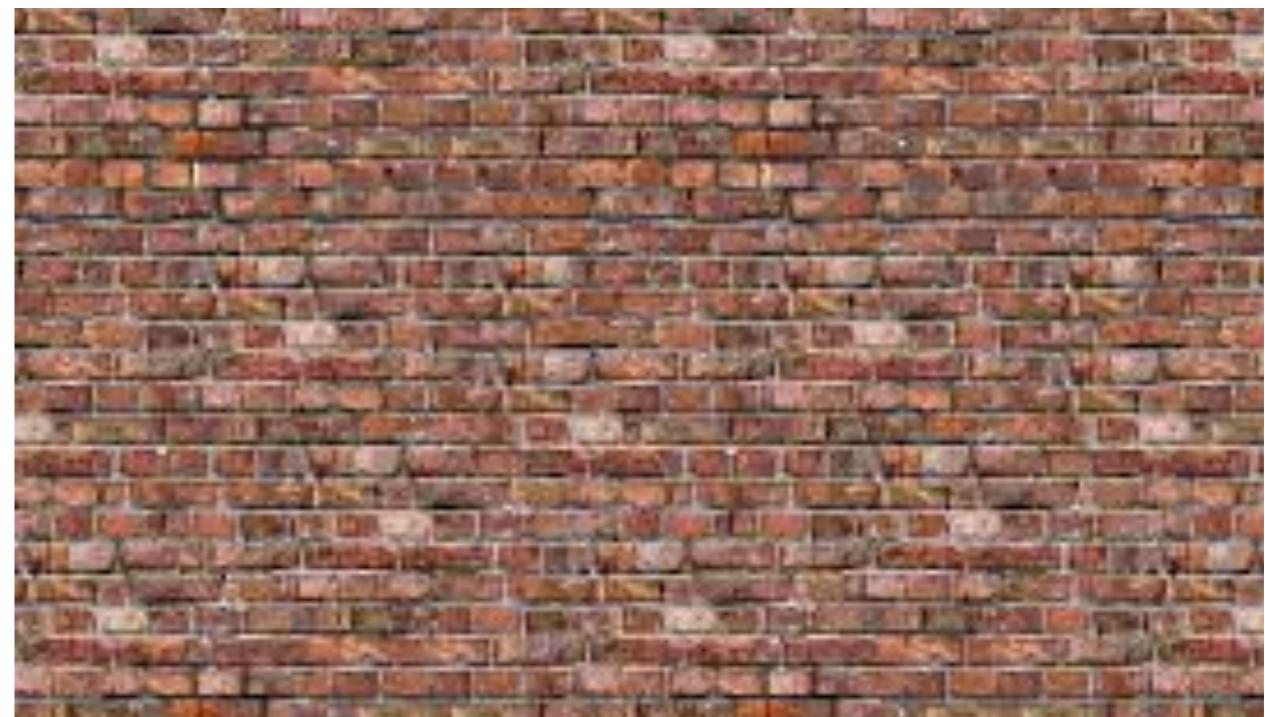


Image Processing II

Computer Vision
Fall 2018
Columbia University

Vision is Repetitive



Joseph Fourier

A bold idea (1807):

Any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.

Don't believe it?

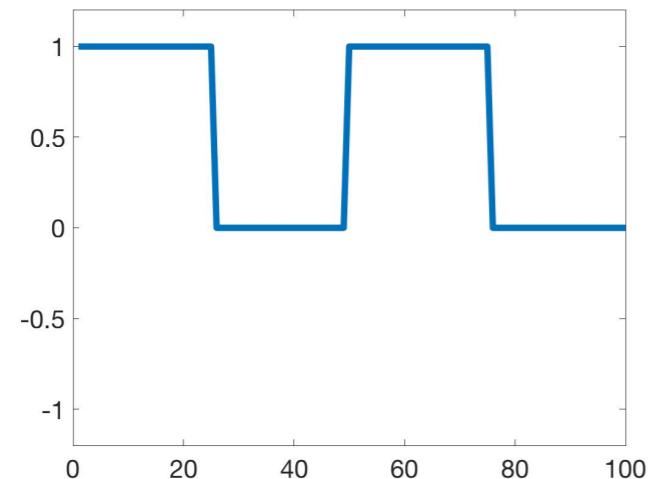
Neither did Lagrange, Laplace, Poisson and other bigwigs

Not translated into English until 1878!

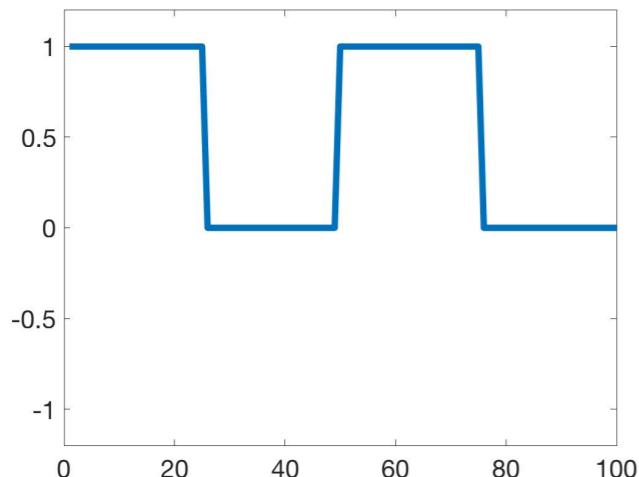


Wikipedia

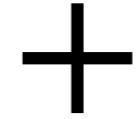
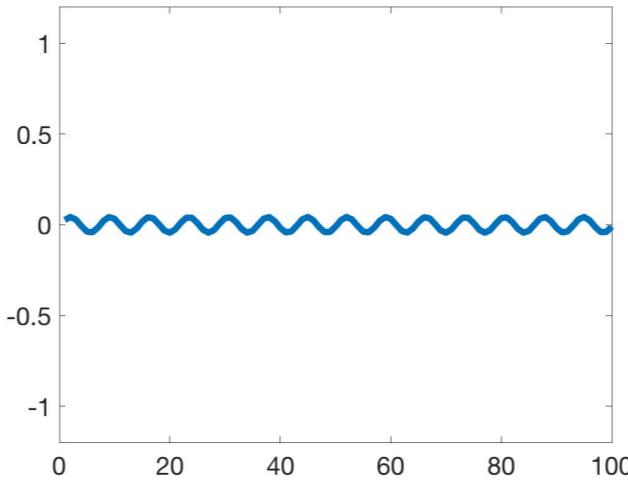
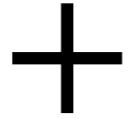
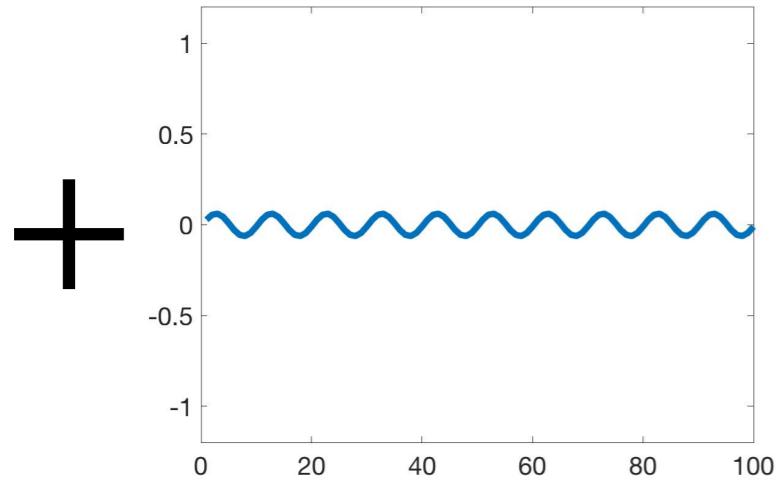
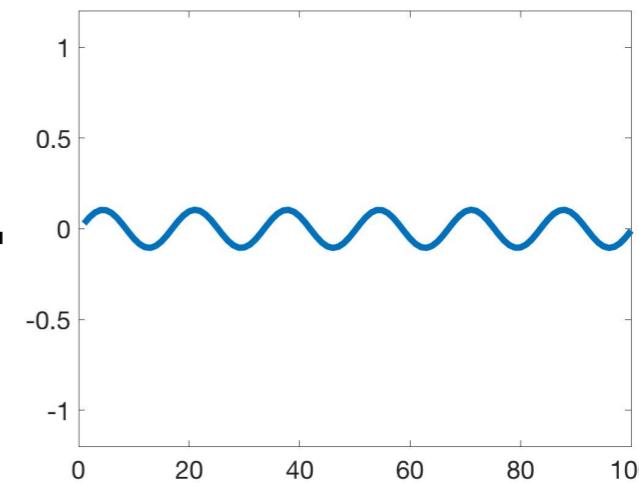
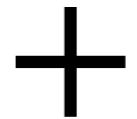
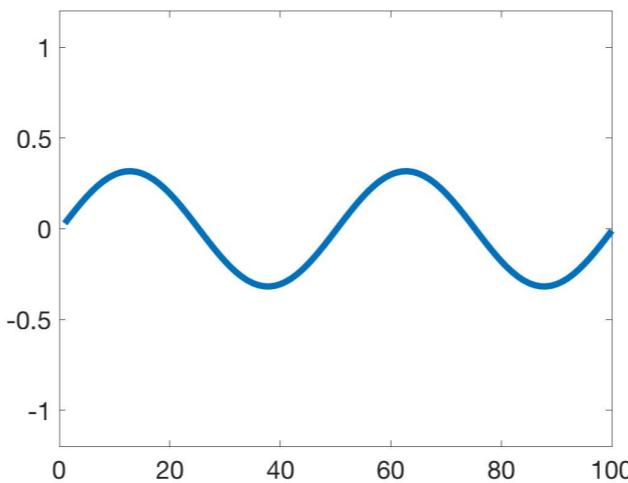
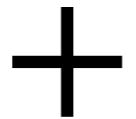
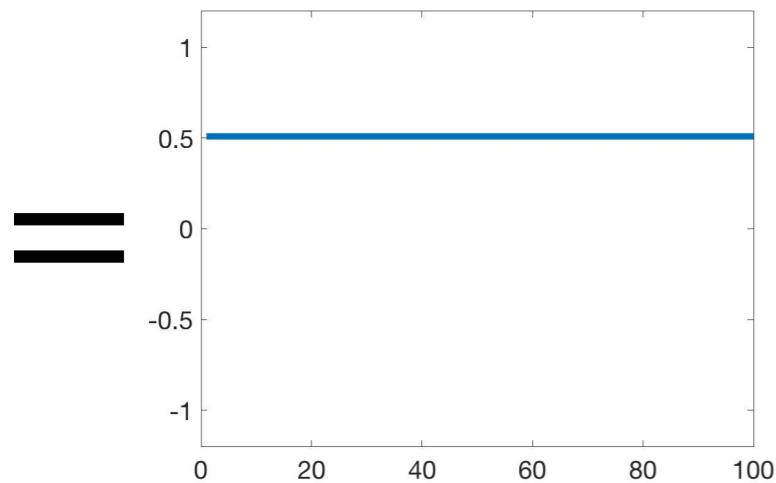
Source: James Hays



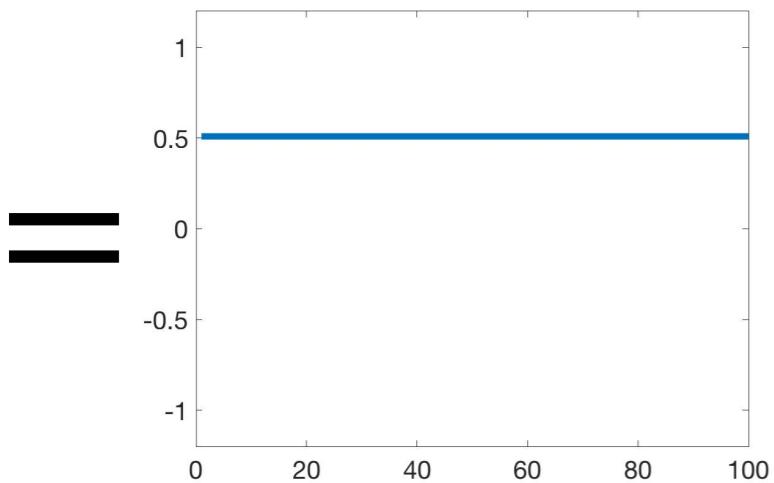
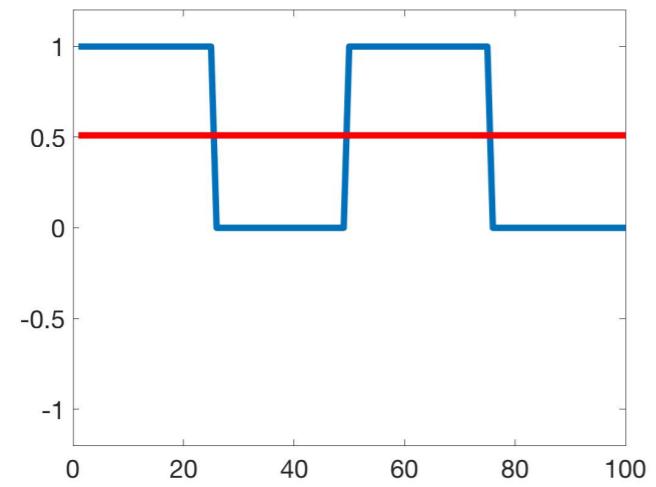
How to build this 1D signal
using sin waves?

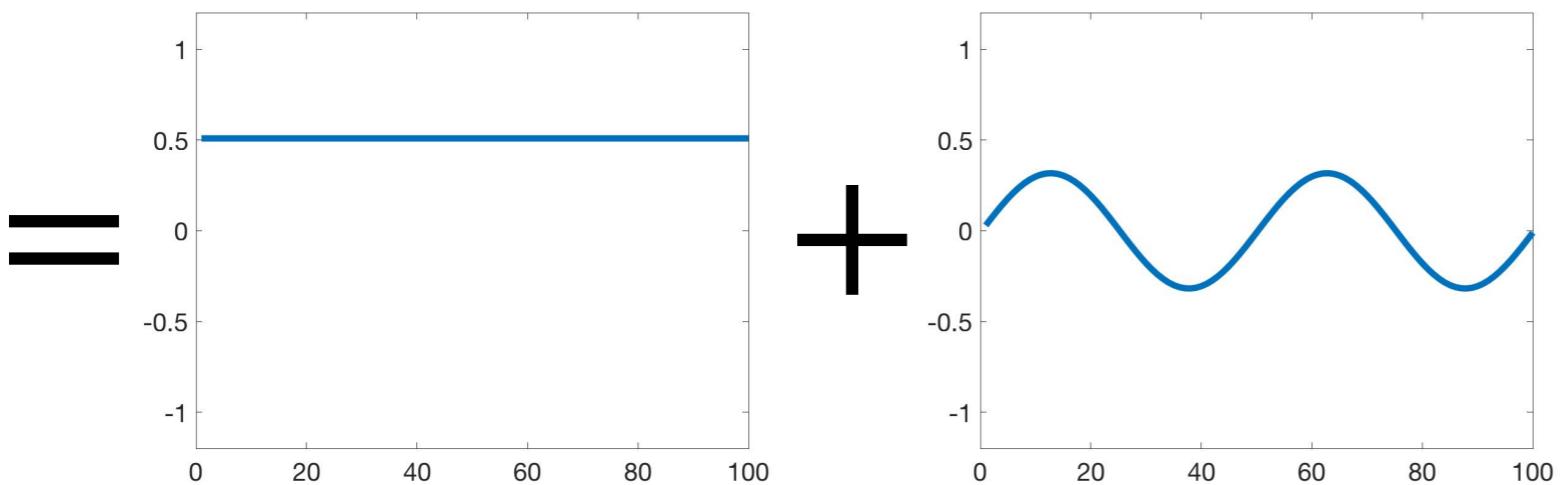
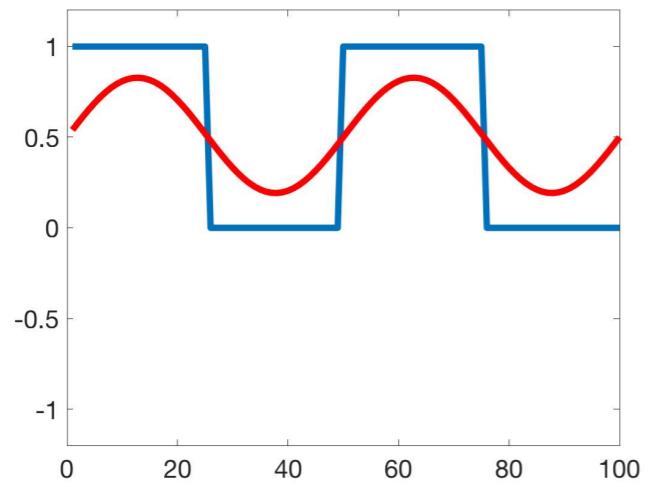


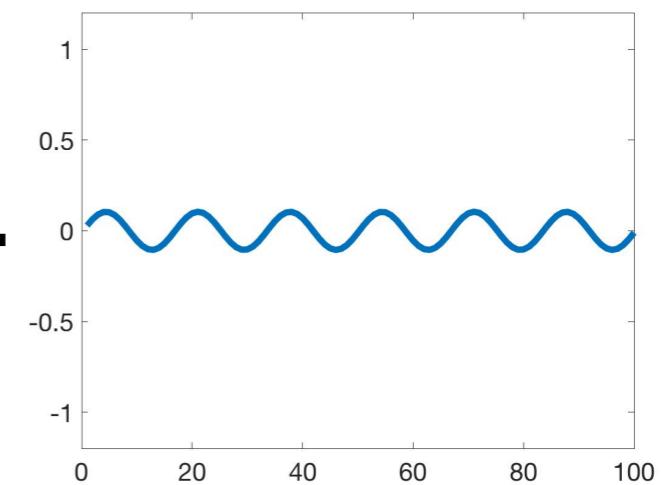
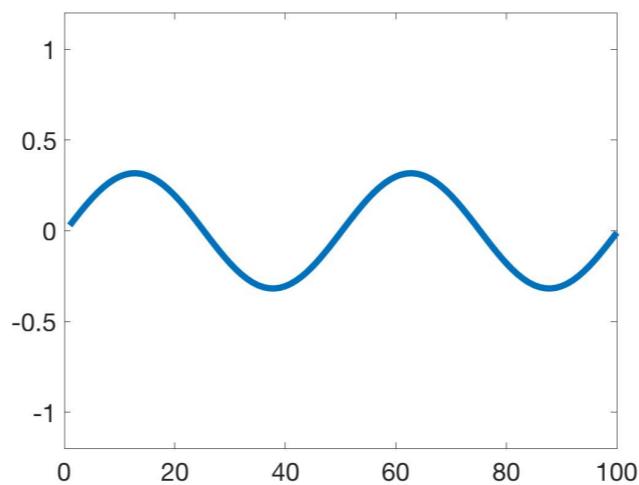
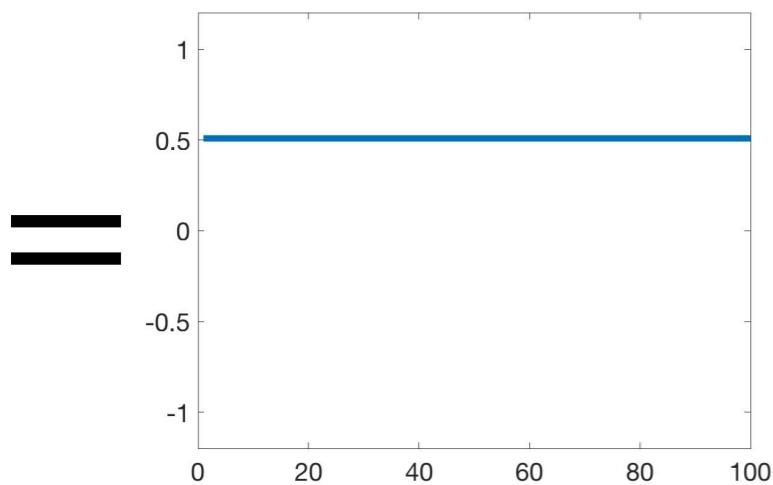
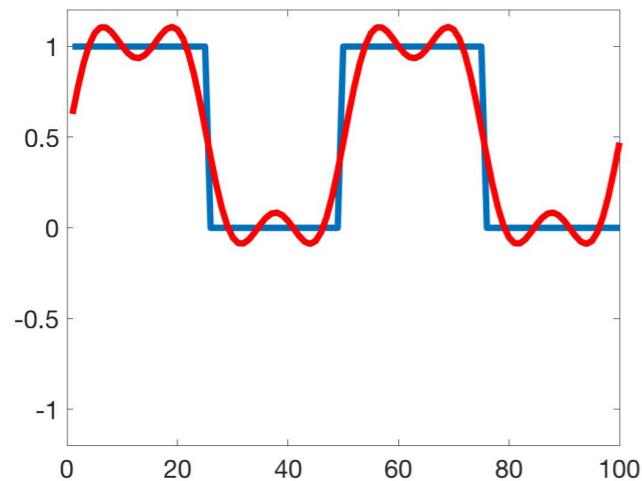
How to build this 1D signal
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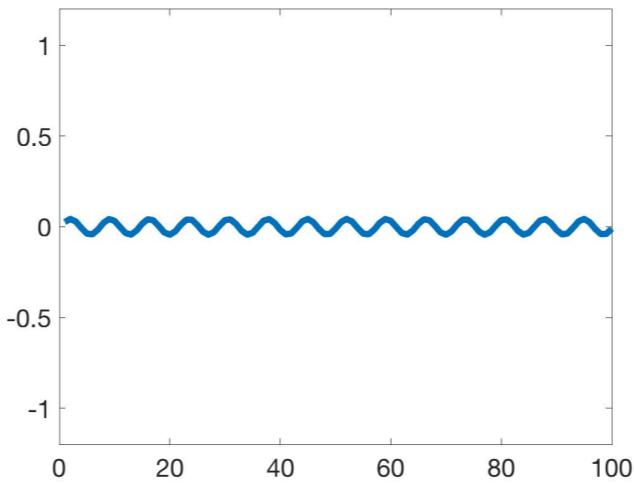
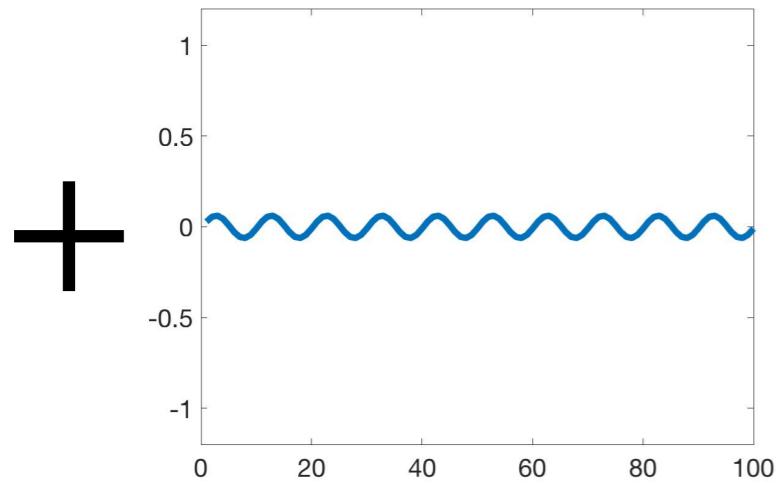
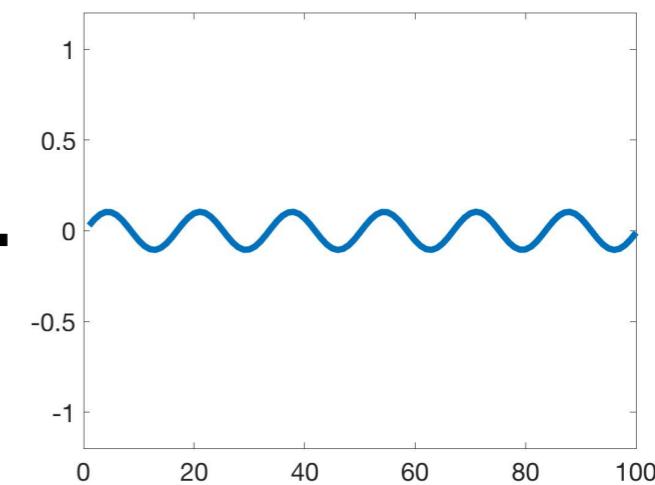
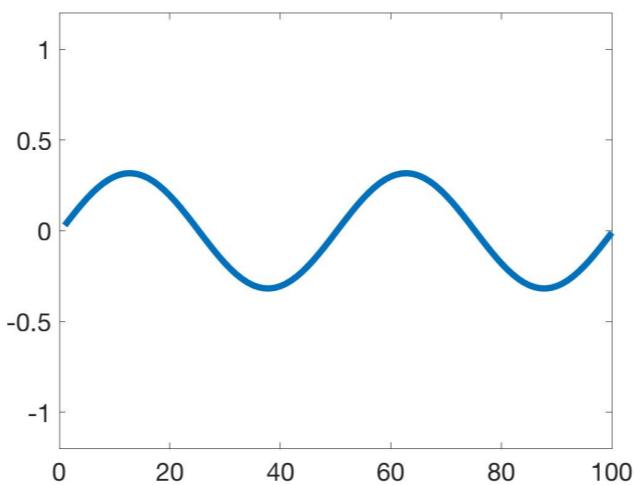
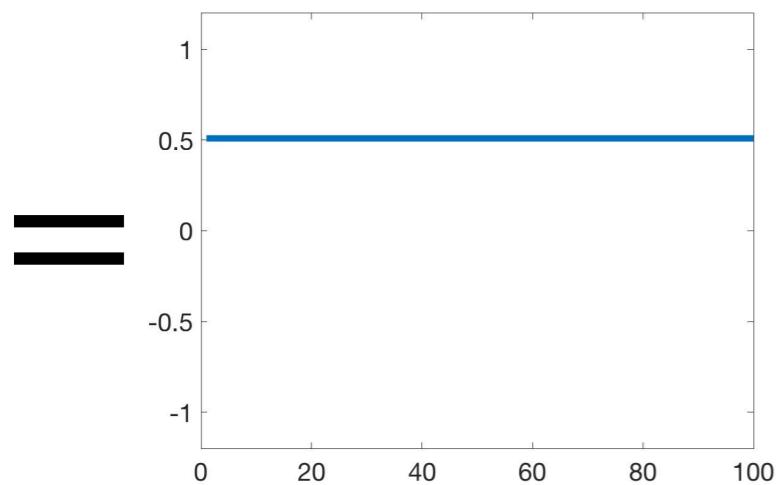
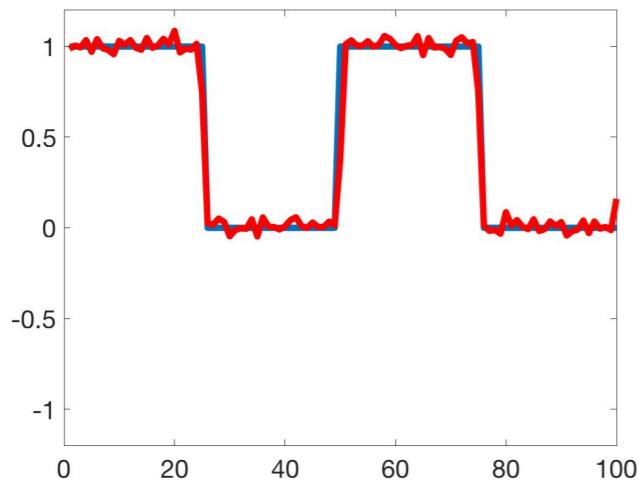


...









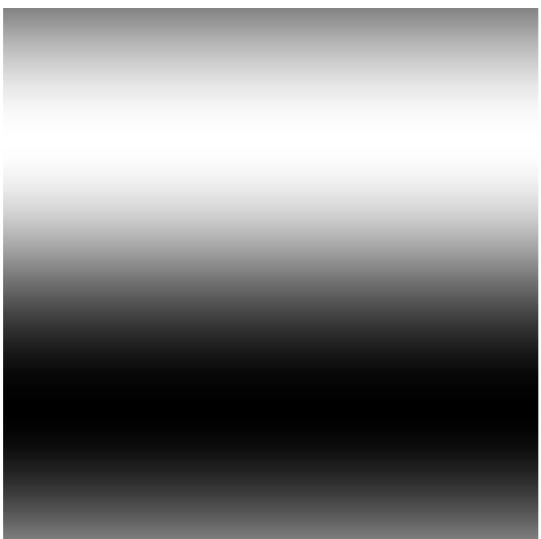
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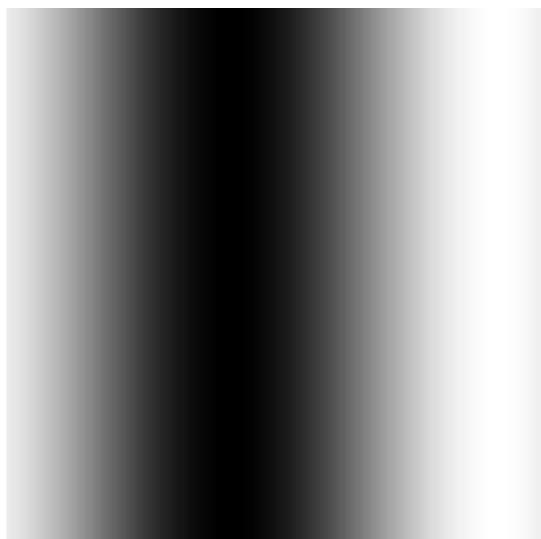


Where we are going...

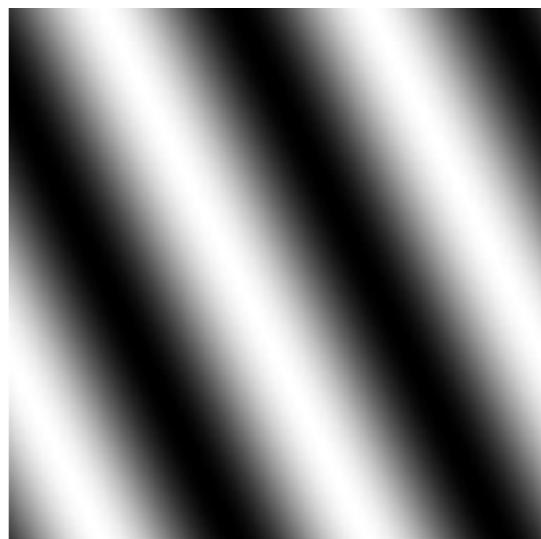
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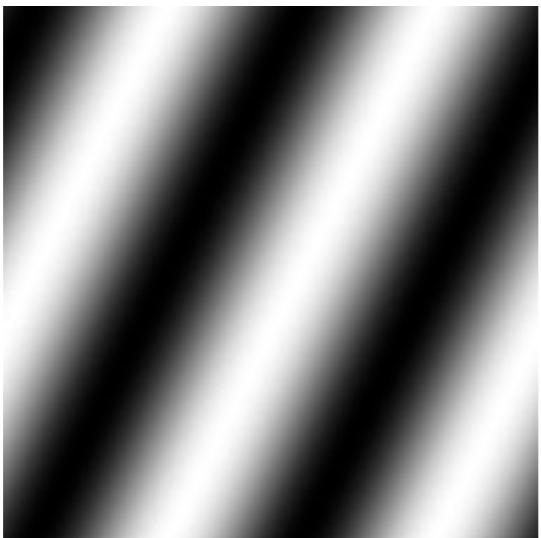
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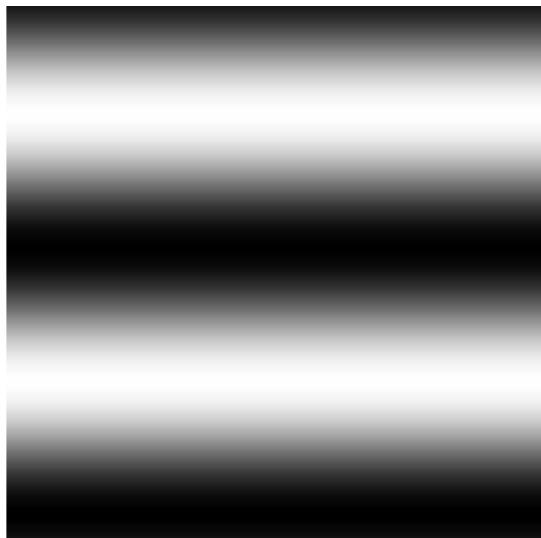
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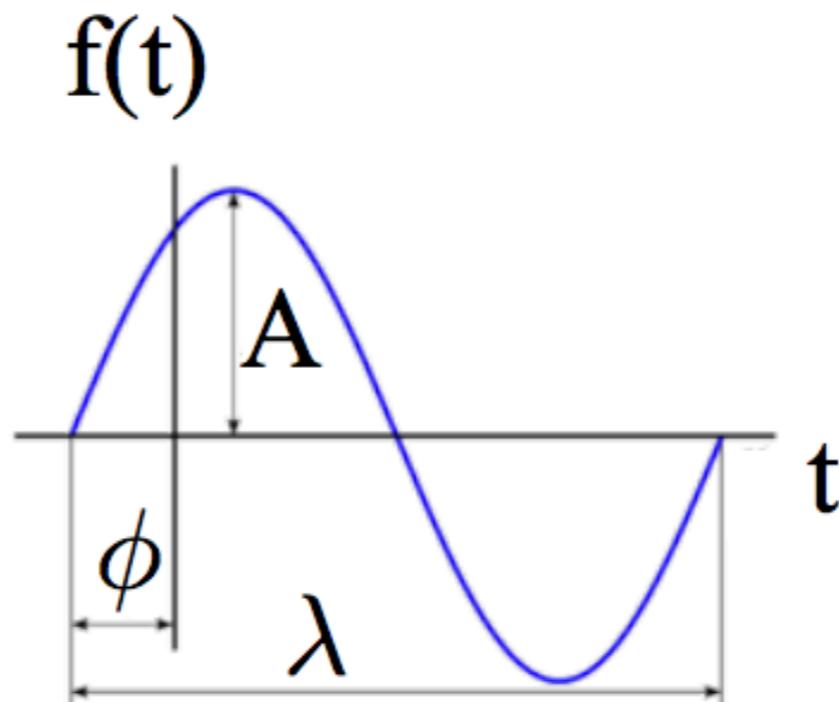
 ...

