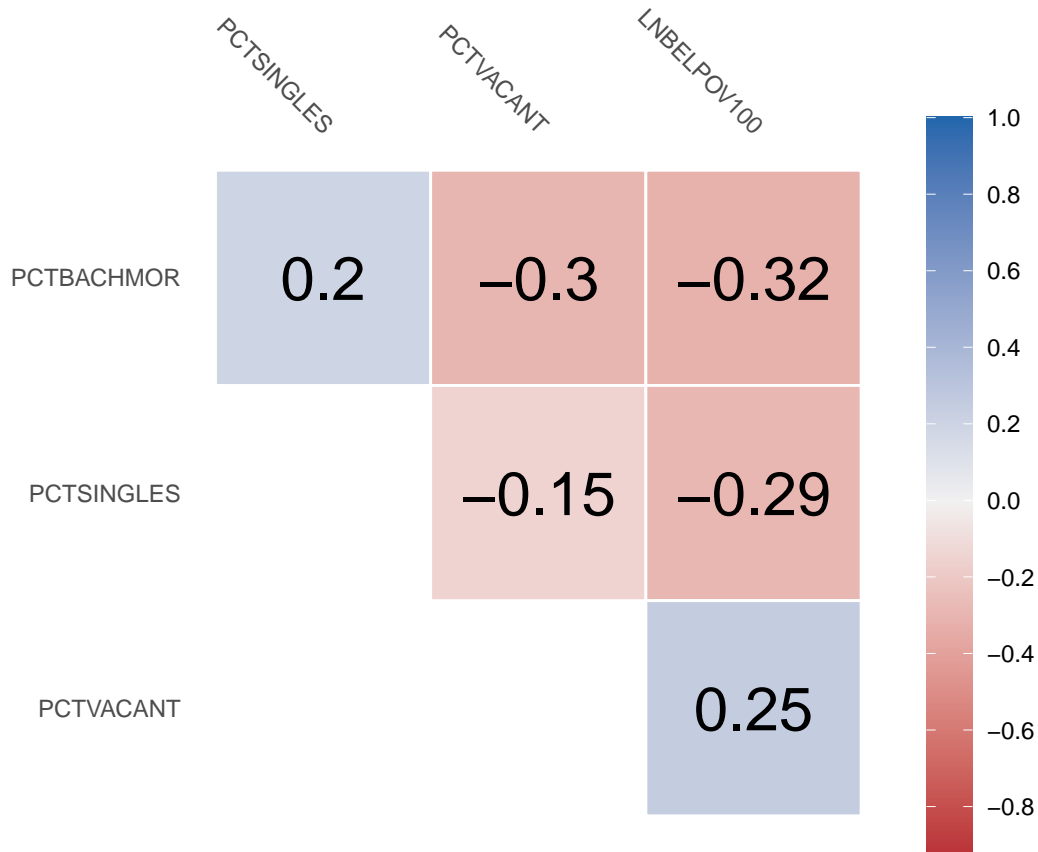


Pearson correlations

The correlation matrix shows the Pearson correlation between each of our dependent variables. For example, PCTBACHMOR is negatively correlated with LNBELPOV100 and PCTVACANT is positively correlated with LNBELPOV100.

Despite the correlations, we can conclude that there is not severe multicollinearity. This is because the Pearson correlation values are all between 0.8 and -0.8, which is considered an acceptable range when considering multicollinearity.

The Pearson correlation value for the relationship between PCTBACHMOR and PCTVACANT is -0.3, supporting our previous conclusion that there is a negative correlation between the variables. However, the Pearson correlation is within the acceptable range -0.8 to 0.8.



Regression Analysis

We first examine the f-ratio of our regression analysis. The f-ratio is 840.9, and the p-value associated with the f-ratio is less than 0.0001. Thus, we can reject the null hypothesis that all β coefficients are zero, and at least one of our independent variables is considered statistically significant.

