# MATH 3431 Maple Assignment 1
# Due: February 9, 2017

## Commands You Will Use, Etc.

Solving PDEs often involves long, purely mechanical calculations where it is easy to make mistakes. Luckily, computers are good at such calculations! In this assignment we look at getting Maple to do tedious calculations for us, and we look at the PDEtools package. You will need to use

1. `D`, which is used to differentiate functions rather than expressions. For example

   ```
   u := (x,y) -> x^2*y;
   D[1](u)(x,z);
   ```

   differentiates $u(x, y) = x^2 y$ with respect to the first variable ($x$), and evaluates the result at $(x, z)$. `D[1,2](u)(x, x^2)` would find $\dfrac{\partial^2 u}{\partial x \partial y}$, and evaluate the result at $(x, x^2)$.

2. The `assume` and `assuming` commands. The `assume` command tells Maple to assume the given variables have the given property for all calculations after the command has been given. The `assuming` command applies only to the line on which it is on. An example of the `assume` command is `assume(x::real, t >= 0, y > -1 and y < 1);`. Note that `t >= 0` and `y > -1 and y < 1` tell Maple that both $t$ and $y$ are real numbers. An example of the `assuming` command is `sqrt(a^2) assuming a>=0;`. If a variable has assumptions on it, Maple follows the variable by a $\sim$. You can find what the assumptions are using the `about` command, and you can remove assumptions, on `a`, for example, by entering `a := 'a';`. See the `assume` help page for details.

   In PDEs, especially, it is good practice to tell Maple explicitly about all assumptions. Some calculations cannot be done over the complex numbers, but can be done over the positive reals. For example, $\sin(n\pi)$ has no specific value if $n$ is assumed to be complex, or even real, but is 0 if $n$ is assumed to be integer.

3. Maple does not always simplify or solve things the way you want it to. In some cases you have to tell Maple explicitly how to simplify. Useful commands are `simplify`, `combine`, `expand`, `factor`, and `eval` (to sub-in for one of the variables). Remember that some simplifications are true over the reals, but not over the complex numbers. For example, $\sqrt{a}\sqrt{b} = \sqrt{ab}$ is true over the positive reals, but false over the complex numbers. Make sure Maple knows what is real!

4. Maple's `solve` command often does not find all solutions. If you include "`AllSolutions`" as an optional argument, Maple will try to find all solutions. For example

   ```
   solve(sin(n*Pi)=0, n, AllSolutions) assuming n::real;
   ```

   will return _Z1~. Check the conditions on the unspecified parameter (the part preceded by "_" without the ~) to make sure Maple has found what you expected! Note that the underscore is part of the name of that parameter.

5. When solving polynomial equations, Maple's `solve` command sometimes returns a "RootOf" in the answer. Often a "RootOf" actually represents several answers. To tell Maple to return all answers, use the `allvalues` command. Check the `allvalues` help page for details.

6. Maple is object-oriented, with the result that certain commands can be applied only to specific objects. For example, you can add vectors but you cannot add lists of equal length, and you differentiate functions with `D` but expressions with `diff`. Sometimes you will need to change the type of an object. In C or Java this is called "casting", but in Maple you "convert" objects. Check the `convert` help page for the types you can convert. Note that you convert an expression to a function with the `unapply` command. `exprAsFunction := unapply(x^2*y*sqrt(z), [x, y]);` makes the expression $x^2y\sqrt{z}$ into the function $(x, y) \to x^2y\sqrt{z}$, which has $\sqrt{z}$ as a parameter.

7. The PDEtools package contains commands that help solve PDEs. The `PDEplot` command plots the solution to first-order PDEs. An example is:

```
PDEplot(diff(u(t,x),t)+3*x*u(t,x)*diff(u(t,x),x)=u(t,x), [0, x, 1/(1+x^2)], x=-3..3)
```

   The second argument is the initial conditions as a parametric curve. The order of the coordinates of the parametric curve is the order of the independent variables, followed by the dependent variable. In this case, the initial condition is $u(0, x) = \dfrac{1}{1 + x^2}$. The last argument is a range, used for plotting. Optional arguments include `animate=true`, which tells Maple to trace out how the initial condition evolves, and `basechar=true` which tells Maple to draw the characteristic curves. Check the PDEplot help page, and the PDEplot,options help page for elaborations and details. The `pdsolve` command tries to solve PDEs (check its help page). If you find typing `diff(u(t,x), t,x)` tedious, check the `diff_table` command. Note that Maple's diff_table help page is incomplete, and using `diff_table` can complicate initial and boundary conditions.

## Assignment

1. What does the following command do?

```
plot([[sin(2*t), cos(3*t)][1], [sin(2*t), cos(3*t)][2], t=0..2*Pi]);
```

What simpler command will do the same? **Moral** Simple solutions are easier to understand!

2. The inttrans package contains several integral transforms, including the Laplace transform (laplace), and the inverse Laplace transform (invlaplace). Find the Laplace transform of $\frac{e^{-\frac{1}{4t}}}{\sqrt{t}}$ and $\frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$. The latter does not work because Maple assumes that $x$ is a complex number. Tell Maple that $x$ is a positive real number, and re-execute the last command (on a separate line, of course). Find $\mathcal{L}^{-1}\left\{\frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}}\right\}$, both without assumptions on $x$, and assuming $x > 0$. **Moral** Make sure Maple knows the assumptions you are making about variables!

3. Use Maple to find $\int \frac{dx}{\sqrt{1-x^2}}$, $\int_2^3 \frac{dx}{\sqrt{1-x^2}}$, and $\sin^{-1}(3) - \sin^{-1}(2)$. In the real world, $\sin^{-1}$ has domain $[-1, 1]$. What has happened? **Moral** Pay attention to the inputs of Maple commands.

4. The generic solution to $y'' + \lambda y = 0$, where $\lambda$ is a nonzero real number, is $y = Ae^{-\sqrt{\lambda}x} + Be^{\sqrt{\lambda}x}$. Find all solutions ($A$, $B$ and $\lambda$) where

   (a) $y(0) = 0$, and $y(1) = 0$

   (b) $y(0) = 0$ and $y'(1) = 0$

   (c) $y(0) - y'(0) = 0$ and $y(1) - y'(1) = 0$

   Note that you can do the first two parts efficiently (with 3-4 lines of code in total) if you use a function rather than an expression. For the third part, you may need to give Maple more guidance (tell it explicitly how to solve the system of equations). You differentiate functions with `D` rather than `diff`. For example,

```
fn := x -> x^2*cos(x);
D(fn)(Pi);
```

   gives the name `fn` to the function $x \to x^2 \cos x$, and the second line differentiates the function, and evaluates it at $\pi$.

5. Use PDEplot in the PDEtools package to plot the solution of $u_t + 2u^2 u_x = 0$ where $u(0, x) = \frac{1}{1+x^2}$. Make sure to choose a range of $x$ that shows the interesting features of the solution. Include a plot of the characteristic curves. What is interesting about those curves? Figure out how to create an animation of the solution traced over time, with the characteristic curves included, but without the full solution surface plotted (check the PDEplot,options help page). Is the solution a function?

6. Use pdsolve to solve $u_{tt} = u_{xx}$ where $u(0, x) = x(x - 1)$, $u_t(0, x) = \sin(\pi x)$. Note that you will need to use `D` at least for the initial conditions.

7. The `pdsolve` command is very limited, and doesn't always recognize PDEs it can solve.

   (a) Try solving $u_{tt} = u_{xx}$ where $u(0, x) = 0$ and $u_t(0, x) = \frac{1}{1+x^2}$.

(b) Try solving $u_{tt} = u_{xx}$ where $u(0,x) = \epsilon$ and $u_t(0,x) = \dfrac{1}{1+x^2}$. Remember that $\epsilon$ is epsilon. What is the solution to part (a)?

(c) Try solving $u_t = u_{xx}$ where $u(0,x) = x(1-x)$, $u(t,0) = 0$ and $u_t(t,1) = 0$.

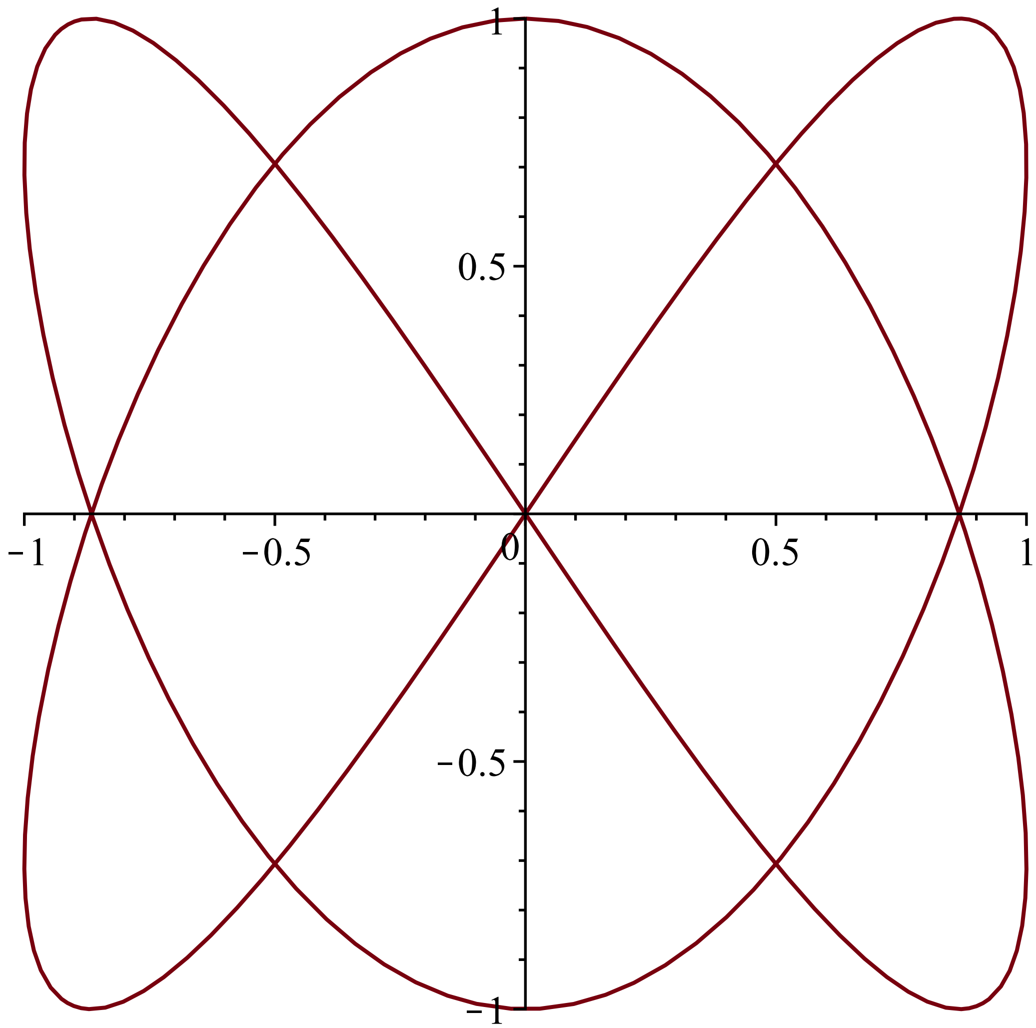
(d) How do you think pdsolve "solves" PDEs? If you execute the command `infolevel[pdsolve] := 3;` before executing `pdsolve`, you may find the output informative.