Background: Sinusoids

A = amplitude

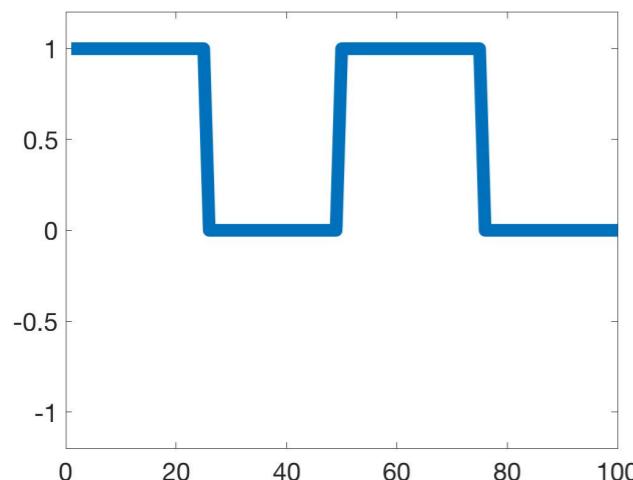
ϕ = phase

f = frequency

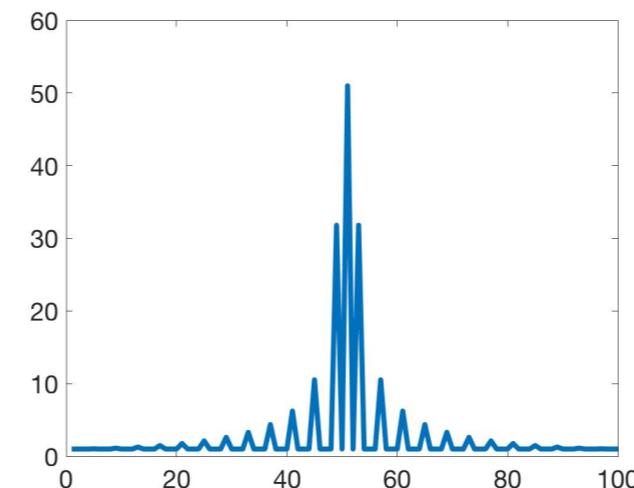
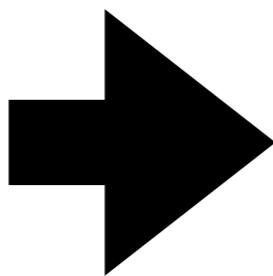


$$f(t) = A \sin(2\pi ft + \phi) = A \sin(\omega x + \phi)$$

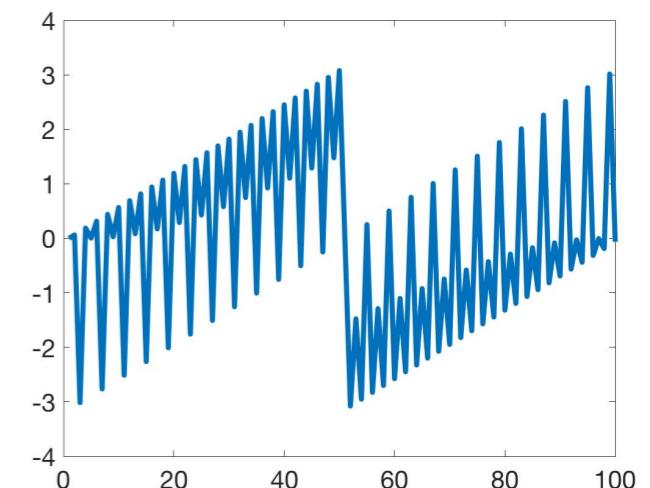
Fourier Transform



Signal



Amplitude

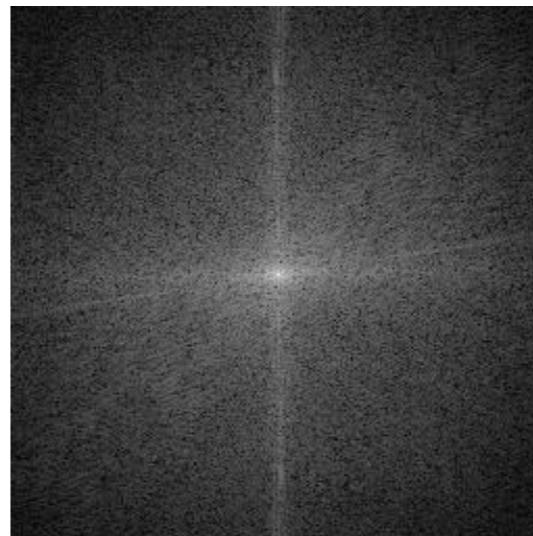
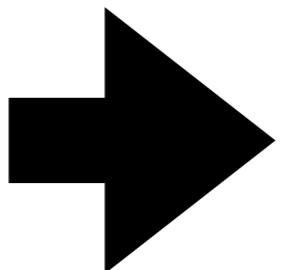


Phase

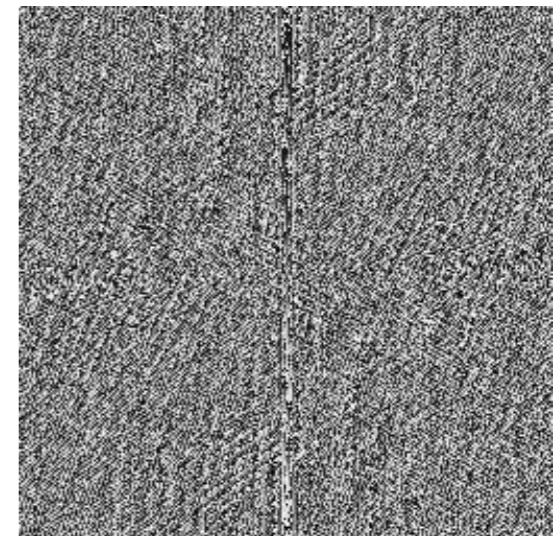
2D Fourier Transform



Signal

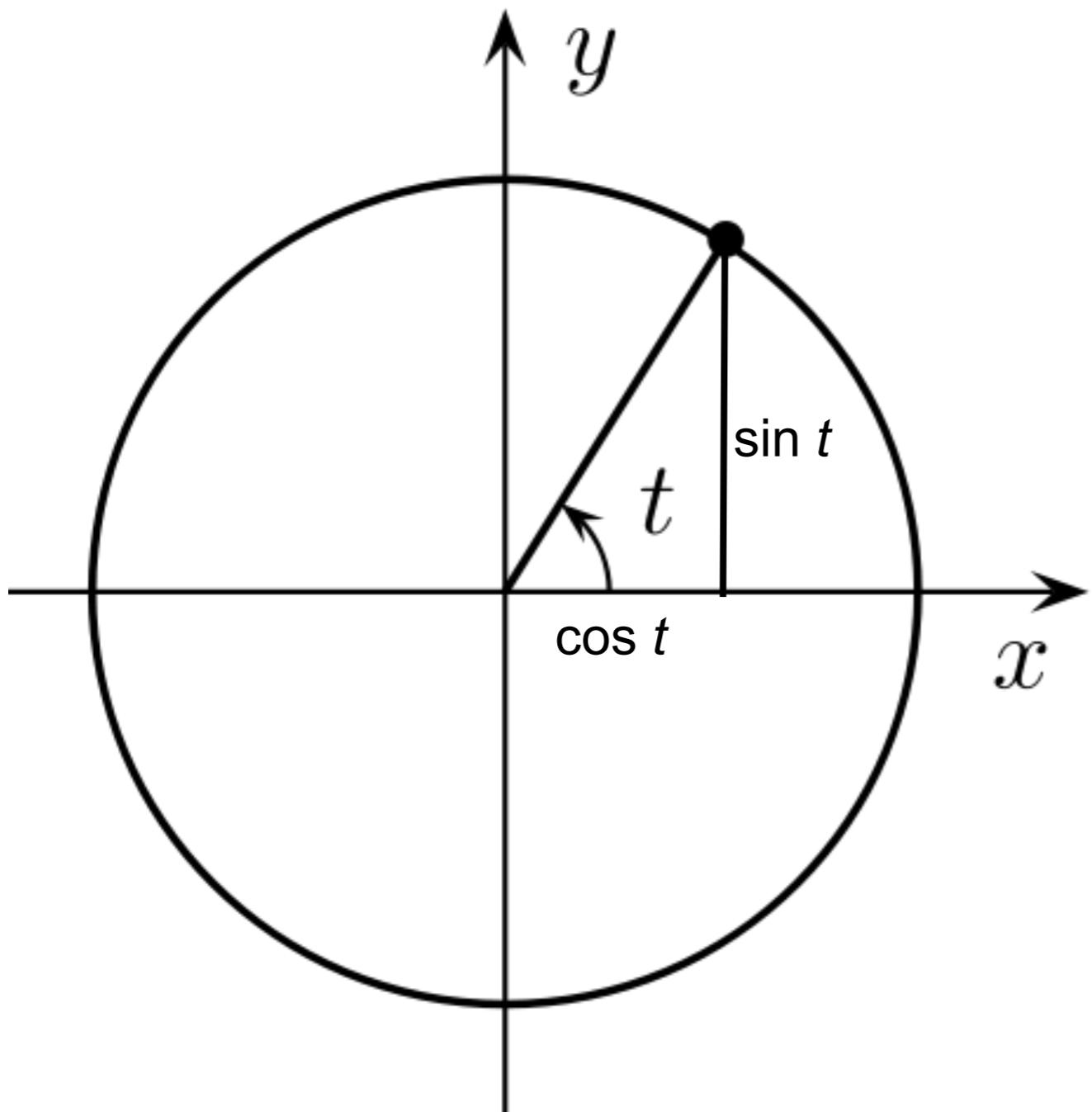


Amplitude



Phase

Sine/cosine and circle

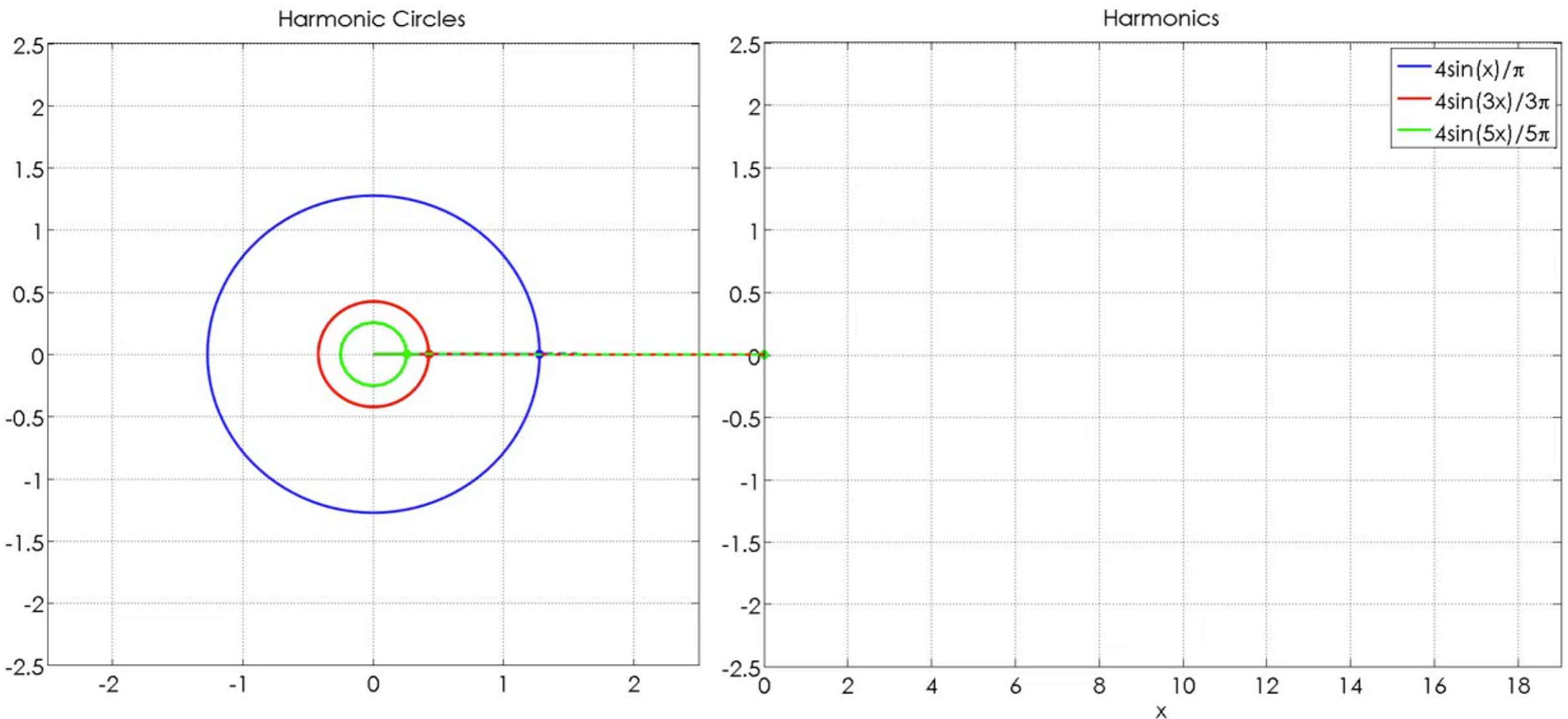


$$e^{ift} = \cos ft + i \sin ft$$

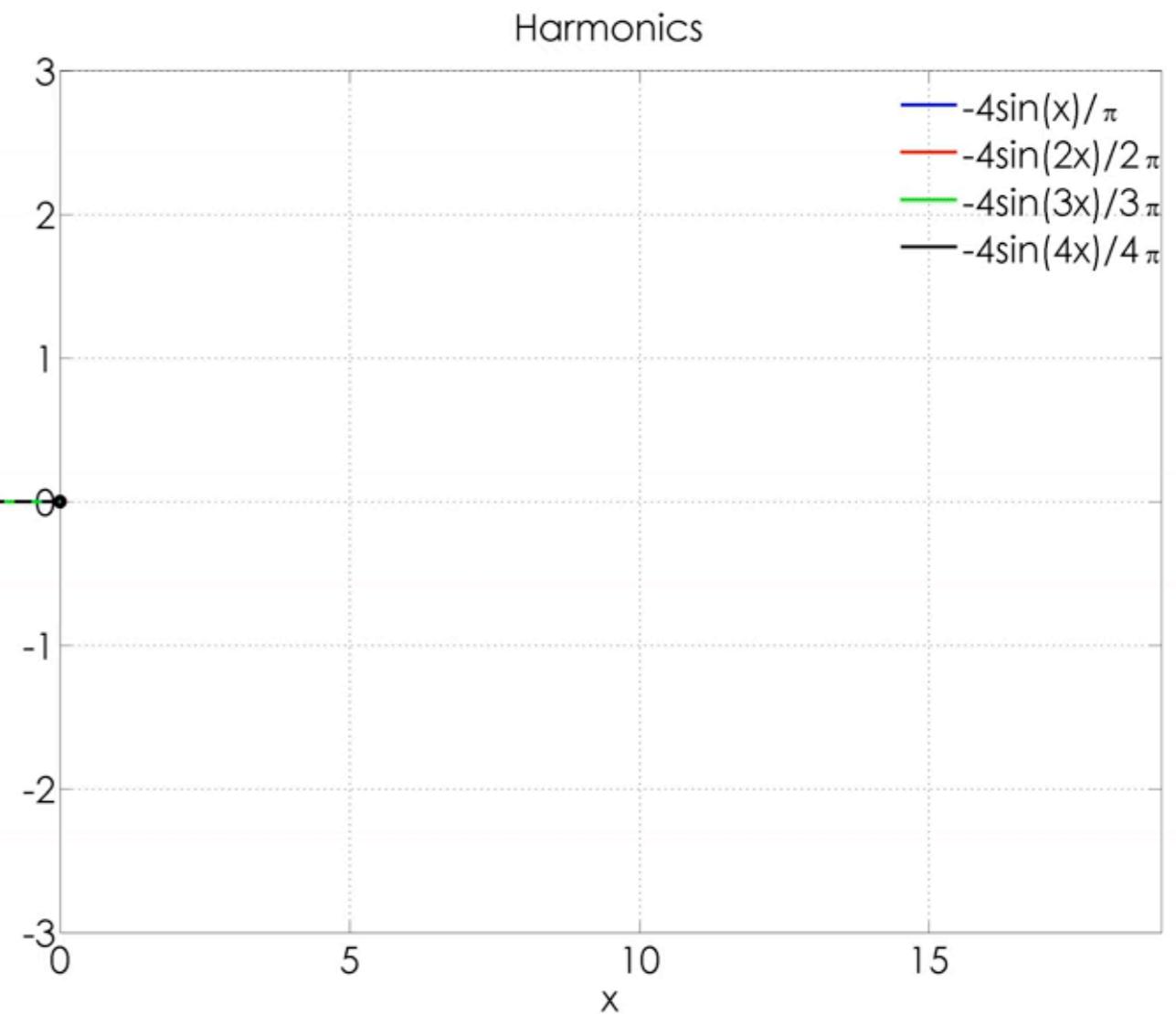
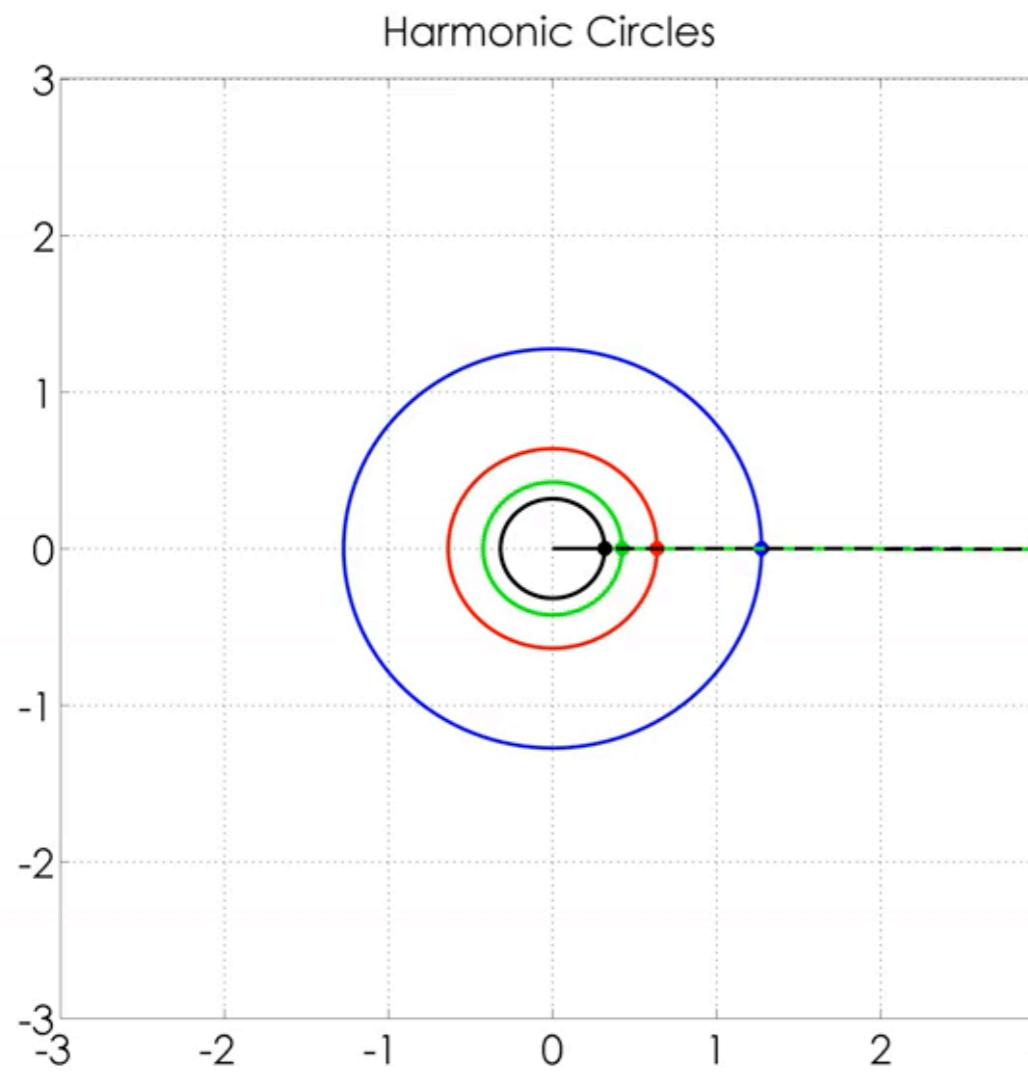
Amplitude: Radius of circle

Frequency: How fast you change t

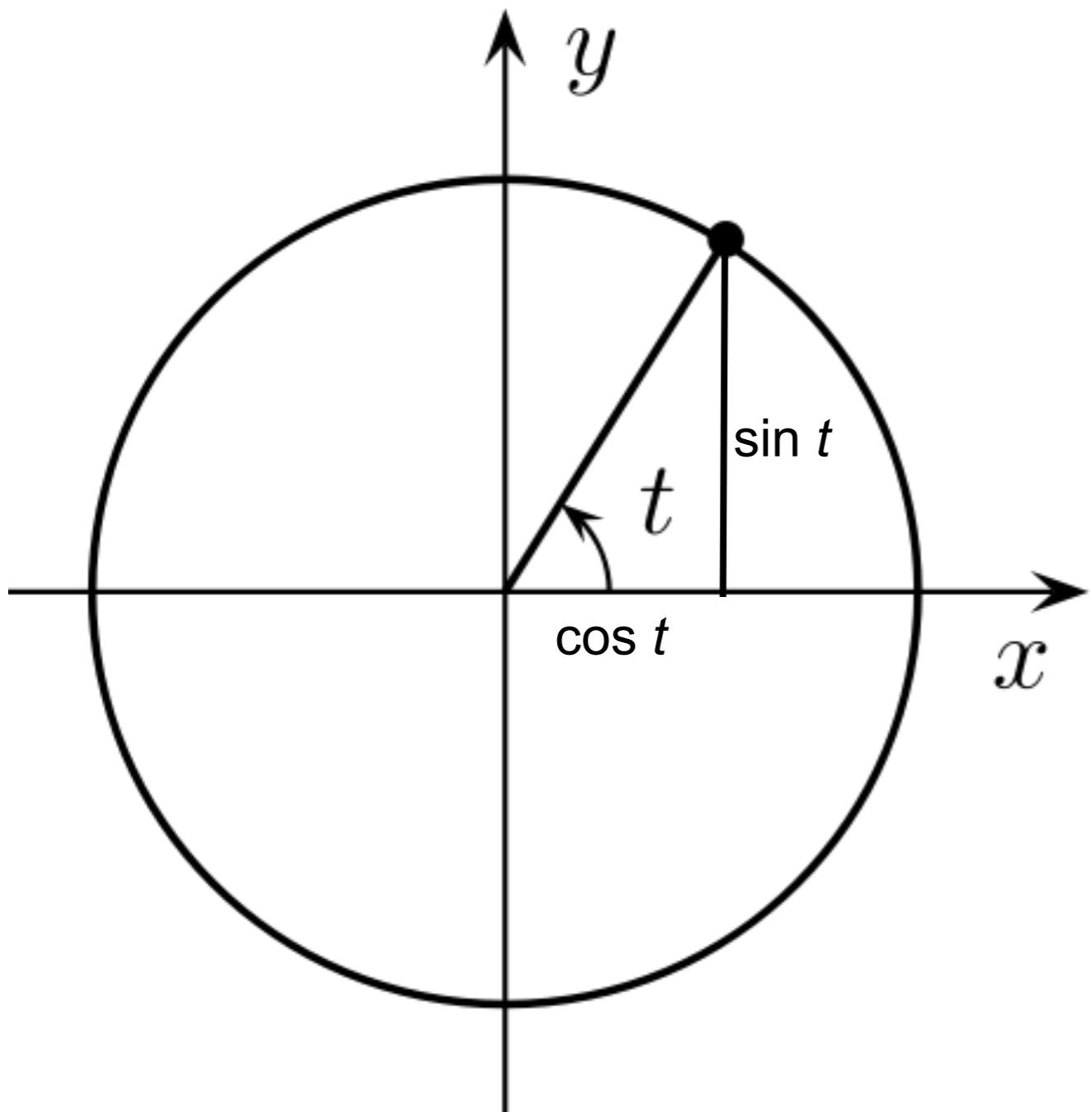
Square wave (approx.)



Sawtooth wave (approx.)



Sine/cosine and circle

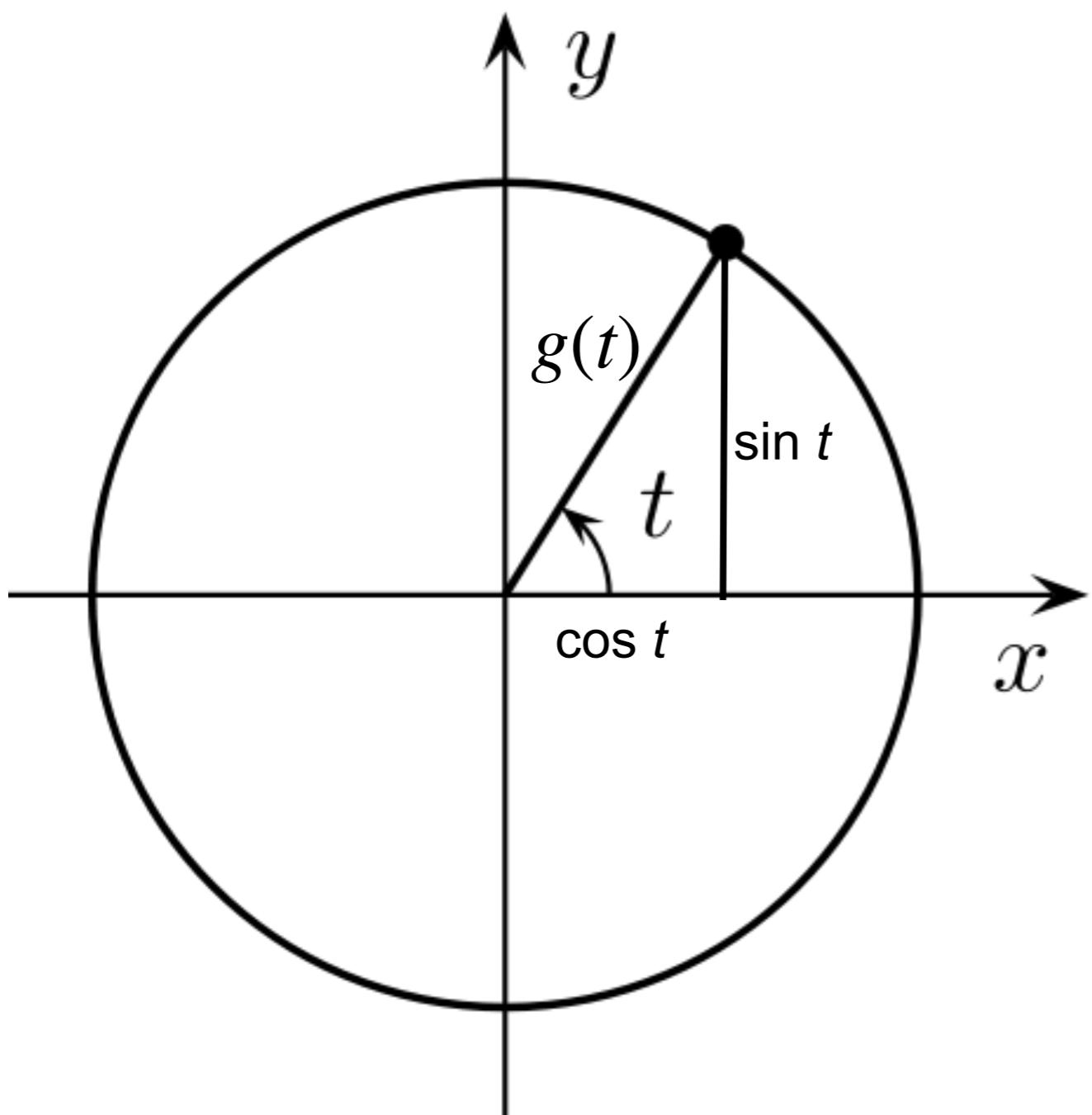


$$e^{ift} = \cos ft + i \sin ft$$

Amplitude: Radius of circle

Frequency: How fast you change t

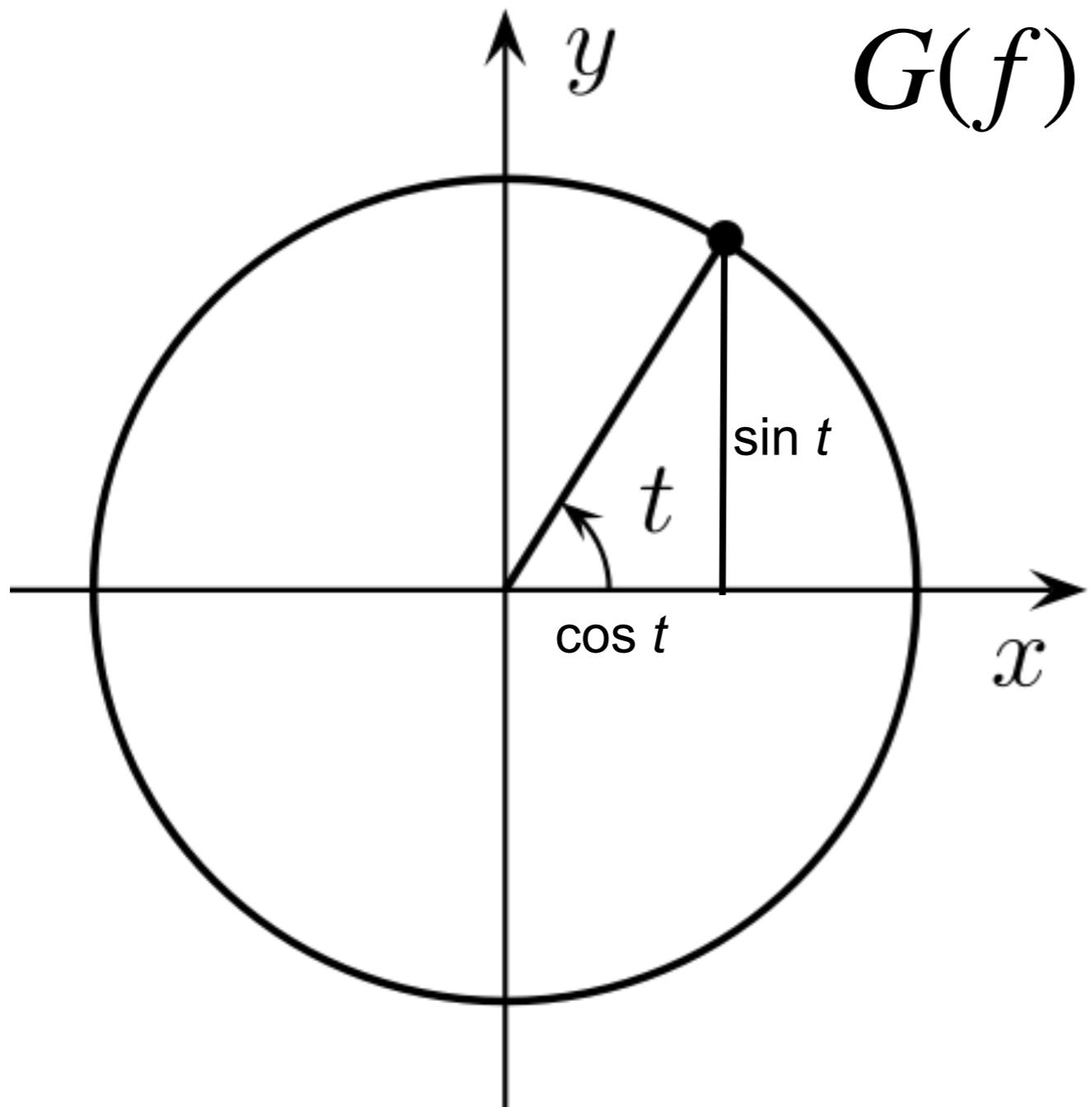
Towards Fourier Transform



$$g(t)e^{-2\pi ift}$$

Maps $g(t)$ on to the unit circle with frequency f

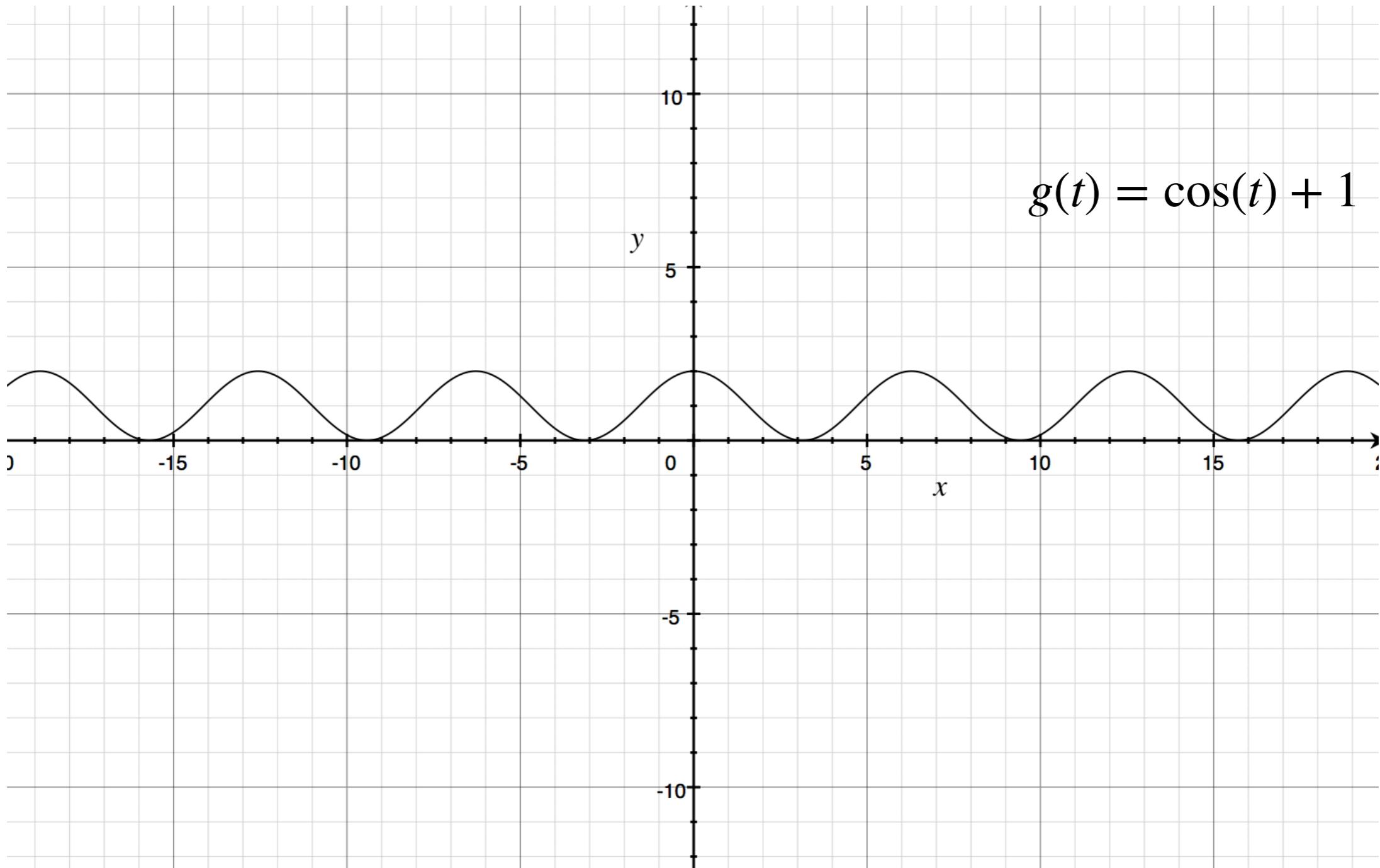
The Fourier Transform



$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\piift} dt$$

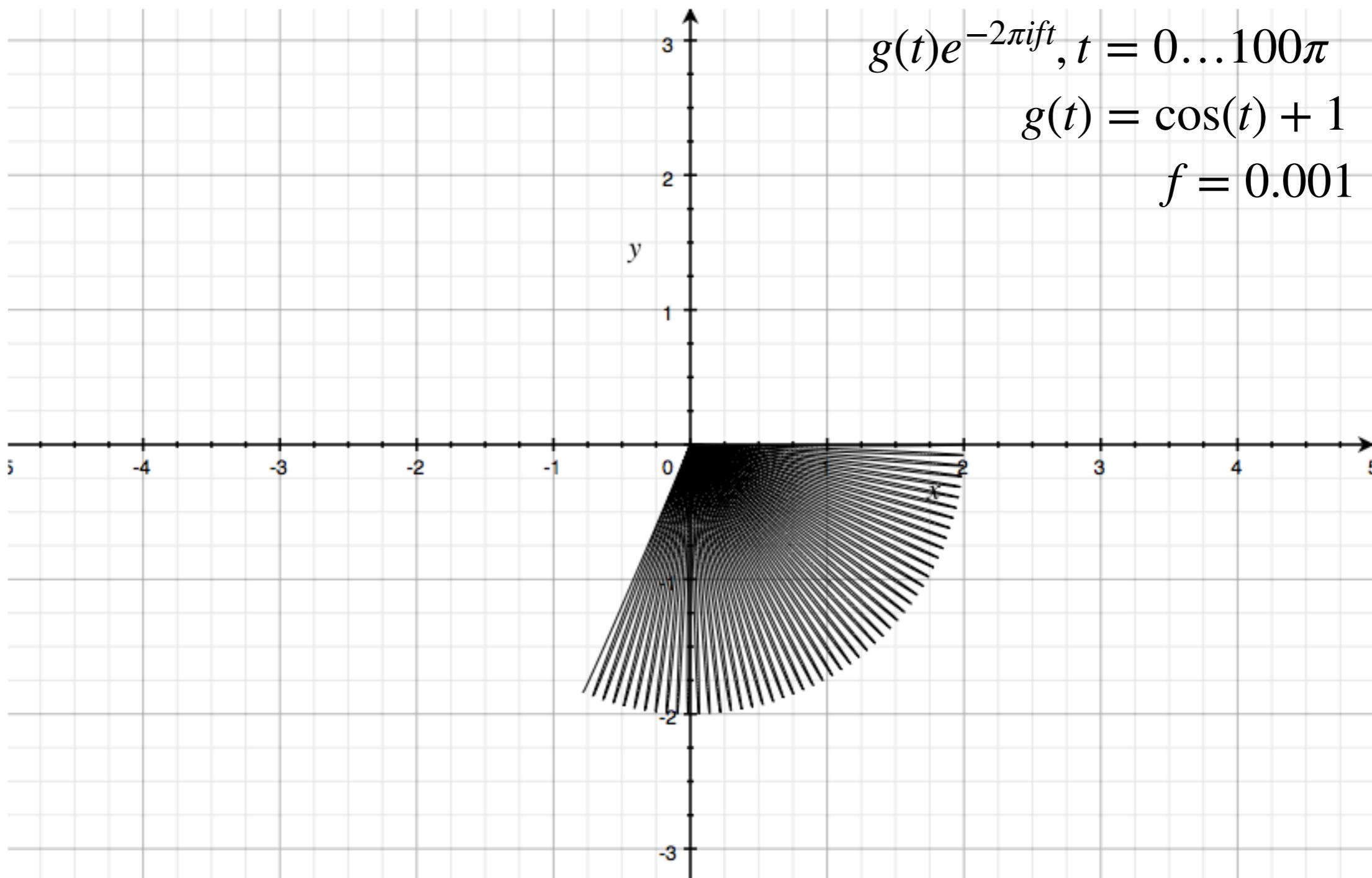
How I think of it:
You wrap $g(t)$ around the circle with frequency f ,
then calculate average position of $g(t)$

Signal that we want to compute FT on



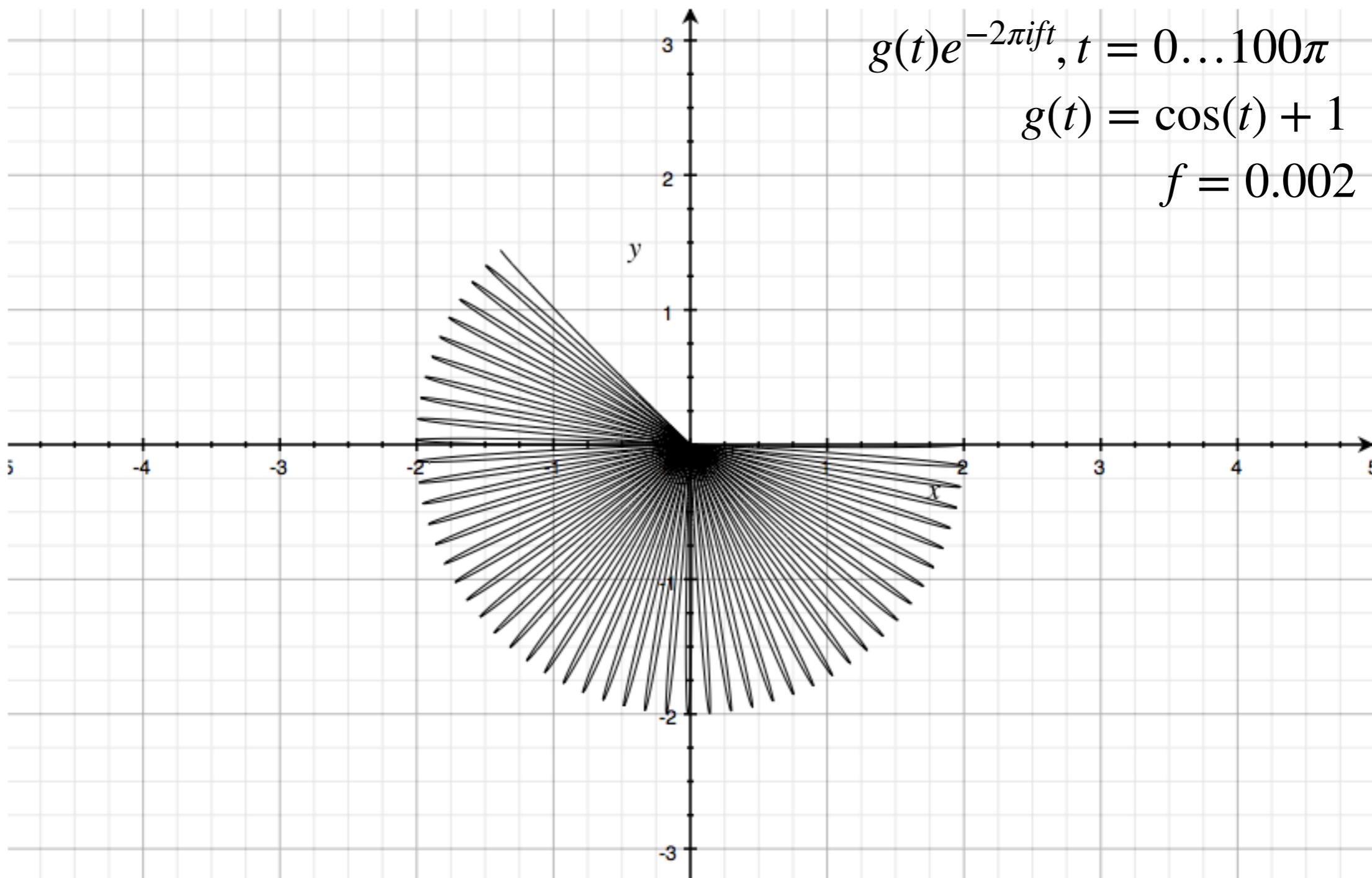
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