After reviewing the f-ratio, we consider the β coefficients, standard errors, t statistics, and p-values for our four independent variables. All four independent variables have a low enough p-value to consider them statistically significant (< 0.05) and we can reject the null hypotheses that any of our β coefficients are equal to 0.

Based on our rejection of the null hypotheses, we can conclude the following for block groups in our model:

- There is a statistically significant negative relationship between the natural log of the median home value and both of our independent variables for the proportion of homes which are vacant and the natural log of the number of households living in poverty.
- There is a statistically significant positive relationship between the natural log of the median home value and both of our independent variables for the proportion of homes which are single family homes and the percent of individuals with a bachelor's degree.

When reviewing our β coefficient, we must consider the effects of the natural log transformation on our dependent variable. For the predictors proportion of homes which are standalone single family homes, proportion of homes which are vacant, and percent of individuals with a bachelor degree, only the dependent variable is natural log transformed. The β coefficients are also less than 0.3. Thus, we can conclude that as our independent variables increase by 1 unit, the expected change in median home value is approximately $100\beta_1$:

- As the proportion of homes which are vacant goes up by 1%, the median home value will decrease by approximately 1.916%.
- As the proportion of homes which are detached single family homes goes up by 1% the median home value will increase by approximately 0.298%.
- As the percentage of individuals who have a bachelor's degree goes up by 1%, the median home value will increase by approximately 2.091%.

For the population below the poverty line, both our independent variable and our dependent variable are natural log-transformed. Thus, as the population below the poverty line increases by 1, the median home value will change by approximately $(1.01_1^\beta - 1) \bullet 100$, i.e: the median home value will decrease by approximately 0.07848%.

The R^2 value is 0.6623, indicating that 66.23% of the variance in our dependent variable is explained by our four independent variables. 33.77% of the variance is not explained by our four independent variables. Our adjusted R^2 value is 0.6615, indicating that 66.15% of the variance in our dependent variable is explained by our four independent variables after adjusting the R^2 to account for the model including more than one independent variable.

```
##
## Call:
## lm(formula = LNMEDHVAL ~ PCTVACANT + PCTSINGLES + PCTBACHMOR +
##     LNBELPOV100, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.25825 -0.20391  0.03822  0.21744  2.24347
##
## Coefficients:
##              Estimate Std. Error t value      Pr(>|t|)
## (Intercept)  11.1137661   0.0465330  238.836 < 0.0000000000000002 ***
## PCTVACANT    -0.0191569   0.0009779  -19.590 < 0.0000000000000002 ***
## PCTSINGLES    0.0029769   0.0007032   4.234    0.0000242 ***
## PCTBACHMOR    0.0209098   0.0005432   38.494 < 0.0000000000000002 ***
## LNBELPOV100  -0.0789054   0.0084569   -9.330 < 0.0000000000000002 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.3665 on 1715 degrees of freedom
## Multiple R-squared:  0.6623, Adjusted R-squared:  0.6615
## F-statistic: 840.9 on 4 and 1715 DF, p-value: < 0.00000000000000022
```

