The conformal Killing spinor initial data equations

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in collaboration with Jarrod L. Williams

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Spinorial notation in a nutshell

Translation spinors and world tensors $a \to AA'$. Given $T_{\boldsymbol{ab}}$ the spinorial counterpart is given by

$$T_{\boldsymbol{AA'BB'}} = T_{\boldsymbol{ab}}\sigma^{\boldsymbol{a}}{}_{\boldsymbol{AA'}}\sigma^{\boldsymbol{b}}{}_{\boldsymbol{BB'}}$$

where $\sigma^{\boldsymbol{a}}{}_{\boldsymbol{AA'}}$ are the Infeld-van der Waerden symbols (Pauli matrices & Identity)

$$(\alpha_{\boldsymbol{0}}, \alpha_{\boldsymbol{1}}, \alpha_{\boldsymbol{2}}, \alpha_{\boldsymbol{3}}) \to \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha_{\boldsymbol{0}} + \alpha_{\boldsymbol{3}} & \alpha_{\boldsymbol{1}} - \mathrm{i}\alpha_{\boldsymbol{2}} \\ \alpha_{\boldsymbol{1}} + \mathrm{i}\alpha_{\boldsymbol{2}} & \alpha_{\boldsymbol{0}} - \alpha_{\boldsymbol{3}} \end{bmatrix}$$

(frame-) metric: $g_{AA'BB'} = \epsilon_{AB}\epsilon_{A'B'}$

Raise and lower indices: $\xi_B = \xi^A \epsilon_{AB}$

Curvature spinors:
Riemann $\to$ Weyl, tracefree Ricci, Ricci scalar.

Ψ_{ABCD}, $\Phi_{ABA'B'}$, Λ

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Killing spinors: Hidden symmetries

Some Killing objects

- Killing vectors

$$\tilde{\nabla}_{(a}\tilde{\xi}_{b)} = 0.$$

- Killing tensors

$$\tilde{K}_{ab} = \tilde{K}_{(ab)}, \quad \tilde{\nabla}_{(a}\tilde{K}_{bc)} = 0,$$

- Killing-Yano tensors

$$\tilde{Y}_{ab} = \tilde{Y}_{[ab]}, \quad \tilde{\nabla}_{(a}\tilde{Y}_{b)c} = 0,$$

$$\tilde{K}_{ab} = \tilde{Y}_a{}^c\tilde{Y}_{cb}.$$

- Killing spinors

$$\tilde{\kappa}_{AB} = \tilde{\kappa}_{(AB)}, \quad \tilde{\nabla}_{A'(A}\tilde{\kappa}_{BC)} = 0$$

$$\tilde{Y}_{AA'BB'} = \mathrm{i}(\tilde{\kappa}_{AB}\tilde{\epsilon}_{A'B'} - \bar{\tilde{\kappa}}_{A'B'}\tilde{\epsilon}_{AB})$$

Killing spinors are more primitive objects!

An example: Kerr spacetime

$$\tilde{\xi}^a = (\partial_t)^a, \; \tilde{\eta}^a = (\partial_\varphi)^a,$$

$$\exists \tilde{K}_{ab}, \qquad \tilde{u}^a\tilde{\nabla}_a\tilde{u}^b = 0.$$

$$\mu = \tilde{u}^a\tilde{u}_a, \quad e = \tilde{\xi}^a\tilde{u}_a,$$

$$\ell = \tilde{\eta}^a\tilde{u}_a, \quad C = \tilde{K}_{ab}\tilde{u}^a\tilde{u}^b$$

Observations

- Geodesic motion completely integrable!
- Hidden symmetry

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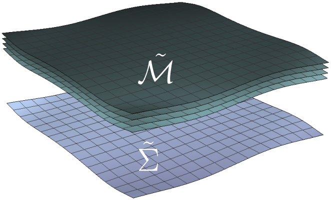
Rigidity thru symmetries. Symmetries thru intial data

Spacetime symmetries

- Black hole uniqueness problems
- Spacetime characterisations

Killing Initial Data KIDs (Chrusciel & Beig 97)

- Given initial data $(\tilde{\Sigma}, \tilde{h}_{ij}, \tilde{\chi}_{ij})$ for the **vacuum <u>Einstein</u> Field Equations** (EFE) if the **Killing <u>vector</u> initial data equations** (KID) are satisfied on $\tilde{\Sigma}$ then the spacetime developement admits a Killing vector ξ^a.