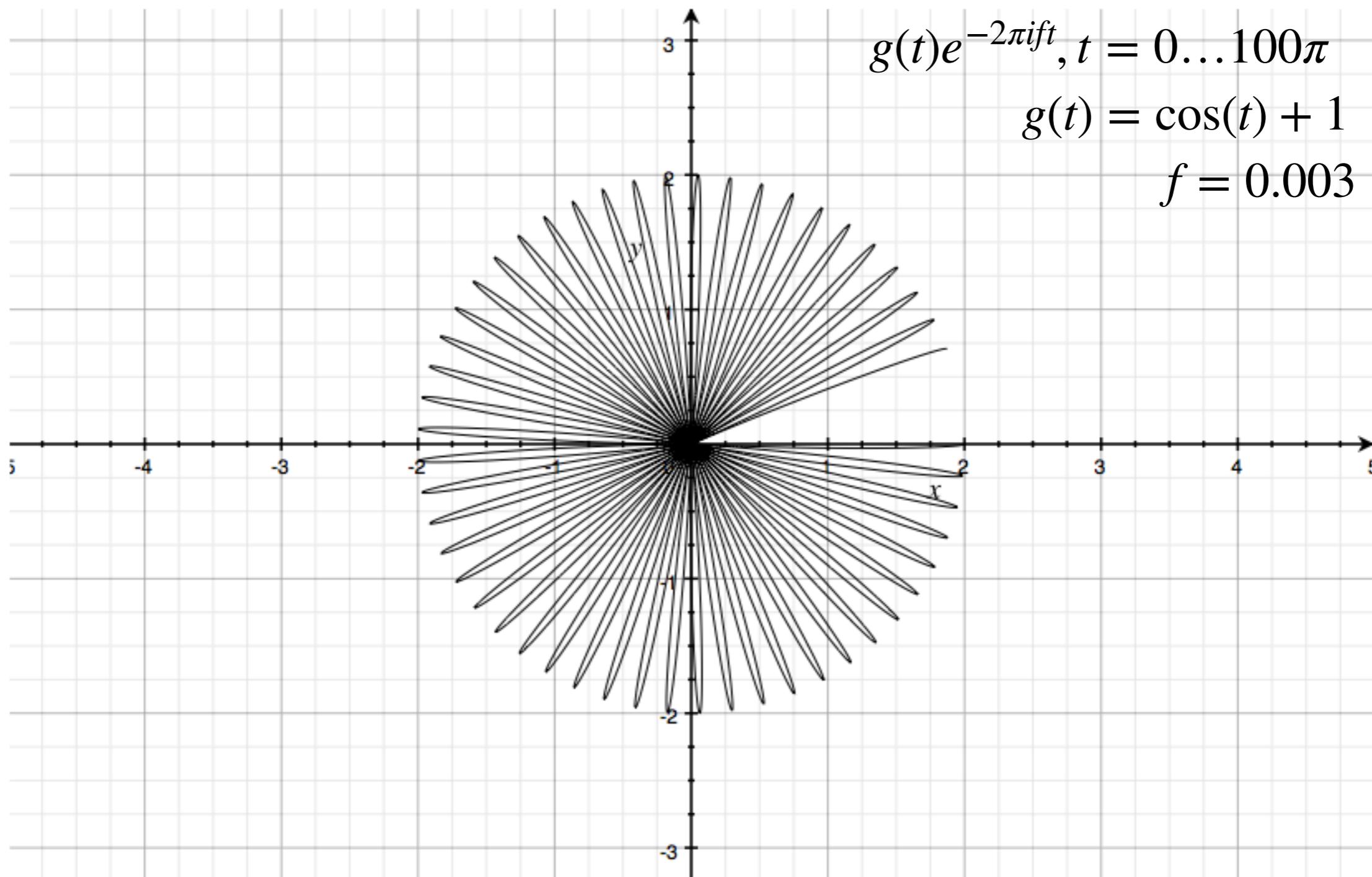
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



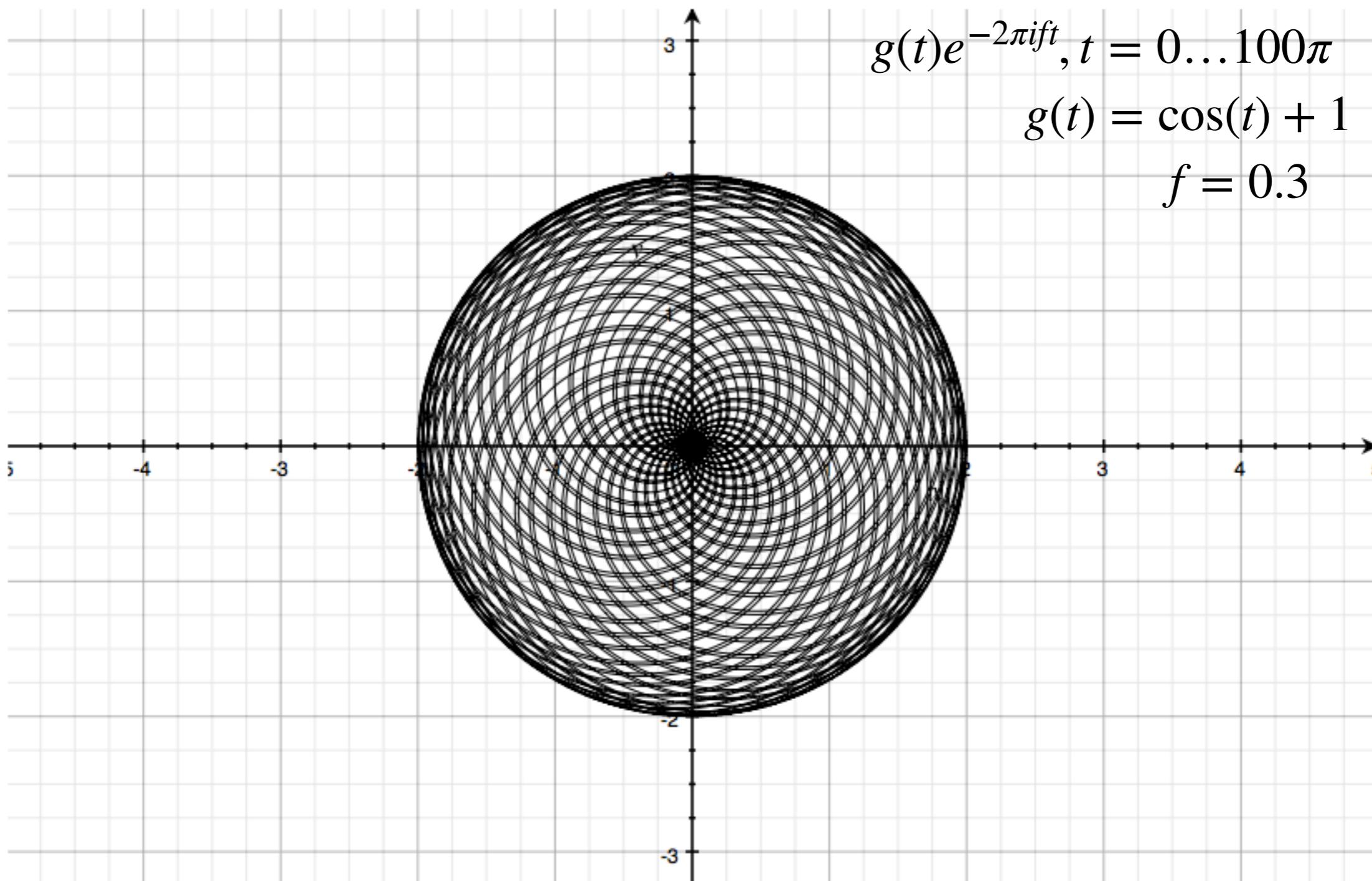
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



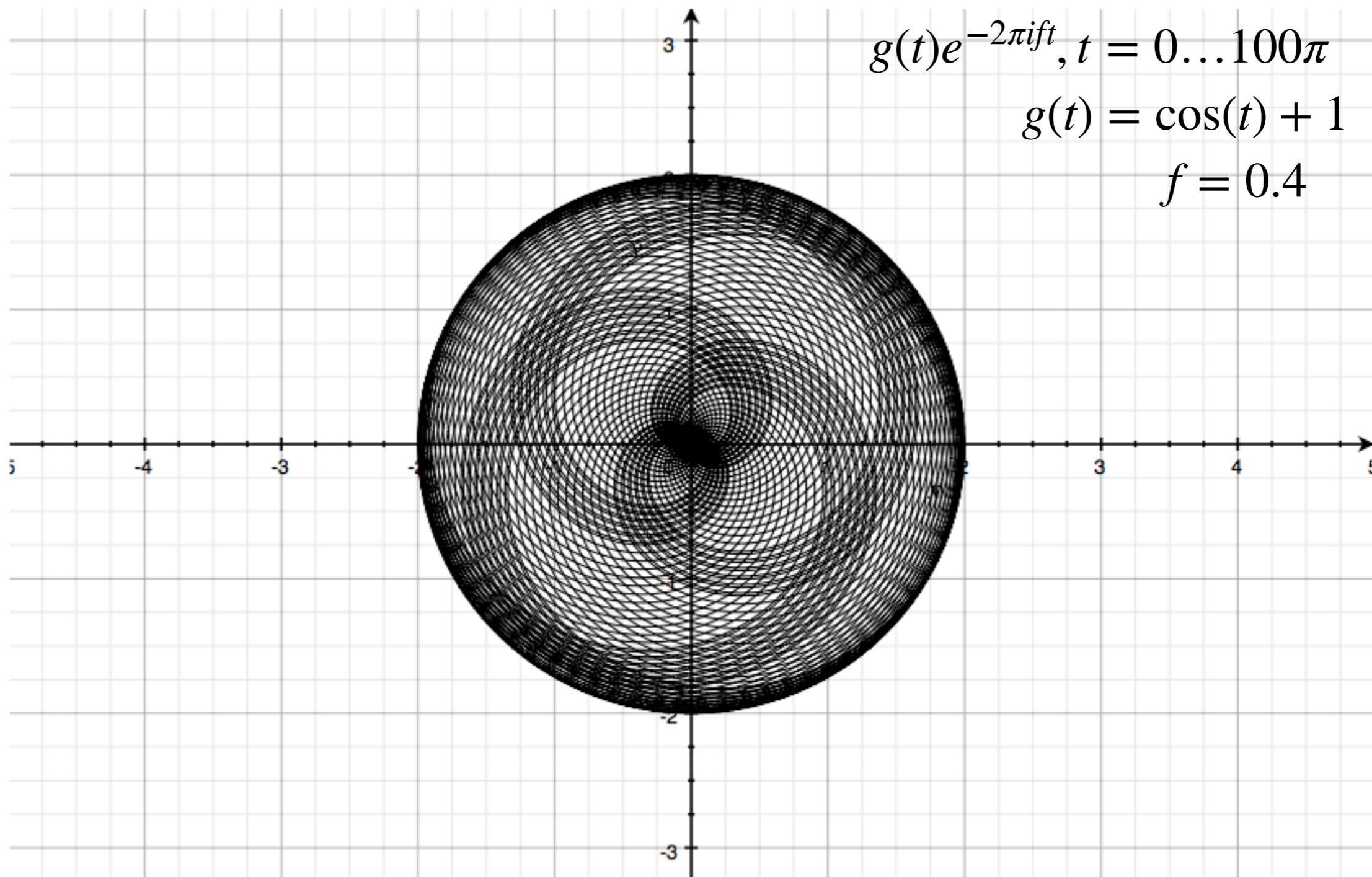
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



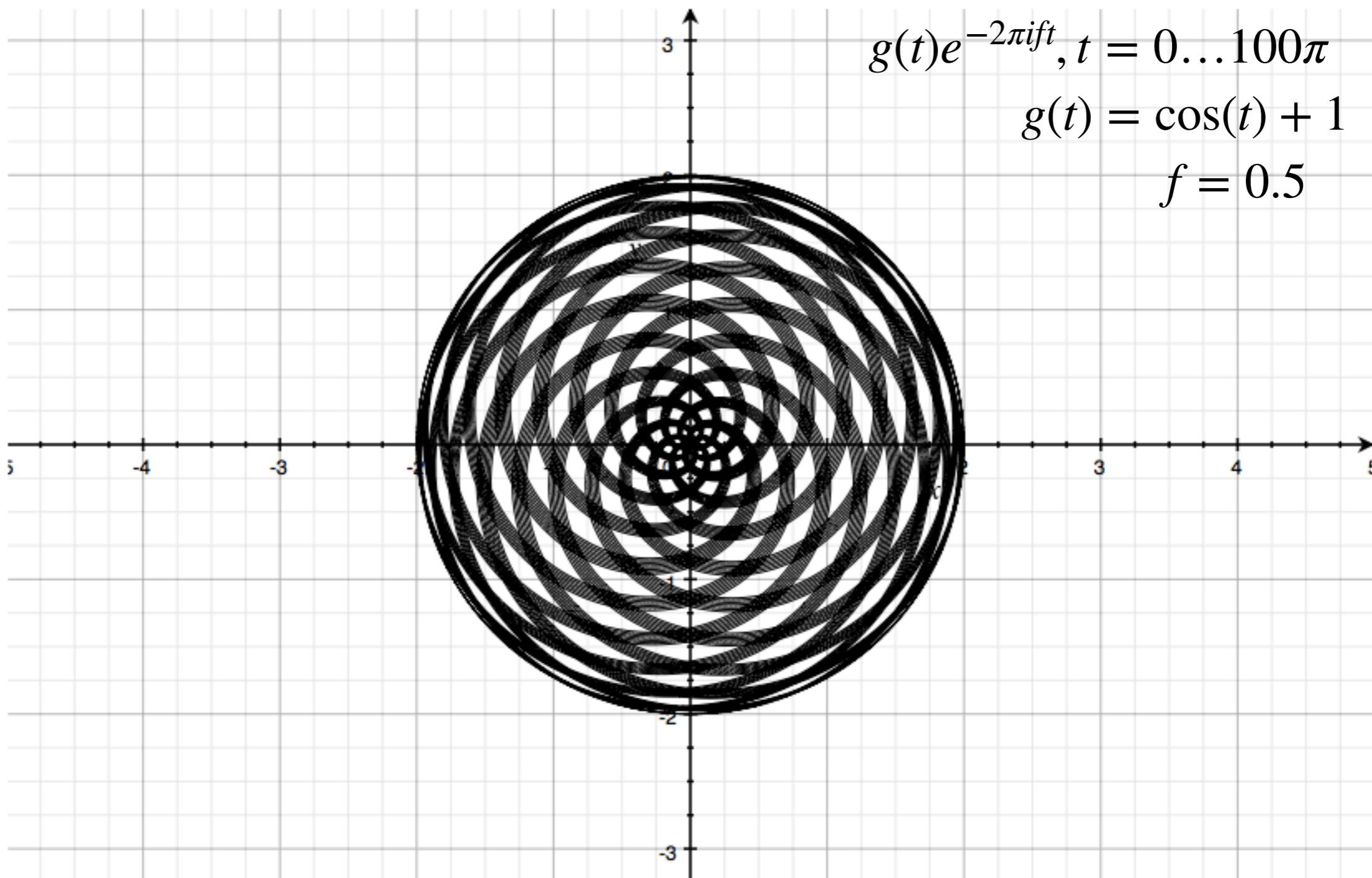
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



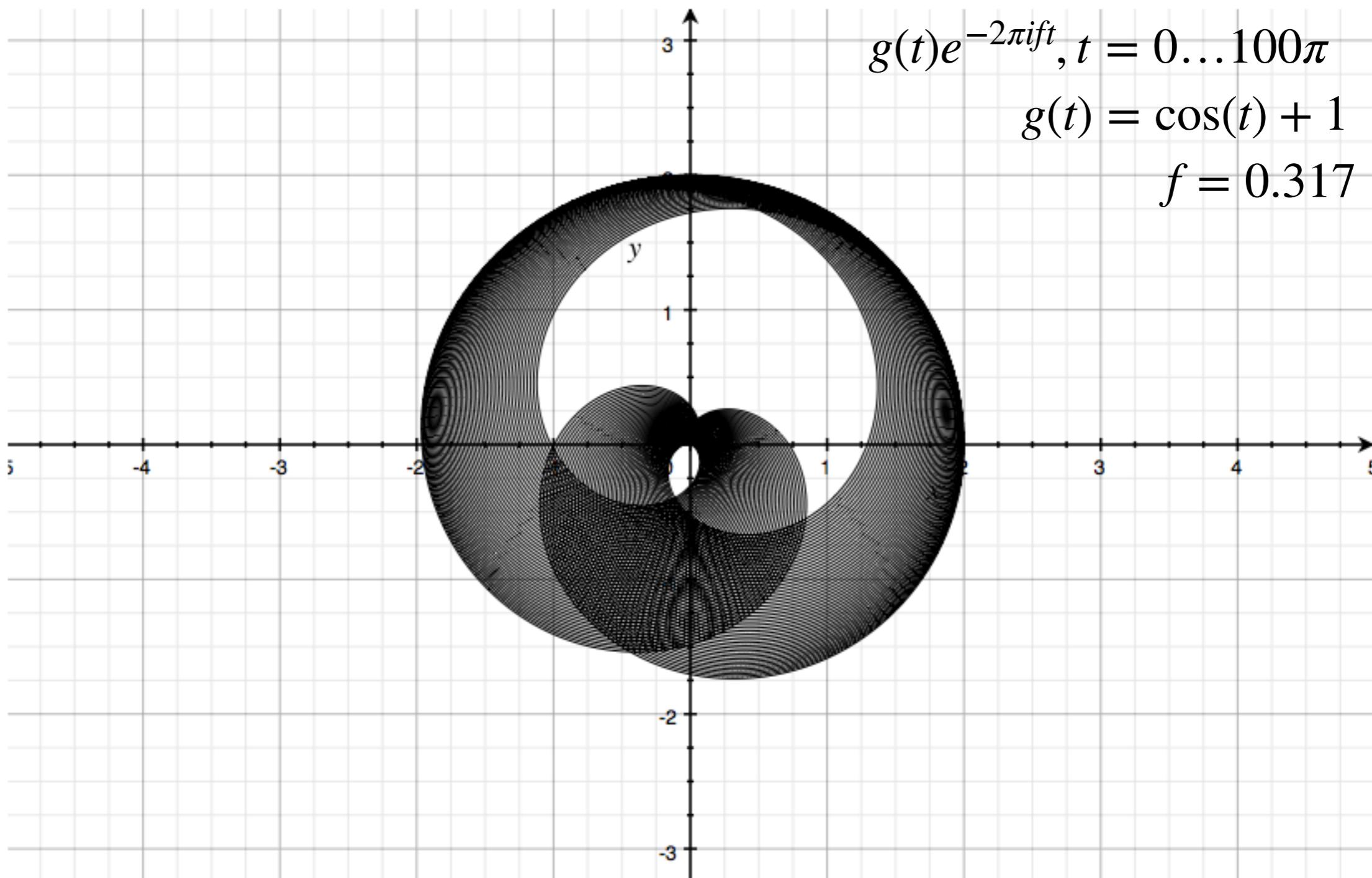
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



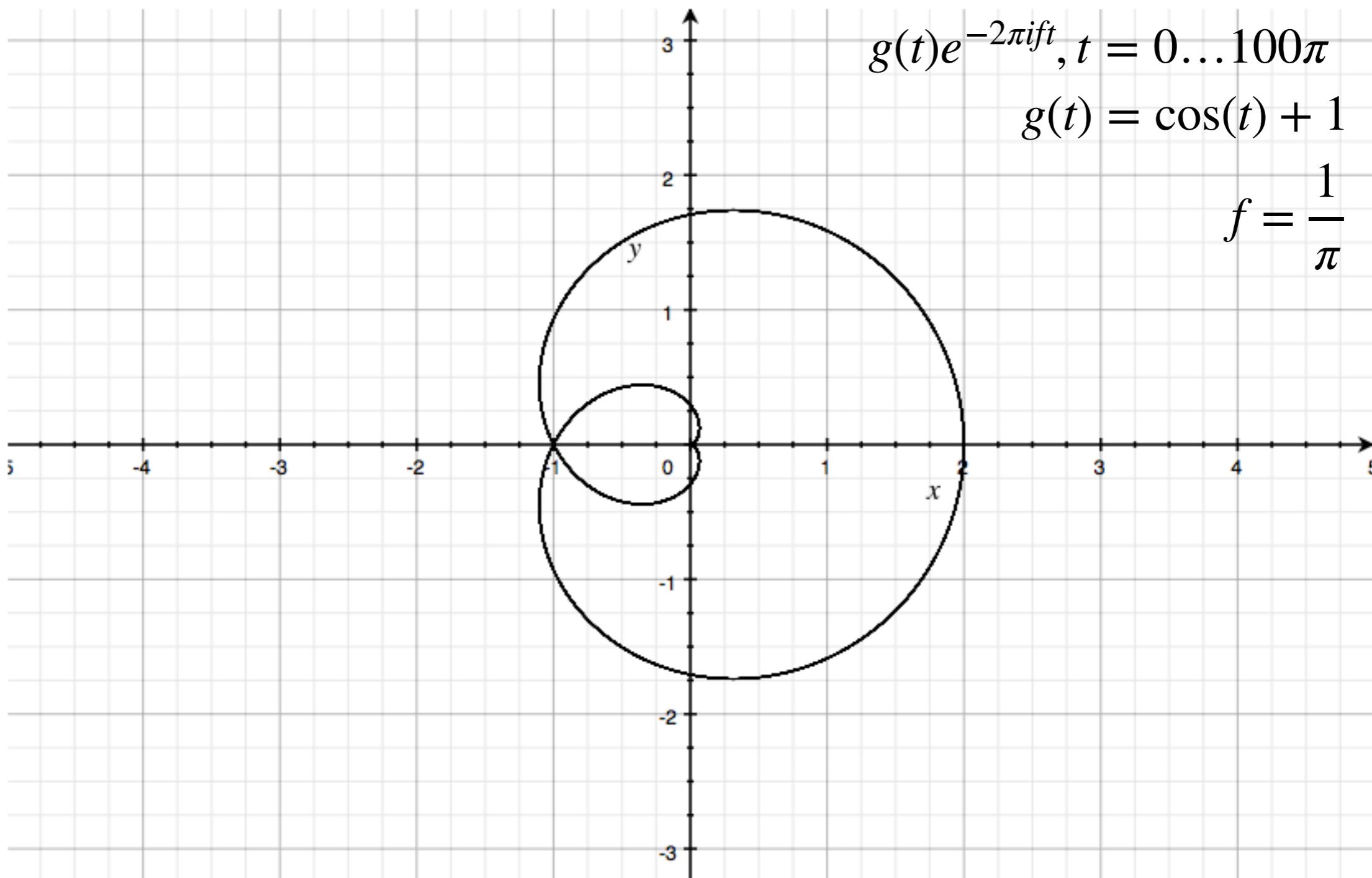
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



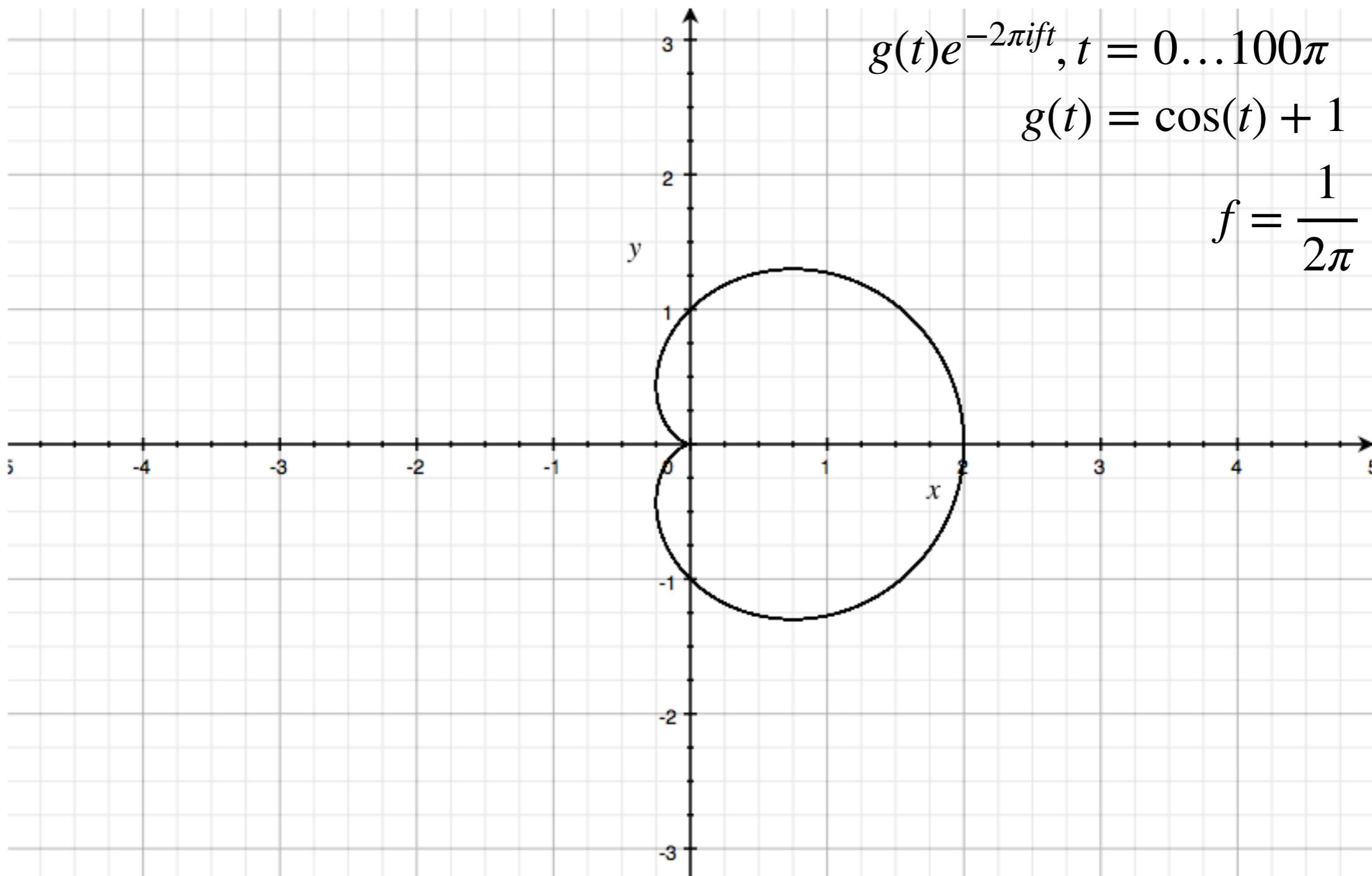
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



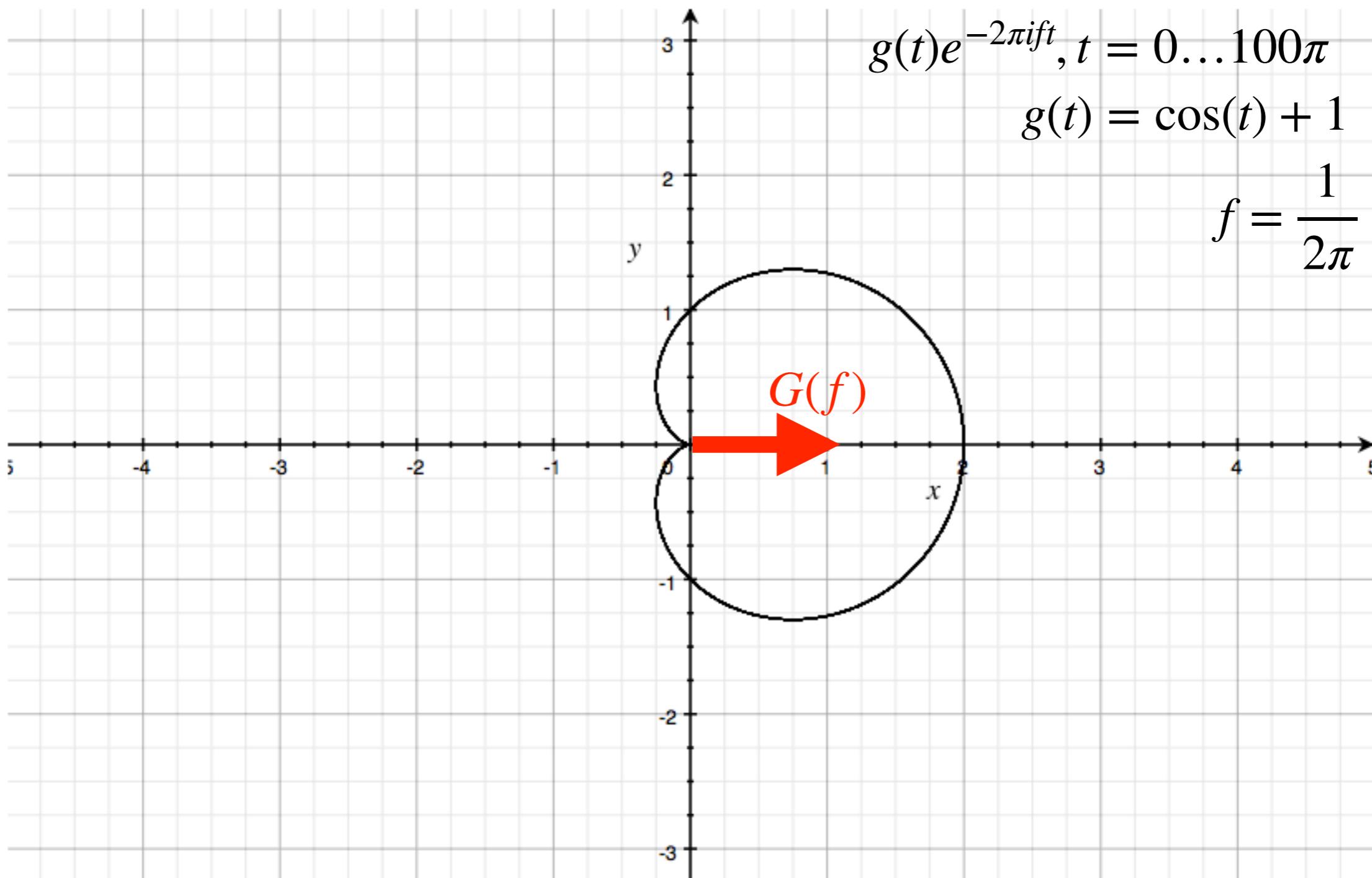
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



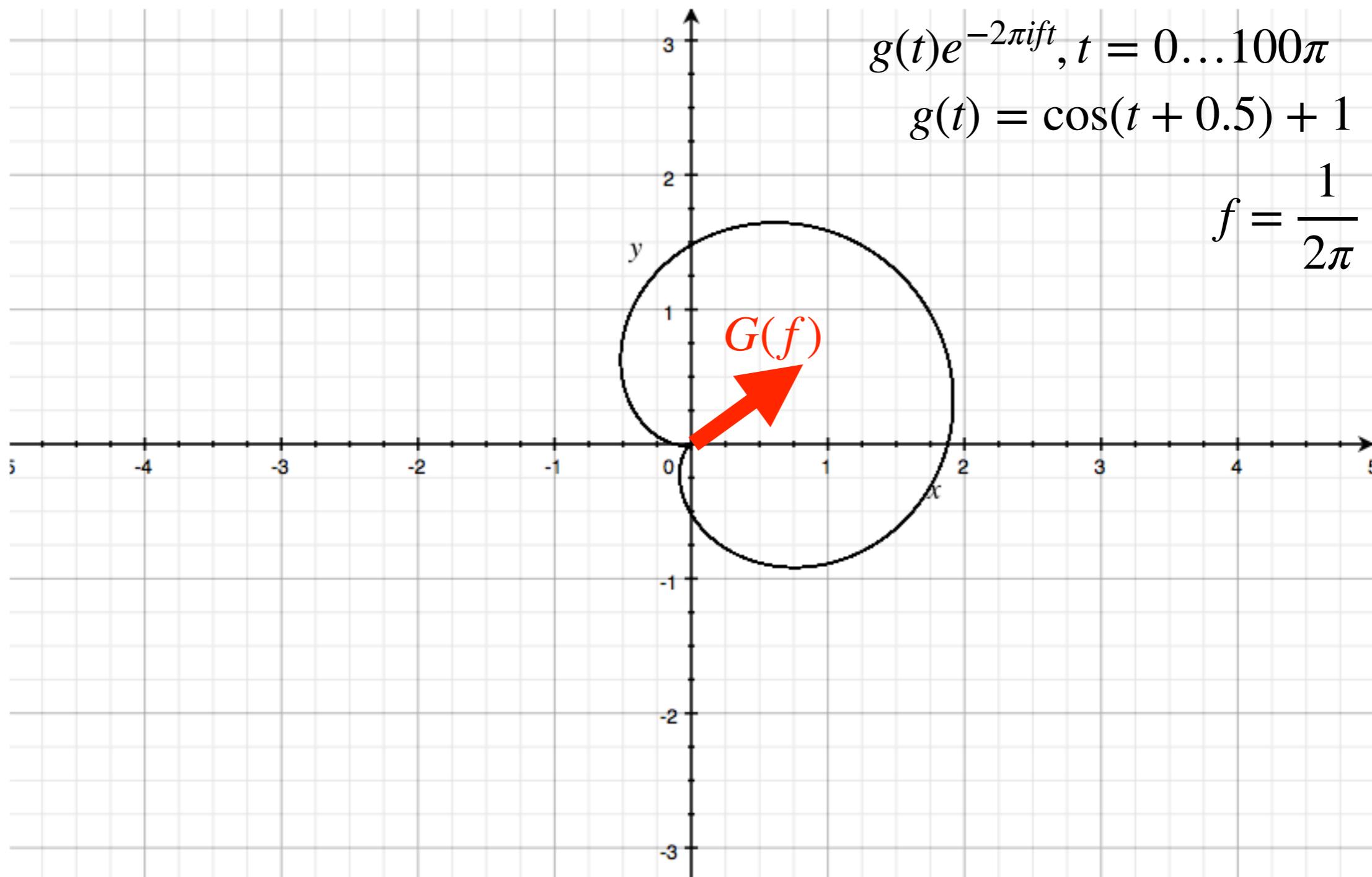
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



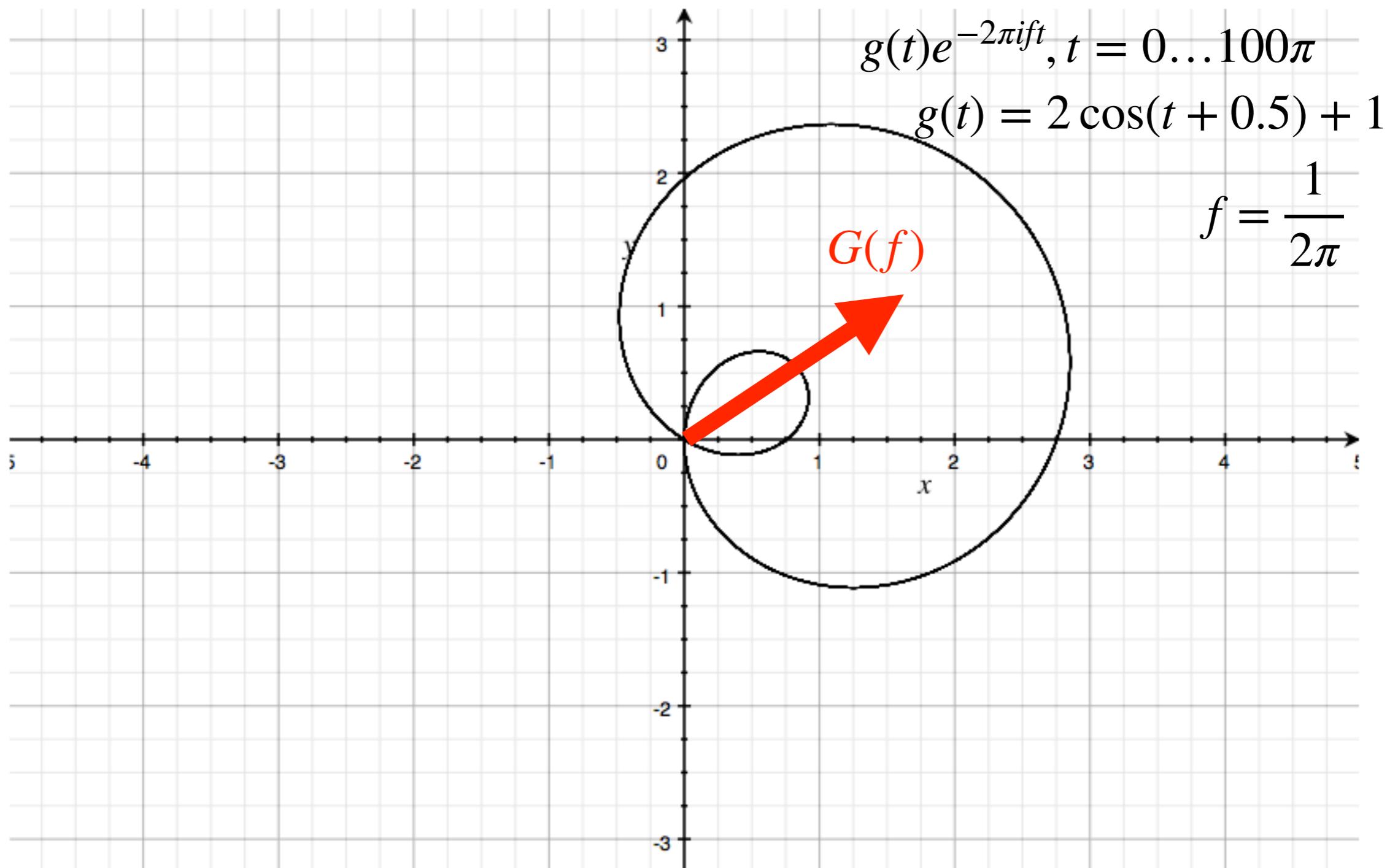
The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi i f t} dt$$



The Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-2\pi ift} dt$$

Amplitude:

$$\sqrt{\Re[G(f)]^2 + \Im[G(f)]^2}$$

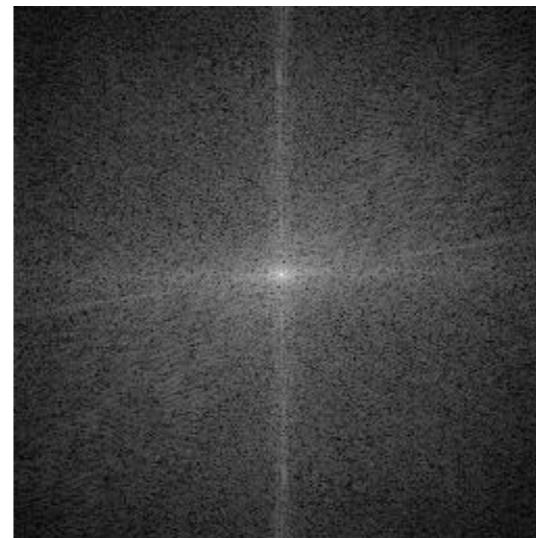
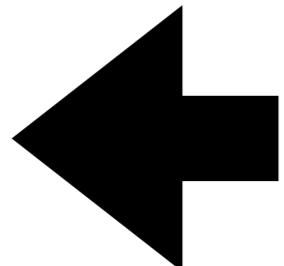
Phase:

$$\tan^{-1} \frac{\Im[G(f)]}{\Re[G(f)]}$$

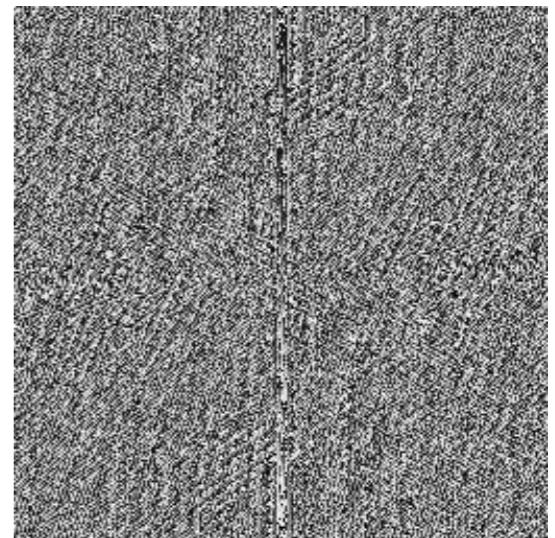
Inverse Fourier Transform



Signal



Amplitude



Phase

Fourier Transform:

