QKD, qCrypto and LDPC

03 05 **Quantum Key** What's next for qCrypto Distribution qCrypto SpeQtral's software Taking qCrypto A description stack forward 04 **Open Source** LDPC Solutions Protocol procedure & simulation results

QKD

Alice & Bob want to talk.

QKD

Why?



"I like you"

"Sorry, social distancing, we cannot be together"



Alice is shy and doesn't want you to read this

Bob

Alice

"Sorry, social distancing, we cannot be together"

"I like you"

Bob doesn't want you to read this either

QKD

And there may be people who *really* want to know what they are talking about... We call these people Eve

Alice Bob "I like you" Serry social distancing, we cannot be together"

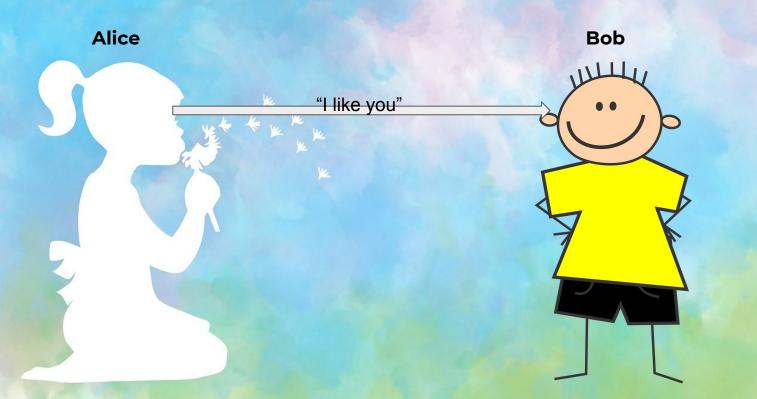
QKD

Eve eavesdrops on their communications by usually by listening,

By "listening" we really mean "listening" and then "forwarding".



QKD



QKD



And even if they have a direct connection, Eve can just cut the wire and sit in the middle and they won't know.

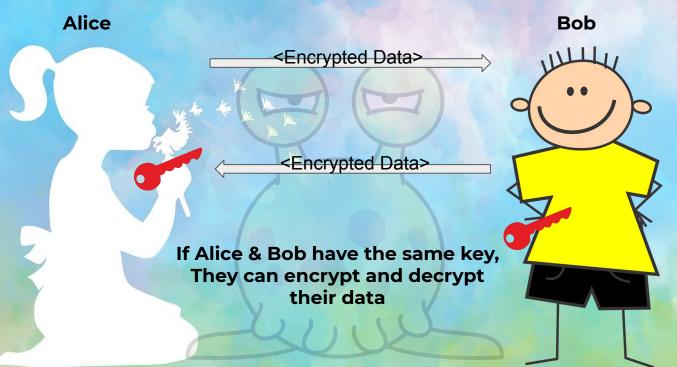
(all they see is the input & output)

Alice Bob "I like you" "I like you" "Oh my... Alice likes Bob?!?!"

QKD

So how security is usually done is to assume Eve is always there, and just encrypt the data

Now even if Eve can "eat" and "spit" the data, she can't understand it



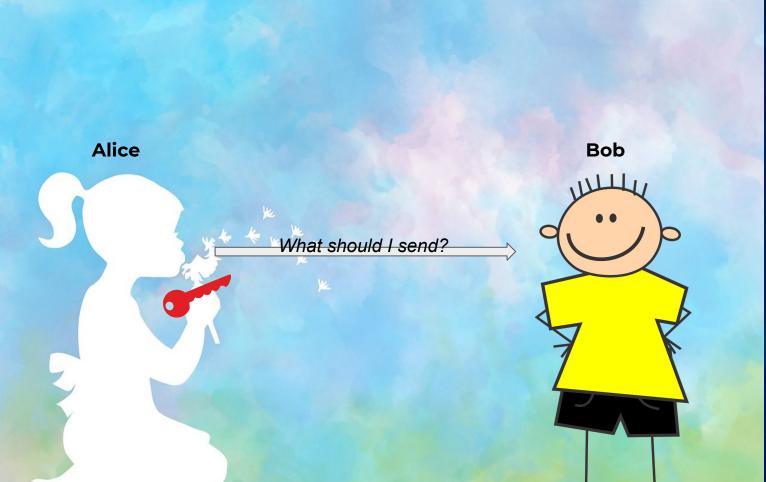
QKD

Here's the catch... How to make sure both of them have the same key?