Maple Assignment # 1
Jarren Ralf

```
> with (PDEtools):
> with (inttrans):
```

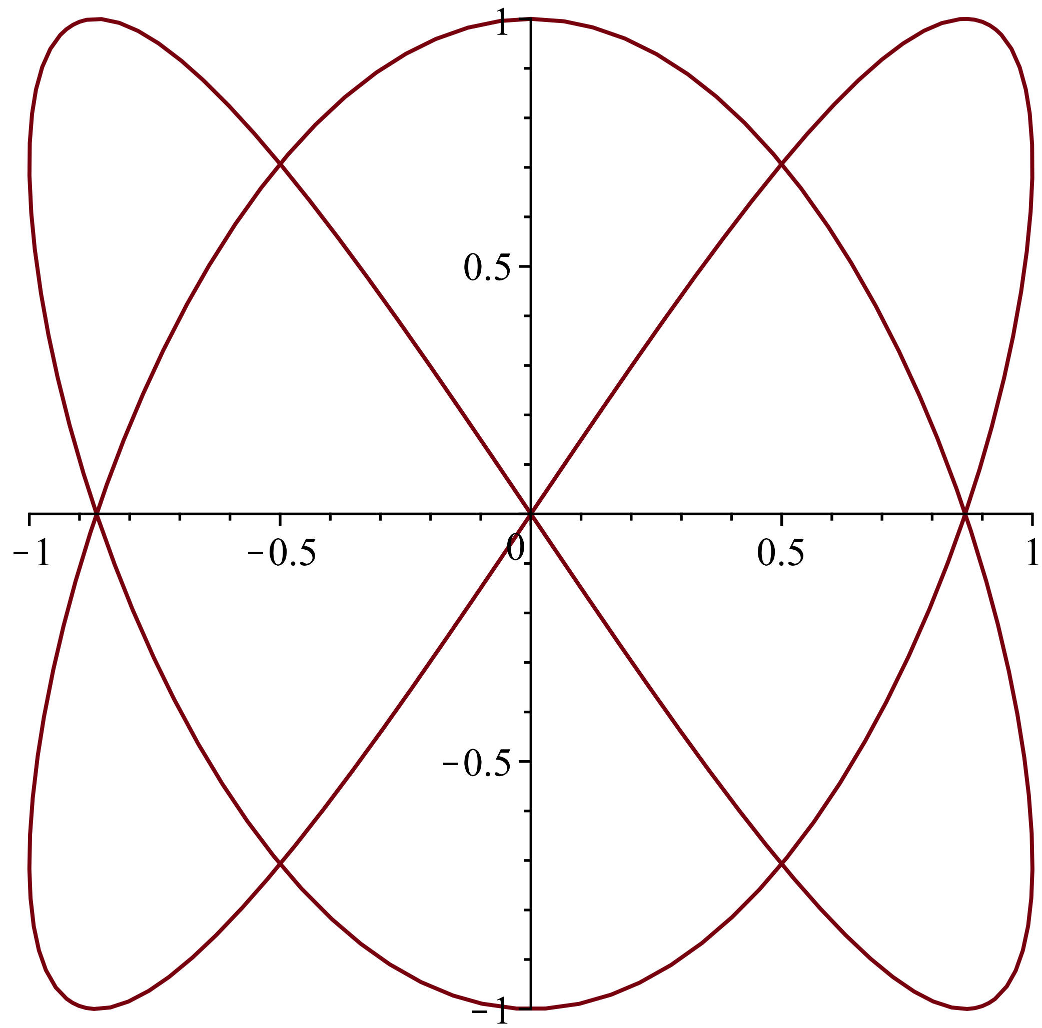
1. What does the following command do? What simpler command will do the same?

```
> plot([[sin(2*t), cos(3*t)][1], [sin(2*t), cos(3*t)][2], t=0..2*
  Pi]);
```




This command is just plotting the parametric equations $x = \sin(2\,t)$ and $y = \cos(3\,t)$ from $[0, 2\,\pi]$. The $[1]$ corresponds to plotting the first component. Similarily, $[2]$ corresponds to the second component.

```
> plot([sin(2*t), cos(3*t), t=0..2*Pi]); # This is the more simple
  function
  # It is a list of 3 arguments; the x-coordinate, the y-
  coordinate, and the range for the parameter
```




2. Find the laplace transform of $\frac{e^{-\frac{1}{4t}}}{\sqrt{t}}$ and $\frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$. Find the inverse laplace transform of $\frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}}$.

```
> functionTwo := exp(-1/(4*t))/sqrt(t);
  functionThree := exp(-1/(4*x*t))/sqrt(x*t);
  functionFour := exp(-1/(4*x*s))/sqrt(x*s);
```

$$functionTwo := \frac{e^{-\frac{1}{4t}}}{\sqrt{t}}$$

$$functionThree := \frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$$

$$functionFour := \frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}} \qquad \textbf{(1)}$$

```
> laplace(functionTwo, t, s);
```

**(2)**

$$\sqrt{\frac{\pi}{s}}\, e^{-\sqrt{s}} \tag{2}$$

```
> laplace(functionThree, t, s);
```

$$\frac{laplace\left(\frac{e^{-\frac{1}{4xt}}}{\sqrt{t}}, t, s\right)}{\sqrt{x}} \tag{3}$$

```
> assume(x>0);
  laplace(functionThree, t, s);
```

$$\sqrt{\frac{\pi}{s\, x\sim}}\, e^{-\sqrt{\frac{s}{x\sim}}} \tag{4}$$

```
> x := 'x': # clear all assumptions and assignments
  invlaplace(functionFour, s, t);
```

$$invlaplace\left(\frac{e^{-\frac{1}{4xs}}}{\sqrt{x\, s}}, s, t\right) \tag{5}$$

```
> assume(x>0);
  invlaplace(functionFour, s, t);
```

$$\frac{\cos\left(\sqrt{\frac{t}{x\sim}}\right)}{\sqrt{\pi\, t\, x\sim}} \tag{6}$$

3. Use maple to find $\int \frac{dx}{\sqrt{1-x}}$, $\int_2^3 \frac{dx}{\sqrt{1-x}}$, and $\sin^{-1}(3) - \sin^{-1}(2)$. What has happened?

```
> x := 'x': # clear all assumptions and assignments
  functionFive := 1/sqrt(1-x^2);
```

$$functionFive := \frac{1}{\sqrt{-x^2+1}} \tag{7}$$

```
> Int(functionFive, x)=int(functionFive, x);
```

$$\int \frac{1}{\sqrt{-x^2+1}}\, dx = \arcsin(x) \tag{8}$$

```
> Int(functionFive, x=2..3)=int(functionFive, x=2..3);
```

$$\int_2^3 \frac{1}{\sqrt{-x^2+1}}\, dx = \arccos(2) - \arccos(3) \tag{9}$$

```
> arcsin(3)-arcsin(2)=evalf(arcsin(3)-arcsin(2));
```

$$\arcsin(3) - \arcsin(2) = 0. - 0.445789277\, I \tag{10}$$

What has happened is maple is trying to evaluate arcsin outside of it's defined domain $[-1, 1]$. It seems to be computing this expression over the complex numbers.

4. The generic solution to $y'' + \lambda y = 0$, where $\lambda$ is a nonzero real number, is $y = Ae^{-\sqrt{\lambda}\, x} + Be^{\sqrt{\lambda}\, x}$. Find all solutions (A, B, and $\lambda$) where

(a) $y(0)=0$, and $y(1)=0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  functionSix := diff(y(x),x$2)+lambda*y(x)=0;
  dsolve({functionSix, y(0)=0, y(1)=0}, y(x));
```

$$functionSix := \frac{d^2}{dx^2}\,y(x) + \lambda\, y(x) = 0$$

$$y(x) = 0 \tag{11}$$

y(0)=0 is a solution, but, however, it is not the only one. We have not figured out how to use maple functions in order to find all of the eigenvalue functions. My strategy is to solve the equation with only one initial condition, and then, solve the resulting function for lambda.