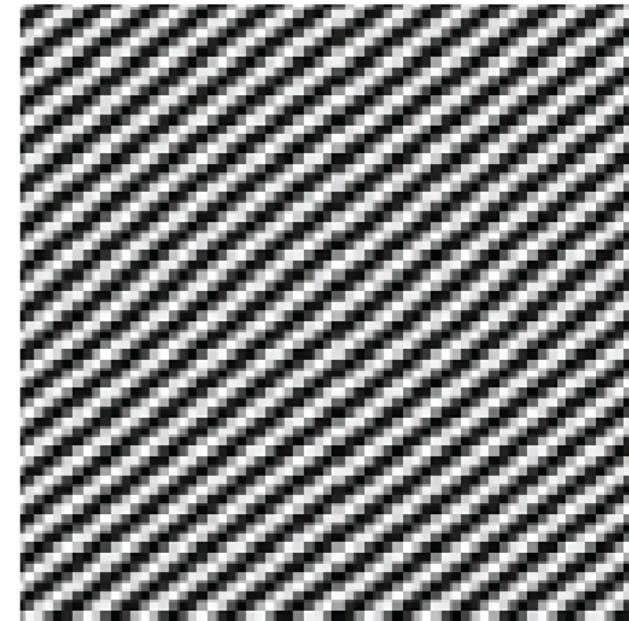
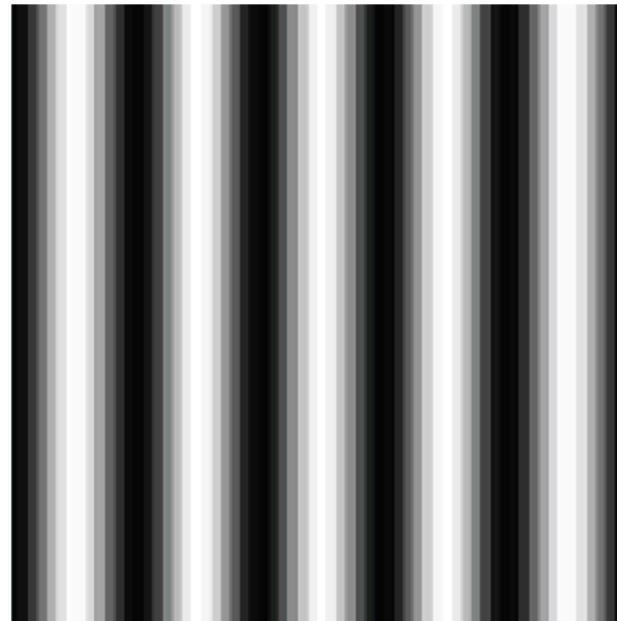
$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i f t} dt$$

Inverse Fourier Transform:

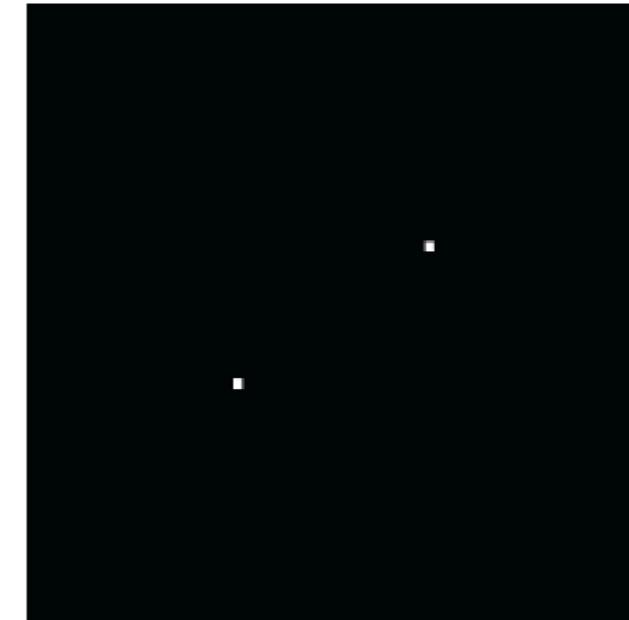
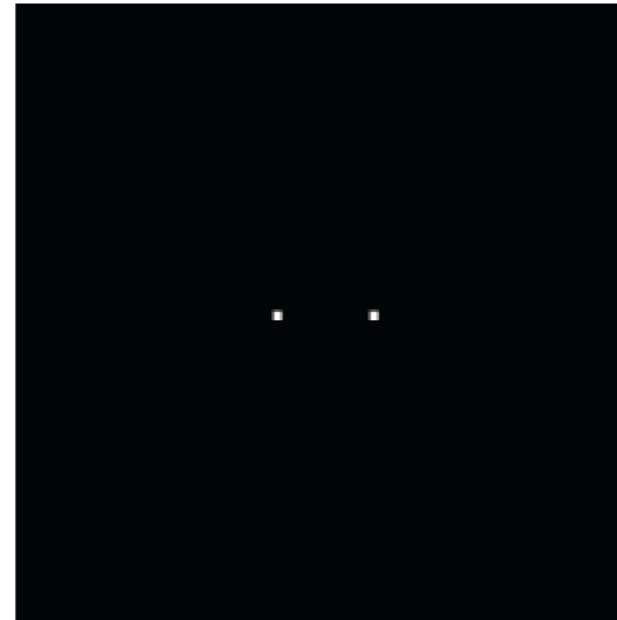
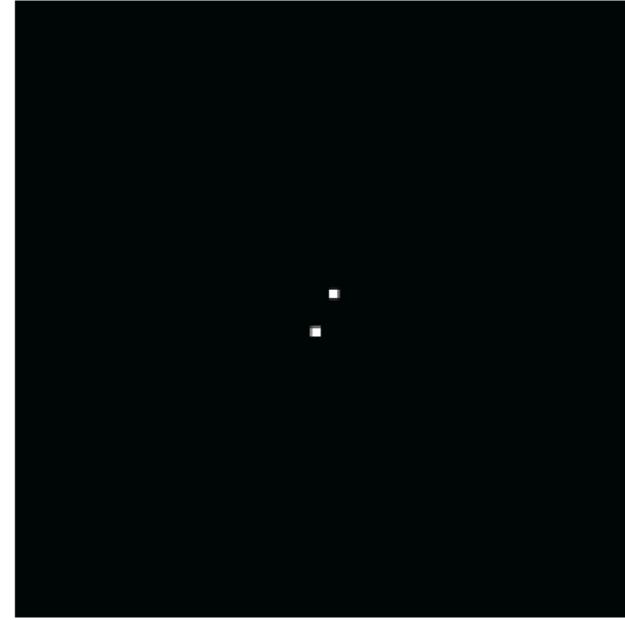
$$g(t) = \int_{-\infty}^{\infty} G(f) e^{2\pi i f t} dt$$

Frequencies

Image

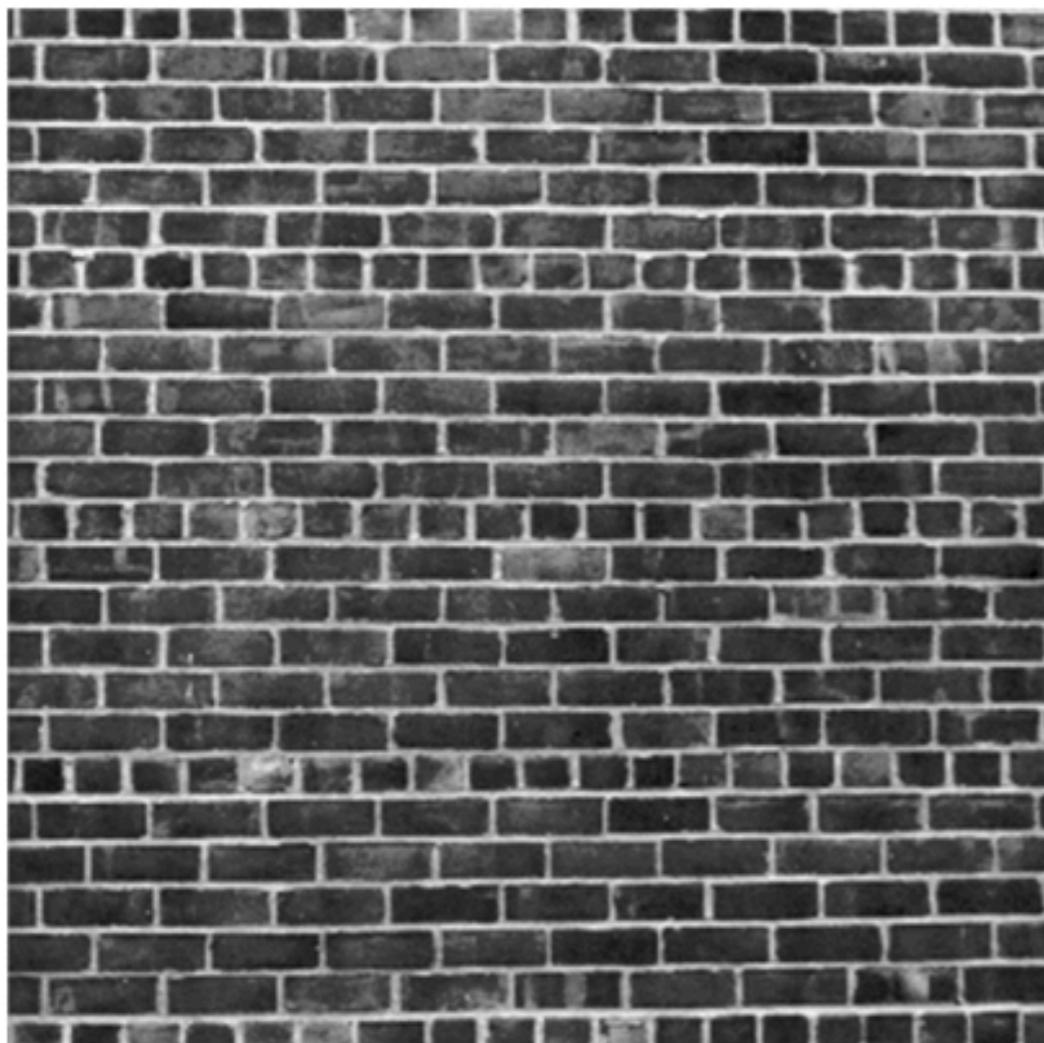


DFT Amplitude

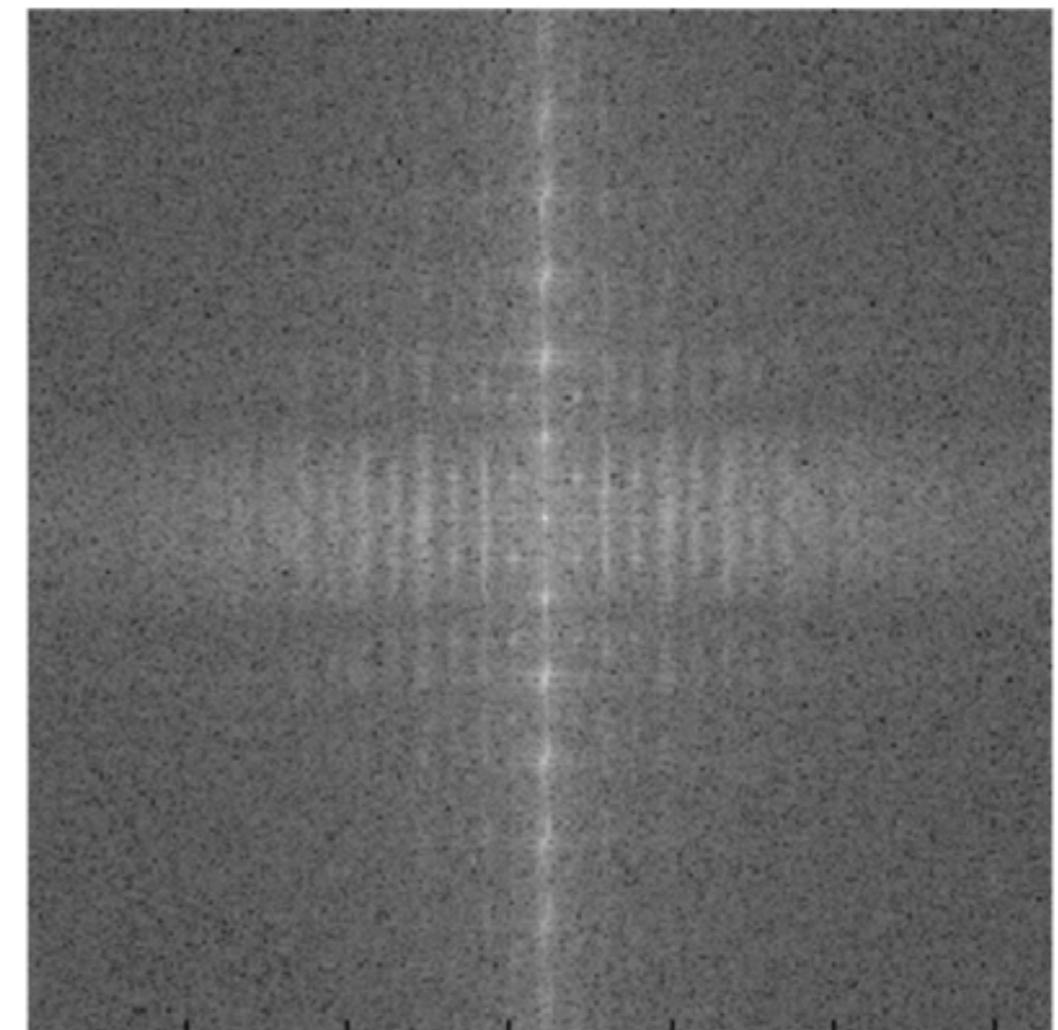


Images are 64x64 pixels. The wave is a cosine (if phase is zero).

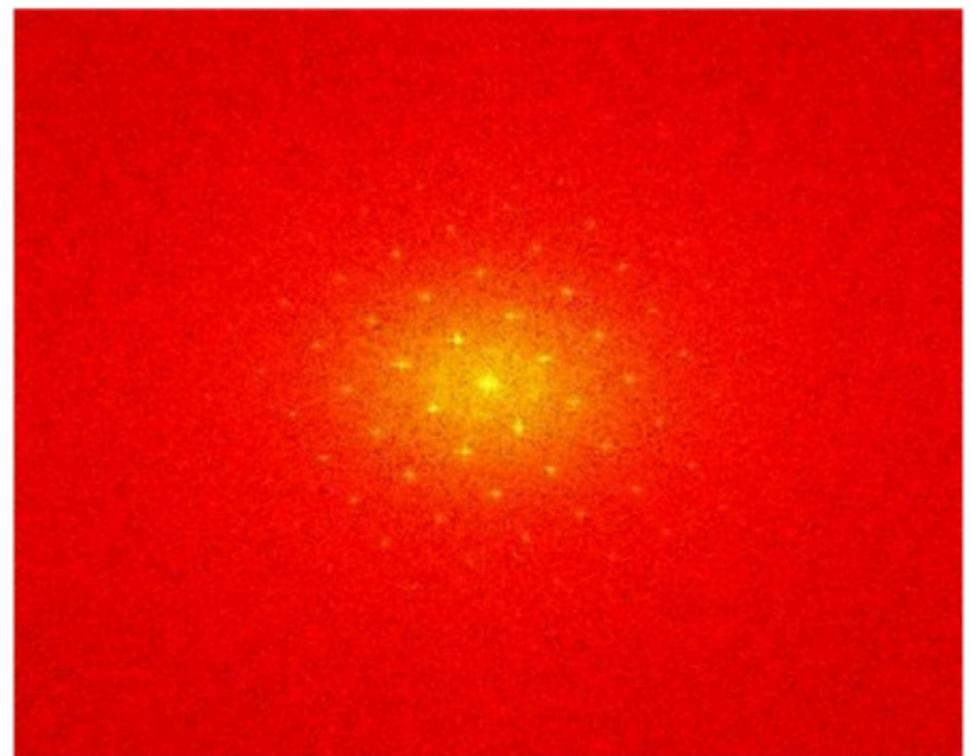
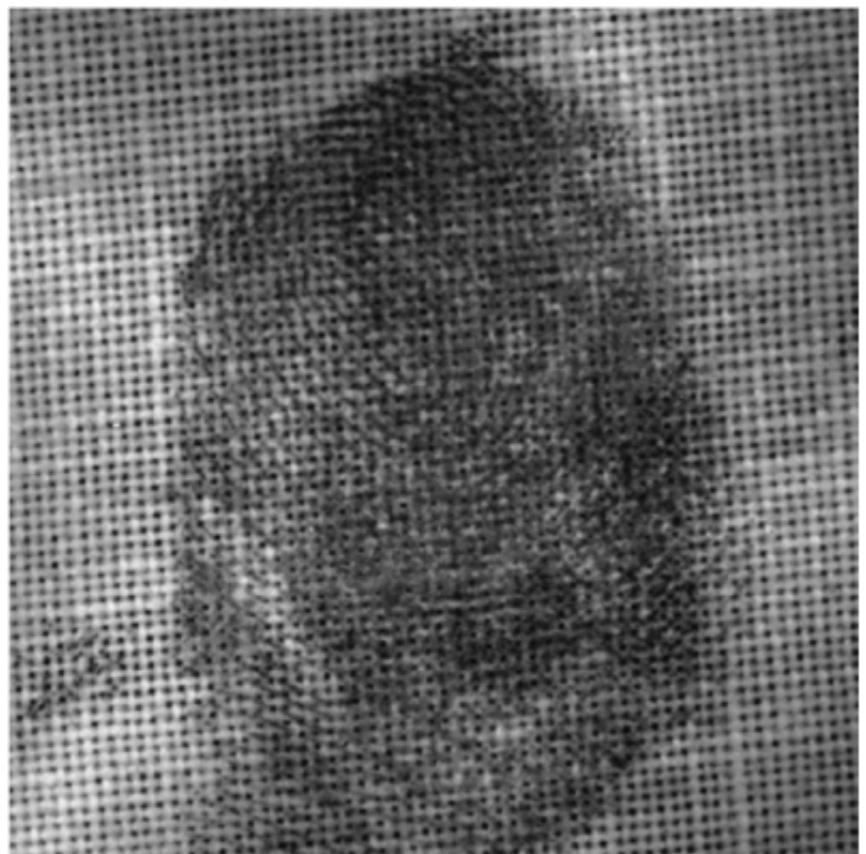
FT has peaks at spatial frequencies of repeated structure

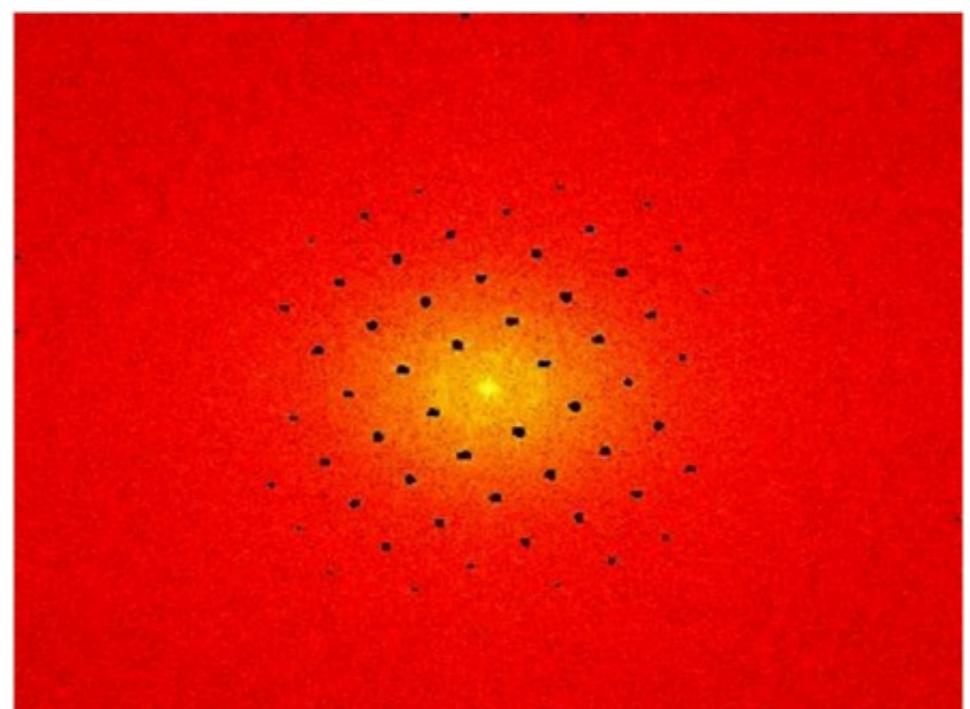
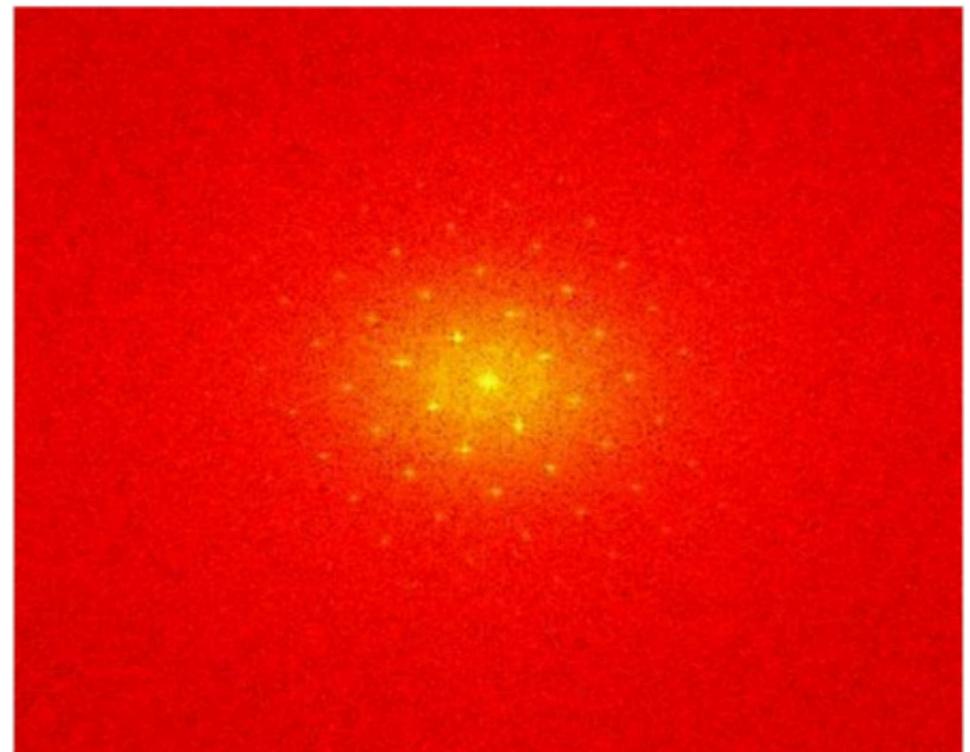
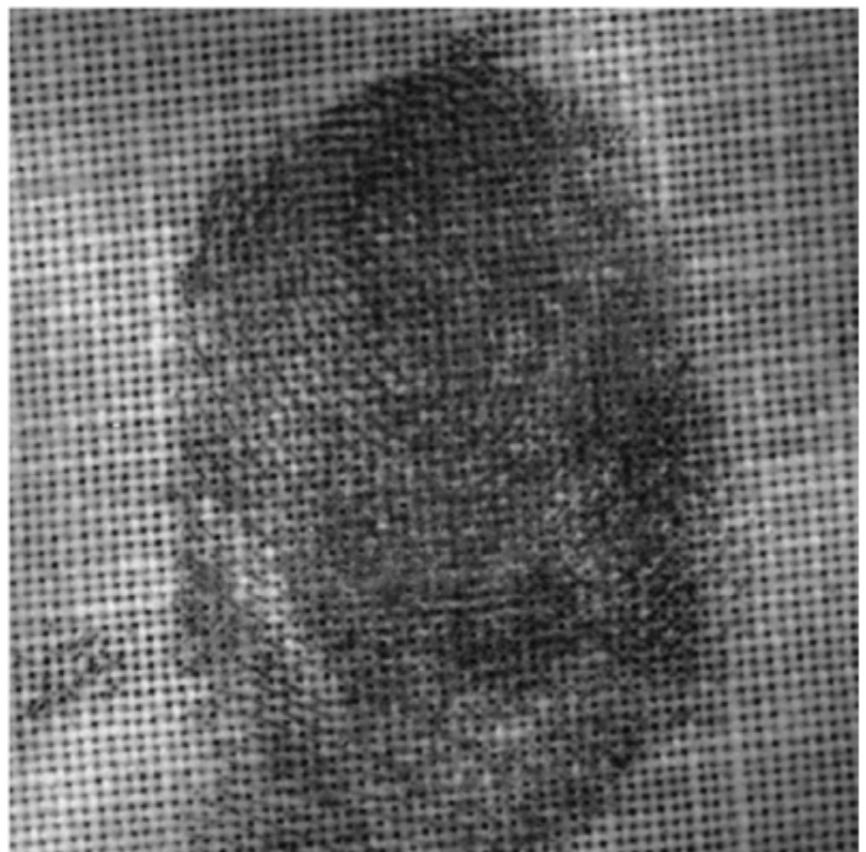


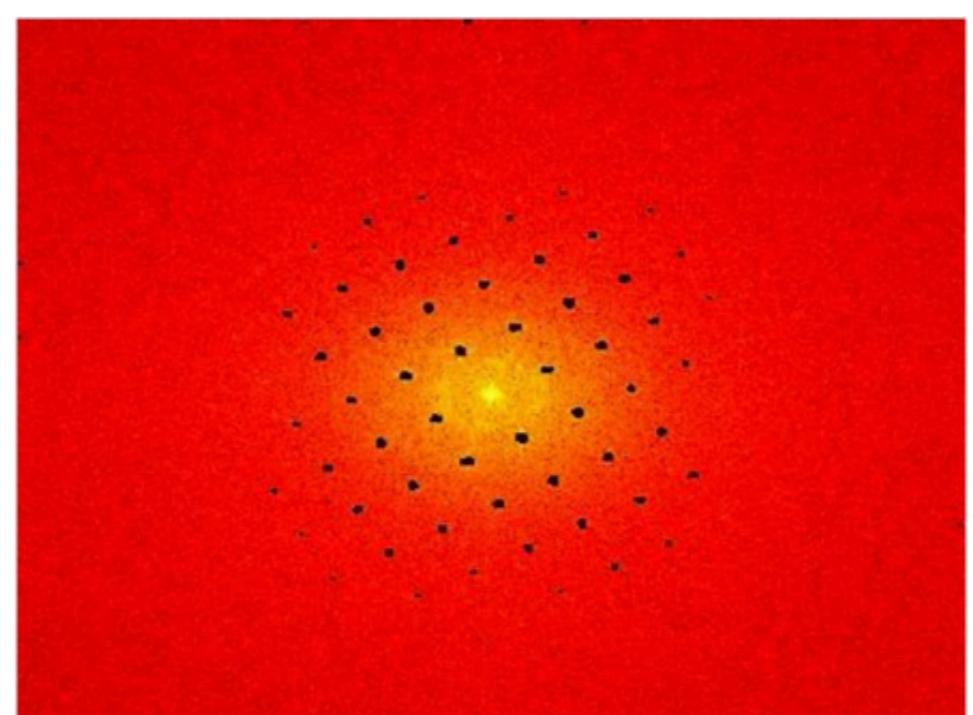
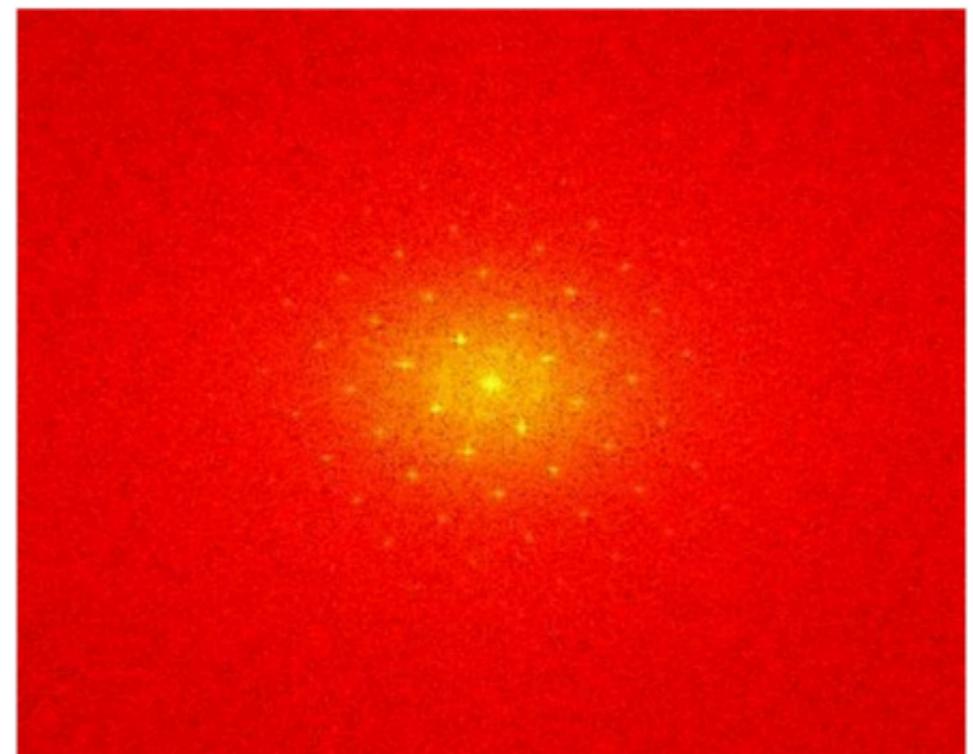
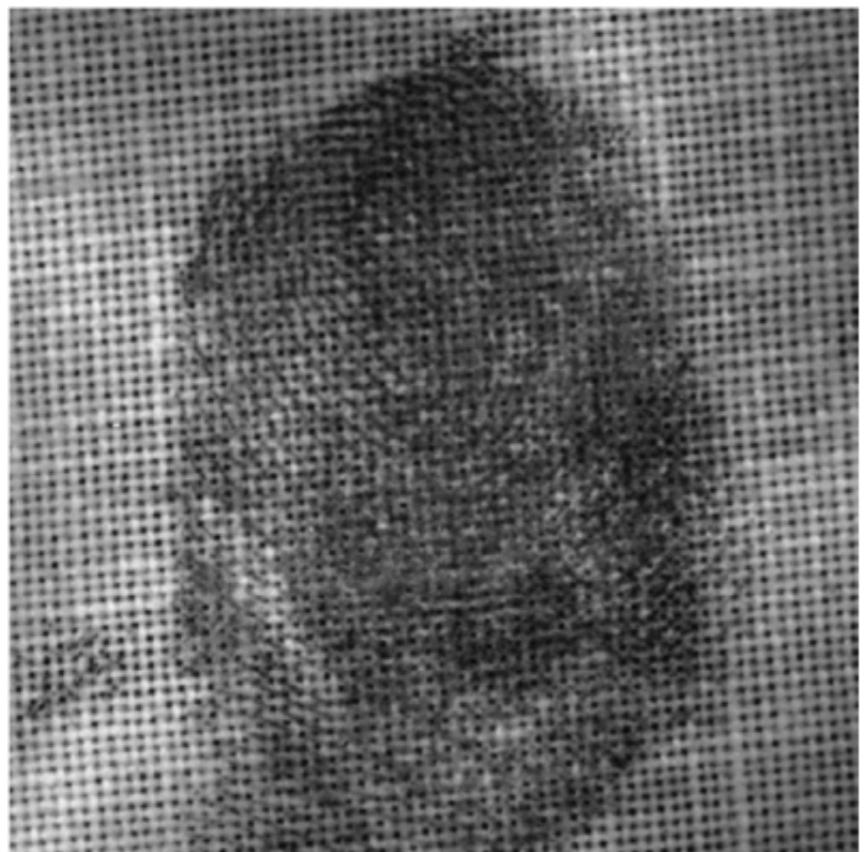
Image



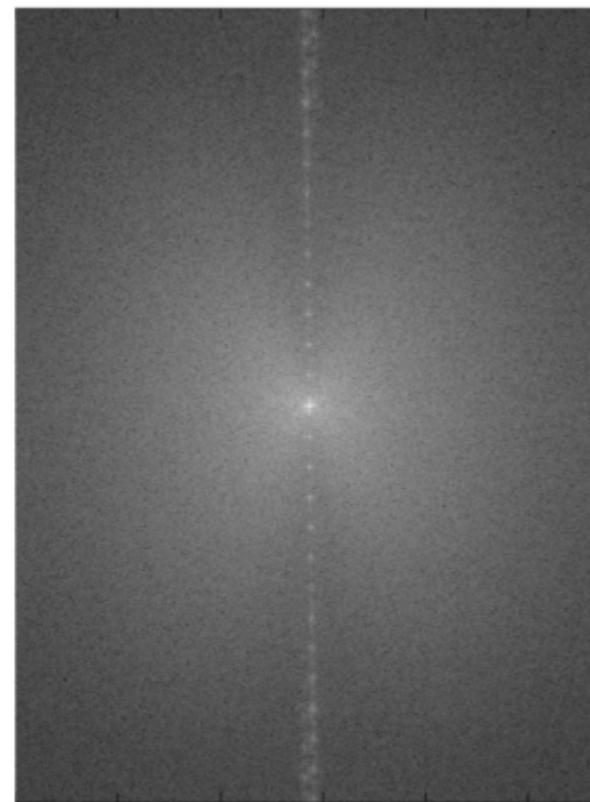
Amplitude





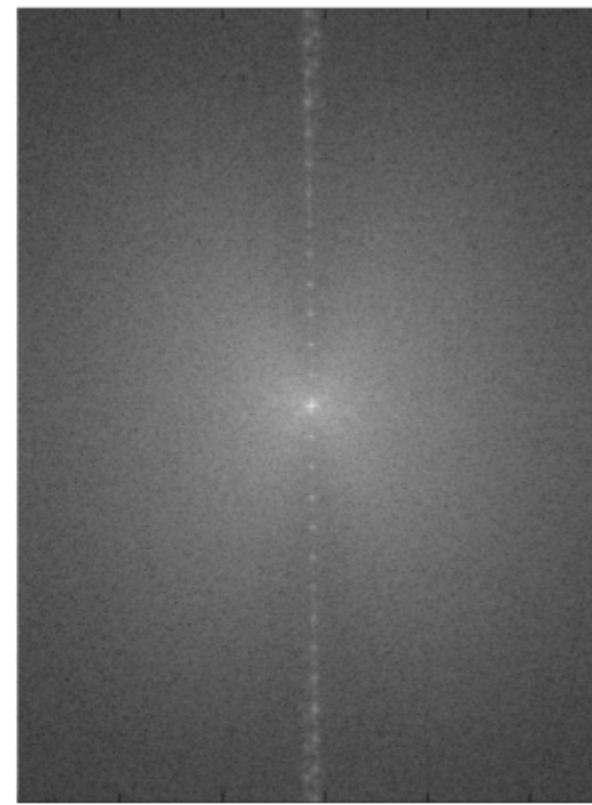


Lunar Orbital Image (1966)

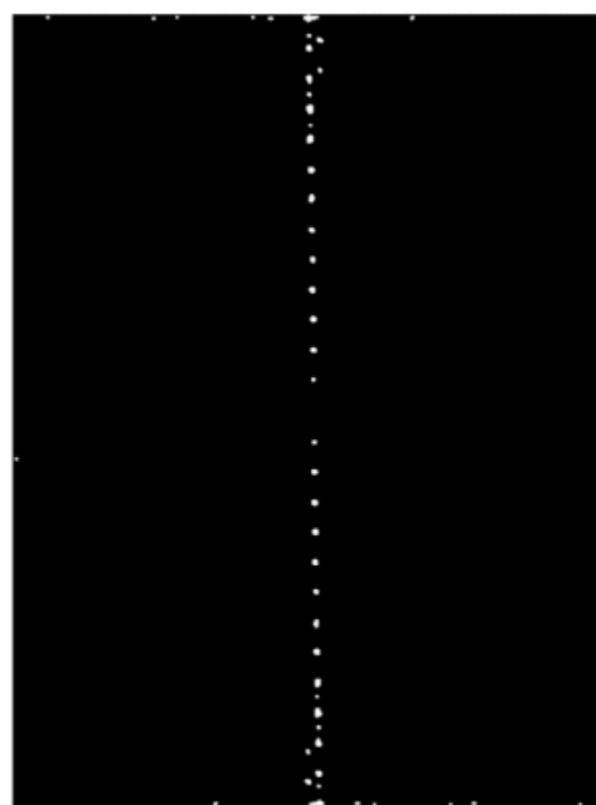


Amplitude

Lunar Orbital Image (1966)



Amplitude



Remove Peaks



Let's Practice

Some important Fourier transforms

Image

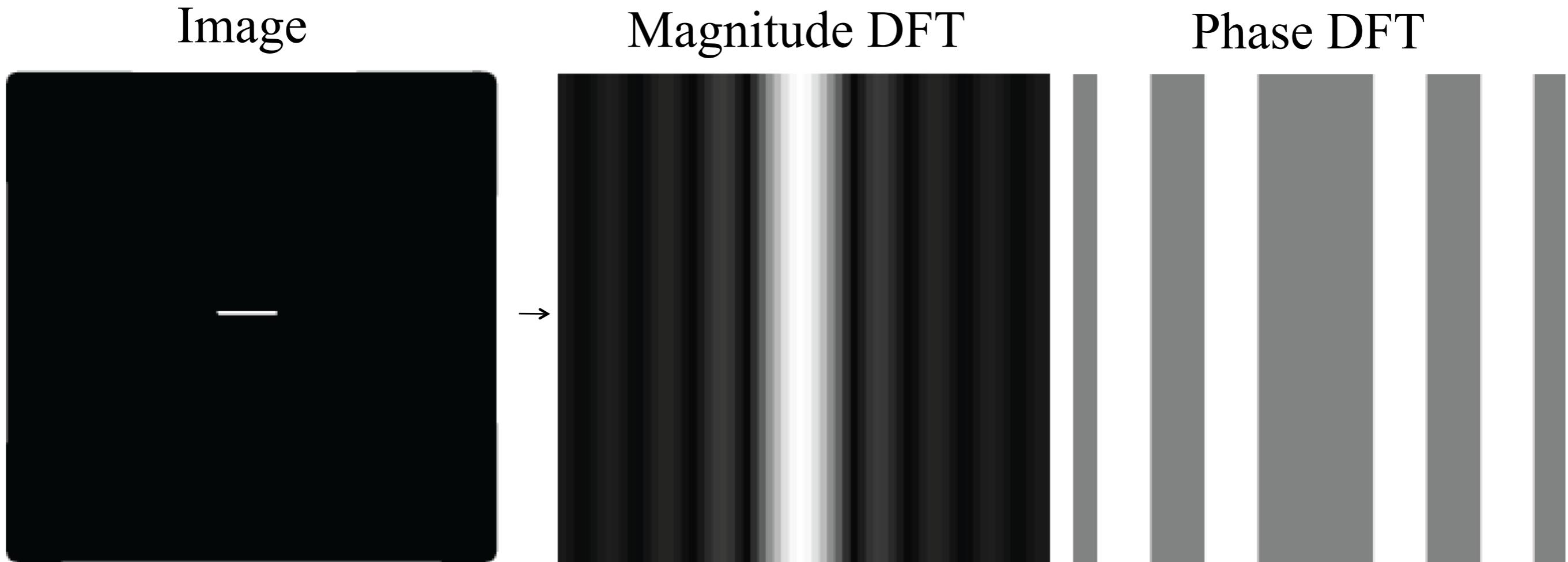
Magnitude DFT

Phase DFT



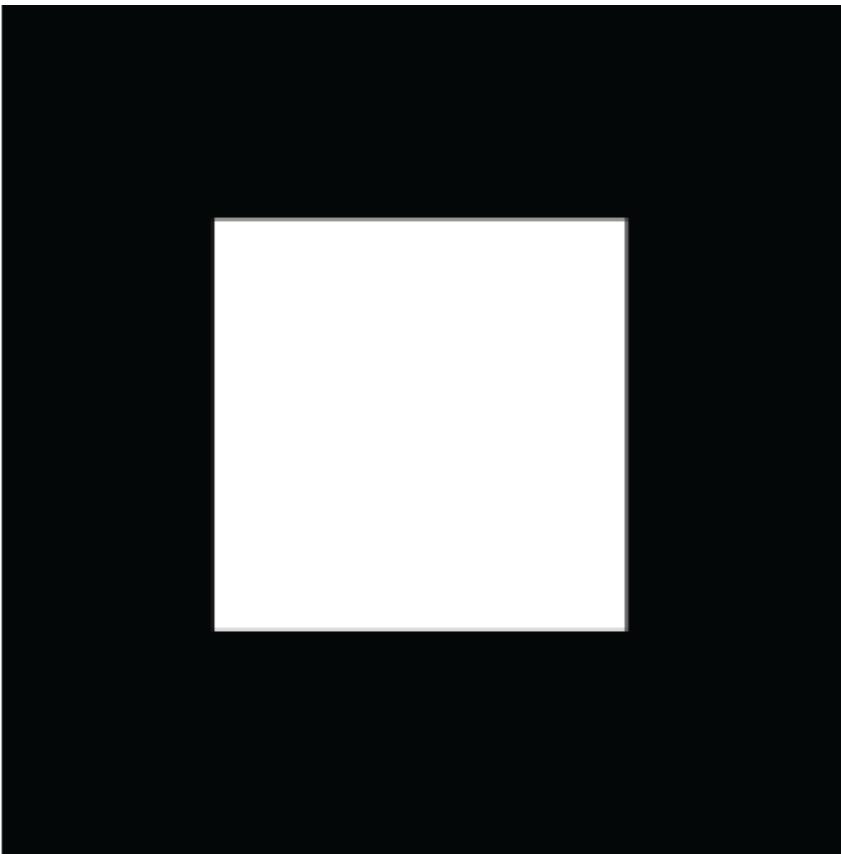
Images are 64x64 pixels.