The table below shows an analysis of the variance table for our linear regression model. The Sum of Square Errors (SSE) for our model is 230.44. The Regression Sum of Squares (SSR) is equal to 451.745, and the Total Sum of Squares (SST) is equal to 672.185. We can calculate the R^2 for our model by dividing the SSR by the SST, i.e: $\frac{451.745}{682.089}$ which equals 0.6623.

```
## Analysis of Variance Table
##
## Response: LNMEDHVAL
##              Df Sum Sq Mean Sq F value      Pr(>F)
## PCTVACANT      1  180.392  180.392  1343.087 < 0.00000000000000022 ***
## PCTSINGLES      1   24.543   24.543  182.734 < 0.00000000000000022 ***
## PCTBACHMOR      1  235.118  235.118 1750.551 < 0.00000000000000022 ***
## LNBELPOV100     1   11.692   11.692   87.054 < 0.00000000000000022 ***
## Residuals    1715  230.344    0.134
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Regression Assumptions Checks

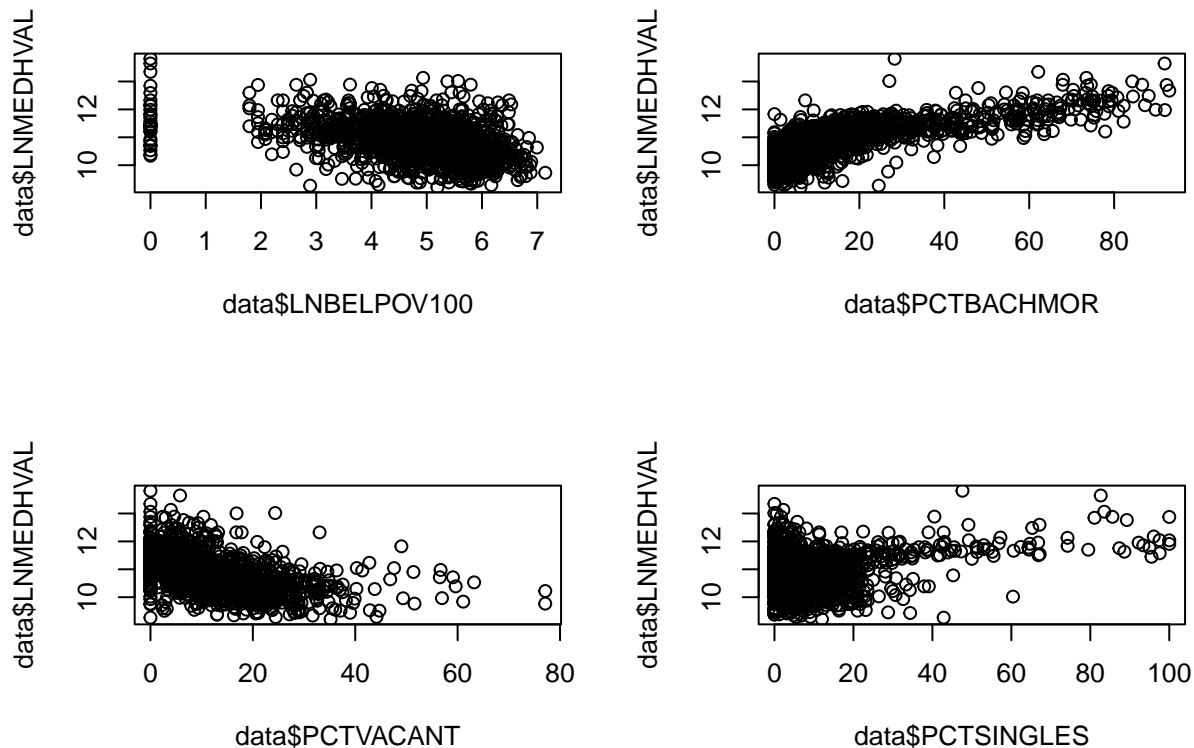
In this section, we will discuss testing model assumptions. We have already examined the variable distributions in a prior section.

Scatter Plots - Linear Relationships Between Variables

When running linear regressions, a core assumption is that there is a linear relationship between the dependent variable and each of the predictor variables. To check this assumption, we plot the dependent variable with each of the predictor variables in a scatter plot.