```
> dsolve({functionSix, y(0)=0}, y(x));
  lambda=solve(sin(sqrt(lambda)*x), lambda, AllSolutions);
```

$$y(x) = \_C1 \sin\left(\sqrt{\lambda}\, x\right)$$

$$\lambda = \frac{\pi^2\, \_Z62{\sim}^2}{x^2} \tag{12}$$

$y(x) = A sin\left(\sqrt{\lambda} x\right)$ for $\lambda$ above.

(b) $y(0) = y'(1) = 0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  dsolve({functionSix, D(y)(1)=0, y(0)=0}, y(x));
```

$$y(x) = 0 \tag{13}$$

```
> dsolve({functionSix, D(y)(1)=0}, y(x));
  lambda=solve((cos(sqrt(lambda))*cos(sqrt(lambda)*x))/sin(sqrt
  (lambda)), lambda, AllSolutions);
```

$$y(x) = \_C1 \sin\left(\sqrt{\lambda}\, x\right) + \frac{\_C1 \cos\left(\sqrt{\lambda}\right) \cos\left(\sqrt{\lambda}\, x\right)}{\sin\left(\sqrt{\lambda}\right)}$$

$$\lambda = \left(\frac{1}{4}\pi^2 (1 + 4\,\_Z63{\sim})^2, \frac{1}{4}\pi^2 (-1 + 4\,\_Z64{\sim})^2, \frac{1}{4}\,\frac{\pi^2 (1 + 2\,\_Z66{\sim})^2}{x^2}\right) \tag{14}$$

(c) $y(0) - y'(0) = 0$ and $y(1) - y'(1) = 0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  dsolve({functionSix, y(0)-D(y)(0)=0, y(1)-D(y)(1)=0}, y(x));
```

$$y(x) = 0 \tag{15}$$

```
> #allvalues function is an attempt to remove the RootOf. But it
  failed.

  allvalues(solve(((1/sqrt(lambda))*sin(sqrt(lambda)*x)+cos(sqrt
  (lambda))*x), lambda, AllSolutions));
```

$$RootOf\left(\cos(\_Z)\sqrt{\_Z^2}\, x + \sin\left(\sqrt{\_Z^2}\, x\right)\right)^2 \tag{16}$$

```
> dsolve({functionSix, y(0)-D(y)(0)=0}, y(x));
```

```
lambda=solve(((1/sqrt(lambda))*sin(sqrt(lambda)*x)+cos(sqrt
(lambda))*x), lambda, AllSolutions);
```

$$y(x) = \frac{\_C1 \sin\left(\sqrt{\lambda}\, x\right)}{\sqrt{\lambda}} + \_C1 \cos\left(\sqrt{\lambda}\, x\right)$$

$$\lambda = RootOf\left(\cos(\_Z)\sqrt{\_Z^2}\, x + \sin\left(\sqrt{\_Z^2}\, x\right)\right)^2 \qquad \textbf{(17)}$$

5. Plot the solution $u_t + 2\,u^2 u_x = 0$ where $u(0,x) = \frac{1}{1+x^2}$. Include a plot of the characteristic curves.

Is the solution a function?

```
> x := 'x': # clear all assumptions and assignments
  t := 't': # clear all assumptions and assignments
  functionSeven := diff(u(t,x),t)+2*u(t,x)^2*diff(u(t,x),x)=0, [0,
  x, 1/(1+x^2)];
  characteristicEquation := diff(x(t),t)=2*u(t,x)^2;
  characteristicCurve := pdsolve(characteristicEquation, [u(0,x)=1/
  (1+x^2)]);
```

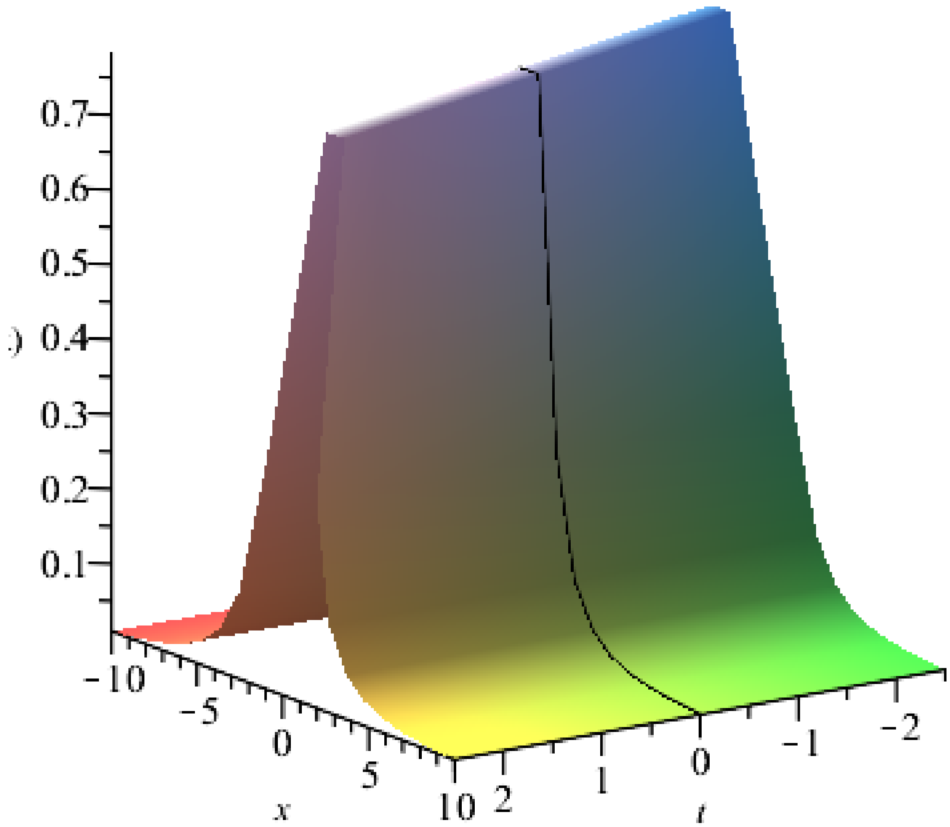
$$functionSeven := \frac{\partial}{\partial t}\, u(t,x) + 2\,u(t,x)^2 \left(\frac{\partial}{\partial x}\, u(t,x)\right) = 0, \left[0, x, \frac{1}{x^2+1}\right]$$

$$characteristicEquation := \frac{d}{dt}\, x(t) = 2\,u(t,x)^2$$

```
Error, (in pdsolve/info) the indeterminate function, x(t),
depends on only one variable. Use solve or dsolve
```

I could not figure out the characteristic curve.

```
> PDEplot(functionSeven, x=-10..10);
```




```
> PDEplot(characteristicEquation, x=-10..10);
Error, (in PDEtools:-PDEplot) invalid input: op expects 1 or 2 arguments, but received 3
```

6. Solve $u_{tt} = u_{xx}$ where $u(0,x) = x(x-1)$, $u_t(0,x) = \sin(\pi x)$.

```
> functionEight := diff(u(t,x),t$2)=diff(u(t,x),x$2);
```

$$functionEight := \frac{\partial^2}{\partial t^2} u(t,x) = \frac{\partial^2}{\partial x^2} u(t,x) \tag{18}$$

```
> pdsolve([functionEight, u(0,x)=x*(x-1), D[1](u)(0,x)=sin(Pi*x)]);
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
First set of solution methods (general or quasi general solution)
Trying differential factorization for linear PDEs ...
differential factorization successful.
First set of solution methods successful
   <- trying "linear_in_xt" successful
<- methods for class "_Fn" for 2nd order PDEs successful
```

$$u(t,x) = \frac{1}{2}\, \frac{\cos(\pi (t-x)) - \cos(\pi (x+t)) + (2t^2 + 2x^2 - 2x)\pi}{\pi} \tag{19}$$

7. (a) Try solving $u_{tt} = u_{xx}$ where $u(0, x) = 0$ and $u_t(0, x) = \dfrac{1}{1+x^2}$.

```
> pdsolve([functionEight, u(0,x)=0, D[1](u)(0,x)=1/(1+x^2)]);
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
differential factorization successful.
First set of solution methods successful
   <- trying "linear_in_xt" successful
<- methods for class "_Fn" for 2nd order PDEs successful
```

$$u(t, x) = \frac{1}{2} \arctan(t - x) + \frac{1}{2} \arctan(x + t) \tag{20}$$

(b) Try solving $u_{tt} = u_{xx}$ where $u(x, 0) = \epsilon$ and $u_t(0, x) = \dfrac{1}{1+x^2}$.

```
> pdsolve([functionEight, u(0,x)=epsilon, D[1](u)(0,x)=1/(1+x^2)]);
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
differential factorization successful.
First set of solution methods successful
   <- trying "linear_in_xt" successful
<- methods for class "_Fn" for 2nd order PDEs successful
```

$$u(t, x) = \epsilon + \frac{1}{2} \arctan(t - x) + \frac{1}{2} \arctan(x + t) \tag{21}$$

(c) Try solving $u_t = u_{xx}$ where $u(0, x) = x(1 - x)$, $u(t, 0) = 0$, and $u_t(t, 1) = 0$.

```
> functionNine := diff(u(t,x),t)=diff(u(t,x),x$2);
```

$$functionNine := \frac{\partial}{\partial t} u(t, x) = \frac{\partial^2}{\partial x^2} u(t, x) \tag{22}$$

```
> infolevel[pdsolve] := 3;
  pdsolve([functionNine, u(0,x)=x*(1-x), u(t,0)=0, D[1](u)(t,1)=0])
  ;
```

$$infolevel_{pdsolve} := 3$$

```
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
   -> trying "BC_equal_0"
* trying methods for class "_Cn_cn" for 2nd order PDEs
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
```