Some important Fourier transforms

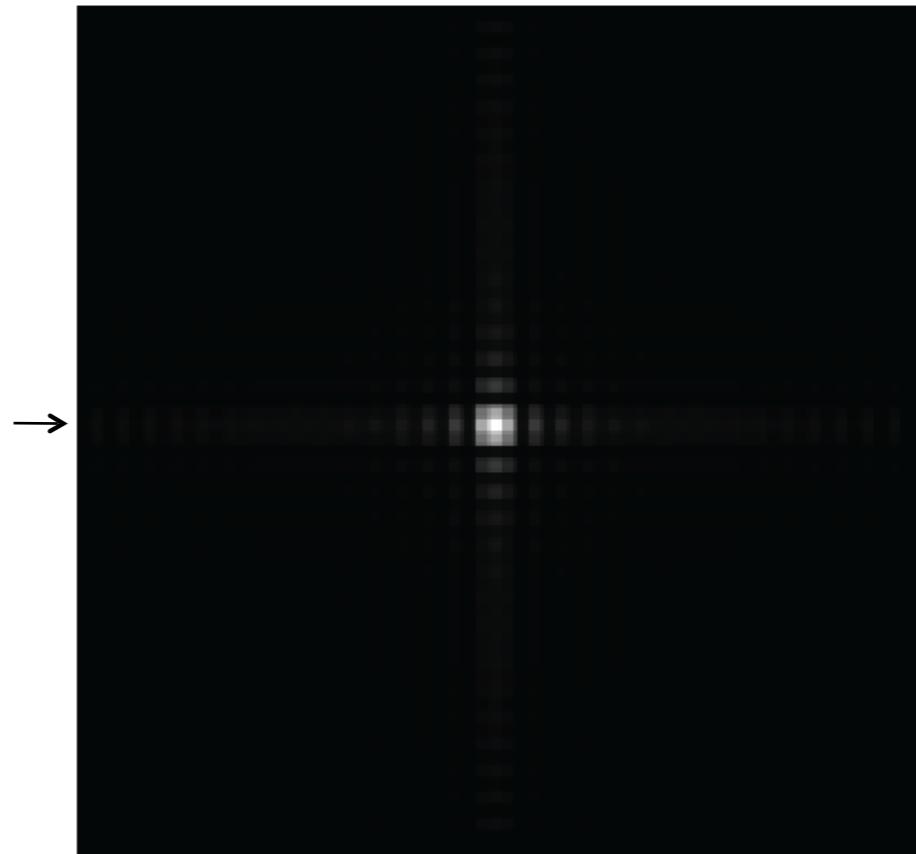


Some important Fourier transforms

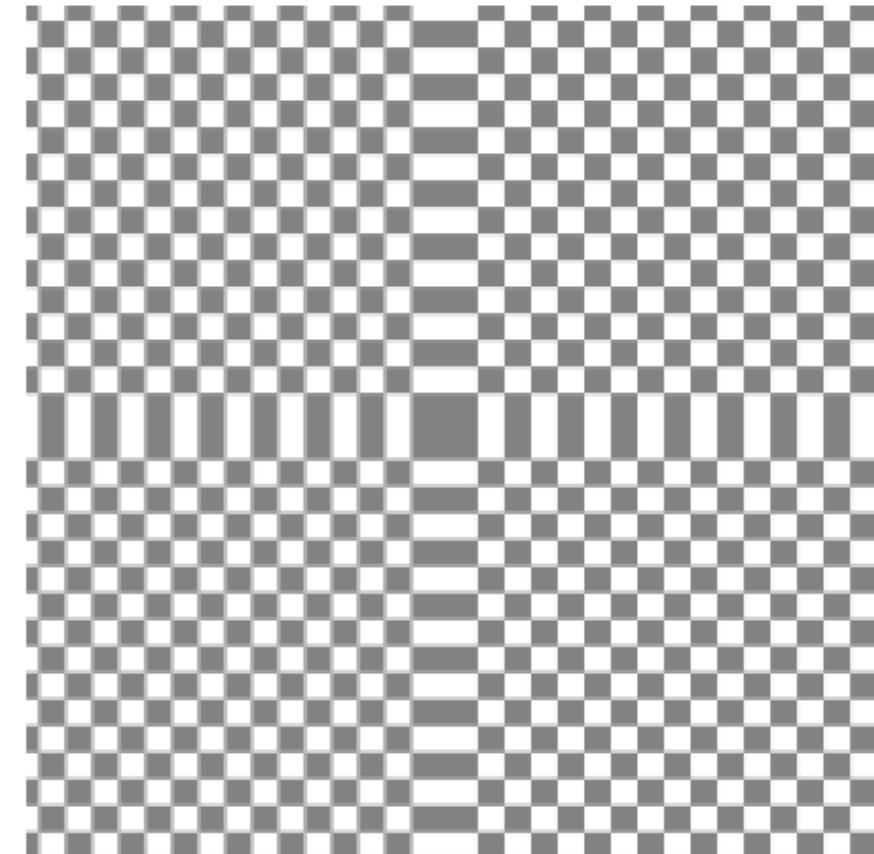
Image



Magnitude DFT

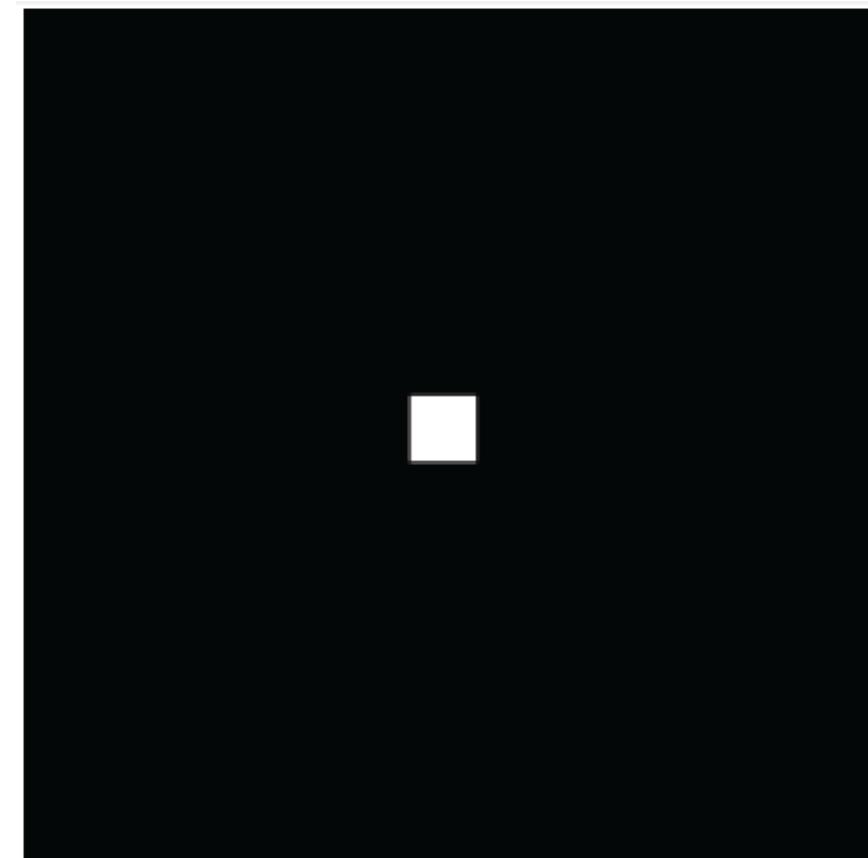


Phase DFT

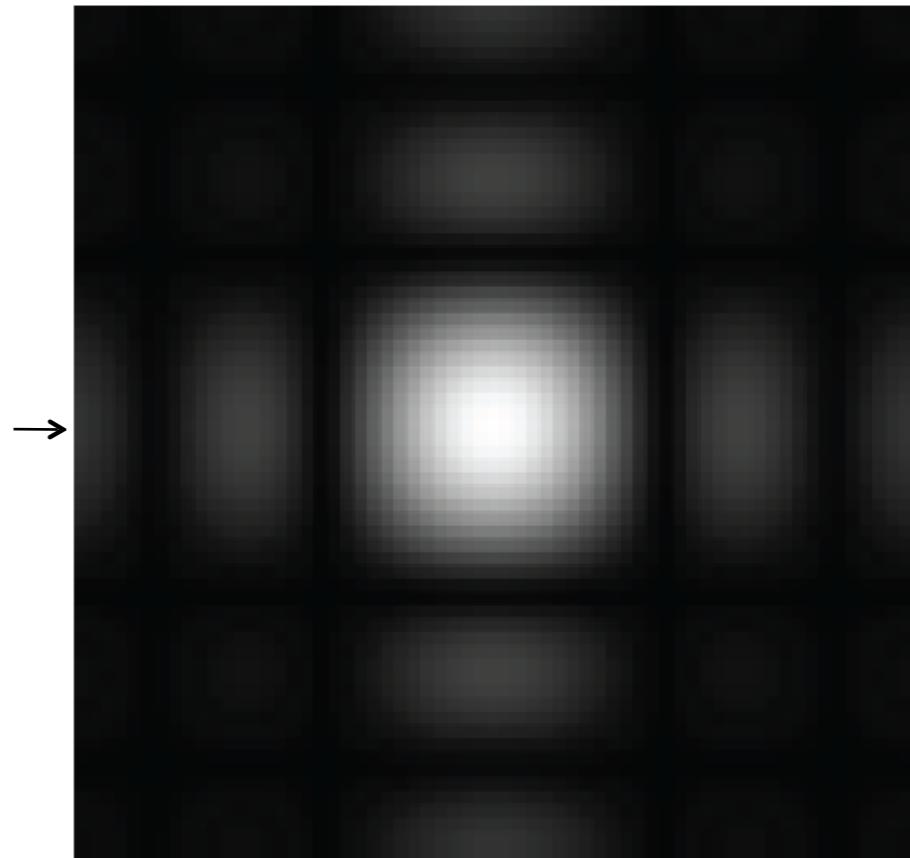


Some important Fourier transforms

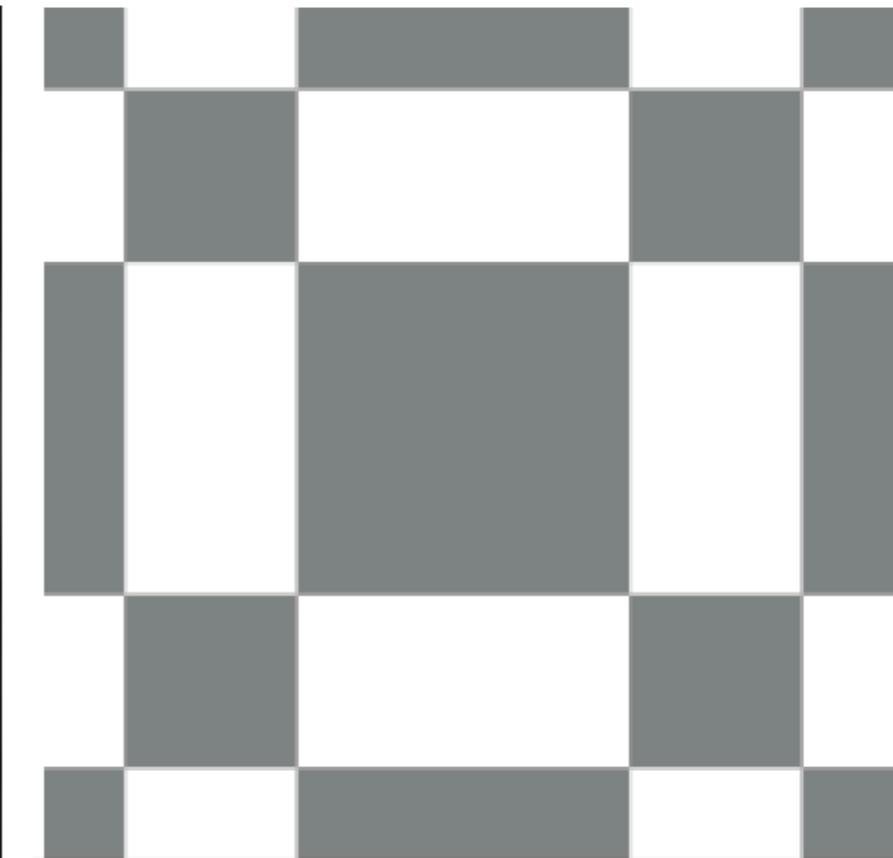
Image



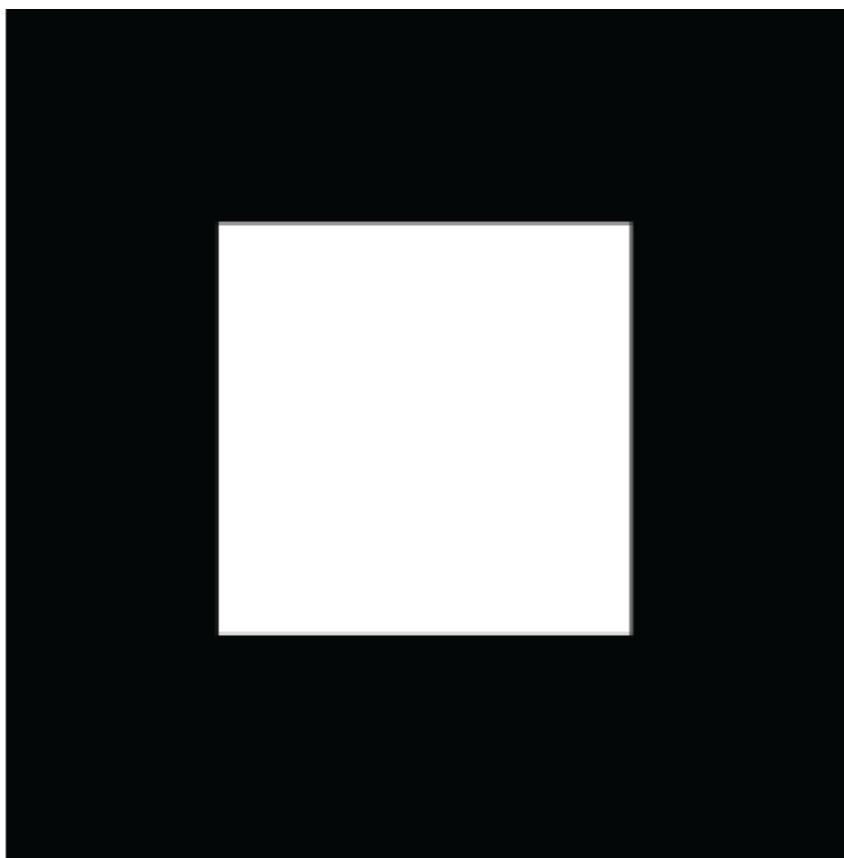
Magnitude DFT



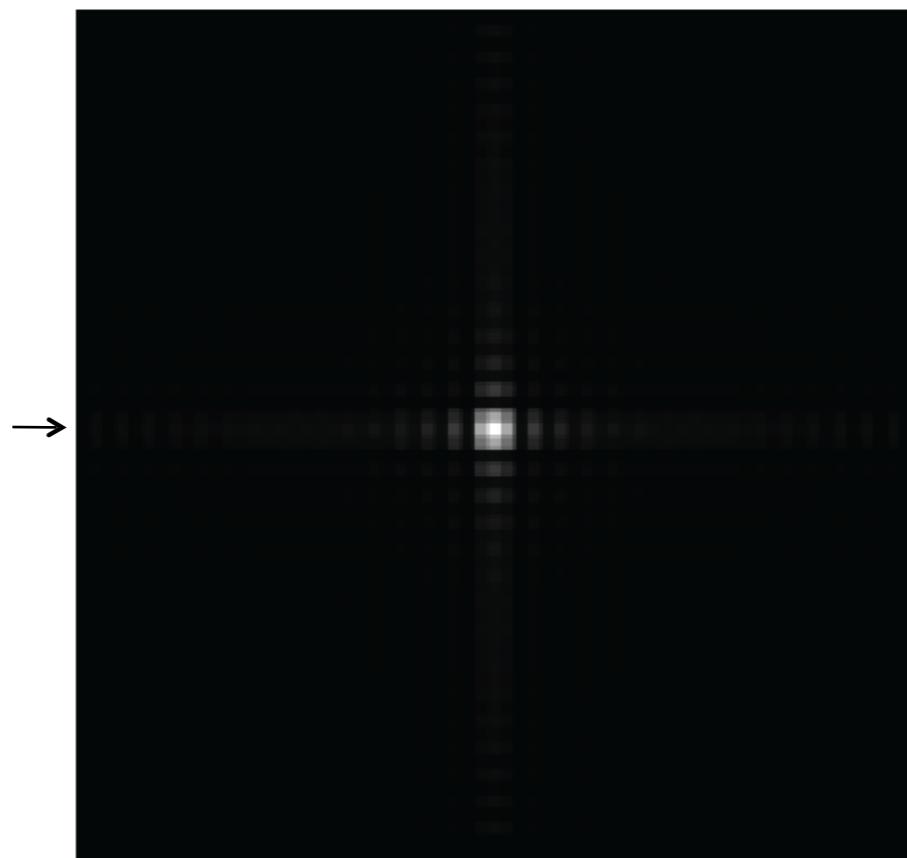
Phase DFT



Image

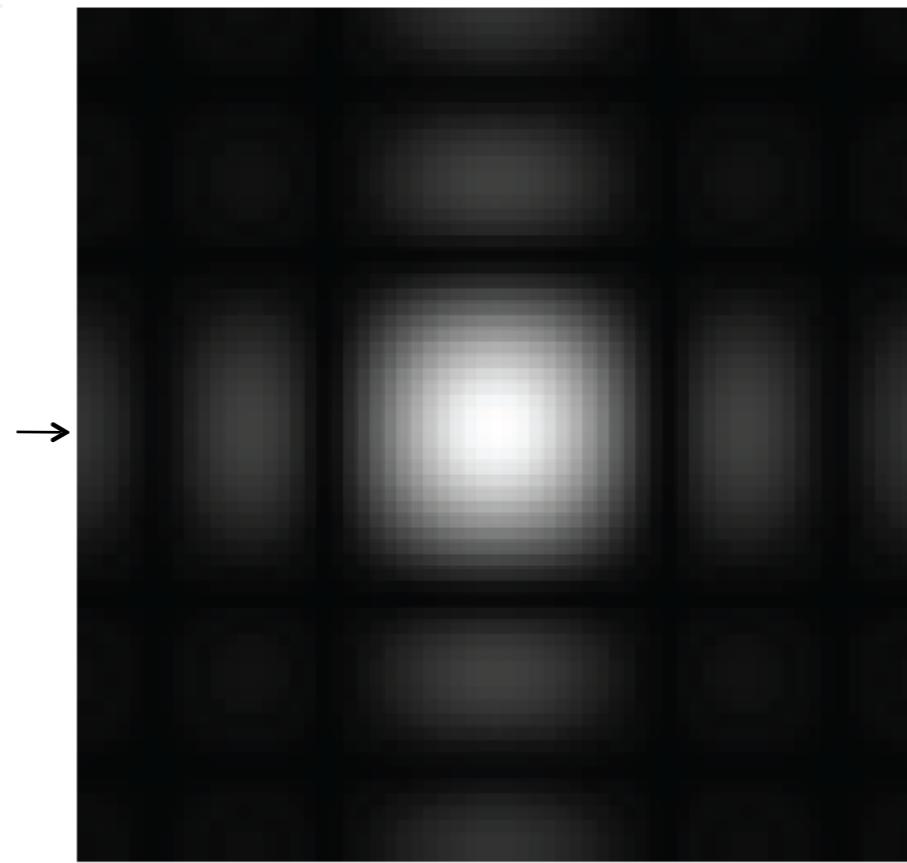
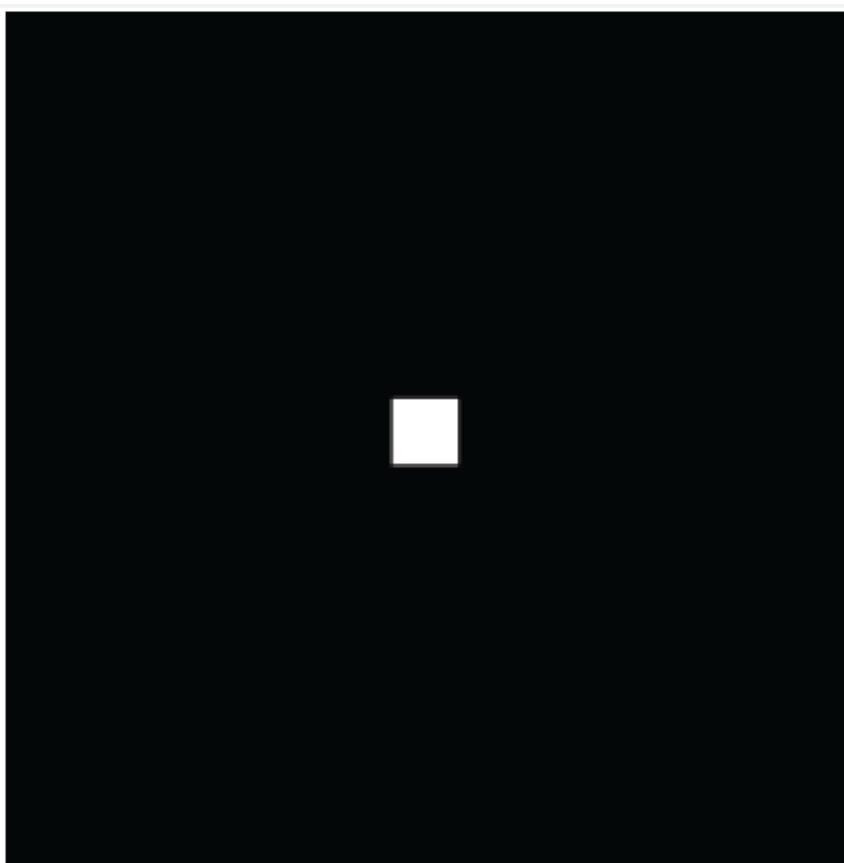


Magnitude DFT

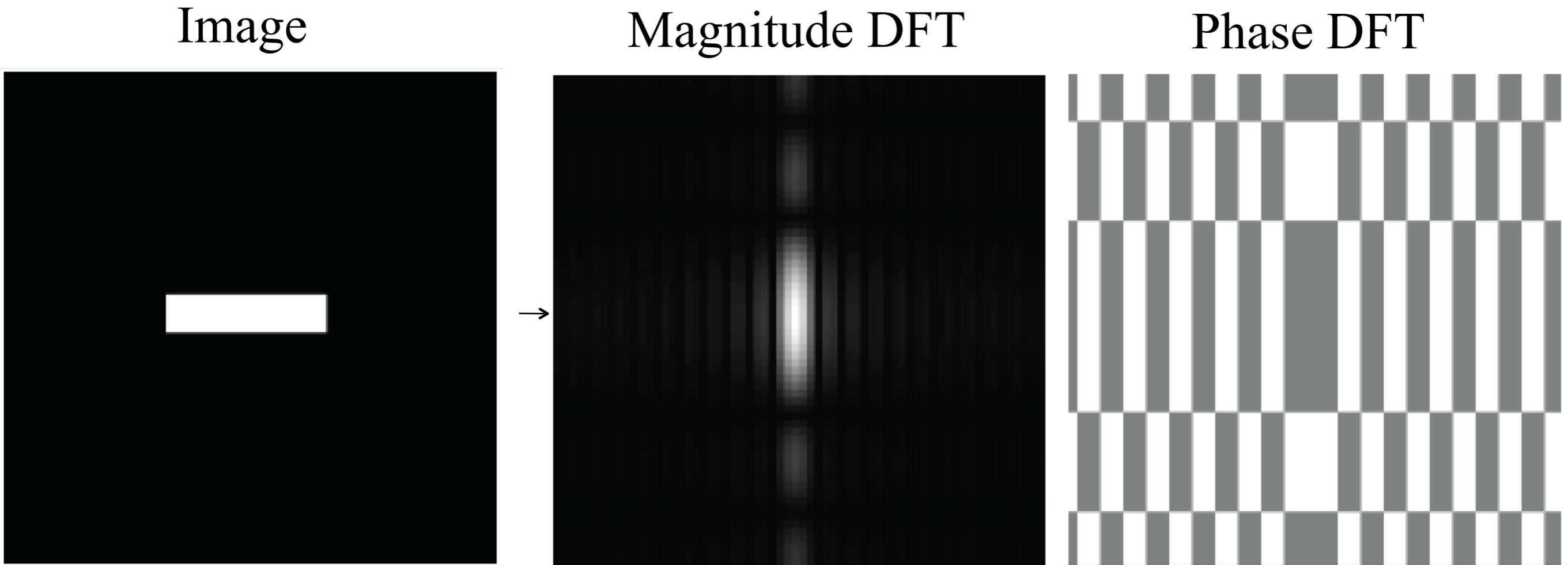


Scale

Small image details produce content in high spatial frequencies

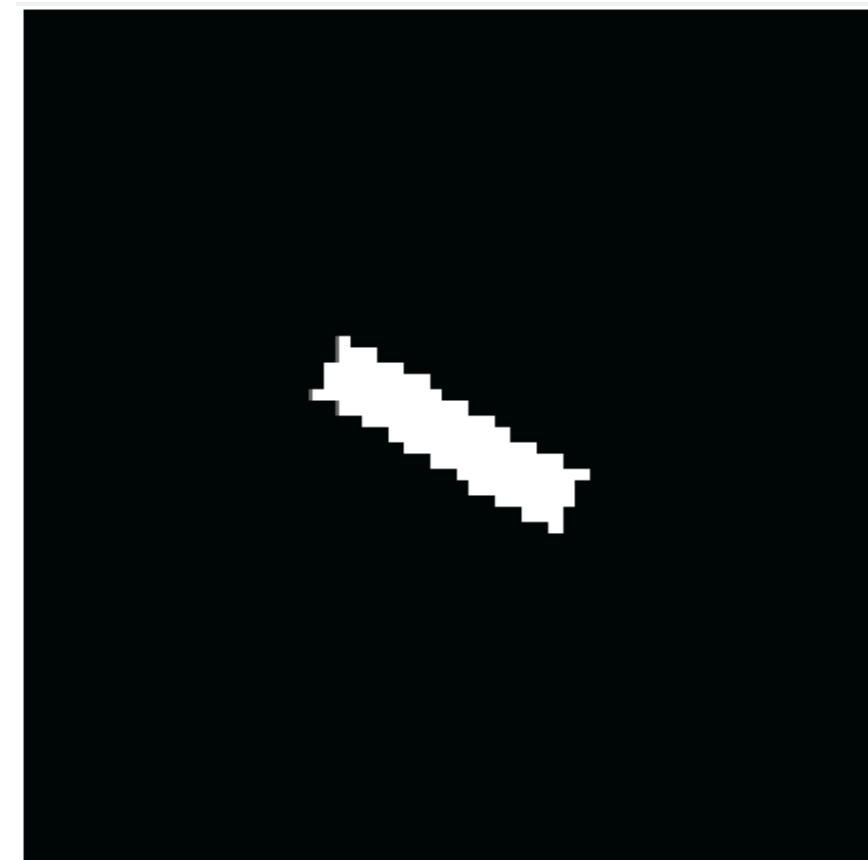


Some important Fourier transforms

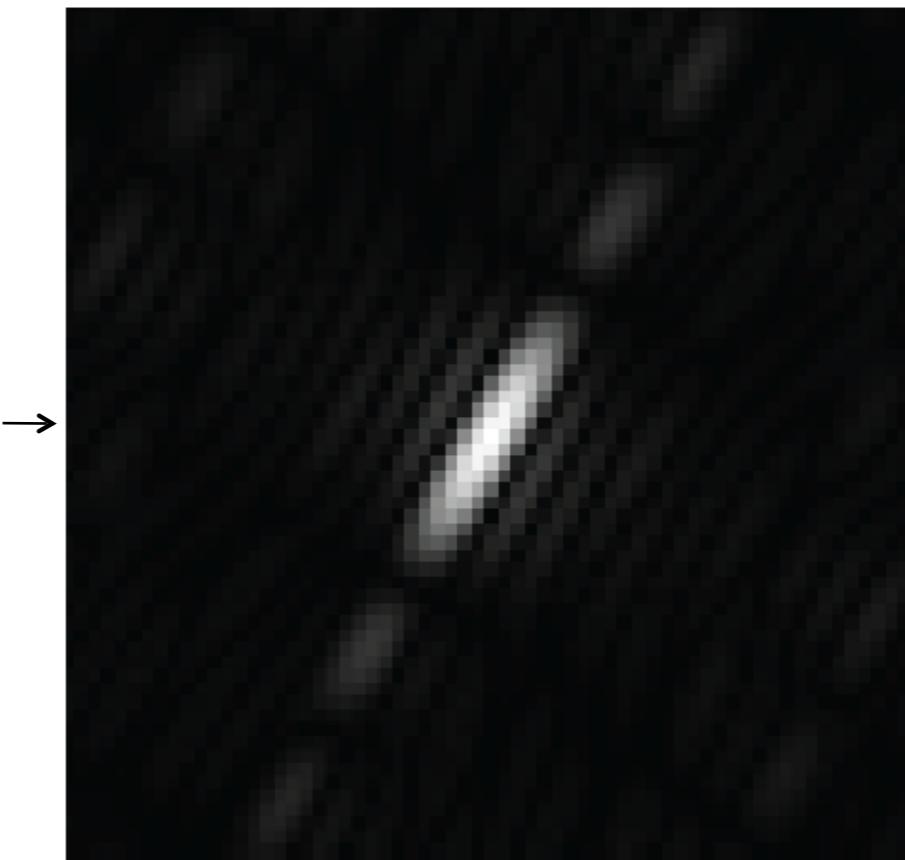


Some important Fourier transforms

Image



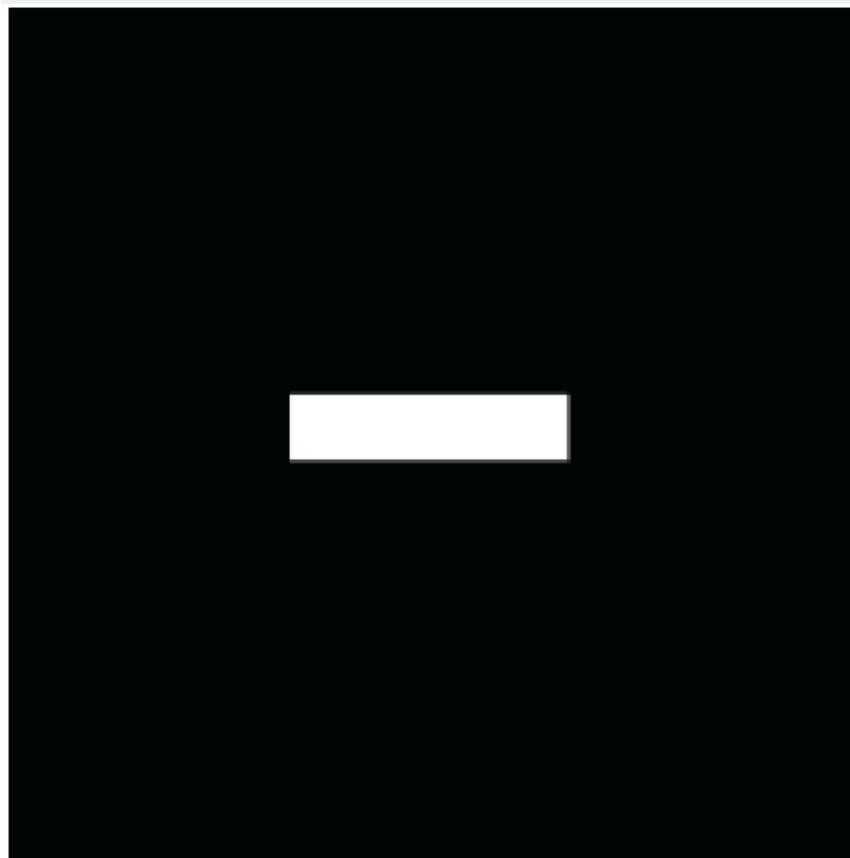
Magnitude DFT



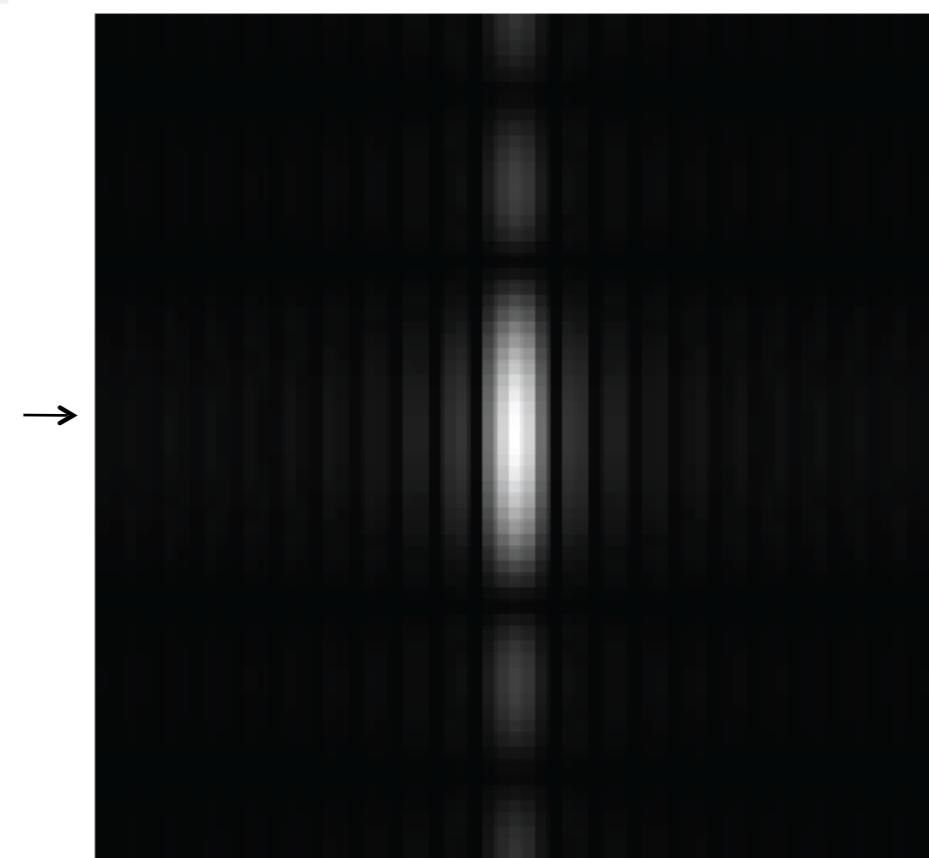
Phase DFT



Image

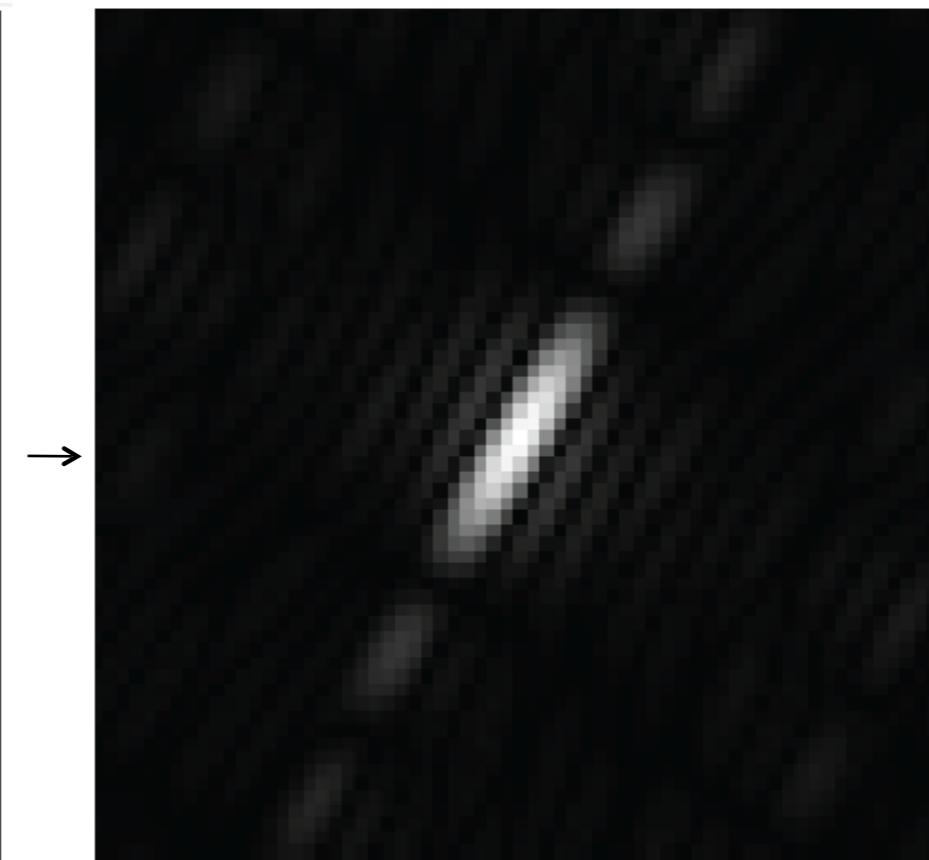
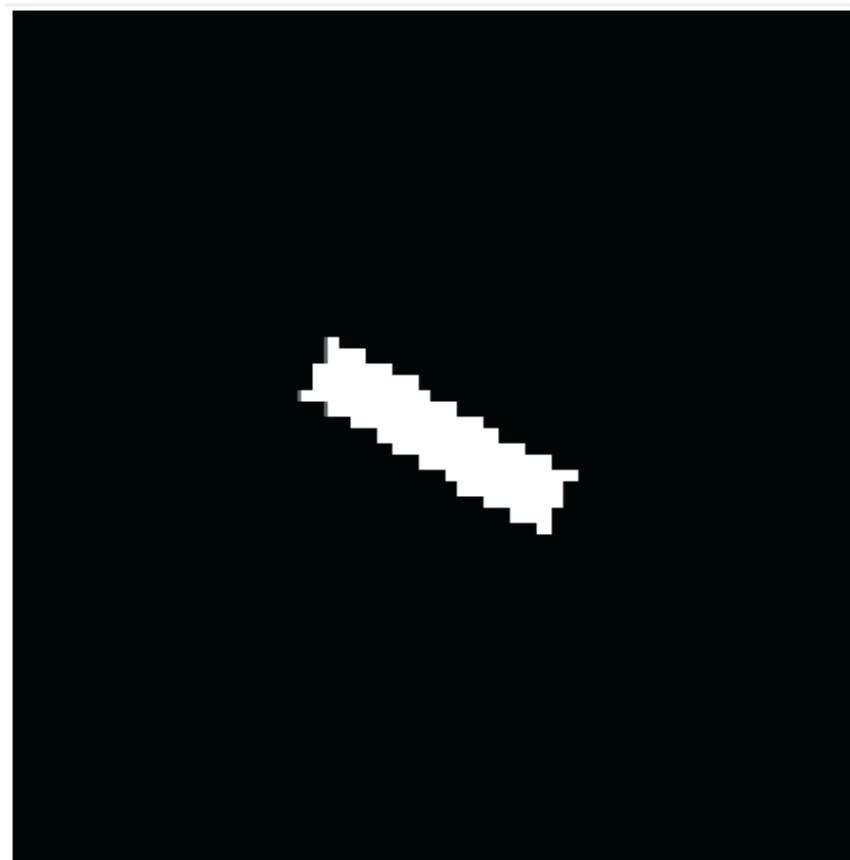


Magnitude DFT



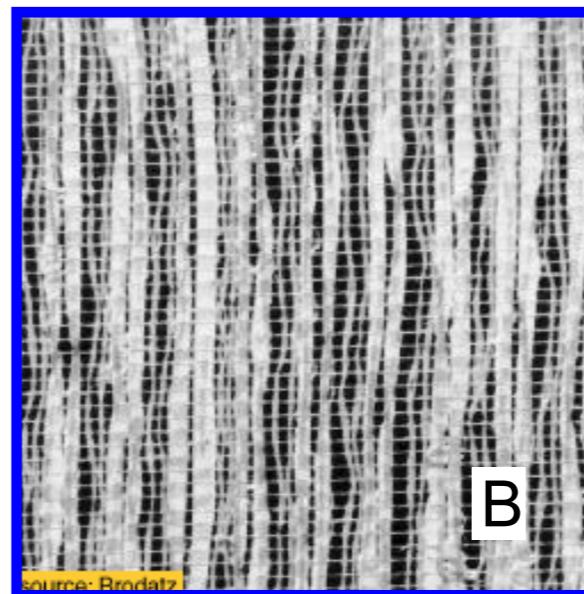
Orientation

A line transforms to a line oriented perpendicularly to the first.

