COMS 4731 Fall 2019 Homework 1

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1 Problem 1

a) Answer: Circle

b) Answer: 0.25 mm^2

Assume r_d is the radius of the disk itself, r_i is the radius of disk in the image, d_d is the distance from the pinhole to the disk itself, d_i is the distance from the pinhole to the image plane. We know that

$$\frac{r_d}{d_d} = \frac{r_i}{d_i}$$

Since the new distance from the pinhole to the disk itself $D_d = 2d_d$, the new radius of disk in the image R_i will be

$$\begin{aligned} R_i &= \frac{r_d}{D_d} d_i \\ &= \frac{1}{2} \frac{r_d}{d_d} d_i \\ &= \frac{1}{2} r_i \end{aligned}$$

Since the radius is half of the original radius, the area is 0.25 mm^2

c) Answer: Ellipse.

Because the locus of the lines of sight tangent to a sphere is a cone of revolution. The intersection of this cone of revolution with the image plane is the contour of the image of the sphere. If the sphere is one side of the image plane and the center of projection is on the other side, the shape of the image pf the sphere can only be an ellipse or a circle (which is a special ellipse).

2 Problem 2

a) Answer: $\frac{kf}{k-f}$ when $k > f$

$$\frac{1}{f} = \frac{1}{d} + \frac{1}{k}$$
$$\frac{1}{d} = \frac{k-f}{kf}$$
$$d = \frac{kf}{k-f}$$

Since d should be positive, only when $k > f$, the focused image of the scene line would be formed.

b) Answer:

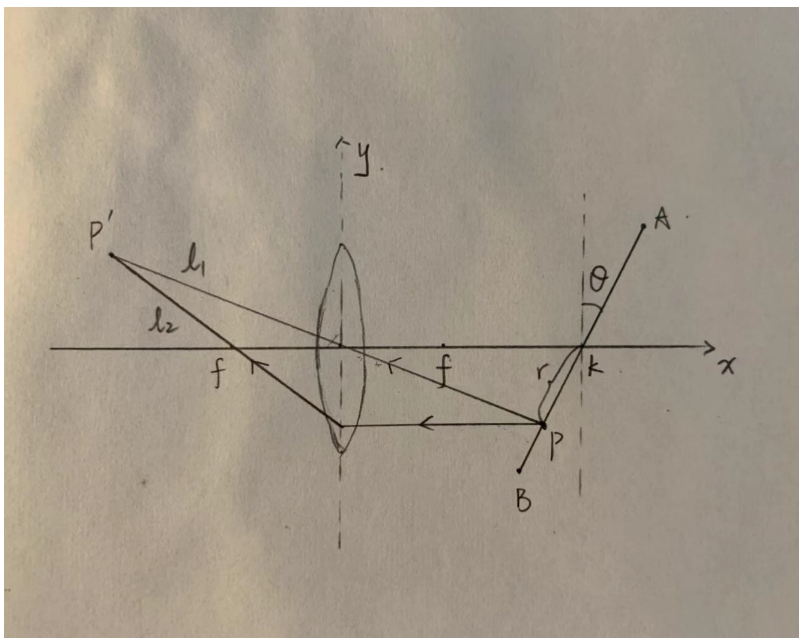

Figure 1: Coordinate system diagram

Assume the coordinate system shown in Figure 1 was set up. r is any real number which is less than $\frac{1}{2}AB$ and the coordinate of P can be written as $(k - r\sin\theta, -r\cos\theta)$

The function of l_1:

$$y = \frac{-r\cos\theta}{k - r\sin\theta}x \tag{1}$$

The function of l_2:

$$y = -\frac{r\cos\theta}{f}x - r\cos\theta \tag{2}$$

Combine Equation 1 and Equation 2 and we can solve the coordinate of P':

$$(\frac{k - r\sin\theta}{f - k + r\sin\theta}, \frac{-r\cos\theta}{f - k + r\sin\theta}), \quad r \in [0, \frac{1}{2}AB]$$

$P^{'}$ is on the line whose function is:

$$y = \frac{f-k}{f\tan\theta}x - \frac{k}{f\tan\theta}$$

So, we can prove that the image of the scene line is still a line. Since the slope is different from the slope of the original line, we can prove that it is tilted.

c) Answer:

From b) we can know the slope of the tilted scene is $\frac{f-k}{f\tan\theta}$ which is also equal to $\tan(\phi+\frac{\pi}{2})$

So, we can prove that $\tan\phi = \frac{f}{k-f}\tan\theta$

3 Problem 3

a) Answer:

We can generate the functions of L_1 and L_2 respectively:

$$L_1 = \frac{\rho_d I}{\pi r^2}(\overrightarrow{n}\cdot\overrightarrow{s_1})$$

$$L_2 = \frac{\rho_d I}{\pi r^2}(\overrightarrow{n}\cdot\overrightarrow{s_2})$$

$$\begin{aligned} L_3 &= L_1 + L_2 \\ &= \frac{\rho_d I}{\pi r^2}\overrightarrow{n}\cdot(\overrightarrow{s_1}+\overrightarrow{s_2}) \\ &= \frac{\rho_d}{\pi r^2}|\overrightarrow{s_1}+\overrightarrow{s_2}|I\overrightarrow{n}\cdot\frac{(\overrightarrow{s_1}+\overrightarrow{s_2})}{|\overrightarrow{s_1}+\overrightarrow{s_2}|} \end{aligned}$$

So,

$$\begin{aligned} s_3 &= \frac{(\overrightarrow{s_1}+\overrightarrow{s_2})}{|\overrightarrow{s_1}+\overrightarrow{s_2}|} \\ I_3 &= |\overrightarrow{s_1}+\overrightarrow{s_2}|I \end{aligned}$$

b) Answer:

$$L_1 = \frac{\rho_d I_1}{\pi r^2}(\overrightarrow{n}\cdot\overrightarrow{s_1})$$

$$L_2 = \frac{\rho_d I_2}{\pi r^2}(\overrightarrow{n}\cdot\overrightarrow{s_2})$$

$$
\begin{aligned}
L_3 &= L_1 + L_2 \\
&= \frac{\rho_d}{\pi r^2} \overrightarrow{n} \cdot (I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}) \\
&= \frac{\rho_d}{\pi r^2} |I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}| \overrightarrow{n} \cdot \frac{I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}}{|I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}|}
\end{aligned}
$$

So,

$$
\begin{aligned}
s_3 &= \frac{I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}}{|I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}|} \\
I_3 &= |I_1 \overrightarrow{s_1} + I_2 \overrightarrow{s_2}|
\end{aligned}
$$