In cases where this assumption is not met, log transformations are often used. Based on the results of our variable distributions, we have already conducted log transformations for the dependent variable for median home value, now `LNMEDHVAL`, and for the predictor variable for number of households below poverty, now `LNNBELPOV100`.

The following scatter plots show the relationship between the dependent variable, `LNMEDHVAL`, and each of the predictor variables `LNNBELPOV100`, `PCTBACHMOR`, `PCTVACANT`, and `PCTSINGLES`. There does not appear to be a linear relationship between `LNMEDHVAL` and the predictor variables, even with log transformations used. The variables all appear heavily skewed - the relationship between `LNNBELPOV` and `LNBELPOV100` appears to be negatively skewed, and the individual relationship between the other three predictors and `LNMEDHVAL` appears to be heavily positively skewed.



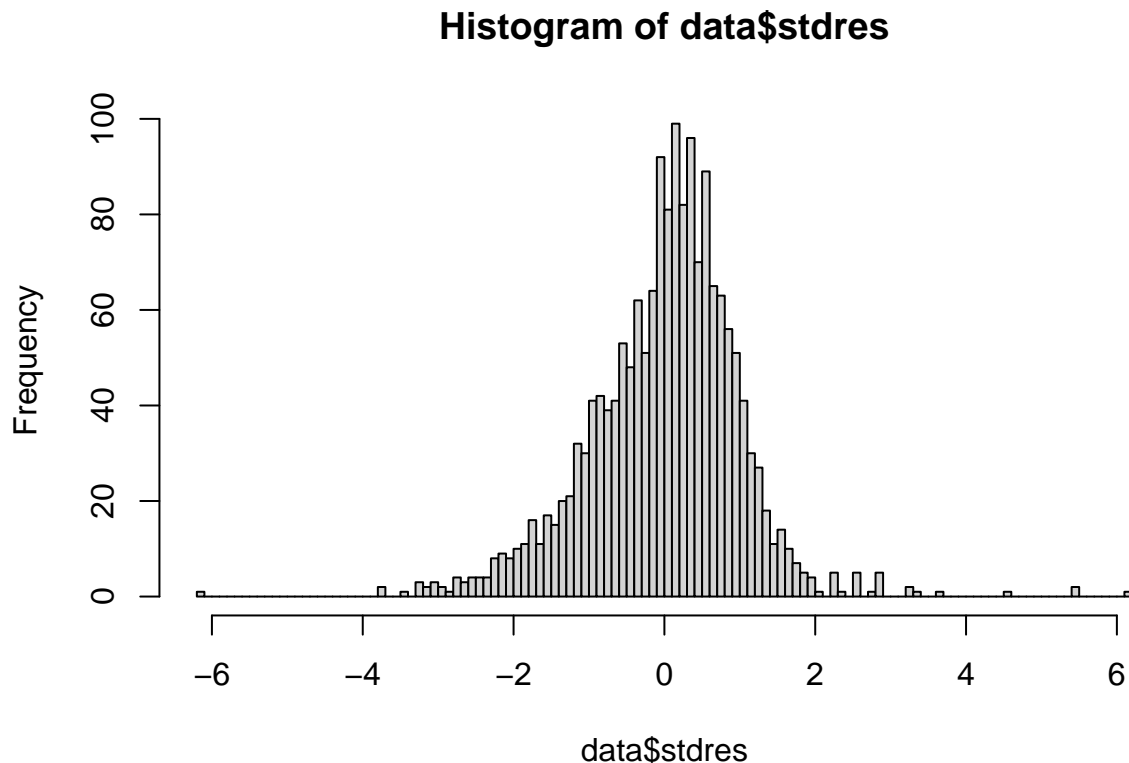
Histogram of the standardized residuals

Another assumption when running linear regression is that regression residuals are distributed normally. However, this assumption of normality is not considered critical in a regression, especially for data sets with a large number of

observations.

In order to compare residuals for different observations, we standardize the residuals through dividing a residual by its standard error. Standardizing allows us to observe how many standard deviations a residual is from our model's estimate

The following histogram of standardized residuals shows that residuals appear normally distributed.



Scatter Plot - Standardized Residual by Predicted Value

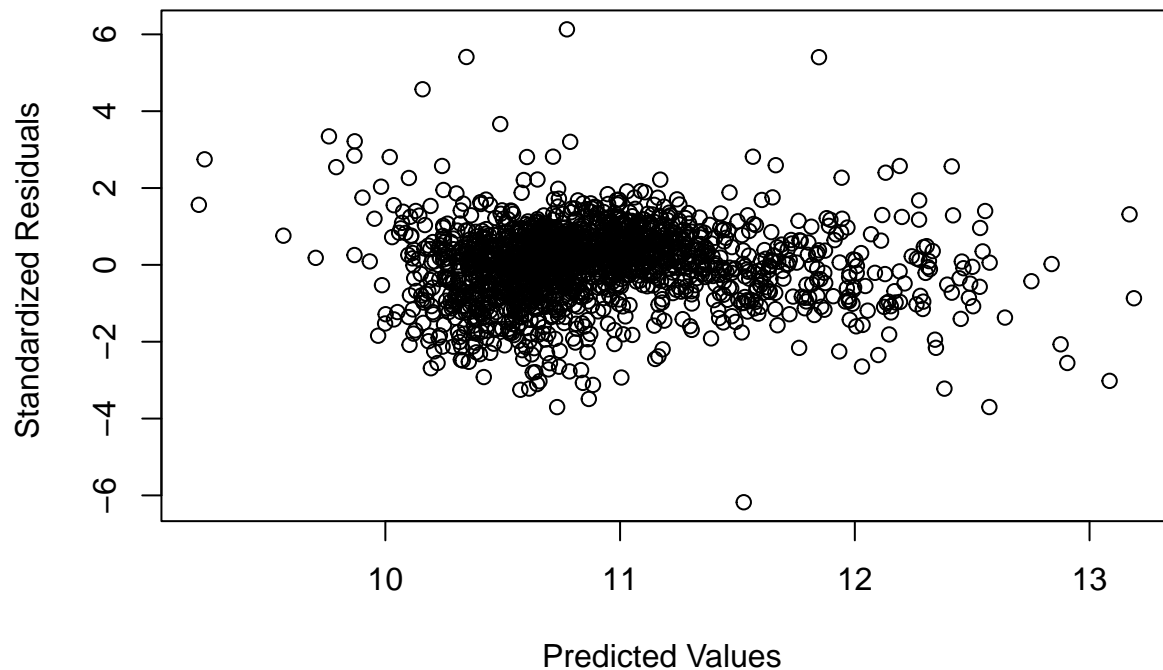