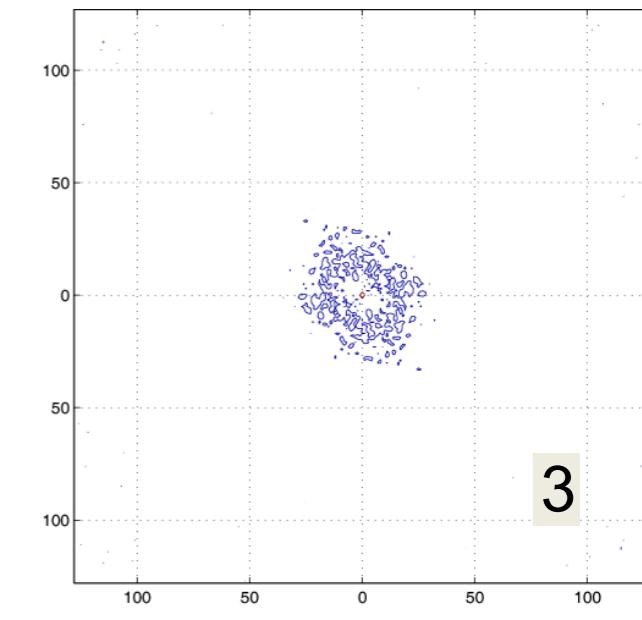
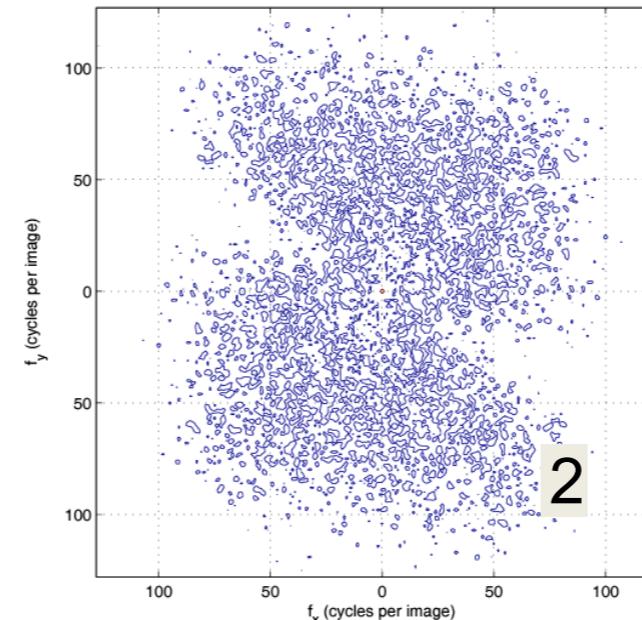
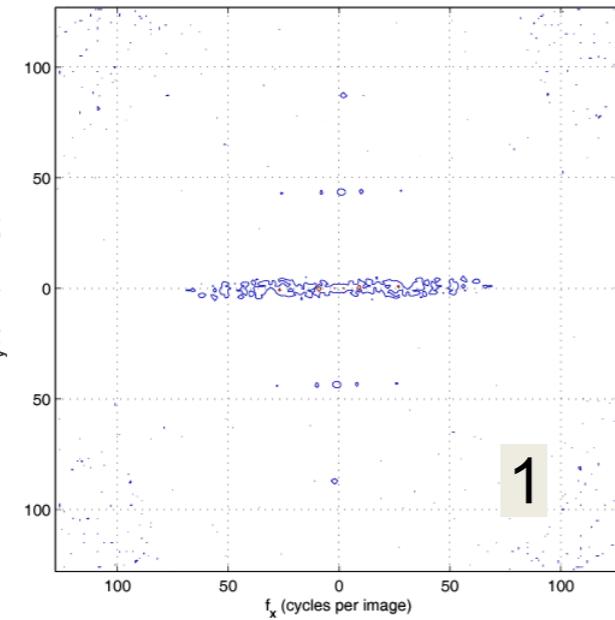


Game: find the right pairs

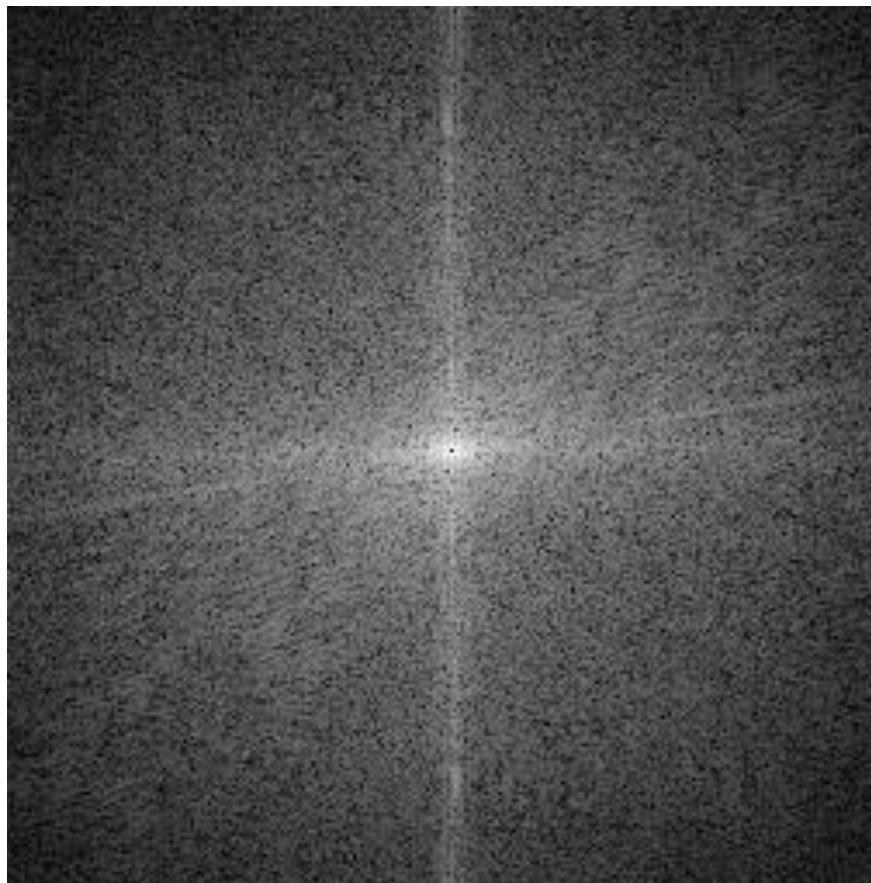
Images



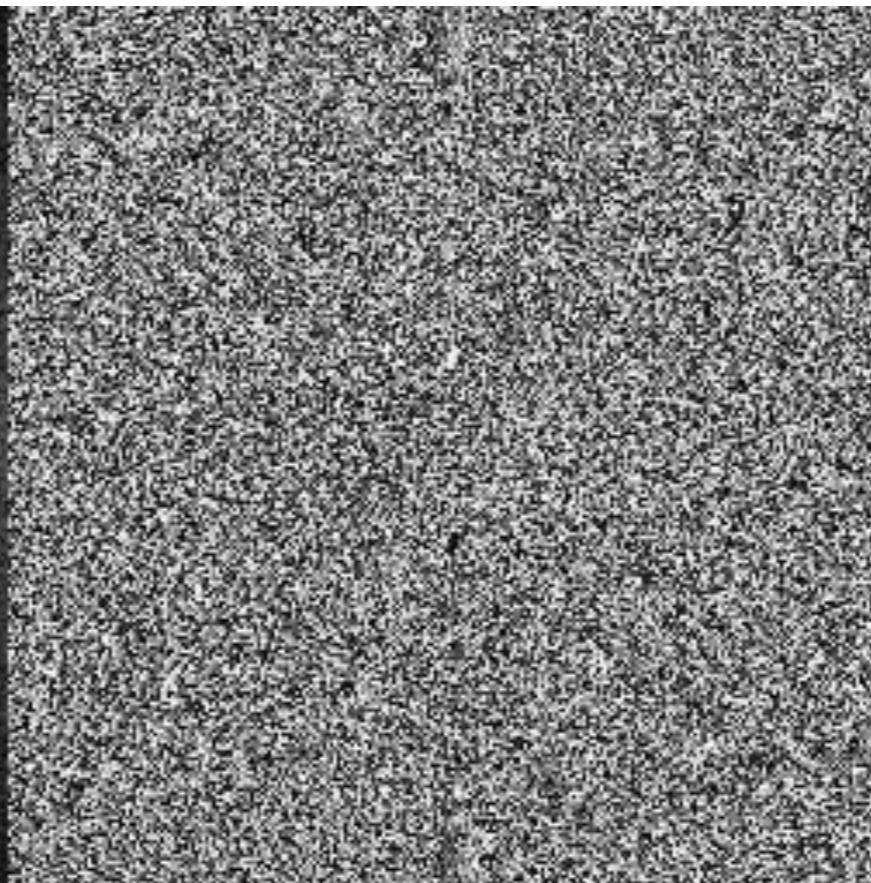
DFT
magnitude



Magnitude



Phase

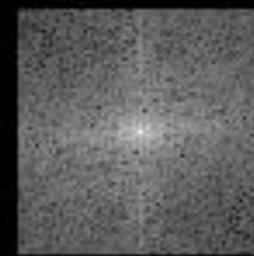


Reconstruction

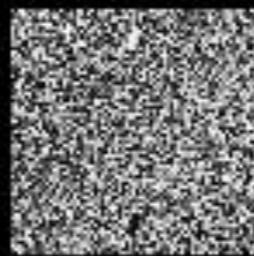


Removing High Frequencies

Magnitude



Phase



Reconstruction

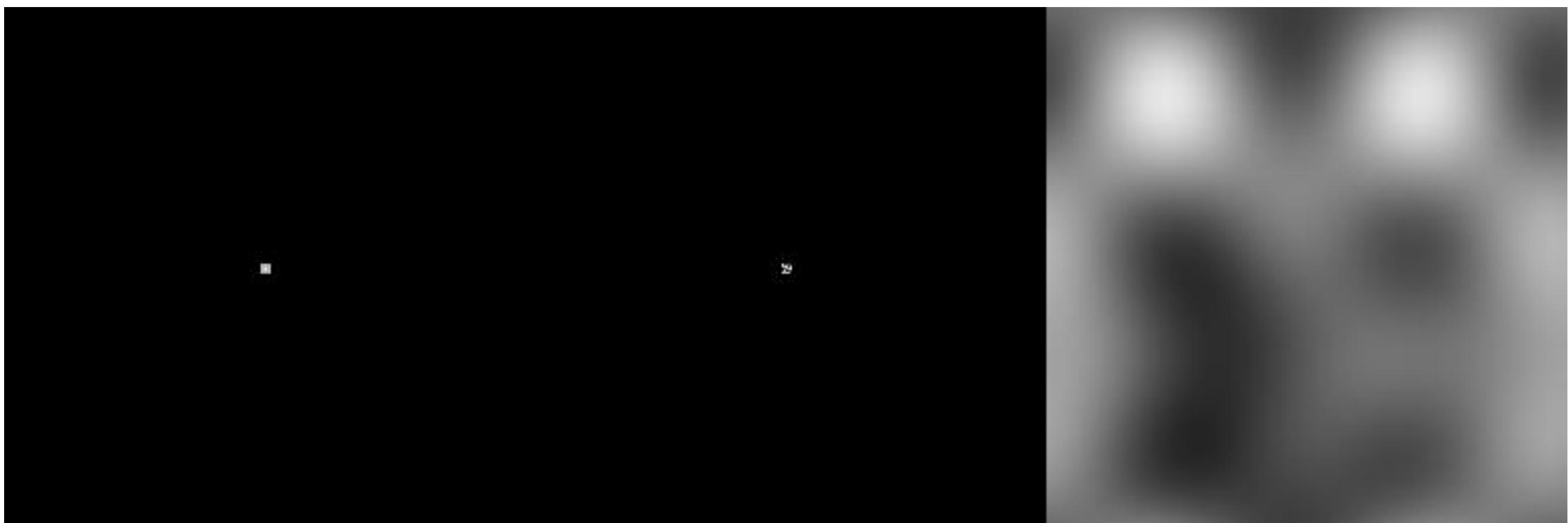


Removing High Frequencies

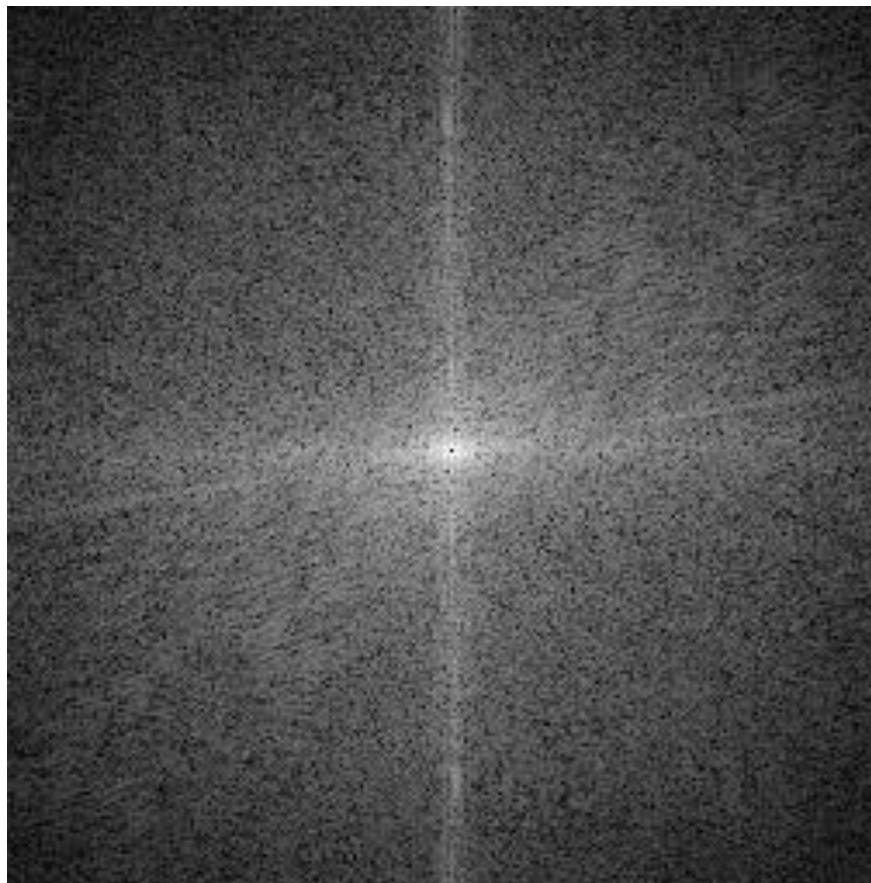
Magnitude

Phase

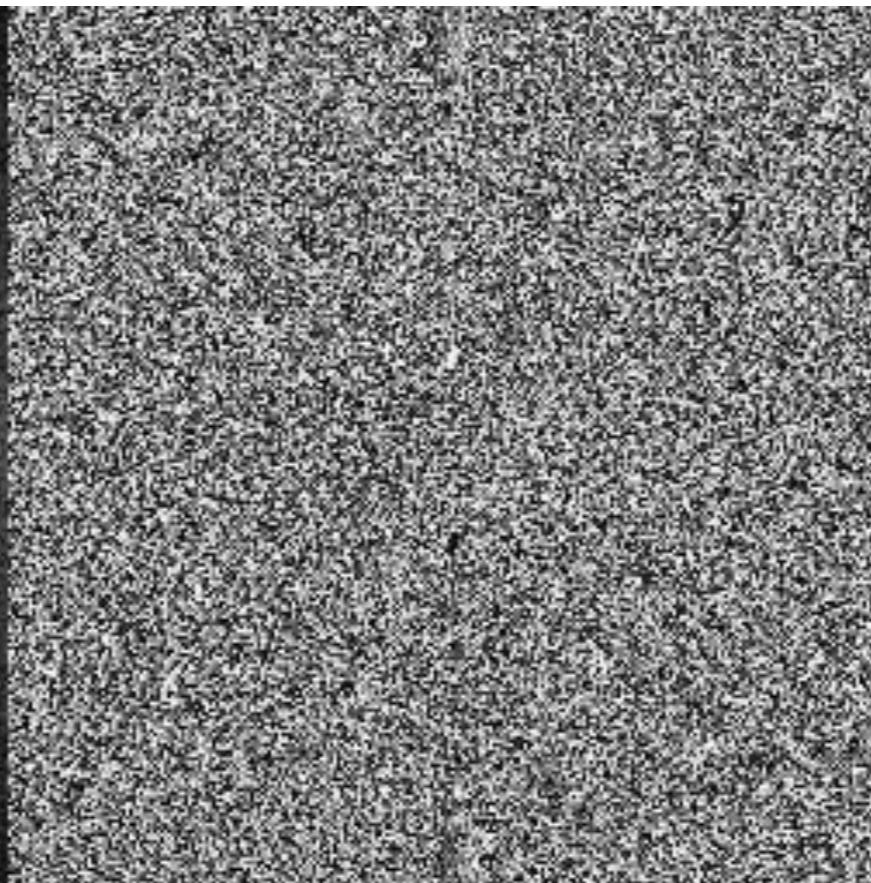
Reconstruction



Magnitude



Phase

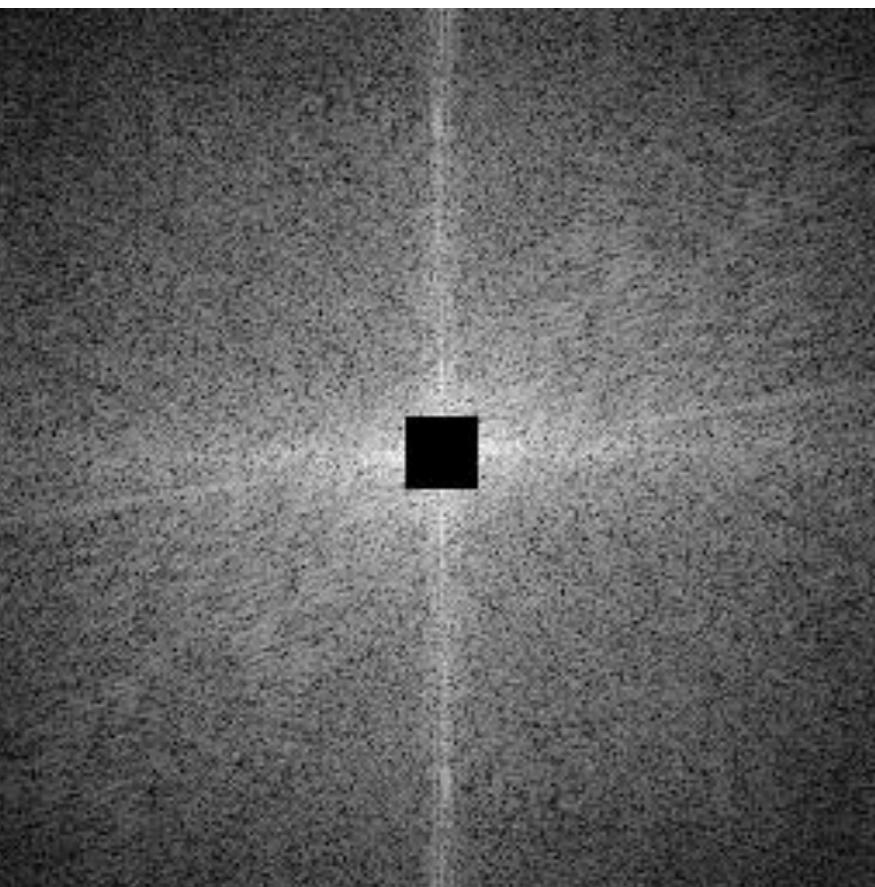


Reconstruction

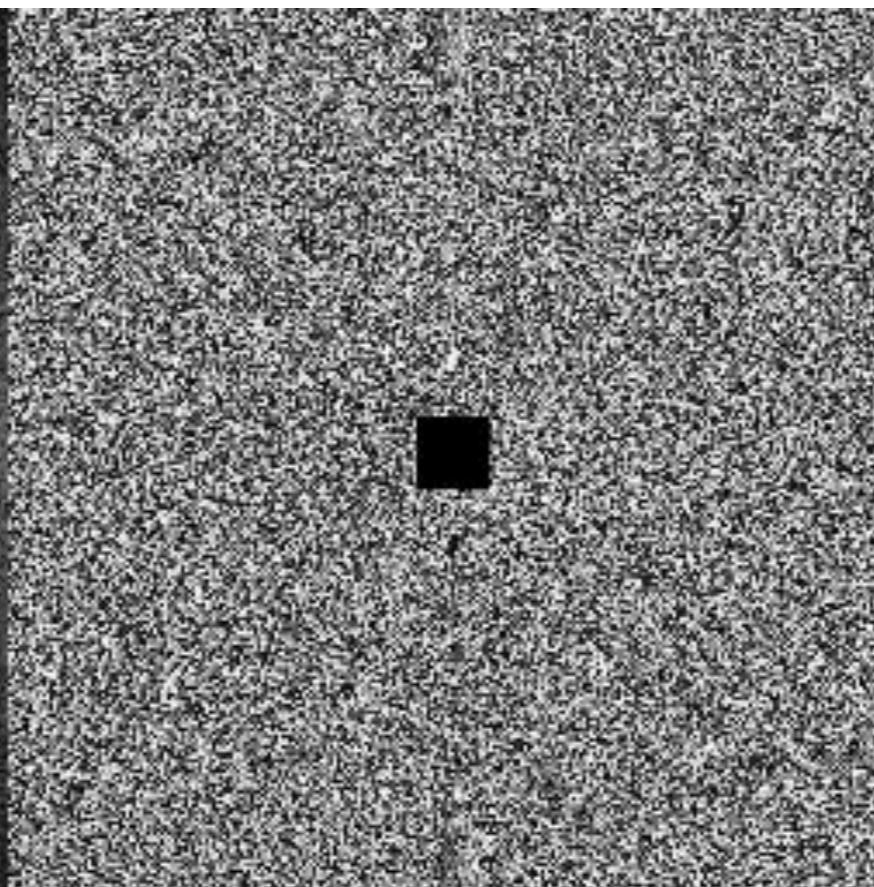


What will happen?

Magnitude



Phase

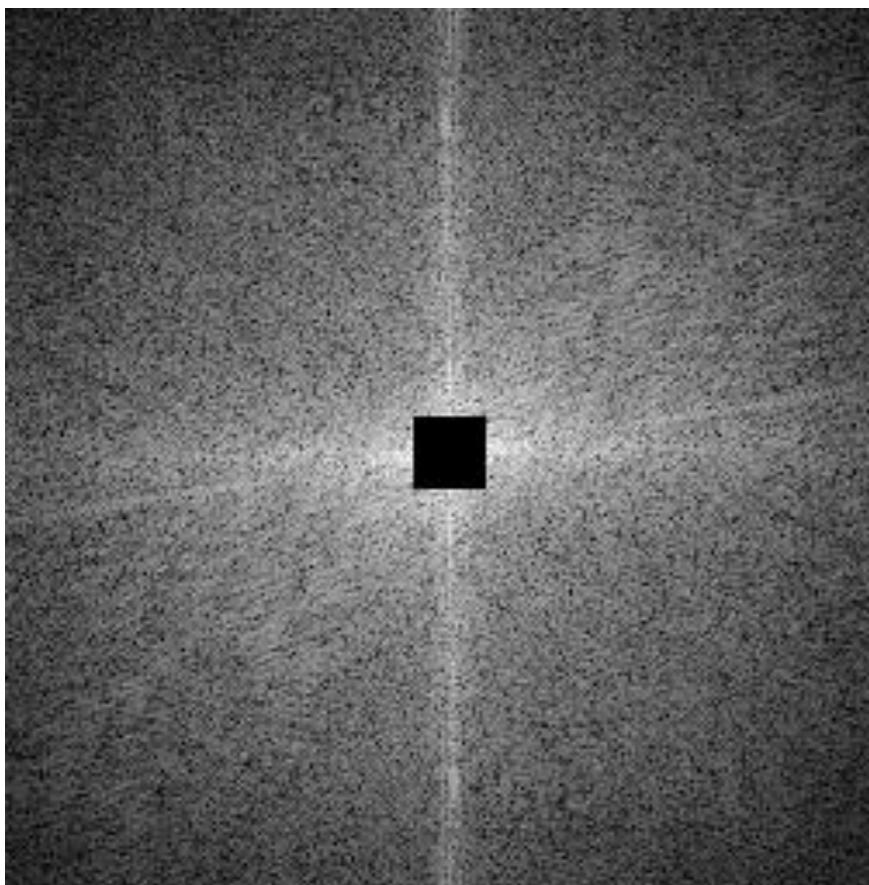


Reconstruction

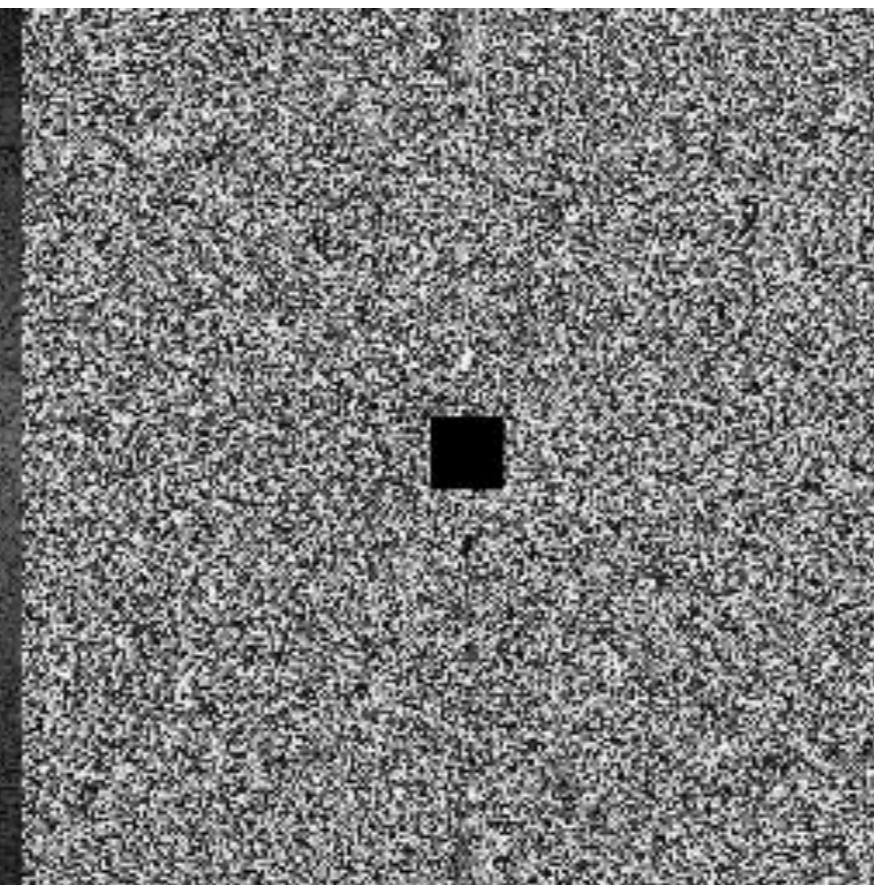
?

Removing Low Frequencies

Magnitude



Phase

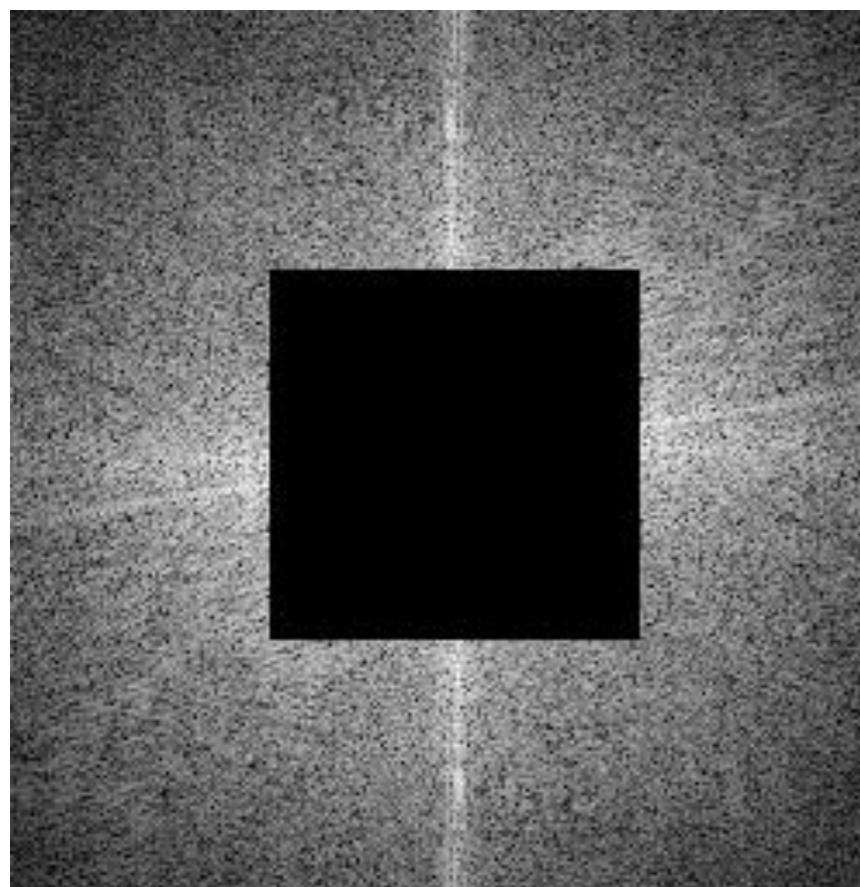


Reconstruction

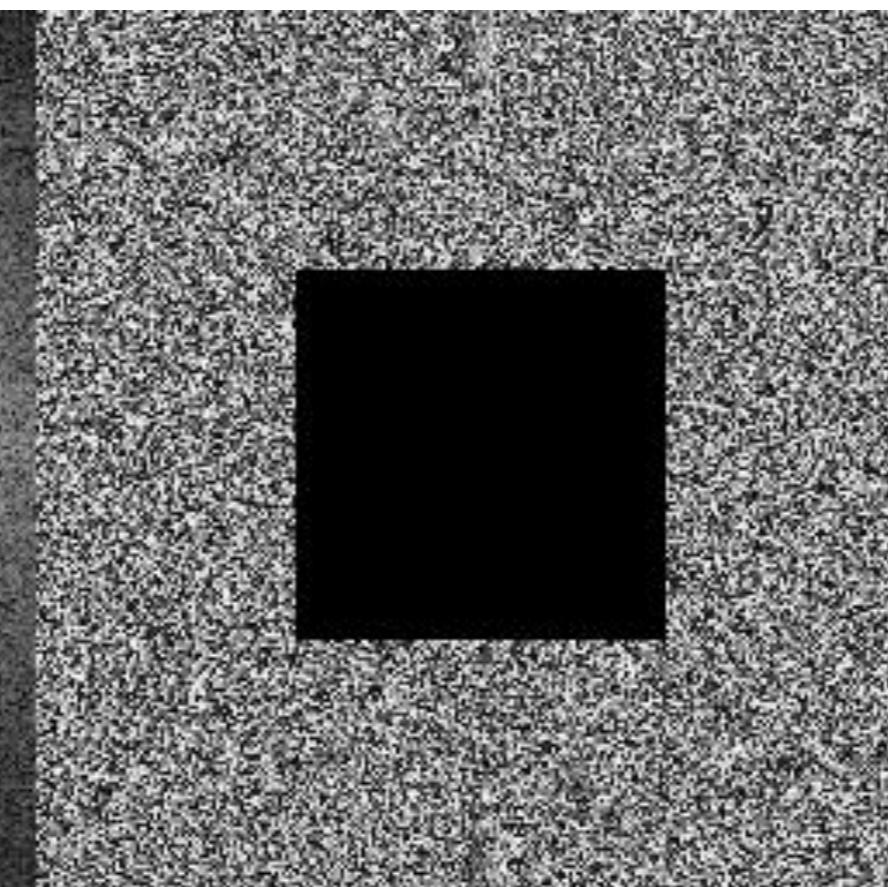


Removing Low Frequencies

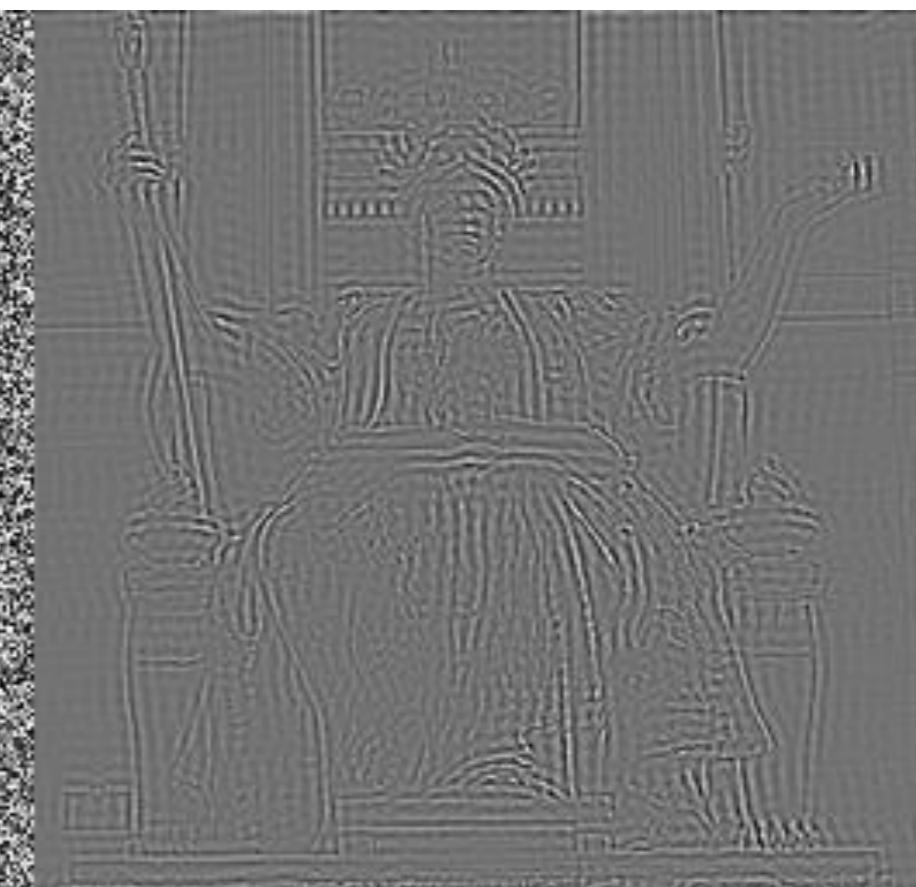
Magnitude



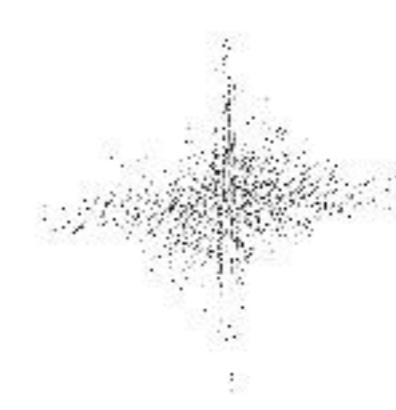
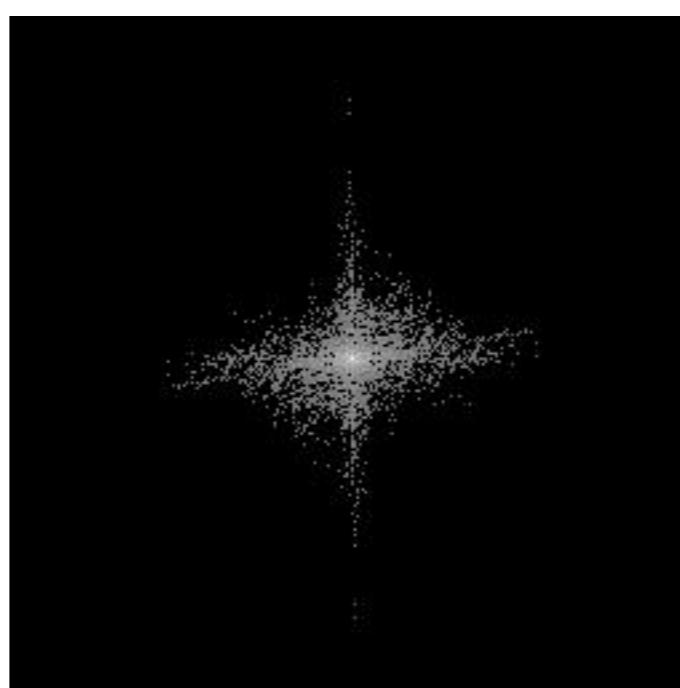
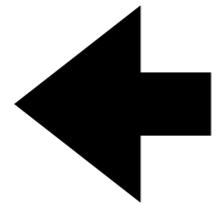
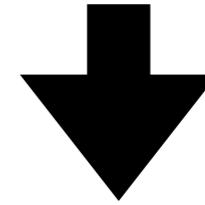
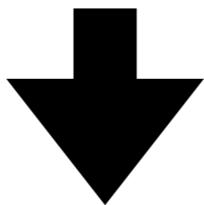
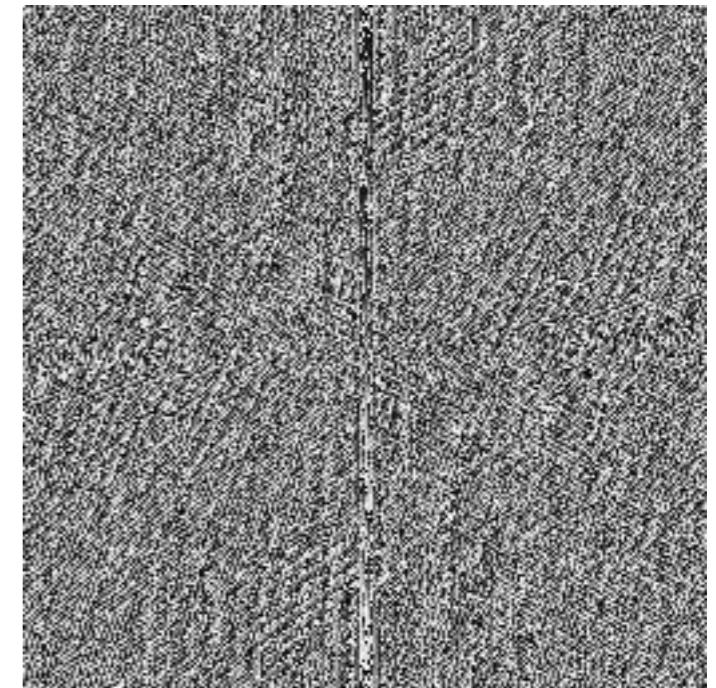
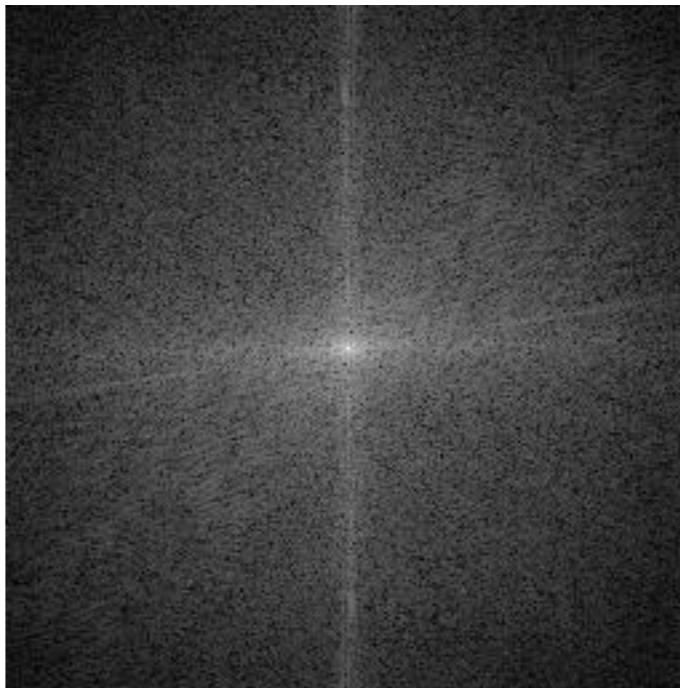
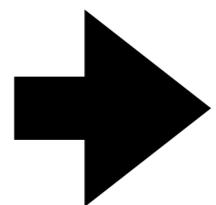
Phase