```
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
* trying methods for class "Wave" for 2nd order PDEs
   -> trying "Cauchy"
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Heat" for 2nd order PDEs
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Laplace" for 2nd order PDEs
   -> trying a Laplace transformation
* trying methods for class "Fourier" for 2nd order PDEs
   -> trying a fourier transformation
* trying methods for class "Series" for 2nd order PDEs
   -> trying "sincos"
   -> trying "2"
   -> trying "3"
   -> trying "4"
   -> trying "WithSourceTerm"
* trying methods for class "Generic" for 2nd order PDEs
   -> trying a solution in terms of arbitrary constants and
functions to be adjusted to the given initial conditions
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
* trying methods for class "Linear_diff_op" for 2nd order PDEs
```

(d) How do you think the pdsolve "solves" PDEs?

As you can read above maple runs an algorithm that checks several different methods of solving the PDE.

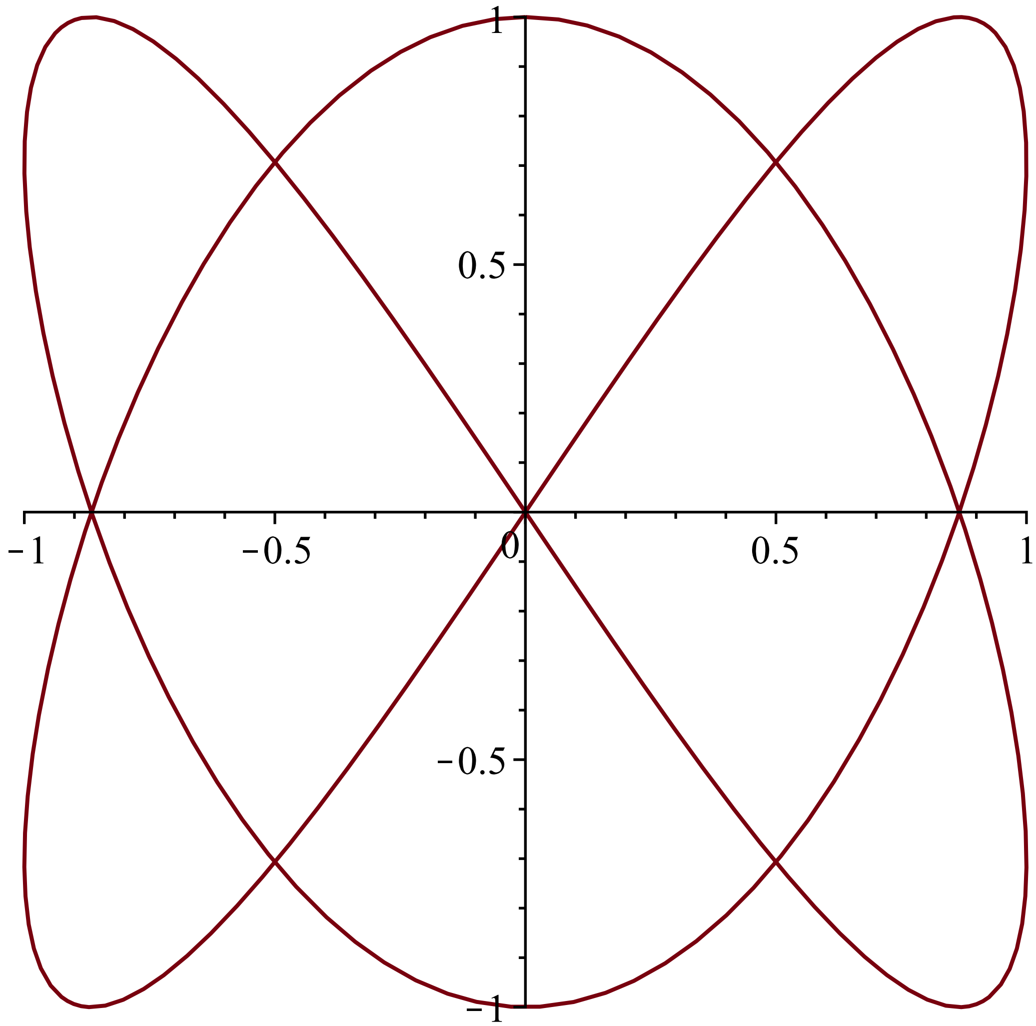
Maple Assignment # 1
Jarren Ralf

**15/24**

```
> with (PDEtools):
> with (inttrans):
```

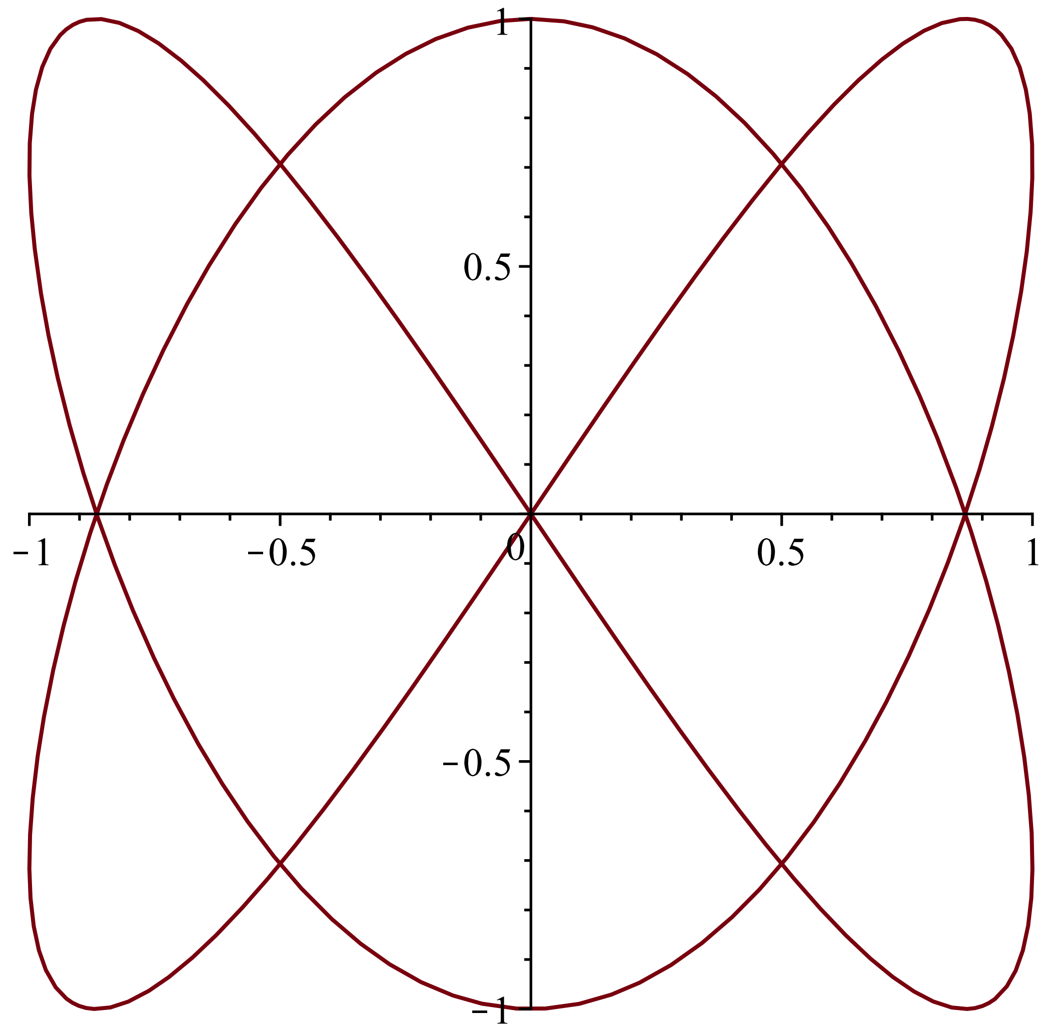
1. What does the following command do? What simpler command will do the same?

```
> plot([[sin(2*t), cos(3*t)][1], [sin(2*t), cos(3*t)][2], t=0..2*
  Pi]);
```




This command is just plotting the parametric equations $x = \sin(2\,t)$ and $y = \cos(3\,t)$ from $[0, 2\,\pi]$. The $[1]$ corresponds to plotting the first component. Similarily, $[2]$ corresponds to the second component.
**2/2**

```
> plot([sin(2*t), cos(3*t), t=0..2*Pi]); # This is the more simple
  function
  # It is a list of 3 arguments; the x-coordinate, the y-
  coordinate, and the range for the parameter
```




2. Find the laplace transform of $\frac{e^{-\frac{1}{4t}}}{\sqrt{t}}$ and $\frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$. Find the inverse laplace transform of $\frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}}$. **2/2**

```
> functionTwo := exp(-1/(4*t))/sqrt(t);
  functionThree := exp(-1/(4*x*t))/sqrt(x*t);
  functionFour := exp(-1/(4*x*s))/sqrt(x*s);
```

$$functionTwo := \frac{e^{-\frac{1}{4t}}}{\sqrt{t}}$$

$$functionThree := \frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$$

$$functionFour := \frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}} \quad \textbf{(1)}$$

```
> laplace(functionTwo, t, s);
```

**(2)**

$$\sqrt{\frac{\pi}{s}}\, e^{-\sqrt{s}} \tag{2}$$

```
> laplace(functionThree, t, s);
```

$$\frac{laplace\left(\frac{e^{-\frac{1}{4 x t}}}{\sqrt{t}}, t, s\right)}{\sqrt{x}} \tag{3}$$

```
> assume(x>0);
  laplace(functionThree, t, s);
```

$$\sqrt{\frac{\pi}{s\, x\sim}}\, e^{-\sqrt{\frac{s}{x\sim}}} \tag{4}$$

```
> x := 'x': # clear all assumptions and assignments
  invlaplace(functionFour, s, t);
```

$$invlaplace\left(\frac{e^{-\frac{1}{4 x s}}}{\sqrt{x s}}, s, t\right) \tag{5}$$

```
> assume(x>0);
  invlaplace(functionFour, s, t);
```

$$\frac{\cos\left(\sqrt{\frac{t}{x\sim}}\right)}{\sqrt{\pi\, t\, x\sim}} \tag{6}$$

3. Use maple to find $\int \frac{dx}{\sqrt{1-x}}$, $\int_2^3 \frac{dx}{\sqrt{1-x}}$, and $\sin^{-1}(3) - \sin^{-1}(2)$. What has happened?

```
> x := 'x': # clear all assumptions and assignments
  functionFive := 1/sqrt(1-x^2);
```

$$functionFive := \frac{1}{\sqrt{-x^2+1}} \tag{7}$$

```
> Int(functionFive, x)=int(functionFive, x);
```

$$\int \frac{1}{\sqrt{-x^2+1}}\, dx = \arcsin(x) \tag{8}$$

```
> Int(functionFive, x=2..3)=int(functionFive, x=2..3);
```

$$\int_2^3 \frac{1}{\sqrt{-x^2+1}}\, dx = I \ln\left(2+\sqrt{3}\right) - I \ln\left(3+2\sqrt{2}\right) \tag{9}$$

```
> arcsin(3)-arcsin(2)=evalf(arcsin(3)-arcsin(2));
```

$$\arcsin(3) - \arcsin(2) = 0. - 0.445789277\, I \tag{10}$$

What has happened is maple is trying to evaluate arcsin outside of it's defined domain $[-1, 1]$. It seems to be computing this expression over the complex numbers. **2/2**

4. The generic solution to $y'' + \lambda y = 0$, where $\lambda$ is a nonzero real number, is $y = Ae^{-\sqrt{\lambda}\, x} + Be^{\sqrt{\lambda}\, x}$. Find all solutions (A, B, and $\lambda$) where

(a) $y(0)=0$, and $y(1)=0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  functionSix := diff(y(x),x$2)+lambda*y(x)=0;
  dsolve({functionSix, y(0)=0, y(1)=0}, y(x));
```

$$functionSix := \frac{d^2}{dx^2}\, y(x) + \lambda\, y(x) = 0$$

$$y(x) = 0 \tag{11}$$

y(0)=0 is a solution, but, however, it is not the only one. We have not figured out how to use maple functions in order to find all of the eigenvalue functions. My strategy is to solve the equation with only one initial condition, and then, solve the resulting function for lambda.

```
> dsolve({functionSix, y(0)=0}, y(x));
  lambda=solve(sin(sqrt(lambda)*x), lambda, AllSolutions);
```

$$y(x) = \_C1 \sin\left(\sqrt{\lambda}\, x\right)$$

$$\lambda = \frac{\pi^2\, \_Z1{\sim}^2}{x^2} \tag{12}$$

$y(x) = A sin\left(\sqrt{\lambda}\, x\right)$ for $\lambda$ above.