Killing vector initial data

KID: Killing vector initial data

- Let $(\tilde{\mathcal{M}}, \tilde{\boldsymbol{g}})$ solution to $\tilde{R}_{ab} = 0$.

$$\tilde{S}_{ab} := \tilde{\nabla}_a \tilde{\xi}_b + \tilde{\nabla}_b \tilde{\xi}_a$$

- $\tilde{\xi}_a$ is KV iff $\tilde{S}_{ab} = 0$ (zero-quantity)
- Does $\tilde{S}_{ab} = 0$ propagate?

Identities to eqs

- Assume **vacuum EFE**
$$\tilde{R}_{ab} = 0$$
- Assume **candidate** eqn $\tilde{\square}\tilde{\xi}_a = 0$ on $(\tilde{\mathcal{M}}, \tilde{g})$
- $\Longrightarrow$ Propagation equation
$$\tilde{\square}\tilde{S}_{ab} = 2\tilde{R}^c{}_{ab}{}^d \tilde{S}_{cd}$$
- $\tilde{S}_{ab} = 0$ & $\partial_t \tilde{S}_{ab} = 0$ on $\tilde{\Sigma}$ $\Longrightarrow$ $\tilde{S}_{ab} = 0$ on $\mathcal{W} \subseteq \mathcal{D}^+(\tilde{\Sigma})$

Sketch of the KIDs derivation & proof

- Identity:
$$\tilde{\nabla}^a \tilde{S}_{ab} - \frac{1}{2}\nabla_b S_a{}^a = \tilde{\square}\tilde{\xi}_b + \tilde{R}_{cb}$$
- Definition: A **candidate** KV $\tilde{\zeta}^a$ is a vector satisfying $\tilde{\square}\tilde{\zeta}_a = -\tilde{R}_{ab}\tilde{\zeta}^b$
- Strategy: find ID $(\zeta_a, \partial_t \zeta_a)$ on $\tilde{\Sigma}$ ensuring a **candidate** gives a **true KV**.
- Indentity:
$$\tilde{\square}\tilde{S}_{ab} = 2\tilde{R}^c{}_{ab}{}^d \tilde{S}_{cd} - \mathcal{L}_{\tilde{\xi}}\tilde{R}_{ab} + 2\tilde{\nabla}_{(a}\left\{\tilde{\square}\tilde{\xi}_{b)} + \tilde{R}_{b)c}\tilde{\xi}^c\right\}$$

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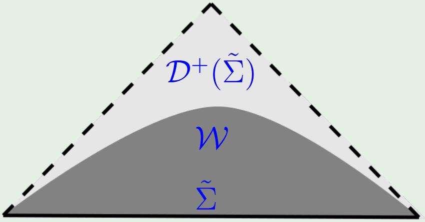

Trivial data: $\tilde{S}_{ab} = 0$ & $\partial_t \tilde{S}_{ab} = 0$ on $\tilde{\Sigma}$
$\Longrightarrow$ $(\tilde{\xi}_a, \partial_t \tilde{\xi}_a)$ on $\tilde{\Sigma}$ aka KIDs.

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"Physical" Killing Spinor ID

Killing spinor

- EFE: $\tilde{R}_{ab} = \lambda \tilde{g}_{ab} \implies \tilde{\Phi}_{AA'BB'} = 0$
- $\tilde{\nabla}_{A'(A}\tilde{\kappa}_{BC)} = 0$
- $\tilde{\xi}_a \mapsto \tilde{\xi}_{AA'} := \tilde{\nabla}^B{}_{A'}\tilde{\kappa}_{AB}$ is a KV if EFE hold.