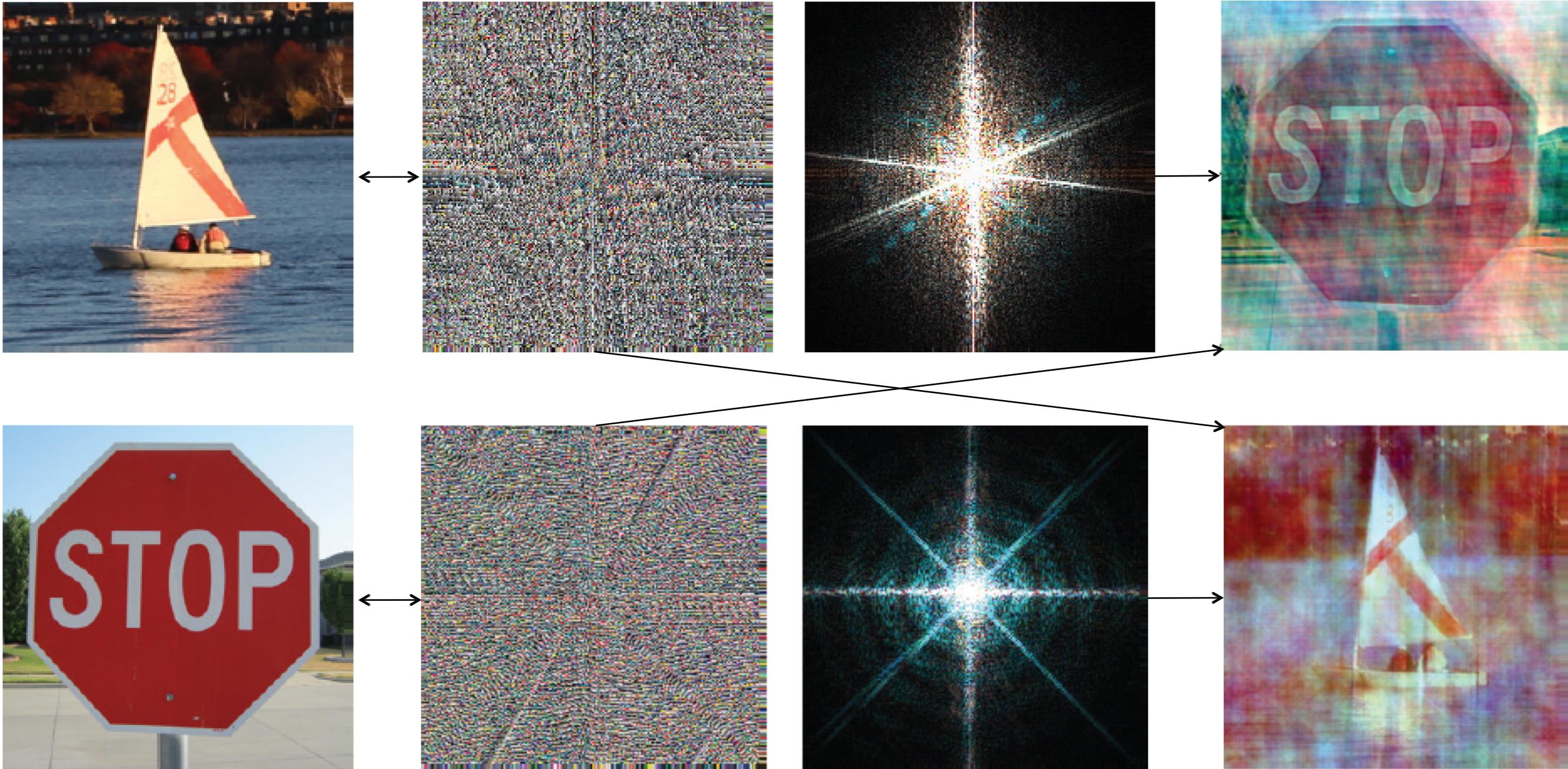
Reconstruction



Compression

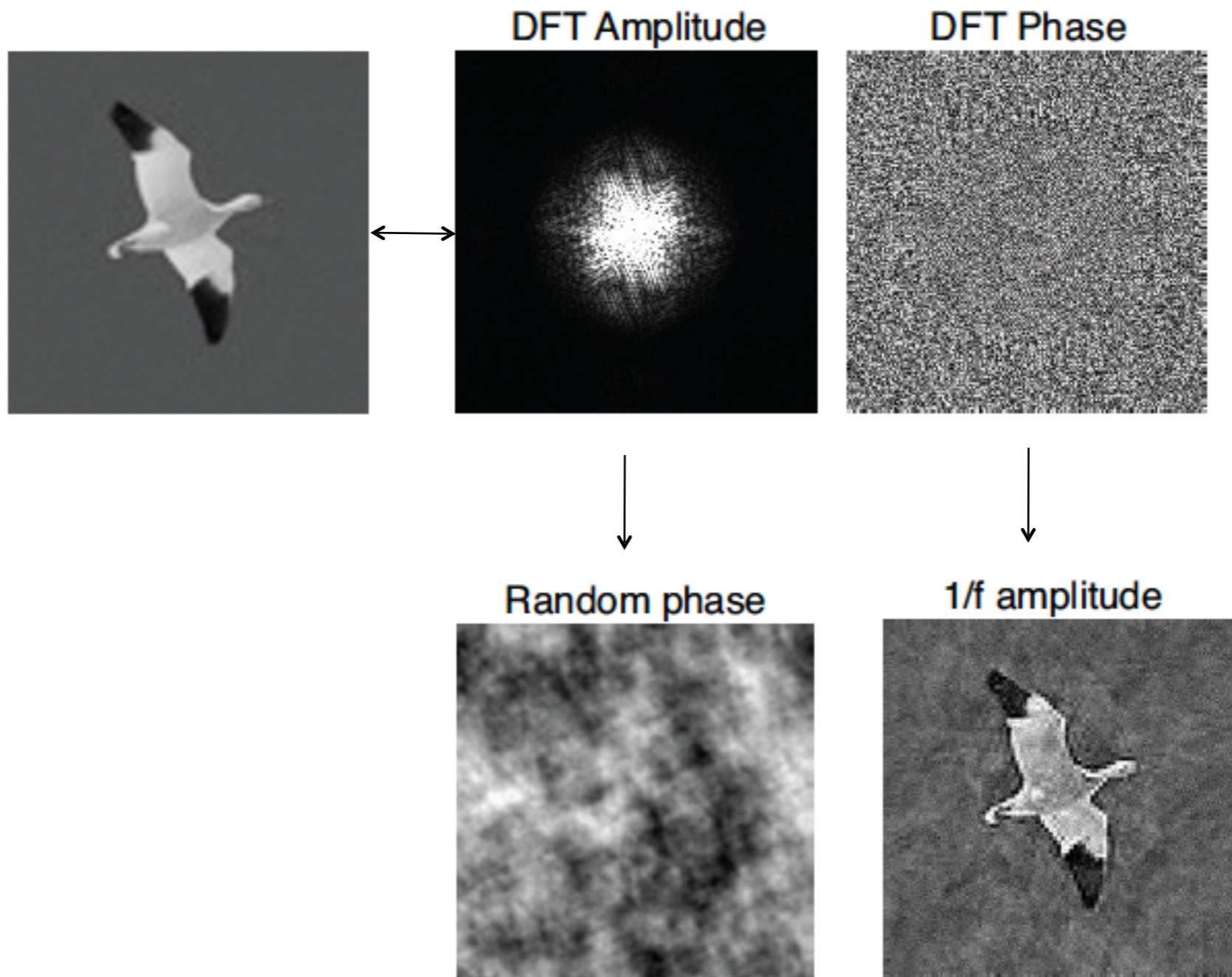


Phase and Magnitude

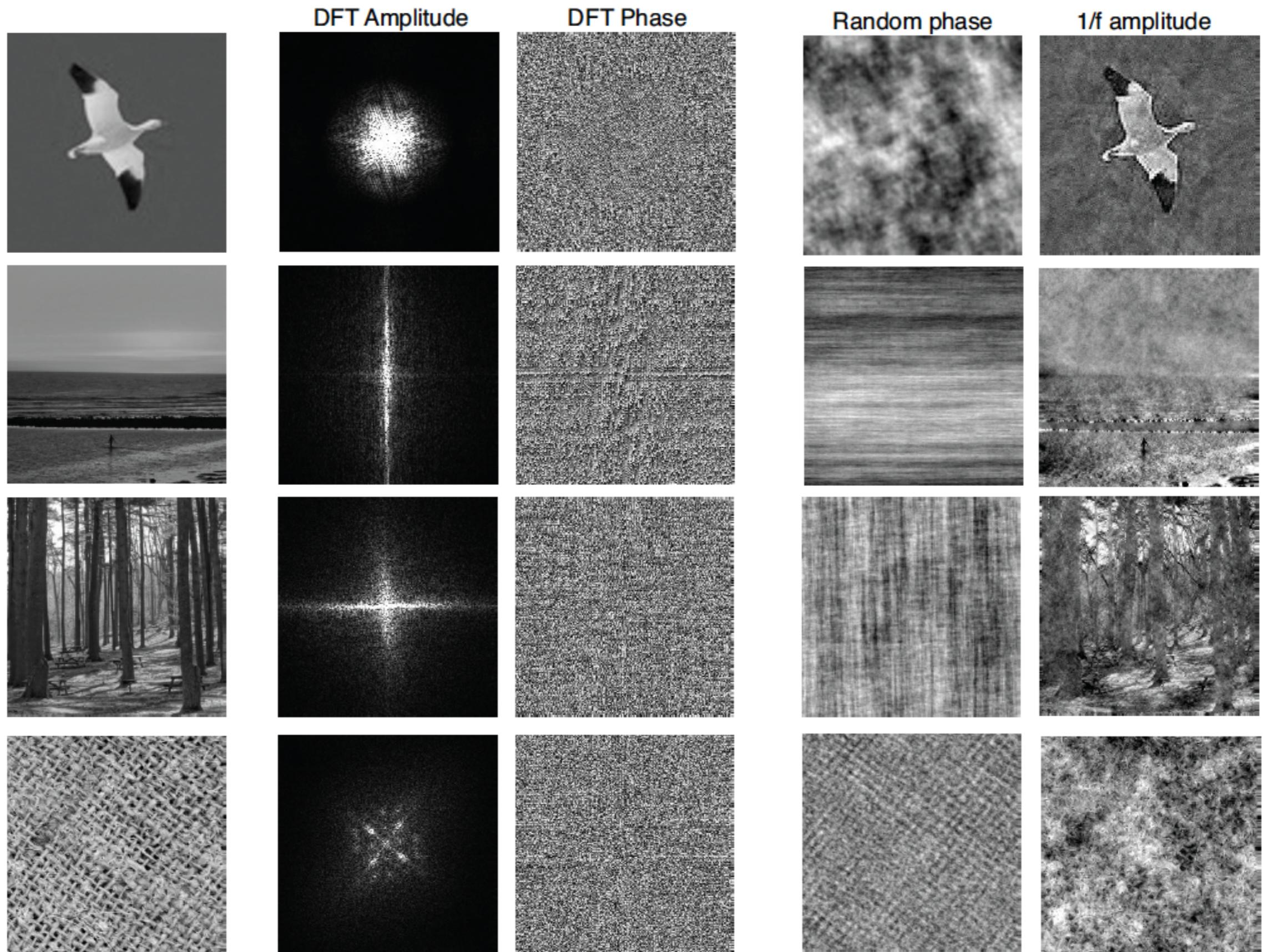


Each color channel is processed in the same way.

Phase and magnitude



Does phase always win?



Convolution

- Convolution in time space is multiplication in frequency space:

$$g(x) * h(x) = \mathcal{F}^{-1} [\mathcal{F}[g(x)] \cdot \mathcal{F}[h(x)]]$$

- Convolution in frequency space is multiplication in time space:

$$\mathcal{F}[g(x)] * \mathcal{F}[h(x)] = \mathcal{F} [g(x) \cdot h(x)]$$

Which is more computationally efficient?

FFT in Practice

**Fast Training of Convolutional Networks through
FFTs**

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Yann LeCun
Courant Institute of Mathematical Sciences
New York University
yann@cs.nyu.edu



Convolutional Neural Networks

Correlation Filters (600 FPS on CPU)

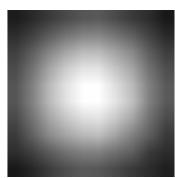
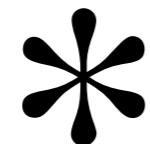
Image



Filter Response



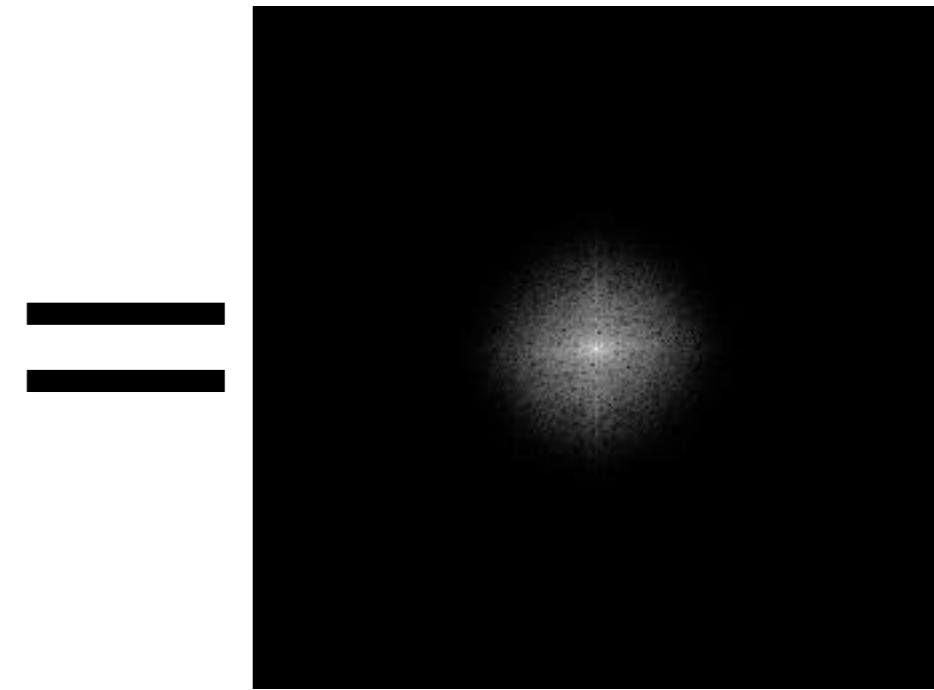
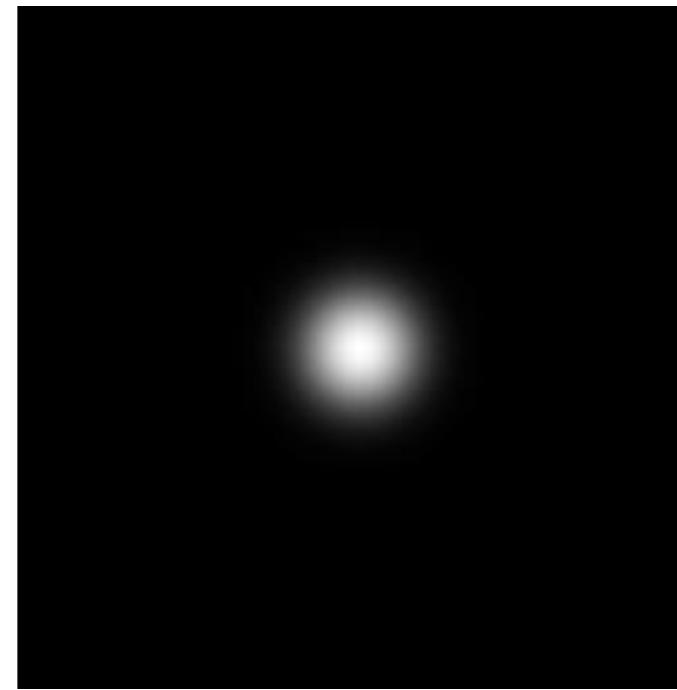
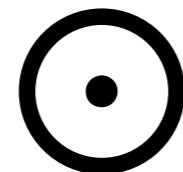
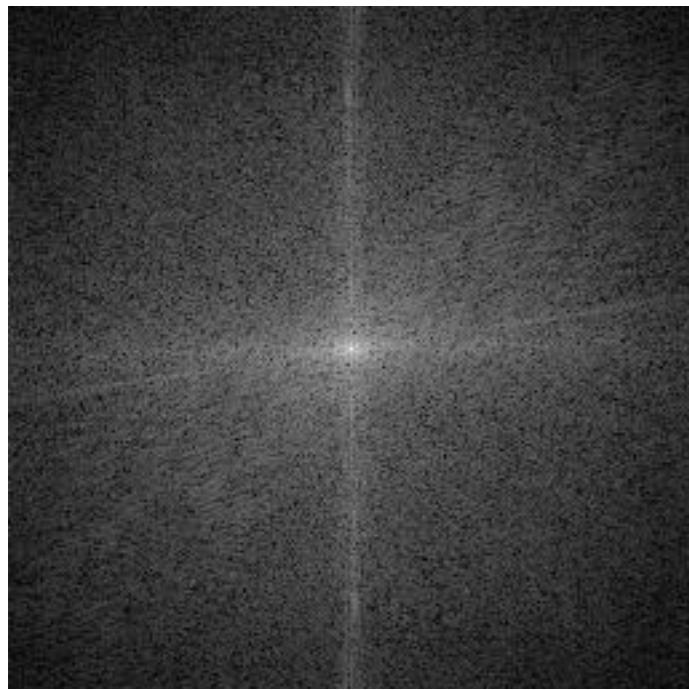
Gaussian
Filter



\mathcal{F}

\mathcal{F}

\mathcal{F}^{-1}

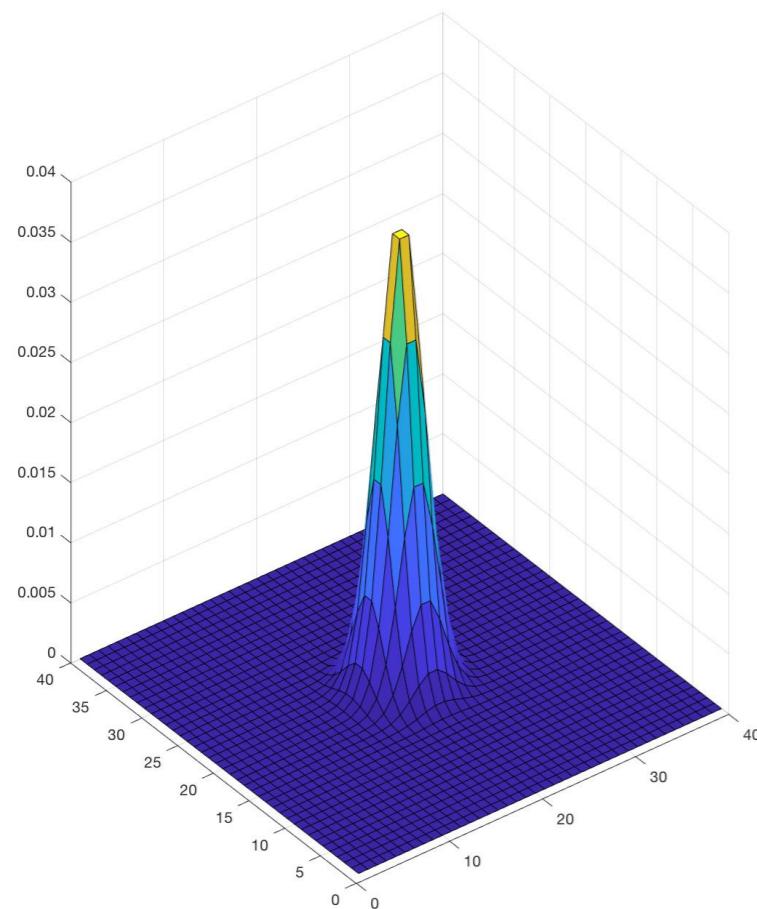


Amplitude

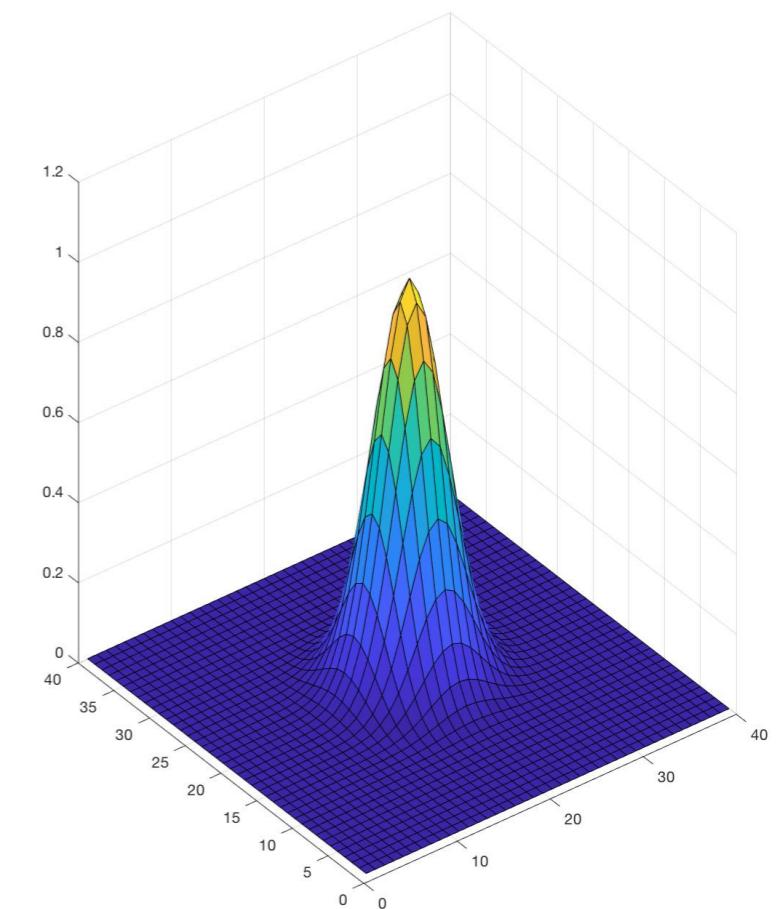
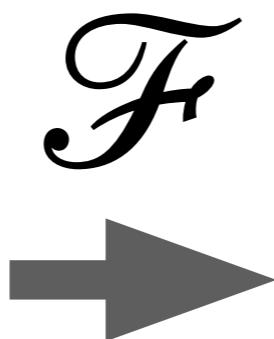
Amplitude

Amplitude

FT of Gaussian is Gaussian

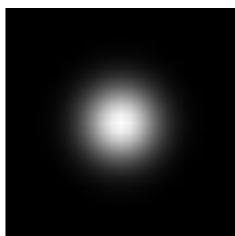


Gaussian ($\sigma = 2$)



Gaussian

Remember box filters?



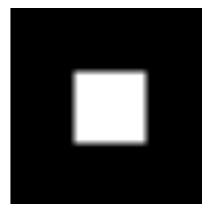
Image



Filter Response



Box Filter

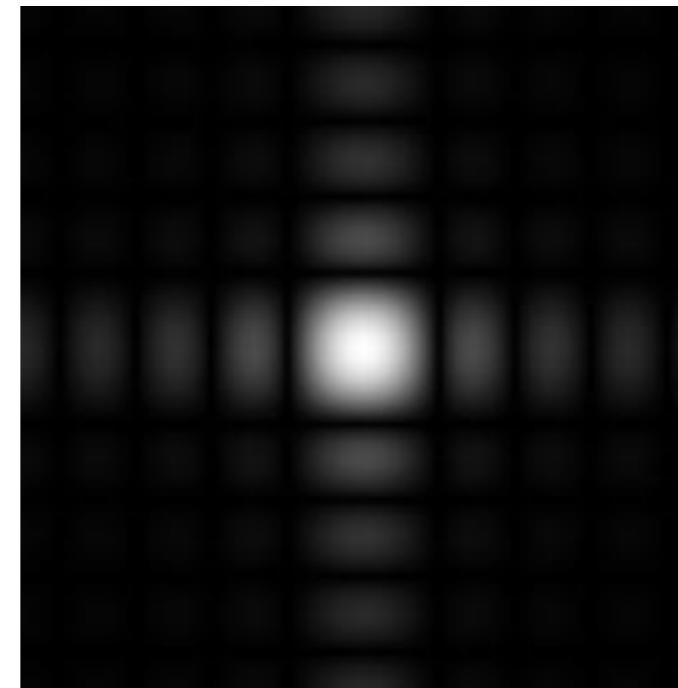
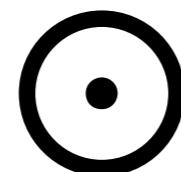
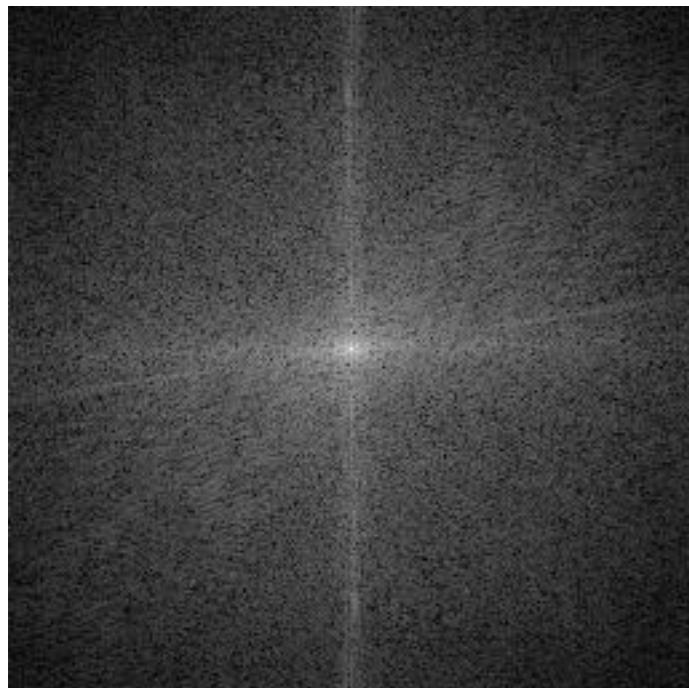


=

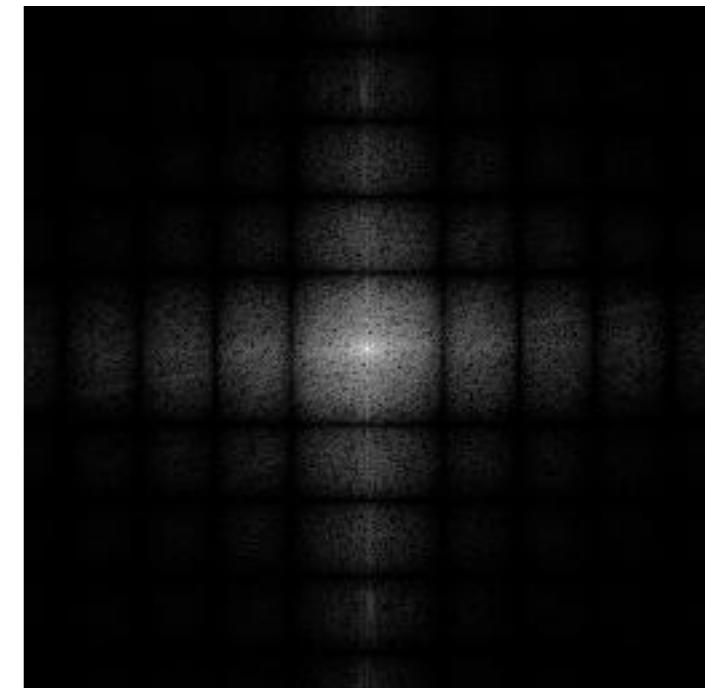
\mathcal{F} ↓

\mathcal{F} ↓

\mathcal{F}^{-1} ↑



=

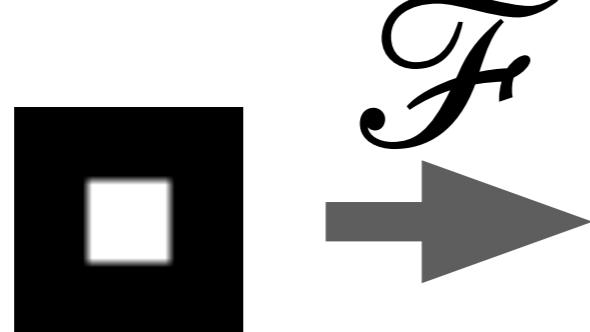


Amplitude

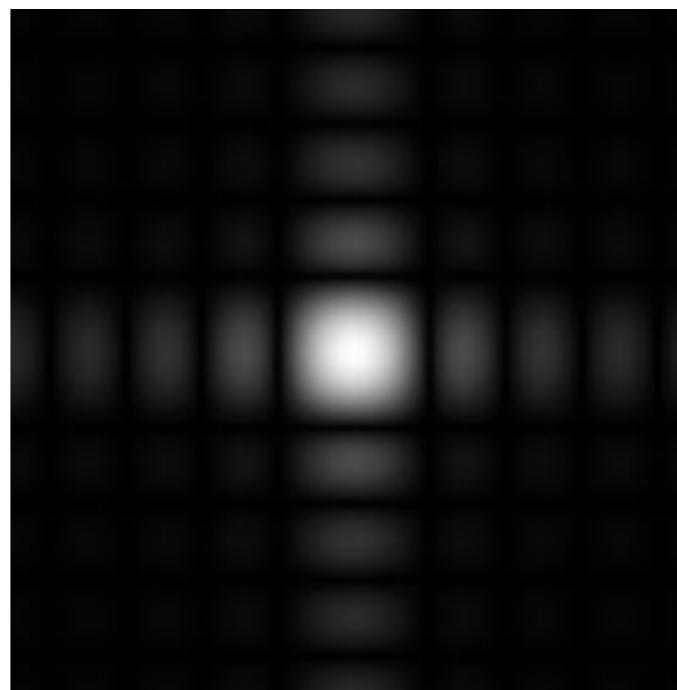
Amplitude

Amplitude

FT of Box Filters



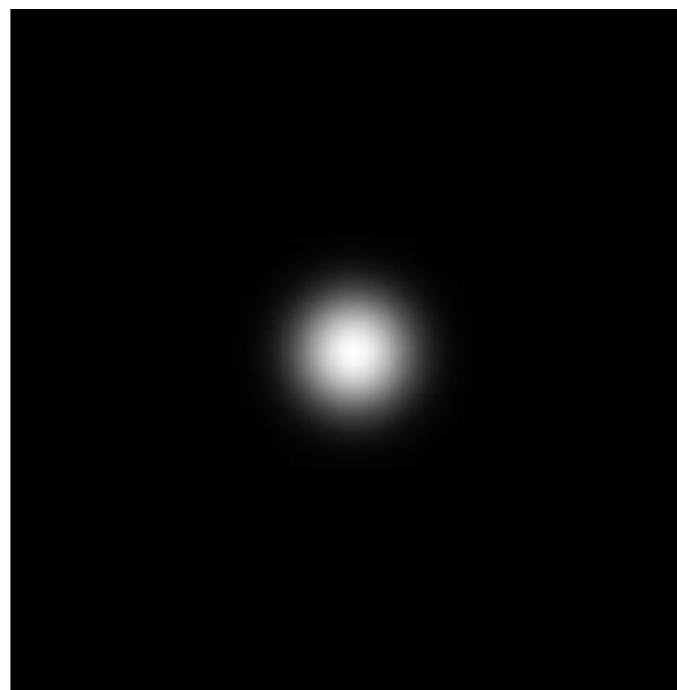
\mathcal{F}
→

A gray arrow pointing right, indicating the transformation from the input image to the Fourier domain.

Amplitude



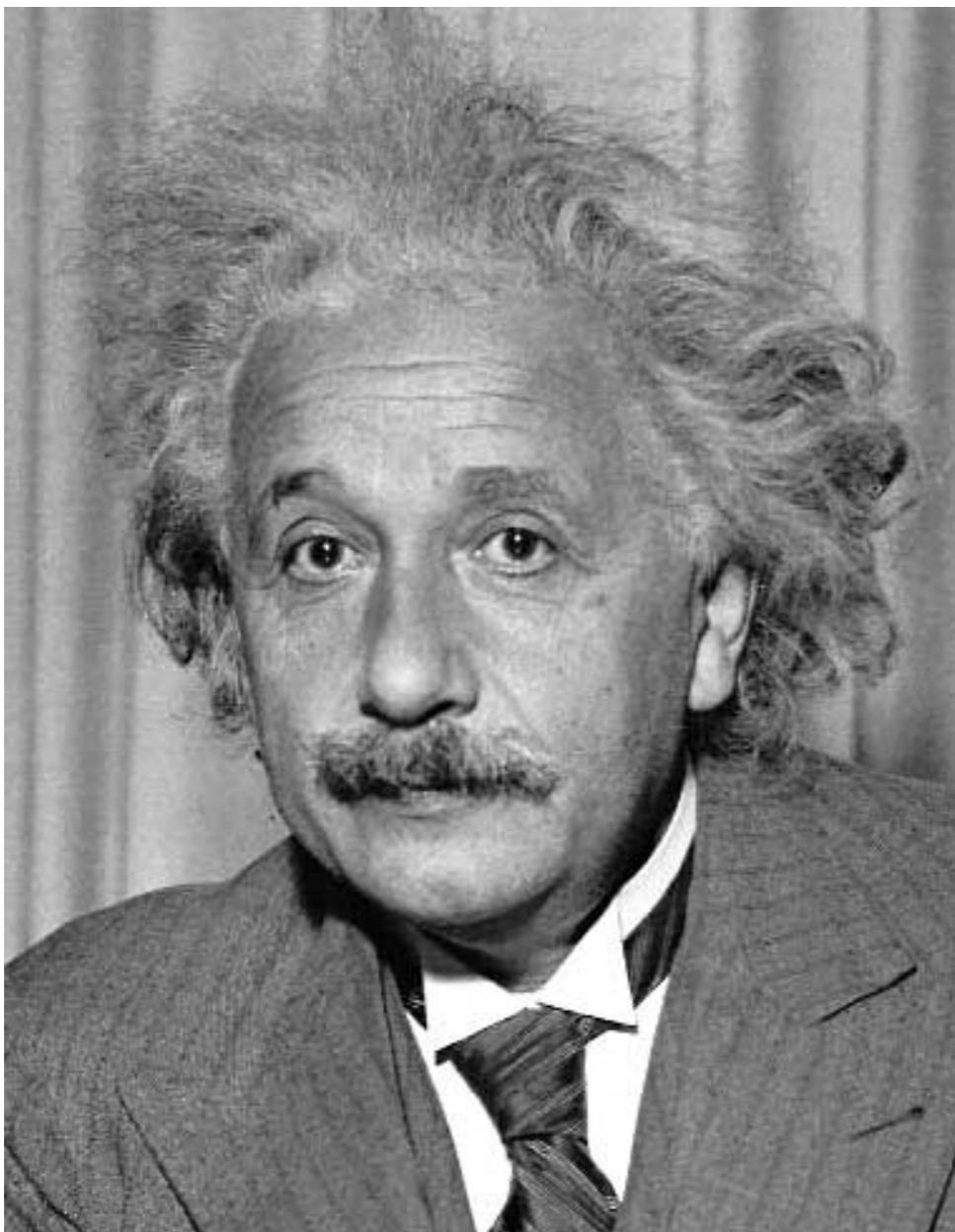
\mathcal{F}
→

A gray arrow pointing right, indicating the transformation from the input image to the Fourier domain.