**You've solved for the constant, $\lambda$, in terms of the variable, $x$. Here, you know the form of the solution, and want to find $\lambda$ that solves the boundary conditions. Try**

**functionSix := x -> A*exp(-sqrt(lambda)*x)+B*exp(sqrt(lambda)*x);**
**solve([functionSix(0)=0, functionSix(1)=0], [A, B, lambda], AllSolutions); 1/6**

(b) $y(0) = y'(1) = 0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  dsolve({functionSix, D(y)(1)=0, y(0)=0}, y(x));
```

$$y(x) = 0 \tag{13}$$

```
> dsolve({functionSix, D(y)(1)=0}, y(x));
  lambda=solve((cos(sqrt(lambda))*cos(sqrt(lambda)*x))/sin(sqrt
  (lambda)), lambda, AllSolutions);
```

$$y(x) = \_C1 \sin\left(\sqrt{\lambda}\, x\right) + \frac{\_C1 \cos\left(\sqrt{\lambda}\right) \cos\left(\sqrt{\lambda}\, x\right)}{\sin\left(\sqrt{\lambda}\right)}$$

$$\lambda = \left(\frac{1}{4}\, \pi^2\, (1 + 4\, \_Z63{\sim})^2, \frac{1}{4}\, \pi^2\, (-1 + 4\, \_Z64{\sim})^2, \frac{1}{4}\, \frac{\pi^2\, (1 + 2\, \_Z66{\sim})^2}{x^2}\right) \tag{14}$$

(c) $y(0) - y'(0) = 0$ and $y(1) - y'(1) = 0$.

```
> x := 'x': # clear all assumptions and assignments
  lambda := 'lambda': # clear all assumptions and assignments
  y := 'y': # clear all assumptions and assignments
  dsolve({functionSix, y(0)-D(y)(0)=0, y(1)-D(y)(1)=0}, y(x));
```

$$y(x) = 0 \tag{15}$$

```
> #allvalues function is an attempt to remove the RootOf. But it
  failed.

  allvalues(solve(((1/sqrt(lambda))*sin(sqrt(lambda)*x)+cos(sqrt
  (lambda))*x), lambda, AllSolutions));
```

$$RootOf\left(\cos(\_Z)\sqrt{\_Z^2}\,x+\sin\left(\sqrt{\_Z^2}\,x\right)\right)^2 \tag{16}$$

```
> dsolve({functionSix, y(0)-D(y)(0)=0}, y(x));
  lambda=solve(((1/sqrt(lambda))*sin(sqrt(lambda)*x)+cos(sqrt
  (lambda))*x), lambda, AllSolutions);
```

$$y(x)=\frac{\_C1\,\sin\left(\sqrt{\lambda}\,x\right)}{\sqrt{\lambda}}+\_C1\,\cos\left(\sqrt{\lambda}\,x\right)$$

$$\lambda=RootOf\left(\cos(\_Z)\sqrt{\_Z^2}\,x+\sin\left(\sqrt{\_Z^2}\,x\right)\right)^2 \tag{17}$$

5. Plot the solution $u_t+2\,u^2u_x=0$ where $u(0,x)=\frac{1}{1+x^2}$. Include a plot of the characteristic curves.

Is the solution a function?

**Use PDEplot to plot the solution. The optional argument, basechar=true plots characteristic curves. Check the PDEplot,options help page for more details. 1/5**

```
> x := 'x': # clear all assumptions and assignments
  t := 't': # clear all assumptions and assignments
  functionSeven := diff(u(t,x),t)+2*u(t,x)^2*diff(u(t,x),x)=0, [0,
  x, 1/(1+x^2)];
  characteristicEquation := diff(x(t),t)=2*u(t,x)^2;
  characteristicCurve := pdsolve(characteristicEquation, [u(0,x)=1/
  (1+x^2)]);
```

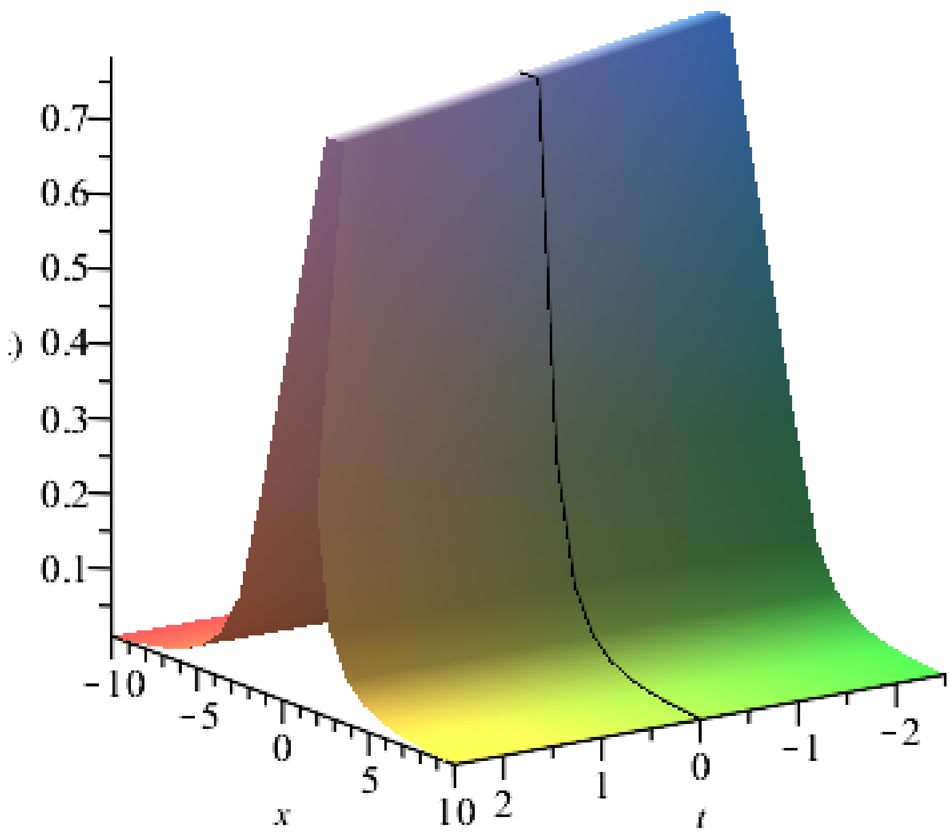
$$functionSeven := \frac{\partial}{\partial t}\,u(t,x)+2\,u(t,x)^2\left(\frac{\partial}{\partial x}\,u(t,x)\right)=0,\left[0,x,\frac{1}{x^2+1}\right]$$

$$characteristicEquation := \frac{d}{dt}\,x(t)=2\,u(t,x)^2$$

```
Error, (in pdsolve/info) the indeterminate function, x(t),
depends on only one variable. Use solve or dsolve
```

I could not figure out the characteristic curve.

```
> PDEplot(functionSeven, x=-10..10);
```




```
> PDEplot(characteristicEquation, x=-10..10);
Error, (in PDEtools:-PDEplot) invalid input: op expects 1 or 2 arguments, but received 3
```

6. Solve $u_{tt} = u_{xx}$ where $u(0, x) = x(x-1)$, $u_t(0, x) = \sin(\pi x)$. **2/2**

```
> functionEight := diff(u(t,x),t$2)=diff(u(t,x),x$2);
```

$$functionEight := \frac{\partial^2}{\partial t^2}\, u(t, x) = \frac{\partial^2}{\partial x^2}\, u(t, x) \tag{18}$$

```
> pdsolve([functionEight, u(0,x)=x*(x-1), D[1](u)(0,x)=sin(Pi*x)]);
```

$$u(t, x) = \frac{1}{2}\, \frac{\cos(\pi(t-x)) - \cos(\pi(x+t)) + (2t^2 + 2x^2 - 2x)\pi}{\pi} \tag{19}$$

7. (a) Try solving $u_{tt} = u_{xx}$ where $u(0, x) = 0$ and $u_t(0, x) = \frac{1}{1+x^2}$.

```
> pdsolve([functionEight, u(0,x)=0, D[1](u)(0,x)=1/(1+x^2)]);
```

(b) Try solving $u_{tt} = u_{xx}$ where $u(x, 0) = \epsilon$ and $u_t(0, x) = \frac{1}{1+x^2}$.

```
> pdsolve([functionEight, u(0,x)=epsilon, D[1](u)(0,x)=1/(1+x^2)]);
```

**(20)**

$$u(t,x) = \epsilon + \frac{1}{2}\arctan(t-x) + \frac{1}{2}\arctan(x+t) \tag{20}$$

(c) Try solving $u_t = u_{xx}$ where $u(0,x) = x(1-x)$, $u(t,0) = 0$, and $u_t(t,1) = 0$.