KSID zero-quantities definitions

$$\tilde{H}_{A'ABC} := 3\tilde{\nabla}_{A'(A}\tilde{\kappa}_{BC)},$$

$$\tilde{S}_{AA'BB'} := \tilde{\nabla}^Q{}_{A'}\tilde{H}_{B'QAB},$$

- $\exists$ Killing spinor $\iff \tilde{H}_{A'ABC} = 0.$

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$\tilde{\Phi} = 0$ & $\tilde{\boldsymbol{H}} = 0 \implies \tilde{\xi}$ is a Killing vector!

KSID (Valiente Kroon & Garcia-Parrado 08)

- Candidate KS eq for $\tilde{\kappa}$, Candidate KV eq for $\tilde{\xi}$.
- Assume $\tilde{\Phi} = 0$.
- Closed system of homogeneous wave eqs

$$\tilde{\square}\tilde{\boldsymbol{H}} = \mathcal{F}_1(\boldsymbol{H}, \boldsymbol{S})$$
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The Einstein equations under conformal transformations

- The Einstein field equations in vacuum

$$\tilde{R}_{ab} = \lambda \tilde{g}_{ab}$$

are not conformally invariant!.

- A calculation shows that

$$g_{ab} = \Omega^2 \tilde{g}_{ab}, \quad \Longrightarrow$$

$$R_{ab} = \tilde{R}_{ab} - 2\Omega^{-1}\nabla_a\nabla_b\Omega - g_{ab}\big(\Omega^{-1}\nabla^c\nabla_c\Omega - 3\Omega^{-2}\nabla_c\Omega\nabla^c\Omega\big),$$

where R_{ab}, R and ∇_a are associated to g_{ab}.

- Formally **singular** whenever $\Omega = 0$!.

- Friedrich's regularisation trick ... and use the curvature as a variable

$$\nabla_a\nabla_b\Omega = -\frac{\Omega}{2}R_{ab} + \ldots$$

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The conformal Einstein field equations (H. Friedrich 81)

Equations:

$$\nabla_a \nabla_b \Omega = -\Omega L_{ab} + s g_{ab},$$
$$\nabla_a s = -L_{ac} \nabla^c \Omega,$$
$$\nabla_c L_{db} - \nabla_d L_{cb} = d^a{}_{bcd} \nabla_a \Omega,$$
$$\nabla_a d^a{}_{bcd} = 0,$$

Fields: $(\Omega, s, L_{ab}, d^a{}_{bcd})$

$L_{ab} := \frac{1}{2} R_{ab} - \frac{1}{12} R g_{ab}$, (Schouten tensor)

$s := \frac{1}{4} \nabla_a \nabla^a \Omega + \frac{1}{24} R \Omega$, (Friedrich scalar)

$d^a{}_{bcd} := \Omega^{-1} C^a{}_{bcd}$. (Rescaled Weyl tensor)

Key equation: Rescaled Weyl spinor

$$\nabla^{AA'} \underbrace{(\Omega^{-1}\Psi_{ABCD})}_{\phi_{ABCD} \ \text{rescaled Weyl spinor}} = \Omega^{-1} \tilde{\nabla}^{AA'} \Psi_{ABCD} \underbrace{= 0}_{\text{vaccum} \ \tilde{\Phi}=0}$$

$$\boxed{\nabla^{AA'} \phi_{ABCD} = 0} \qquad \Phi_{ABA'B'} \neq 0 \qquad (\Phi \text{ sats diff eq})$$

Applications

- Potentially turn **global** problems into **local** ones.
- $\mathscr{I}$ $(\Omega = 0)$ legit hypersurface to prescribe ID: Asympt. initial value problem.

Combining these ideas?