An additional core assumption of linear regression is that there is constant variance in residuals compared to the predicted values of the model - this relationship is referred to as homoscedastic. If non-constant variance is observed, the relationship is heteroscedastic.

Given that there are multiple predictors, we can plot the standardized residuals of the model by our predicted values of LNMEDHVAL. The scatter plot of standardized residuals appears to show a slight heteroscedastic relationship, based on a small “bow-tie” shape present around the predicted value of 11.5.

Outliers also appear to be present based on our scatter plot - there are several positive standardized residuals above 4 standard deviations above 0 and at least one standardized residual beyond -6 standard deviation below 0.

The standardized residuals also appear to be heavily clustered around between the predicted values of about 10.5 to 11.5.

Predicted Values vs. Standardized Residuals



Spatial Autocorrelation of Variables

Based on the maps of the dependent variable LNMEDHVAL and the predictor variables, we can estimate whether observations of each variable appear to show spatial autocorrelation - defined as observing the degree to which similar values cluster near each other. Our variables may appear to show spatial autocorrelation.

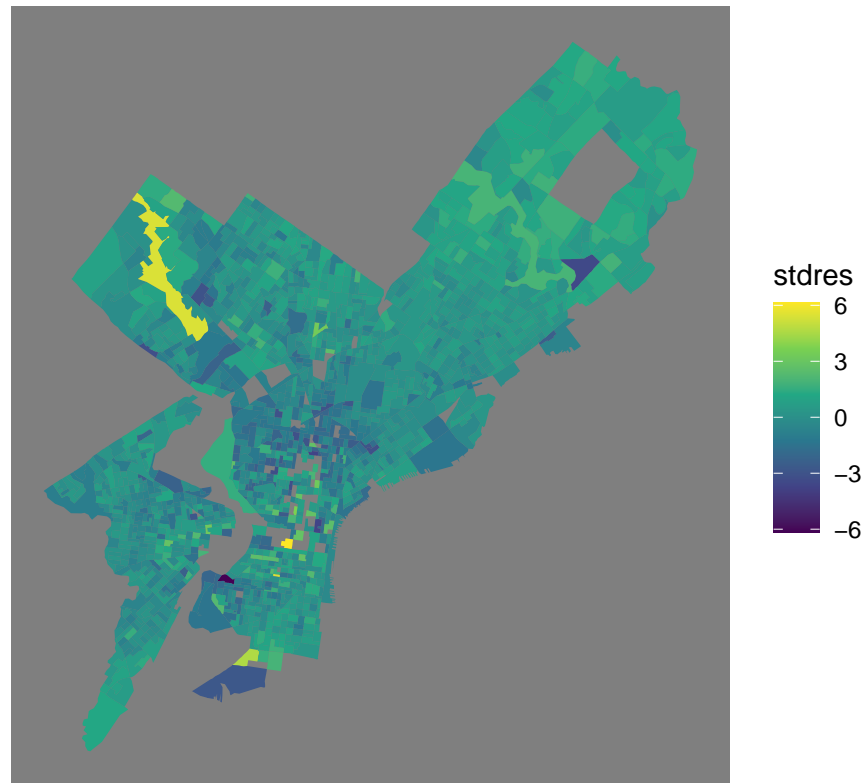