```
> functionNine := diff(u(t,x),t)=diff(u(t,x),x$2);
```

$$functionNine := \frac{\partial}{\partial t}u(t,x) = \frac{\partial^2}{\partial x^2}u(t,x) \tag{21}$$

```
> infolevel[pdsolve] := 3;
  pdsolve([functionNine, u(0,x)=x*(1-x), u(t,0)=0, D[1](u)(t,1)=0])
  ;
```

$$infolevel_{pdsolve} := 3$$

```
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
   -> trying "BC_equal_0"
* trying methods for class "_Cn_cn" for 2nd order PDEs
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
* trying methods for class "Wave" for 2nd order PDEs
   -> trying "Cauchy"
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Heat" for 2nd order PDEs
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Laplace" for 2nd order PDEs
   -> trying a Laplace transformation
* trying methods for class "Fourier" for 2nd order PDEs
   -> trying a fourier transformation
* trying methods for class "Series" for 2nd order PDEs
   -> trying "sincos"
   -> trying "2"
   -> trying "3"
   -> trying "4"
   -> trying "WithSourceTerm"
* trying methods for class "Generic" for 2nd order PDEs
   -> trying a solution in terms of arbitrary constants and
functions to be adjusted to the given initial conditions
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
```

```
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
```

(d) How do you think the pdsolve "solves" PDEs?
As you can read above maple runs an algorithm that checks several different methods of solving the PDE. **5/5**

# MATH 3431 Maple Assignment 1

```
> with(inttrans): with(PDEtools):
```

## Question 1

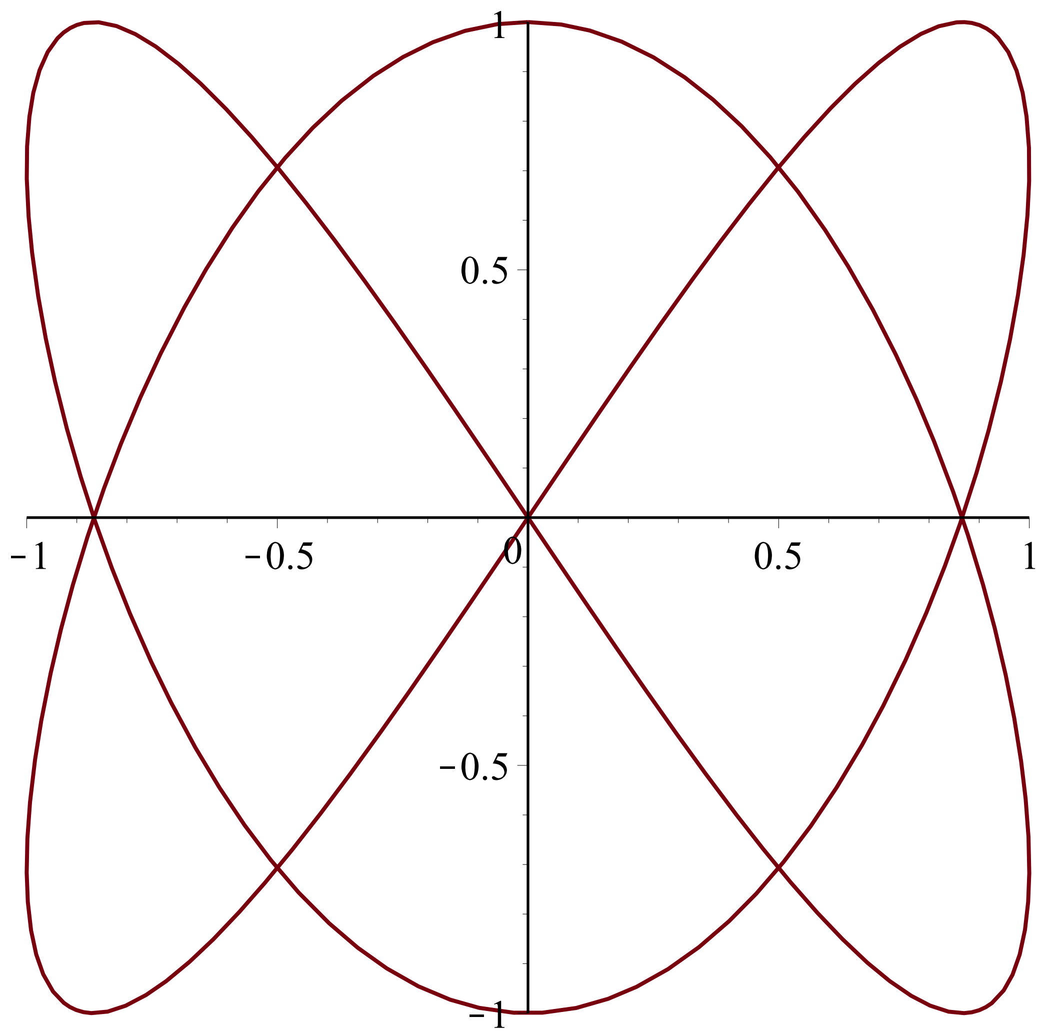
1. The command

```
plot([[sin(2*t), cos(3*t)][1], [sin(2*t), cos(3*t)][2], t=0..2*Pi]
)
```

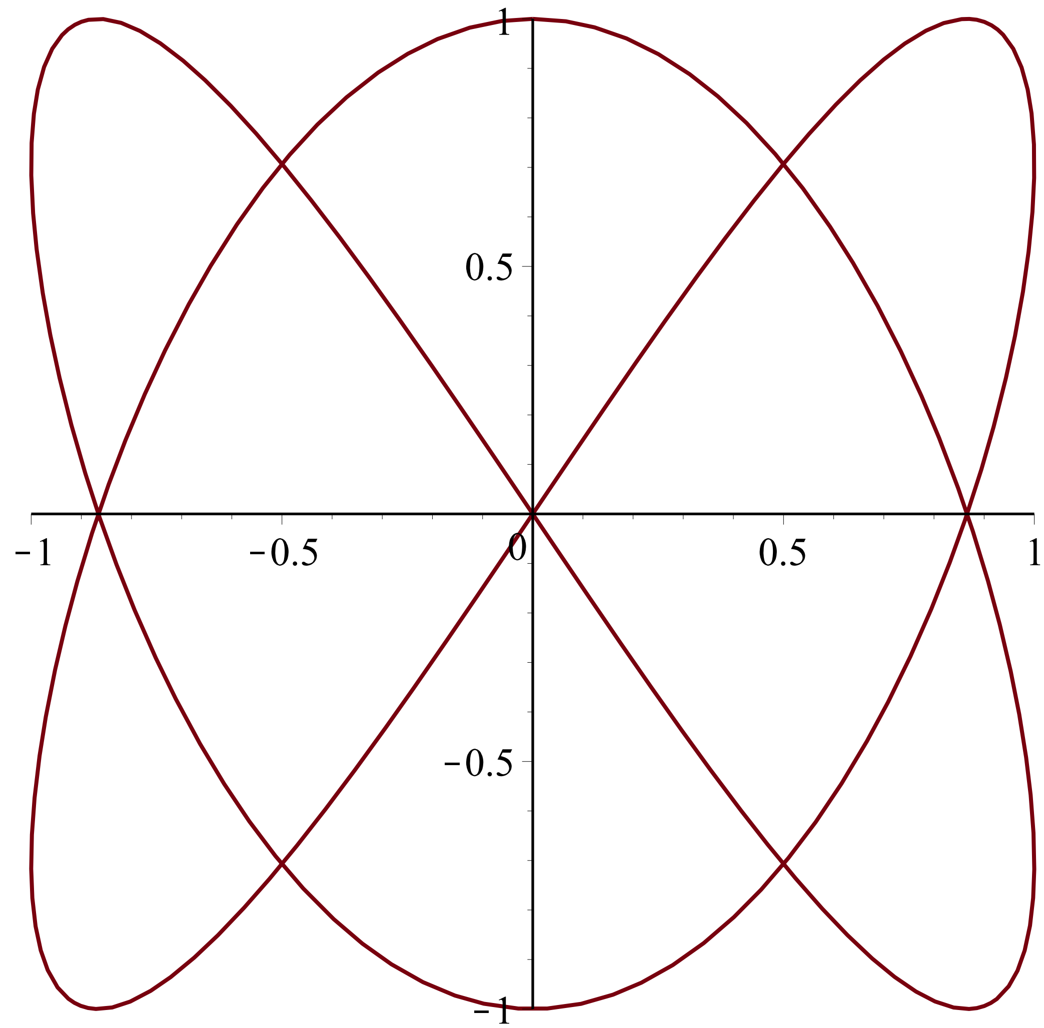
plots the parametric curve $x = \sin(2\,t), y = \cos(3\,t)$. It does so by forming two identical lists, extracting the first element from the first list, and the second from the second list. A simpler command would be

`plot([sin(2*t), cos(3*t), t=0..2*Pi])`.

```
> plot([[sin(2*t), cos(3*t)][1], [sin(2*t), cos(3*t)][2], t=0..2*
  Pi]);
```




```
> plot([sin(2*t), cos(3*t), t=0..2*Pi]);
```




# Question 2

```
> laplace(exp(-1/(4*t))/sqrt(t), t, s);
```

$$\sqrt{\frac{\pi}{s}}\, e^{-\sqrt{s}} \tag{2.1}$$

```
> laplace(exp(-1/(4*x*t))/sqrt(x*t), t, s);
```

$$\frac{\mathit{laplace}\left(\frac{e^{-\frac{1}{4 x t}}}{\sqrt{t}}, t, s\right)}{\sqrt{x}} \tag{2.2}$$

```
> laplace(exp(-1/(4*x*t))/sqrt(x*t), t, s) assuming x>0;
```

$$\sqrt{\frac{\pi}{s x}}\, e^{-\sqrt{\frac{s}{x}}} \tag{2.3}$$

```
> invlaplace(exp(-1/(4*x*s))/sqrt(x*s), s, t);
```

(2.4)

$$\mathit{invlaplace}\left(\frac{e^{-\frac{1}{4xs}}}{\sqrt{xs}}, s, t\right) \tag{2.4}$$

```
> invlaplace(exp(-1/(4*x*s))/sqrt(x*s), s, t) assuming x>0;
```

$$\frac{\cos\left(\sqrt{\frac{t}{x}}\right)}{\sqrt{\pi t x}} \tag{2.5}$$

One would expect the Laplace transform of $\frac{e^{-\frac{1}{4xt}}}{\sqrt{xt}}$ to be evaluated easily by substitution. The substitution, however, assumes that $x$ is positive so that $\sqrt{xt}$ is the same as $\sqrt{x}\sqrt{t}$.

# Question 3

```
> int(1/sqrt(1-x^2), x); q3result2 := int(1/sqrt(1-x^2), x=2..3);
  q3result3 := arcsin(3) - arcsin(2);
```

$$\arcsin(x)$$

$$\mathit{q3result2} := \mathrm{I}\ln\left(2+\sqrt{3}\right) - \mathrm{I}\ln\left(3+2\sqrt{2}\right)$$

$$\mathit{q3result3} := \arcsin(3) - \arcsin(2) \tag{3.1}$$

The first command returns the expected result. Over the reals, the second command should return an error of some sort (the integrand does not exist as a real number when $x$ is between 2 and 3), but returns something complex. The last returns what looks like nonsense. We would expect at least the second result! Getting the decimal approximation to the second and third results shows that Maple evaluates them similarly (at least to 9 decimals).