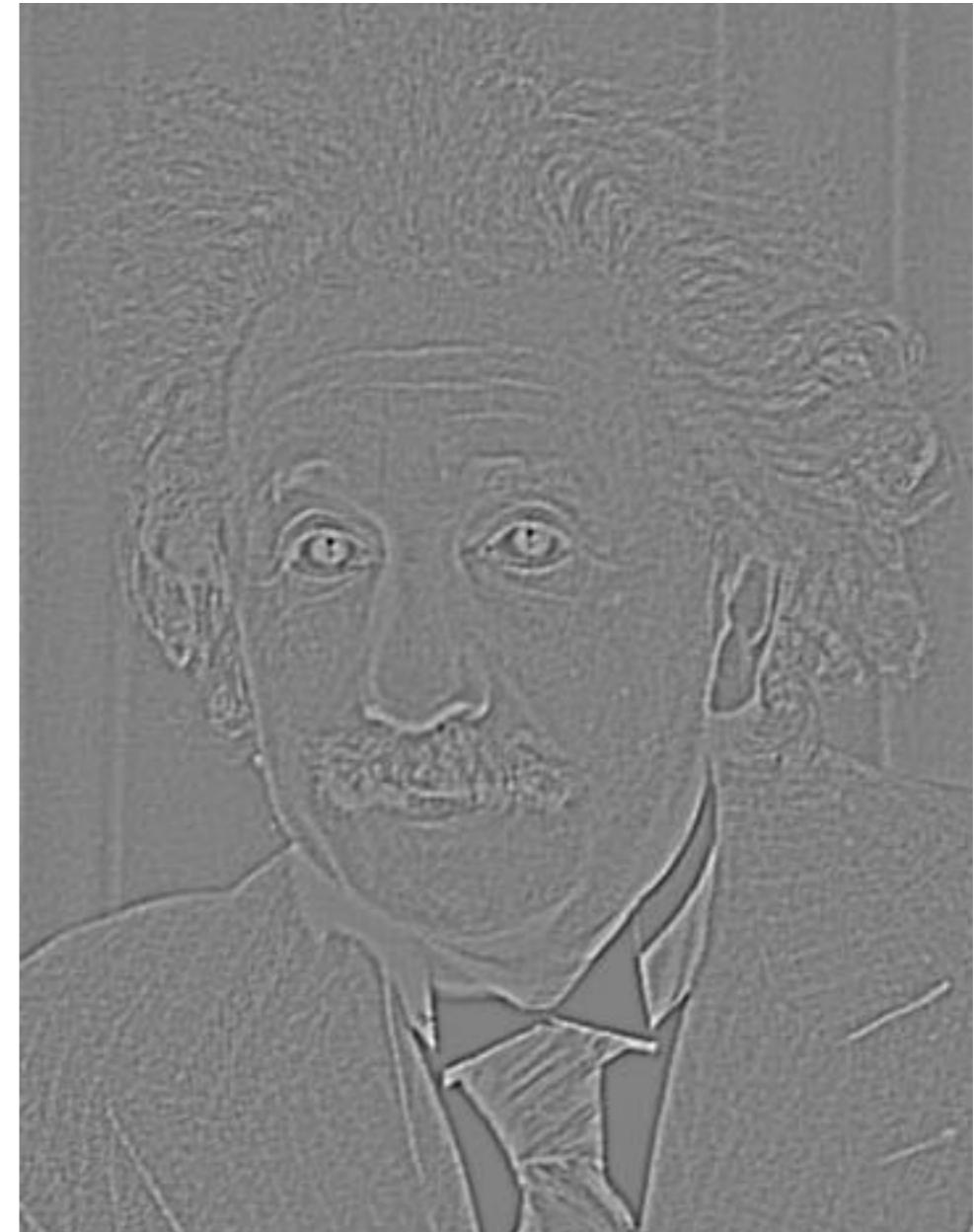
Amplitude

Remember Laplacian filters?

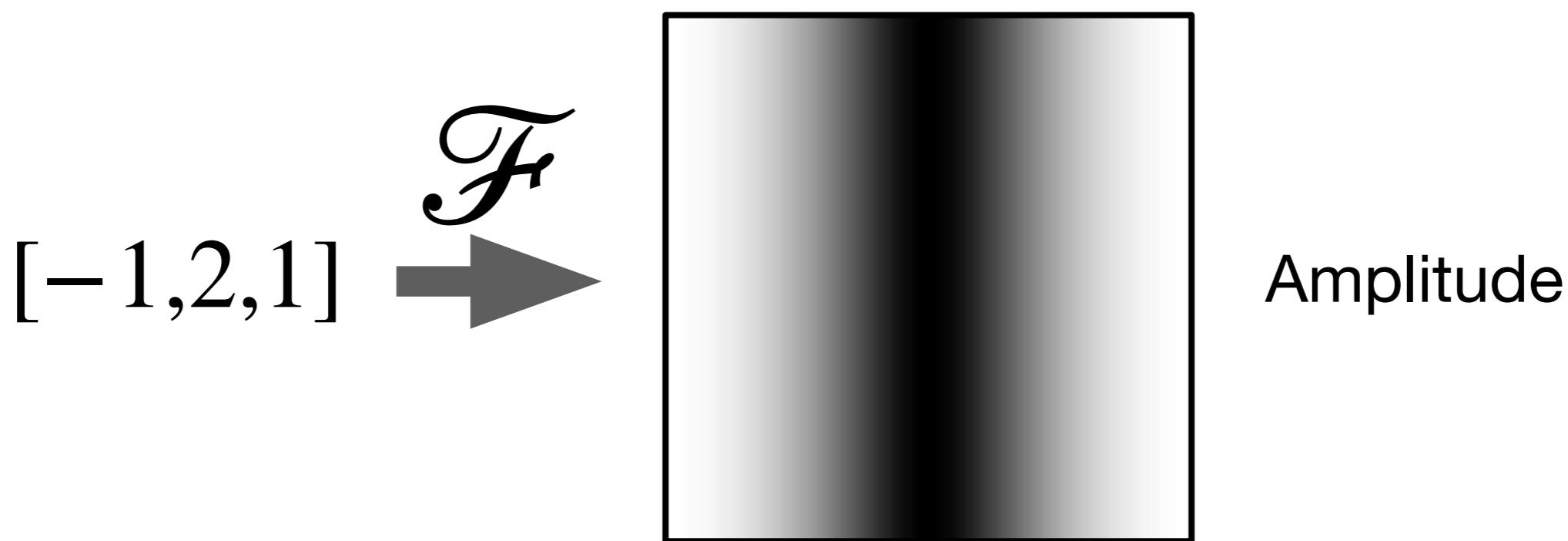
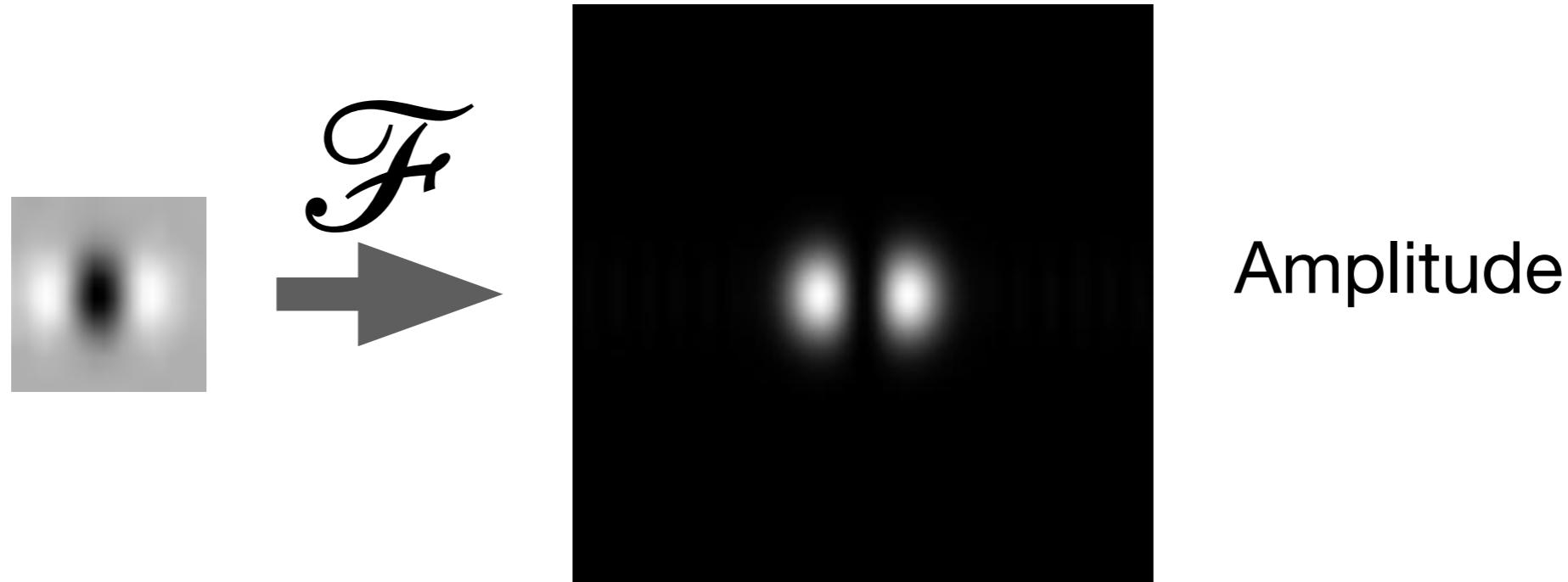
f



$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



FT of Laplacian Filters

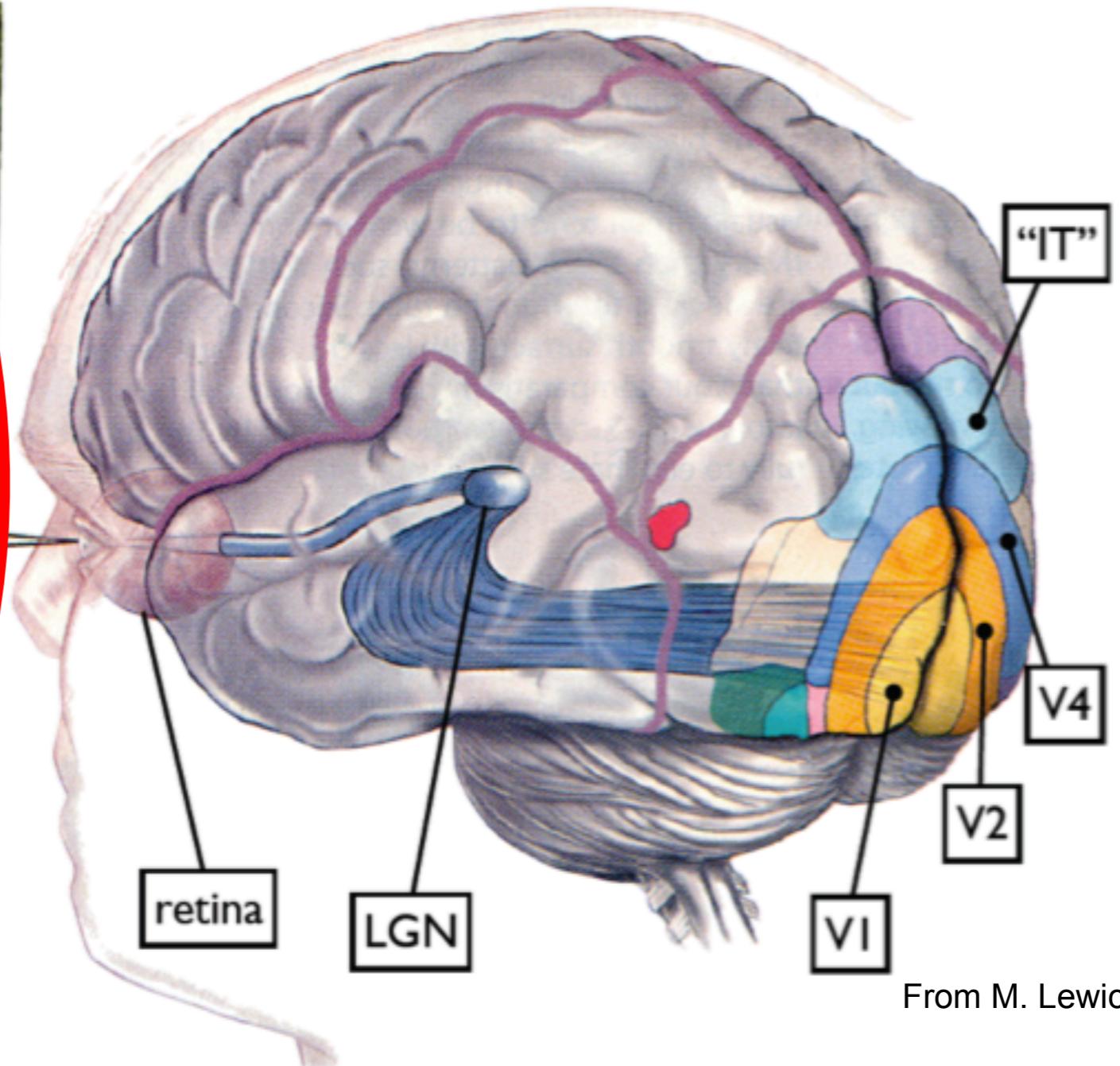


Hybrid Image Illusion





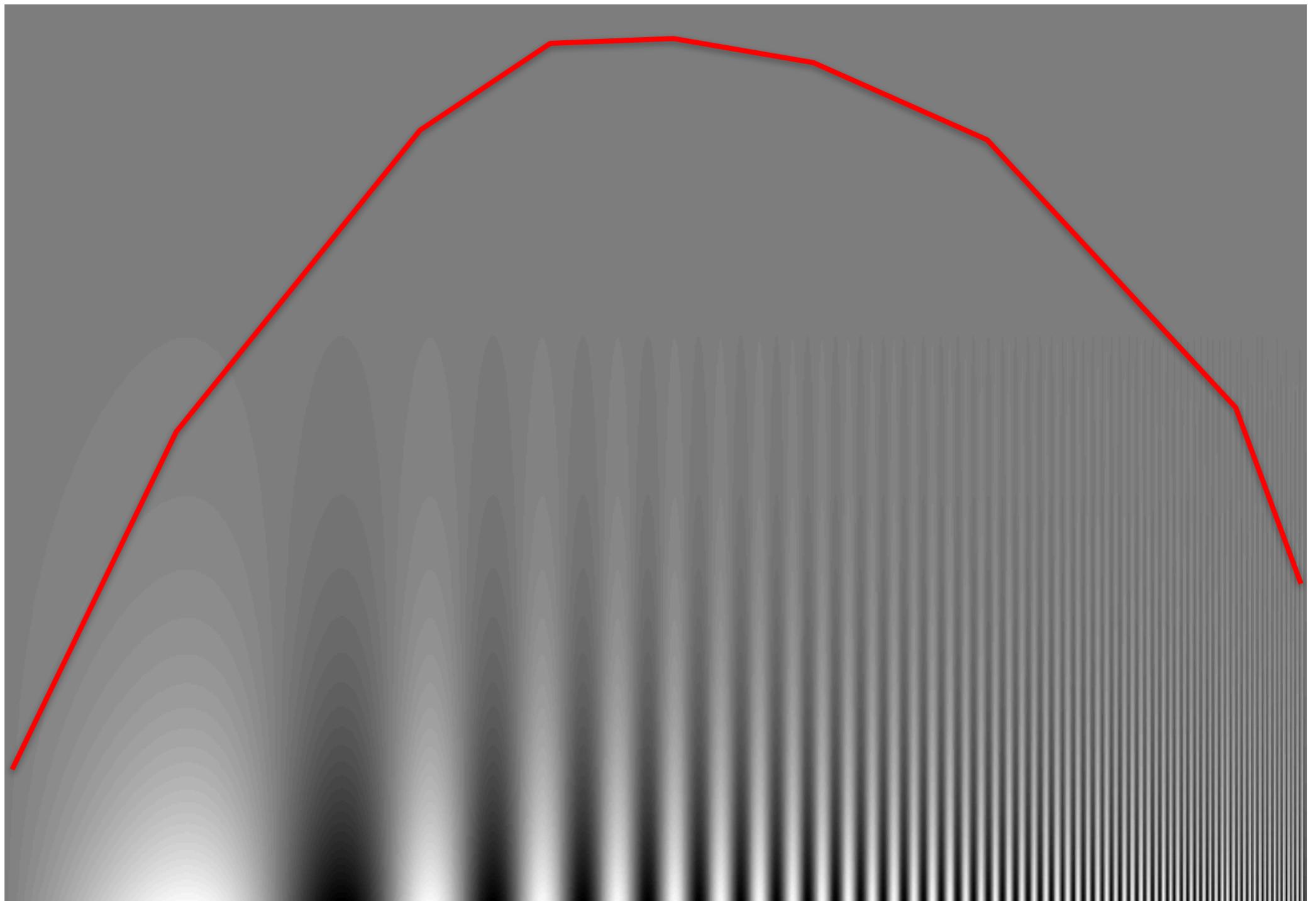
Some visual areas...



From M. Lewicky

Source: Aude Oliva

Contrast



Spatial Frequency

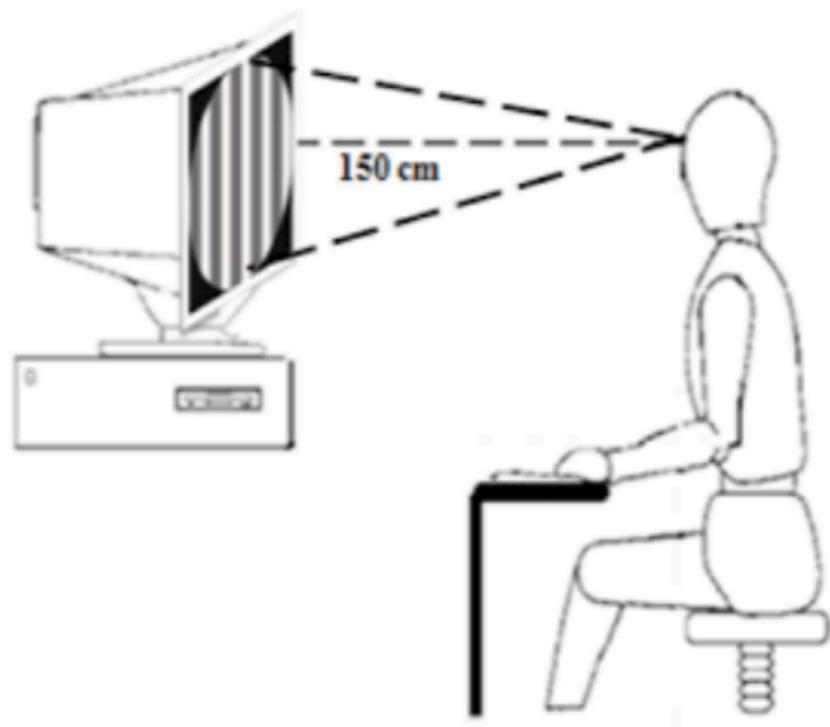
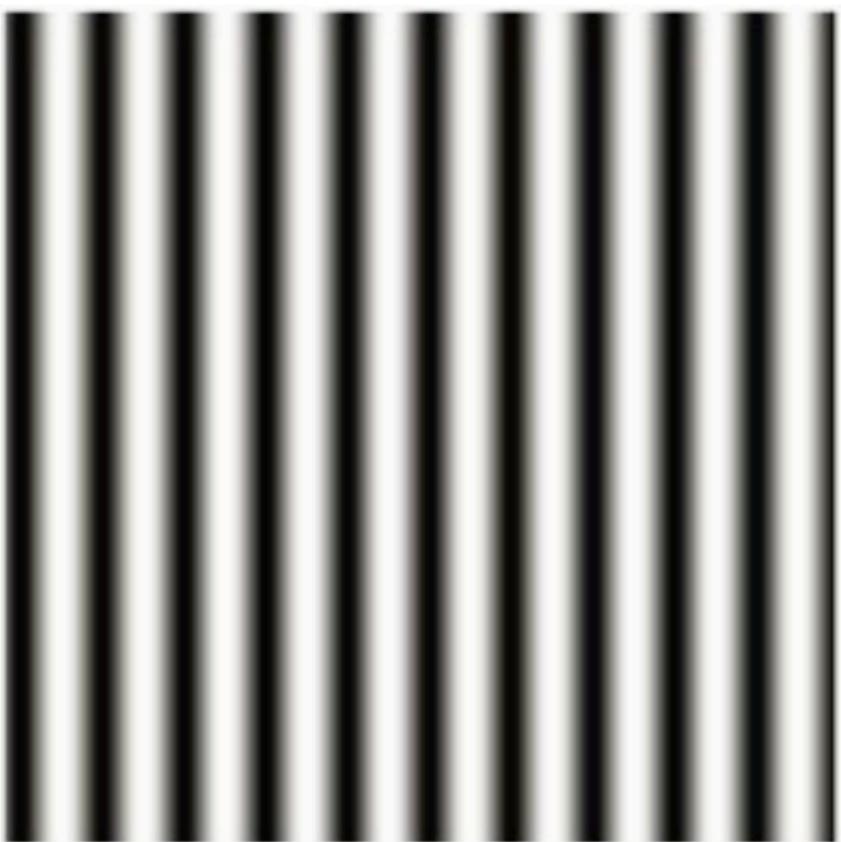
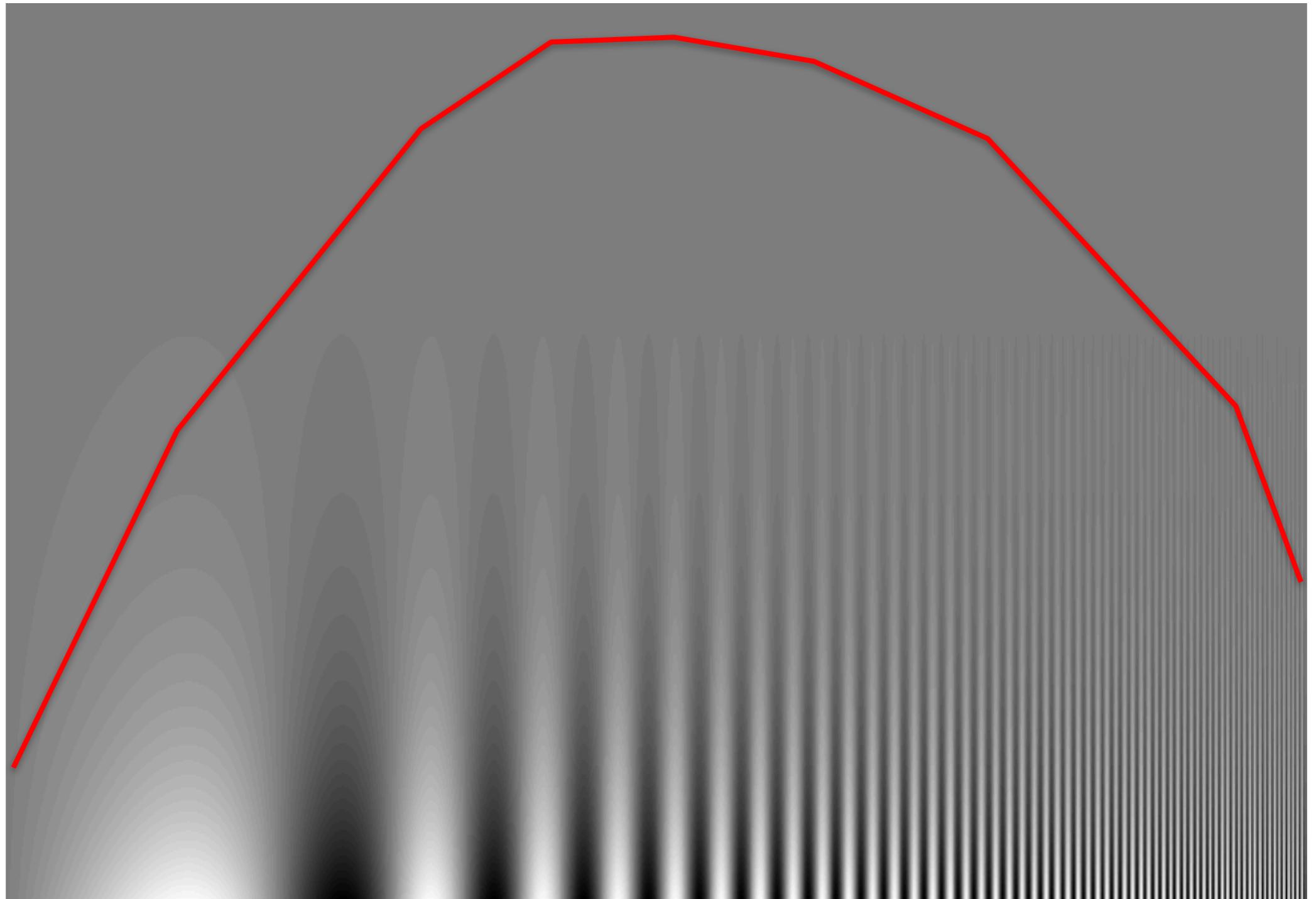


Figure 1. Stimulus presentation scheme. The stimuli were originally calibrated to be seen at a distance of 150 cm in a 19" display.

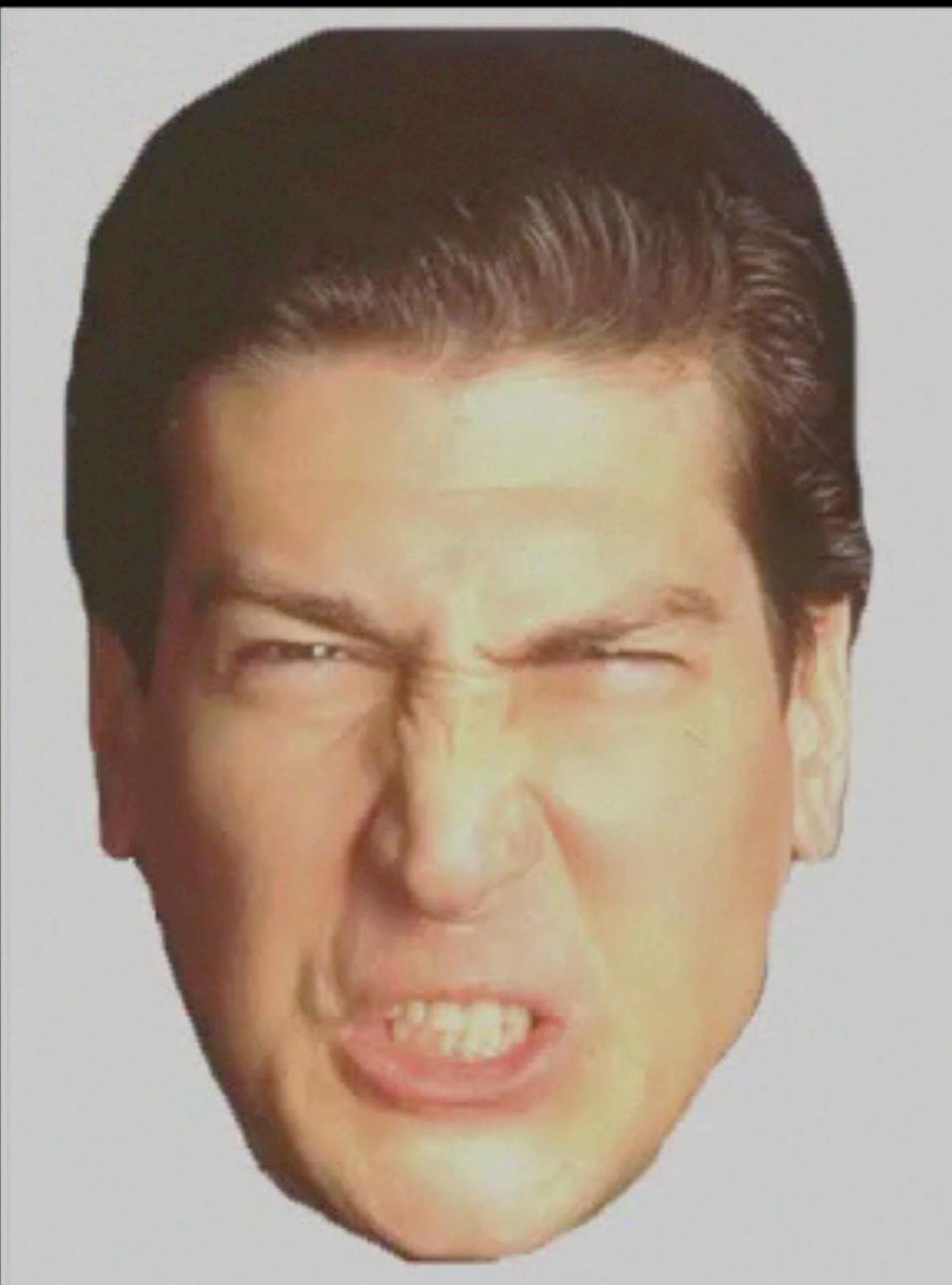
Contrast



Spatial Frequency

Hybrid Images

Oliva & Schyns

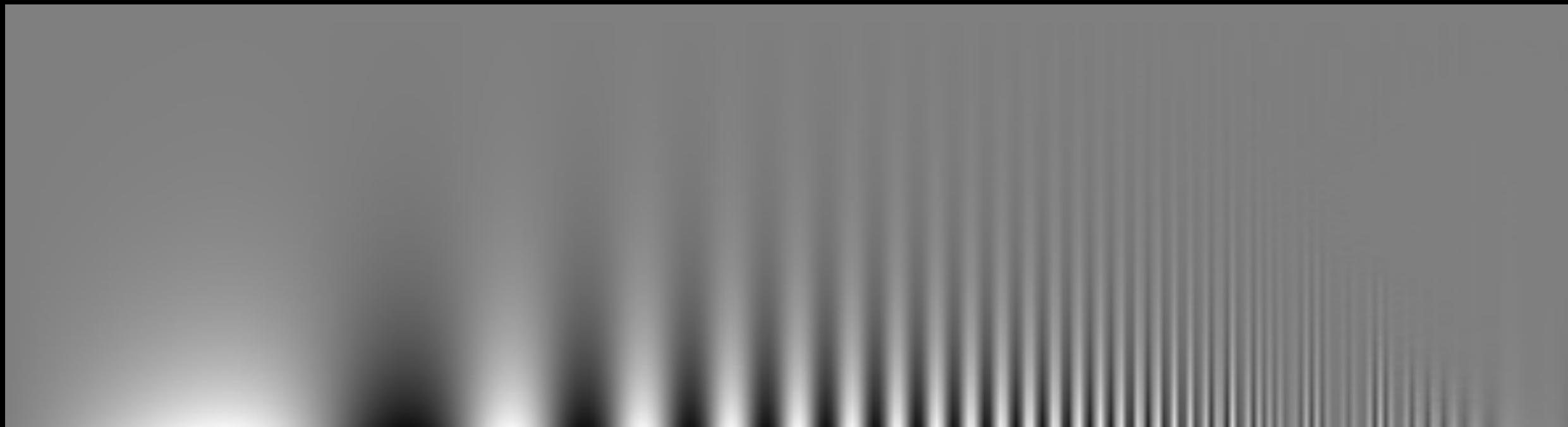




+



=

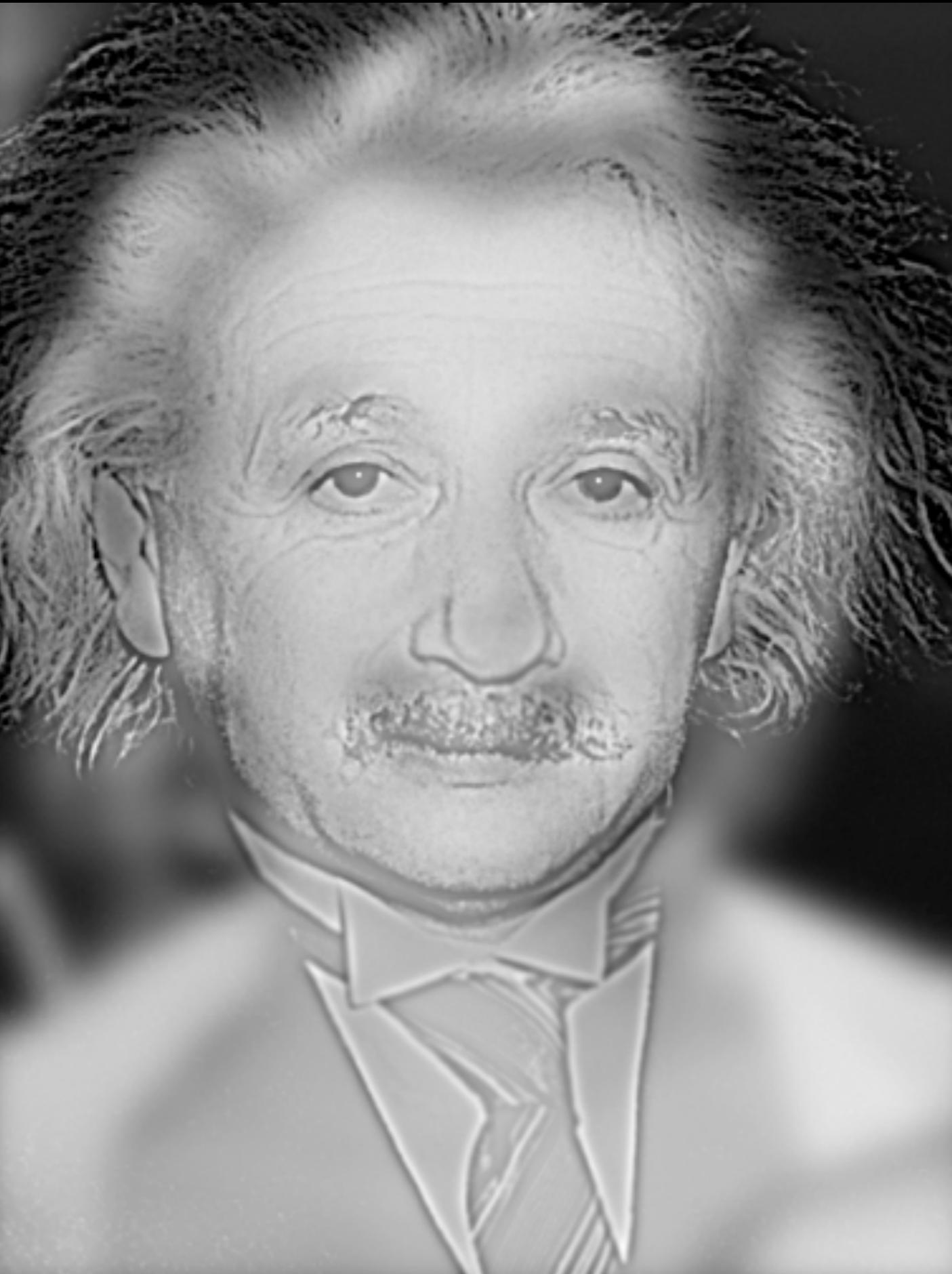


Hybrid Images



Hybrid Images



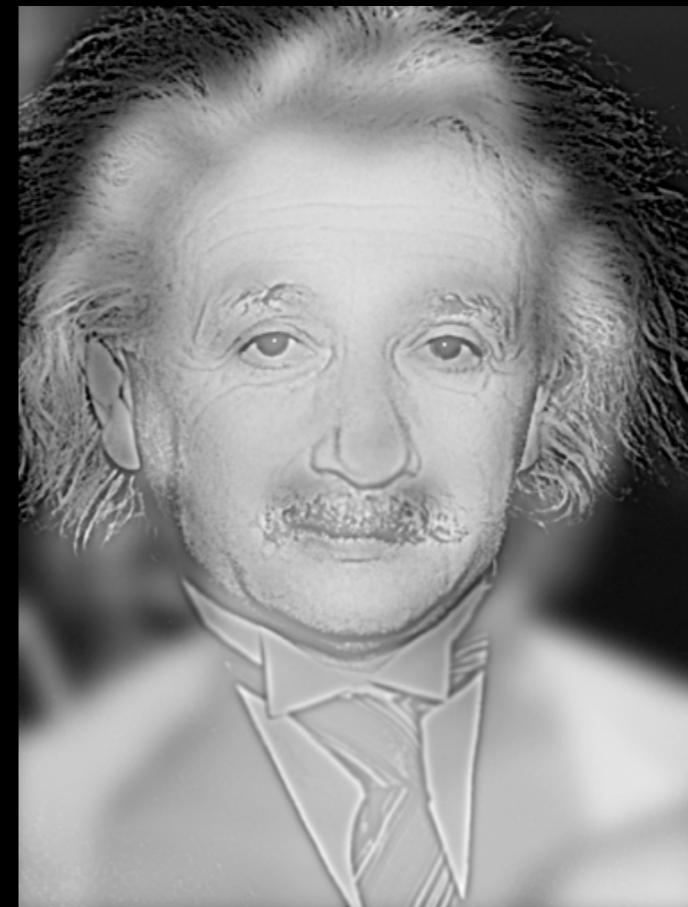


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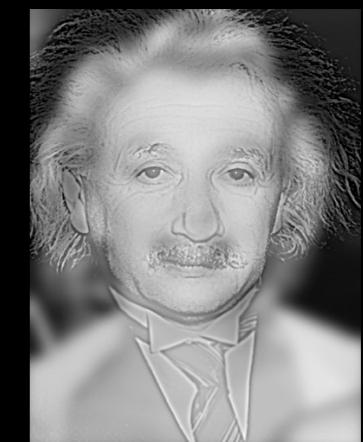
Source: Aude Oliva

[http://cvcl.mit.edu/
hybrid_gallery/gallery.html](http://cvcl.mit.edu/hybrid_gallery/gallery.html)

Copyright © 2007 Aude Oliva, MIT



Copyright © 2007 Aude Oliva, MIT



Source: Aude Oliva

Next Class

A photograph of a person in mid-air, performing a jump or fall from a high, rocky cliff edge. The cliff face is rugged and dark grey. In the background, a deep blue fjord stretches towards a range of mountains under a cloudy sky.

Edges