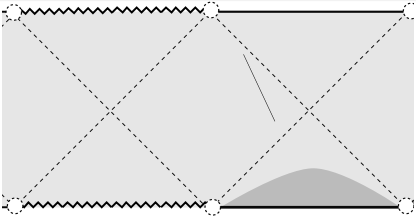
Unphysical Killing vector equation

Given a Killing vector $\tilde{\xi}^a$ on $(\tilde{\mathcal{M}}, \tilde{\boldsymbol{g}})$ and $(\mathcal{M}, \boldsymbol{g})$ with $\boldsymbol{g} = \Omega^2 \tilde{\boldsymbol{g}}$ then $X_a = \Omega^2 \tilde{\xi}_a$ satisfies the **Unphysical Killing vector equations**

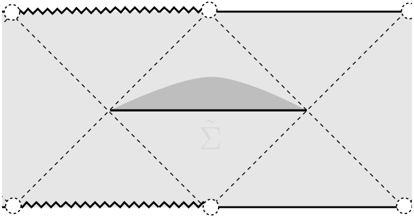
$$\nabla_a X_b + \nabla_b X_a = \frac{1}{2} \nabla^c X_c g_{ab}, \quad X^a \nabla_a \Omega = \frac{1}{4} \nabla_a X^a.$$

CKID: The conformal Killing vector initial data equations (Paetz 14)

- Field equations: CEFE.
- KID's on spacelike conformal boundaries $\mathscr{I}$



$\Sigma = \mathscr{I}$



Apps (Mars Paetz Senovilla 16, 17)

- Characterisations of Kerr-de-Sitter-like spacetimes

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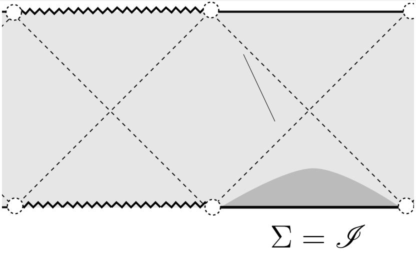

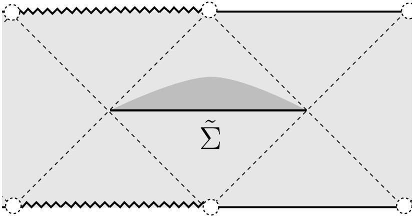

Apps (Mars Paetz Senovilla 16, 17)

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CKSID Conformal Killing Spinor ID (G. & Williams 22)

Killing spinors and conformal transformations

- KS are conf. inv. $\tilde{\nabla}_{A'(A}\tilde{\kappa}_{BC)} = 0 \implies \nabla_{A'(A}\kappa_{BC)} = 0$ with $\kappa_{AB} = \Omega^2\tilde{\kappa}_{AB}$.

Why proof doesn't work in conformal?!

- The EFE are **not** conf. inv.
- $\xi_a \mapsto \xi_{AA'} := \nabla^B{}_{A'}\kappa_{AB}$ is **not** a (C)KV.

Zero-quantities?

$$H_{A'ABC} := 3\nabla_{A'(A}\kappa_{BC)}$$

$$S_{AA'BB'} := \nabla^Q{}_{A'}H_{B'QAB}$$

$$= \nabla_{AA'}\xi_{BB'} + \nabla_{BB'}\xi_{AA'} + 6\kappa_{(A}{}^Q\Phi_{B)QA'B'}$$

- Φ satisfies diff conditions in $(\mathcal{M}, \boldsymbol{g})$.
- S not geom motivated.
- Relate Φ to $\tilde{\Phi}$? $\rightsquigarrow$ Singular eqs Ω^{-1}-terms
- Fuchsian systems theory vs CEFE philosophy

Buchdahl

$\boldsymbol{H} = 0 \implies (\mathcal{M}, \boldsymbol{g})$ Algebraically Special

$$\nabla_{(A}{}^{A'}H_{|A'|BCD)}$$

$$= 6\kappa_{(A}{}^Q\Psi_{BCD)Q} = 0$$

Petrov Type D, N, O.

$\Psi_{ABCD} = \Psi\kappa_{(AB}\kappa_{CD)}$

Buchdahl zero-quantity

$$\phi_{ABCD} = \Omega^{-1}\Psi_{ABCD}$$

Def Buchdahl 0-quant

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KSID (G. & Williams 22)

- Candidate KS eq for $\boldsymbol{\kappa}$.
- Assume vacuum CEFE are satisfied.
- Closed system of homogeneous wave eqs

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- Trivial ID on $\Sigma \implies$ $\boldsymbol{H} = \boldsymbol{B} = \boldsymbol{F} = 0$ on $\mathcal{W} \subseteq \mathcal{D}^{+}(\Sigma)$
- $(\boldsymbol{\kappa}, \partial_t \boldsymbol{\kappa})$ on Σ.