```
> evalf(q3result3); evalf(q3result2);
```

$$0. - 0.445789277\,\mathrm{I}$$

$$-0.445789277\,\mathrm{I} \tag{3.2}$$

# Question 4

In PDEs you often find yourself doing long, mechanical calculations where making errors is very easy. Wouldn't it be nice if computers could do such calculations for you?

## (a)

```
> q4Solution := x -> A*exp(-sqrt(lambda)*x)+B*exp(sqrt(lambda)*
  x);
```

$$\mathit{q4Solution} := x \rightarrow A\,e^{-\sqrt{\lambda}\,x} + B\,e^{\sqrt{\lambda}\,x} \tag{4.1.1}$$

```
> q4PartASolution := solve([q4Solution(0)=0, q4Solution(1)=0],
  [A, B, lambda], AllSolutions);
```

$$\mathit{q4PartASolution} := \left[\left[A=0, B=0, \lambda=\lambda\right], \left[A=-B, B=B, \lambda=-\pi^2\,\_Z1\sim^2\right]\right] \tag{4.1.2}$$

```
> about(_Z1);
Originally _Z1, renamed _Z1~:
```

```
is assumed to be: integer
```

```
> simplify(eval(q4Solution(x), q4PartASolution[2]));
```

$$2\,\mathrm{I}\,B\sin\left(\pi x\,|\_Z1\sim|\right) \tag{4.1.3}$$

The first solution is the trivial solution, and the second corresponds to sines with period dividing 2 (or half-periods dividing 1). Recall that $\sin(\theta) = -\frac{1}{2}\,\mathrm{I}\left(e^{\mathrm{I}\theta} + e^{-\mathrm{I}\theta}\right)$, which explains the $2\,I$ in the simplified form.

## (b)

```
> q4PartBSolution := solve([q4Solution(0)=0, D(q4Solution)(1)=
  0], [A, B, lambda], AllSolutions);
```

$$q4PartBSolution := \left[[A=-B, B=B, \lambda=0], [A=0, B=0, \lambda=\lambda], \left[A=-B, B=B, \lambda=-\frac{1}{4}\pi^2\left(2\,\_Z2\sim+1\right)^2\right]\right] \tag{4.2.1}$$

```
> simplify(eval(q4Solution(x), q4PartBSolution[3]));
```

$$2\,\mathrm{I}\,B\sin\left(\frac{1}{2}\pi\,\mathrm{signum}\left(\_Z2\sim+\frac{1}{2}\right)\left(2\,\_Z2\sim+1\right)x\right) \tag{4.2.2}$$

The first solution assumes that $\lambda = 0$, which we are explicitly *not* assuming. The $\lambda = 0$ case is different from the others, and has to be treated separately. The second solution is the trivial solution, The third solution has odd multiples of $\frac{1}{2}\pi$ as the $x$ coefficient. Because sine is odd, and "B" is arbitrary, negative $x$ coefficients really produce the same wave as the positive coefficients. The $\mathrm{signum}\left(Z+\frac{1}{2}\right)\cdot(2\,Z+1)$ is really $|2\,Z+1|$, so Maple is taking only the positive $x$ coefficients.

## (c)

```
> q4PartCSolution := solve([q4Solution(0)-D(q4Solution)(0)=0,
  q4Solution(1)-D(q4Solution)(1)=0], [A, B, lambda]);
```

$$q4PartCSolution := \left[[A=0, B=0, \lambda=\lambda], [A=0, B=B, \lambda=1], [A=-B, B=B, \lambda=0], \left[A=\frac{B\left(\mathrm{I}\pi-1\right)}{\mathrm{I}\pi+1}, B=B, \lambda=-\pi^2\right]\right] \tag{4.3.1}$$

```
> eval(q4Solution(x), q4PartCSolution[2]);
```

$$B\,e^x \tag{4.3.2}$$

```
> eval(q4Solution(x), q4PartCSolution[3]);
```

$$0 \tag{4.3.3}$$

```
> simplify(q4Solution(x), q4PartCSolution[4]);
```

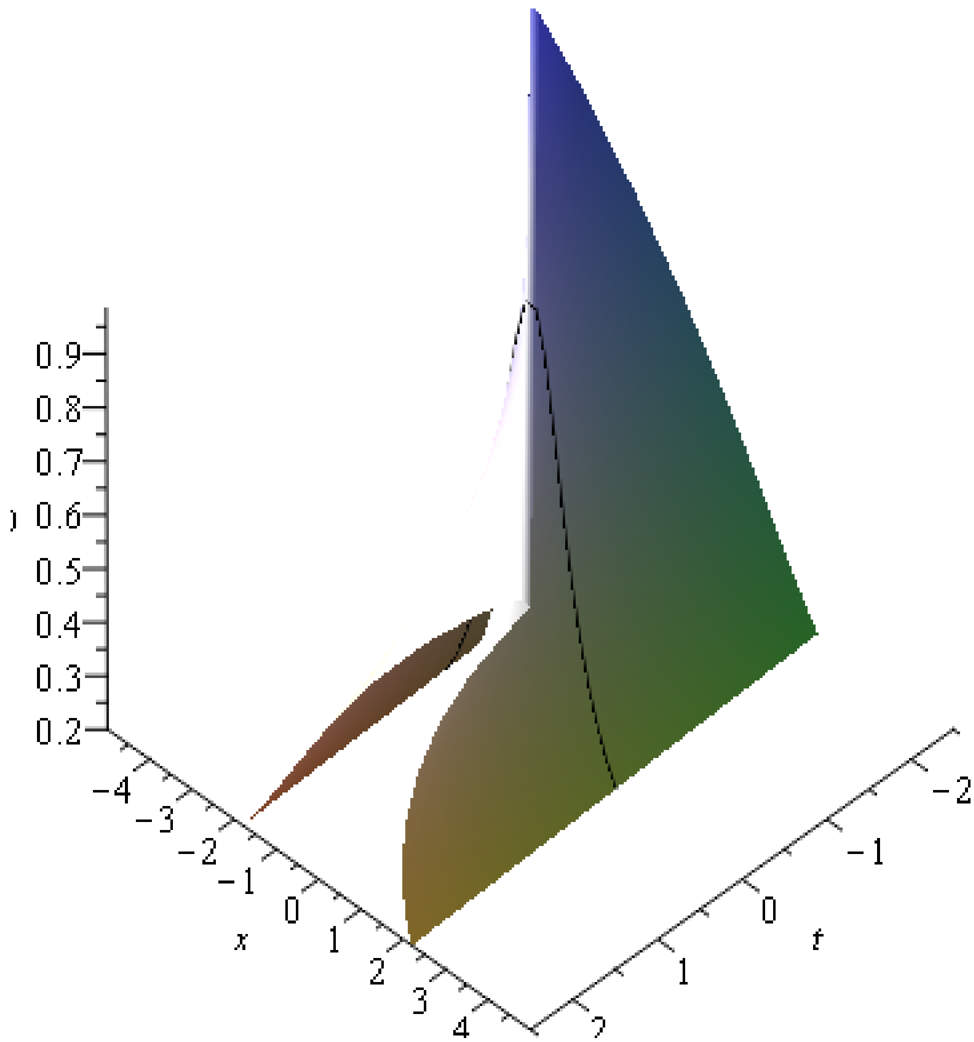
(4.3.4)

$$\frac{B\left(e^{\sqrt{-\pi^2}\,x}\right)^2 + A}{e^{\sqrt{-\pi^2}\,x}} \qquad \textbf{(4.3.4)}$$

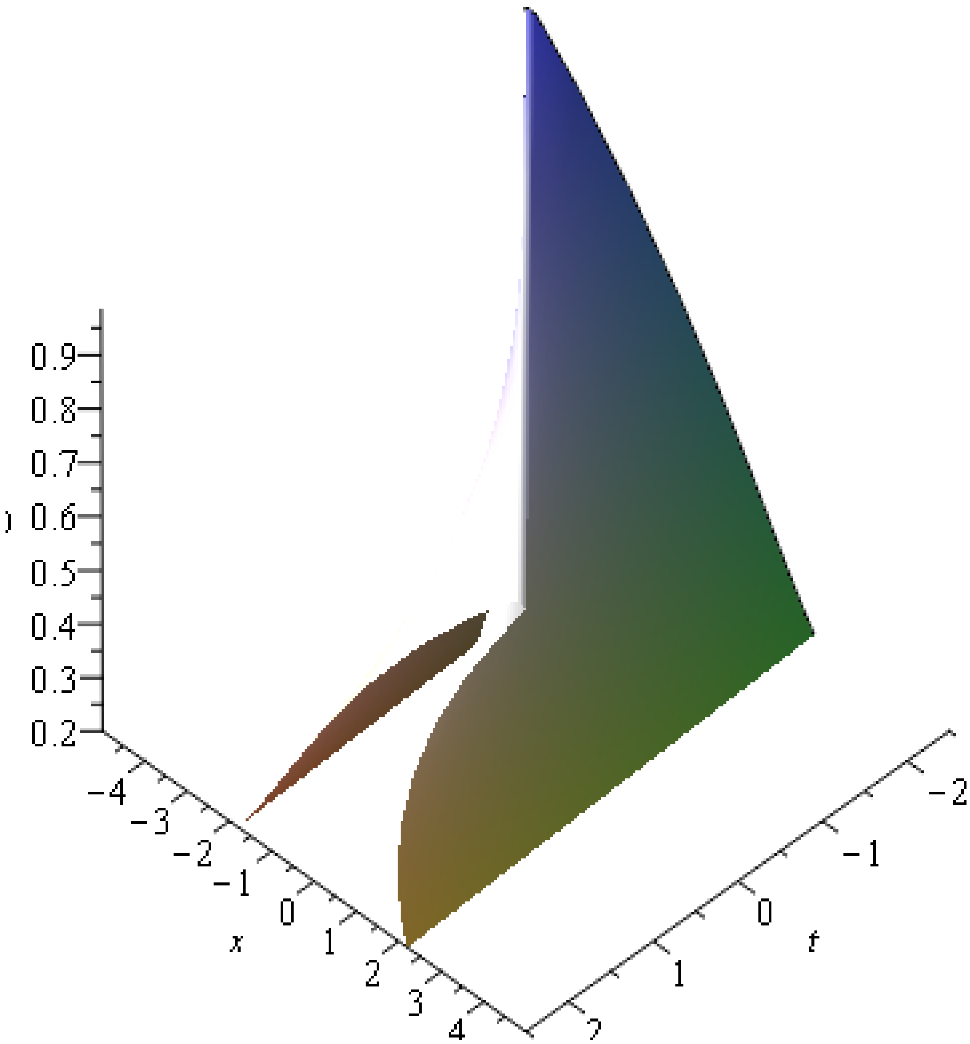
The first and third solutions are the trivial solution, the second is a simple exponential, and the fourth is a sum of $\sin(\pi x)$ and $\cos(\pi x)$.

# Question 5

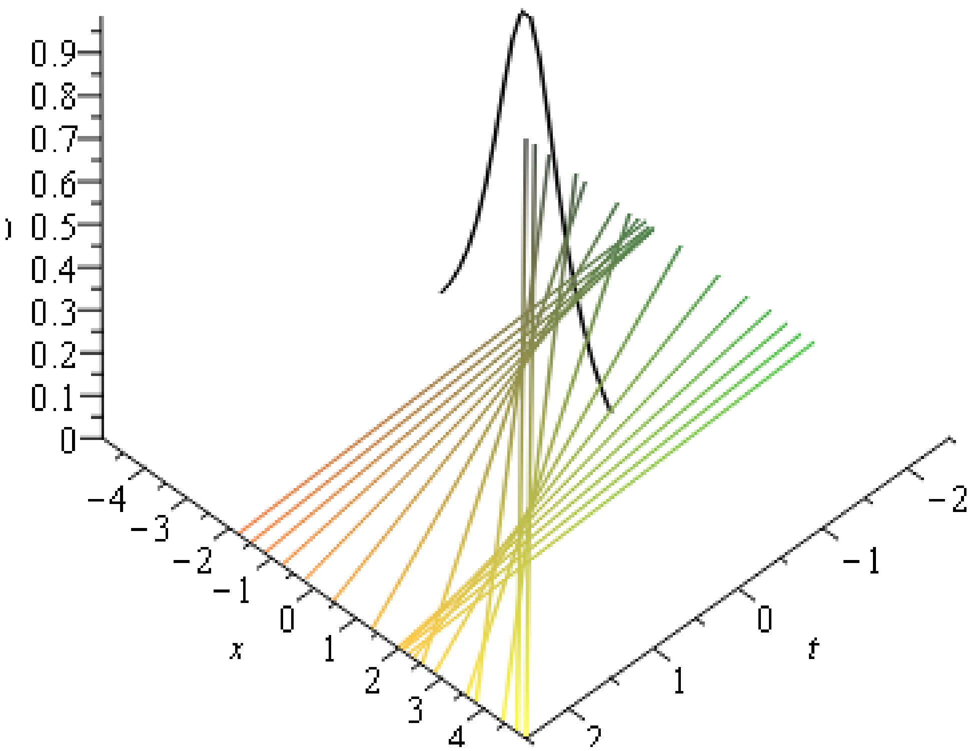
```
> PDEplot(diff(u(t,x), t)+2*u(t,x)^2*diff(u(t,x), x)=0, [0, x, 1/
(1+x^2)], -2..2);
```



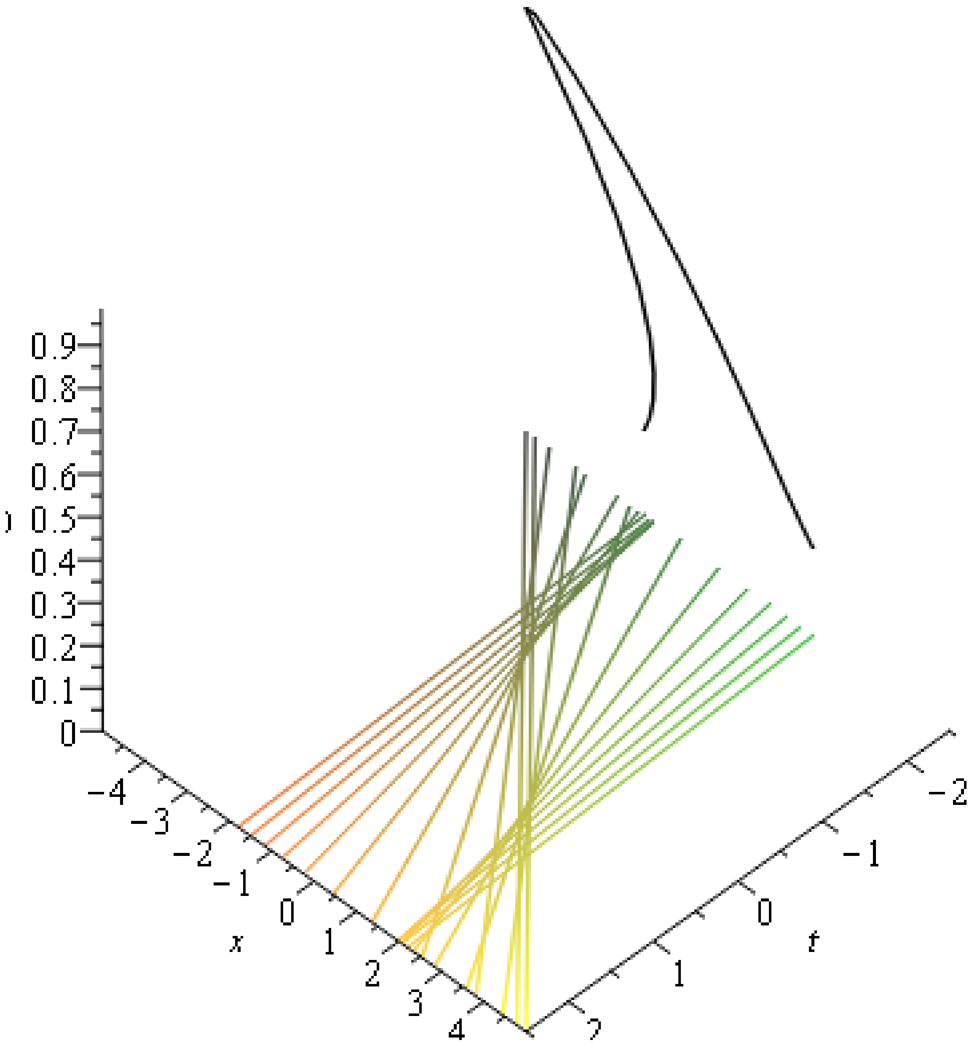