A critical assumption when building a regression model is that each observation of a variable is independent from other observations of that variable which we examine in our variables as spatial dependence, otherwise known as spatial autocorrelation. Based on the maps of the dependent variable LNMEDHVAL and the predictor variables, we can estimate whether observations of each variable appear to show spatial autocorrelation, which would appear as block groups clustering with similar values:

- The log of median home values appears to show spatial autocorrelation. High values appear to cluster in the downtown area and northwest Philadelphia, while low values appear to cluster around North Philadelphia and southwest Philadelphia, among other areas.
- Vacancy percentages appear to cluster in similar areas as the low value clusters of the log of median home values - North Philadelphia and southwest Philadelphia.
- Singles percentages appear to cluster in the northwest and northeast regions of the city.
- Bachelor's degree or more percentages appear to cluster in similar areas to the high values of the log of median home values - the downtown area and northwest Philadelphia.
- The log of the number of households in poverty does not appear to show spatial autocorrelation.

Based on these maps, each of the variables in our regression model appear to show spatial autocorrelation.

Choropleth map of the standardized regression residuals

Standardized Regression Residuals



Additional Models

Stepwise Regression

Inputting the variables of our model into a stepwise regression test results in a final model that retains all four of the original predictor variables. Therefore, all four predictors had sufficiently low p-values and removing predictors did not lower the value of the AIC.

