Homework 1: Optics and Image Sensing

COMS 4731 Computer Vision Fall 2019

Columbia University

Due 18 Sep 2019, 11:55 PM

This homework contains three written problems and one programmming task. Submit a PDF file titled **<UNI>.pdf** containing your solutions to Problems 1, 2 and 3, as well as **<UNI>.ipynb**, which should be a completed version of the IPython notebook that is given in the assignment. Please submit these two files only, rather than a compressed folder (such as .zip).

Please refer to Chapter 2 of the book "Computer Vision: Algorithms and Applications" by Richard Szeliski should you have questions about image formation during this homework.

1 Problem 1

Consider a pinhole camera with perspective projection.

a) Given a circular disk (= the scene) that lies anywhere on a plane parallel to the image plane, what is the shape of the projected image of the disk? (5 pt)

b) Suppose the area of the image of the cirular disk is 1 mm^2 when the distance from the pinhole to the disk itself is 1.35 m. What is the area of the image of the disk if the distance is doubled? (5 pt)

c) Now, replace the disk with a sphere. What is the shape of the image of the sphere? Briefly justify your answer. (10 pt)

2 Problem 2

Consider the imaging system shown in Figure 1a. For this problem, assume that the world is two-dimensional, so that we have image and scene lines instead of planes. The focal length of the lens is f.

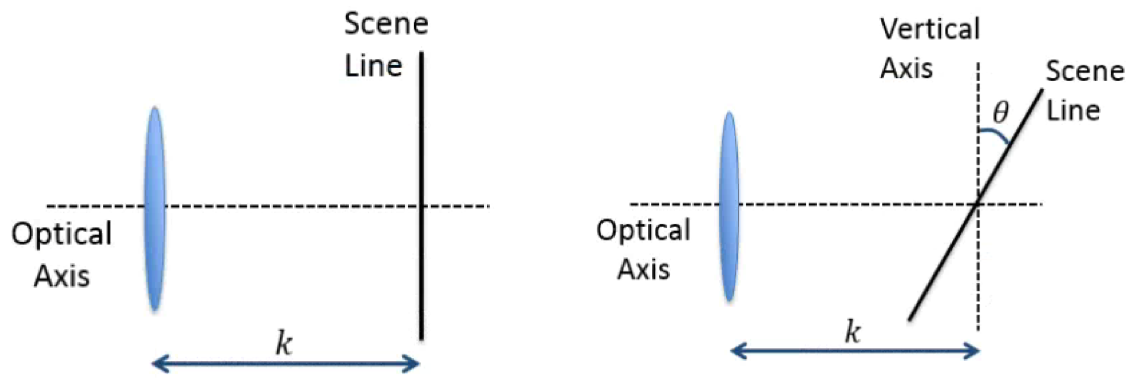
a) Now, consider a scene line that is perpendicular to the optical axis of the lens and at a distance k from the lens. At which distance from the lens would a focused image of the scene line be formed? (5 pt)

b) Suppose the scene line is not perpendicular to the optical axis, but makes an angle of θ with the vertical axis (Figure 1b). Prove that the image of the scene line is still a line, but one that is tilted.

(10 pt)

c) If the image of the tilted scene line is a line making an angle ϕ with the vertical axis, then prove that $\tan(\phi) = \frac{f}{k-f}\tan(\theta)$, where k is the distance between the scene line and the lens along the optical axis. (10 pt)

Note: Although you have derived the above equation for a 2D world with lines as scenes, it holds in a 3D world with planar scenes as well. This equation is known as the Scheimpflug condition.



(a) Problem 2a.

(b) Problem 2b.

Figure 1

3 Problem 3

A Lambertian surface is illuminated simultaneously by two equally distant (but far away) point sources with equal intensities in the directions s_1 and s_2. Note that directions in this context are always unit vectors.

a) Show that for all normals on the surface that are visible to both sources, illumination can be viewed as coming from a single "effective" direction s_3. What is s_3 as a function of s_1 and s_2, and what is the "effective" intensity I_3? (8 pt)

b) Now, if the two equally distant sources have unequal intensities I_1 and I_2 respectively, then what is the direction and intensity of the "effective" source? (8 pt)

4 Problem 4

Please refer to the IPython notebook for the second part of this assignment. (39 pt)

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