```
> PDEplot(diff(u(t,x), t)+2*u(t,x)^2*diff(u(t,x), x)=0, [0, x, 1/
(1+x^2)], -2..2, animate=true);
```




```
> PDEplot(diff(u(t,x), t)+2*u(t,x)^2*diff(u(t,x), x)=0, [0, x, 1/
  (1+x^2)], -2..2, basechar=only);
```




```
> PDEplot(diff(u(t,x), t)+2*u(t,x)^2*diff(u(t,x), x)=0, [0, x, 1/
  (1+x^2)], -2..2, animate=only, basechar=true);
```




The characteristic curves cross, so we have a "shock" wave. The solution is not a function because it becomes multi-valued where the characteristic curves cross.

# Question 6

```
> pdsolve([diff(u(t,x), t$2)=diff(u(t,x), x$2), u(0,x)=x*(x-1), D
  [1](u)(0,x)=sin(Pi*x)]);
```

$$u(t, x) = \frac{1}{2} \frac{\cos(\pi (t - x)) - \cos(\pi (x + t)) + (2 t^2 + 2 x^2 - 2 x) \pi}{\pi} \quad \textbf{(6.1)}$$

# Question 7

```
> pdsolve([diff(u(t,x), t$2)=diff(u(t,x), x$2), u(0,x)=0, D[1](u)
  (0,x)=1/(1+x^2)]);
```

Where's the solution?

```
> pdsolve([diff(u(t,x), t$2)=diff(u(t,x), x$2), u(0,x)=epsilon, D
  [1](u)(0,x)=1/(1+x^2)]);
```

**(7.1)**

$$u(t,x) = \epsilon + \frac{1}{2}\arctan(t-x) + \frac{1}{2}\arctan(x+t) \tag{7.1}$$

Apparently pdsolve can solve the PDE when the initial condition is an unspecified constant, but not when it is 0. Taking the limit as $\epsilon \to 0$ in the second solution does give the expected solution to the first PDE.

```
> pdsolve([diff(u(t,x), t)=diff(u(t,x), x$2), u(0,x)=x*(1-x), u
  (t,0)=0,  D[1](u)(t,1)=0]);
> infolevel[pdsolve] := 3;
  pdsolve([diff(u(t,x), t)=diff(u(t,x), x$2), u(0,x)=x*(1-x), u
  (t,0)=0,  D[1](u)(t,1)=0]);
```

$$infolevel_{pdsolve} := 3$$

```
* trying methods for class "_Fn" for 2nd order PDEs
   -> trying "linear_in_xt"
   -> trying "BC_equal_0"
* trying methods for class "_Cn_cn" for 2nd order PDEs
First set of solution methods (general or quasi general
solution)
Trying differential factorization for linear PDEs ...
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
* trying methods for class "Wave" for 2nd order PDEs
   -> trying "Cauchy"
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Heat" for 2nd order PDEs
   -> trying "SemiInfiniteDomain"
   -> trying "WithSourceTerm"
* trying methods for class "Laplace" for 2nd order PDEs
   -> trying a Laplace transformation
* trying methods for class "Fourier" for 2nd order PDEs
   -> trying a fourier transformation
* trying methods for class "Series" for 2nd order PDEs
   -> trying "sincos"
   -> trying "2"
   -> trying "3"
   -> trying "4"
   -> trying "WithSourceTerm"
* trying methods for class "Generic" for 2nd order PDEs
   -> trying a solution in terms of arbitrary constants and
functions to be adjusted to the given initial conditions
First set of solution methods (general or quasi general
solution)
```

```
Trying differential factorization for linear PDEs ...
Trying methods for PDEs "missing the dependent variable" ...
Second set of solution methods (complete solutions)
Trying methods for second order PDEs
Third set of solution methods (simple HINTs for separating
variables)
PDE linear in highest derivatives - trying a separation of
variables by *
HINT = *
Fourth set of solution methods
Trying methods for second order linear PDEs
Preparing a solution HINT ...
Trying HINT = _F1(t)*_F2(x)
Third set of solution methods successful
```

The pdsolve command seems to have a list of tricks to try on a given PDE. Curiously, even when it solves a PDE, it doesn't always print out the solution.