```
## Start:  AIC=-3448.07
## LNMEDHVAL ~ PCTVACANT + PCTSINGLES + PCTBACHMOR + LNBELPOV100
##
##           Df Sum of Sq   RSS   AIC
## <none>                 230.34 -3448.1
## - PCTSINGLES    1      2.407 232.75 -3432.2
## - LNBELPOV100   1     11.692 242.04 -3364.9
## - PCTVACANT     1     51.546 281.89 -3102.7
## - PCTBACHMOR    1    199.020 429.36 -2379.0

## Stepwise Model Path
## Analysis of Deviance Table
##
## Initial Model:
```

```
## LNMEDHVAL ~ PCTVACANT + PCTSINGLES + PCTBACHMOR + LNBELPOV100
##
## Final Model:
## LNMEDHVAL ~ PCTVACANT + PCTSINGLES + PCTBACHMOR + LNBELPOV100
##
##
##      Step Df Deviance Resid. Df Resid. Dev      AIC
## 1              1715    230.3435 -3448.073
```

Cross-Validation

The following table provides the results of k-fold cross validation, when the number of folds (k) is set to 5. The root mean square error of the original model is represented by `rmse1`, which we compare to a second regression model's root mean square error, `rmse2`. This second model only includes the percent of vacancy and median household income as predictors for median home value. Given that `rmse1` is lower than `rmse2`, our original model is considered more generalizable when using different folds of the data and is thus a better fit for the model.

Discussion and Limitations

Our model indicates that there is a statistically significant relationship between our dependent variable, the natural log of the median home value and our four independent variables. We conclude that block groups in Philadelphia where higher numbers of residents live in poverty, higher proportions of residents do not have higher education degrees, and higher proportions of vacant homes are associated with lower median home values.

This conclusion is not surprising, as poorer households are generally limited to buying homes in areas with lower median home values. The city's history with redlining has also likely played an important role in the Philadelphia housing landscape that may impact the data set. Redlining was a 20th century racist banking practice where predominantly non-white city neighborhoods were deemed "hazardous" and "undesirable" for housing loans, making it impossible to obtain a federally backed home mortgage. Redlining resulted in a lack of investment in neighborhoods identified as undesirable and home values stayed low. Redlining is now illegal, but it had a lasting impact on the Philadelphia housing landscape as redlined neighborhoods continue to have higher vacancy rates and lower income households.

Our model appears to be effective in explaining the variation in median home values across the city of Philadelphia. The results of the f-test indicate that at least one of our dependent variables is a statistically significant predictor of median home value. Our R^2 value of 0.6623 indicates there is a strong linear correlation between our independent variables and the natural log of the median home sales value. We consider an R^2 value above 0.5 to be indicative of a strong correlation for a study in the social sciences domain.

Our stepwise regression model included all four of the original predictor variables. Therefore, all four variables are considered statistically significant predictors of our dependent variable and our model performs best with all four predictors included as opposed to some subset of the independent variables.

Several core assumptions of OLS regressions are violated by our model. Our predictor variables do not all appear to have linear relationships with our dependent variable - namely `LNBELPOV100`, `PCTVACANT`, and `PCTSINGLES`. Natural log transformations were used for the dependent variable and the number of households below poverty, but the non-linear relationships still appear. The variance of the residuals also appears to be heteroscedastic, which likely means that our model systematically underperforms in certain areas of the city based on characteristics not accounted for in our model. The predictor variables also appear to show potential spatial autocorrelation, which our model does not account for and could explain the heteroscedasticity of our residuals.

The predictor variable `NBELPOV100` is the only independent variable that is provided as a raw count and not a percentage. Because the `NELPOV100` variable is not normalized, block groups with smaller population counts experience an outsized impact in the regression model by increasing the count by one household, whereas block groups with larger populations do not experience the same level of impact. Normalizing the `NELPOV100` variable to a percentage would

be beneficial as it would allow us to compare 1 percent increases, representing proportional impacts to block groups with variable populations.

Our cross-validation results show that the root mean square error for the four predictor model was lower than that of the two predictor model, identifying the former model as better and more generalizable for new data.

Ridge regression is a method that offers solutions to issues that can arise in OLS regression. This model allows for

- A large number of predictors relative to the number of observations
- Multicollinearity
- The model overfitting the data by shrinking the coefficients of variables towards 0, which can significantly reduce their variance and the RMSE in the validation set.

Ridge regression functions by minimizing the SSE subject to a bound (i.e., constraint) on the quantity called L2 norm - the square root of the sum of the squared β coefficients. In OLS regression, we minimize the SSE.

Lasso regression (least absolute shrinkage & selection operator) is similar to ridge regression, except that this method sets the values of some coefficients to exactly 0 for different values of λ , minimizing SSE subject to a bound on the quantity called L1 norm - the sum of absolute values of the β coefficients. Both ridge and lasso regression would not be appropriate here because they drop the assumption of no multicollinearity. When this assumption is violated, we can get incorrect estimates of β_i as well as incorrect estimates of p-values of significance.