KS Candidate equation:

$$\Box \kappa_{AB} = -\tfrac{1}{6} R\, \kappa_{AB} + \Omega \phi_{ABCD}\kappa^{CD}$$

Initial data on Σ:

$$\kappa_{AB} = \mathring{\kappa}_{AB}$$
$$\nabla_{\boldsymbol{\tau}}\kappa_{AB} = \mathcal{D}_{Q(A}\mathring{\kappa}_{B)}{}^{Q}$$

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$$\mathcal{D}_{(AB}\mathring{\kappa}_{CD)} = 0, \quad \mathring{\kappa}_{(A}{}^{Q}\phi_{BCD)Q} = 0$$

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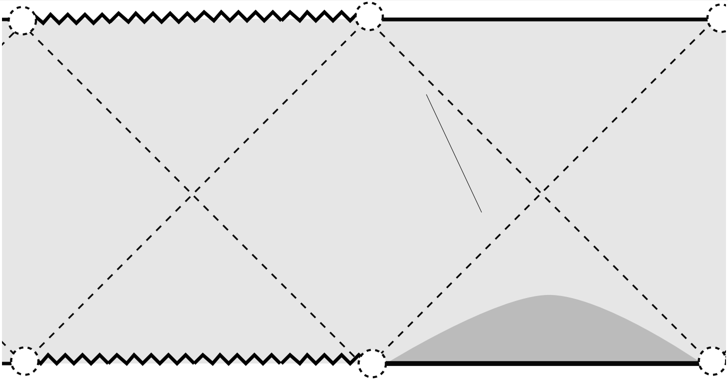
Thm CKSID (G. & Williams 22)

- Given **ID for the vacuum CEFE**, if the CKSID conditions

$$\mathcal{D}_{(AB}\kappa_{CD)} = 0, \qquad \kappa_{(A}{}^{Q}\phi_{BCD)Q} = 0$$

  are satisfied on Σ, then there exists a Killing spinor in $\mathcal{W} \subseteq \mathcal{D}^{+}(\Sigma)$.
- In the asymptotic IVP set-up $\Sigma = \mathscr{I}$



Corollary: Weyl collineation

- If $(\mathcal{M}, \boldsymbol{g})$ is a conformally Einstein manifold
- $\xi_a \mapsto \xi_{AA'} := \nabla^B{}_{A'}\kappa_{AB}$ is a **curvature collineation** of the rescaled Weyl spinor:
- $$\mathcal{L}_{\boldsymbol{\xi}}\phi_{ABCD} = 0$$

Associated conformal Killing vector

$$X_{AA'} = \Omega\xi_{AA'} - 3\kappa_{AQ}\nabla_{A'}{}^{Q}\Omega$$

- X_a is a conformal KV of $(\mathcal{M}, \boldsymbol{g})$
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Future applications

- Characterisation Kerr-de-Sitter through asympt ID.
- Type D initial data characterisation.

Future directions

- Conds on a spacelike $\mathscr{I}$ for simplicity
- Other existence and uniqueness thms for wave eqs $\rightsquigarrow$
  - Characteristic IVP
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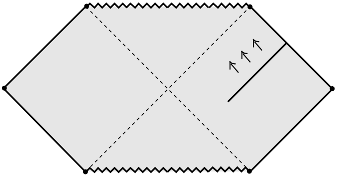
- $\rightsquigarrow$ Data on null or timelike $\mathscr{I}$



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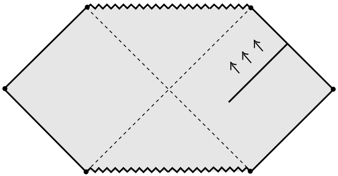

Many thanks for your attention