QKD







QKD

QKD



There are non-quantum ways to do key distribution (e.g. asymmetric key), but that's not what this presentation is about.

Let's talk about quantum key distribution (QKD)

QKD





The classical channel does not provide any info on whether your message was intercepted

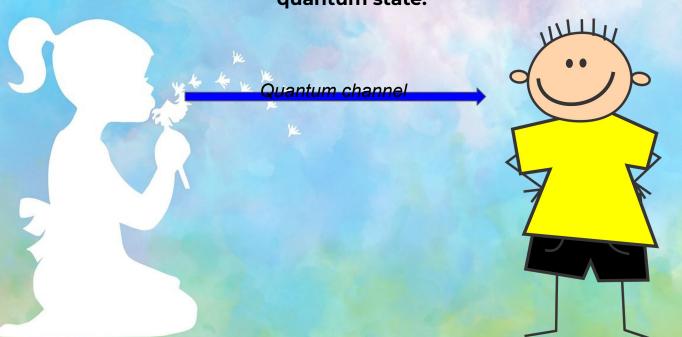
QKD



But the quantum channel is fundamentally different.

"Quantum physics says that measurement in general modifies the state of the measured system" (Scarani et al., 2009, p3)

This is due to the no-cloning theorem which states that it is impossible to create an identical copy of an arbitrary unknown quantum state.



QKD

By reading (measuring) the data from Alice on a quantum channel, Eve will introduce errors into the message which can be detected by Bob.

This is known as an intercept-resend attack



QKD

Since Eve's knowledge of whatever is sent on the quantum channel can be calculated (by the error rate),

we can send the key on the quantum channel,

Then verify that it has has not been read by checking the number of errors in the key. (Quantum Bit Error Rate i.e. QBER)



QKD

So how does QKD work?

4 Phases:

- 1. Detection
- 2. Sifting
- 3. Error Correction
- 4. Privacy Amplification

(Alice sends Bob the key, Bob "detects" it)
(Bob "sifts through" the basis of the key bits)
(Alice & Bob correct errors in the key)
(Alice & Bob make their key more "private" by



QKD

Detection

The key is actually a random sequence of '0' and '1'



Each '0' or '1' is known as a bit



QKD

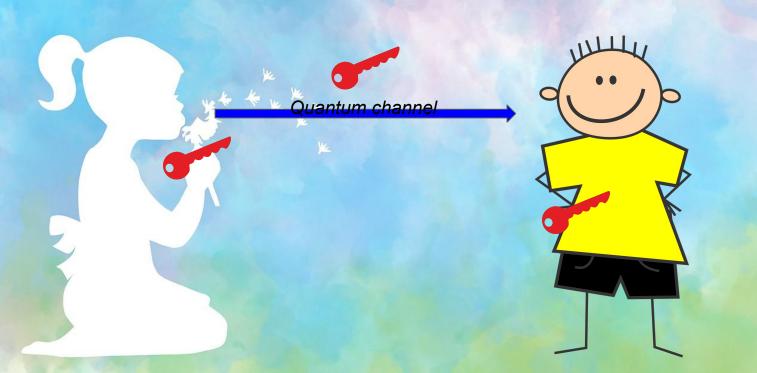
Detection

Alice sends the key bit by bit. Simple, right?

If only it were that simple. We're talking about quantum physics!

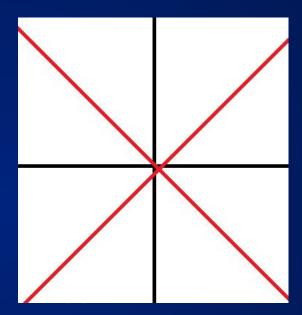
Alice converts each bit into a qubit, then sends the qubit, and Bob measures these qubits.

QKD



Alice Qubit sender (Photon source)

- Alice chooses between 2 basis randomly
- Based on her ith bit's value, she represents the bit as a straight line
- Basis 1: Plus
- 0 = Vertical line
- 1 = Horizontal line
- Basis 2: Cross
- 0 = Diagonal \ line
- 1 = Antidiagonal / line



Basis + in black

Basis x in red

Alice's photon source / qubit "gun"

Alice has a gun

2 Angles

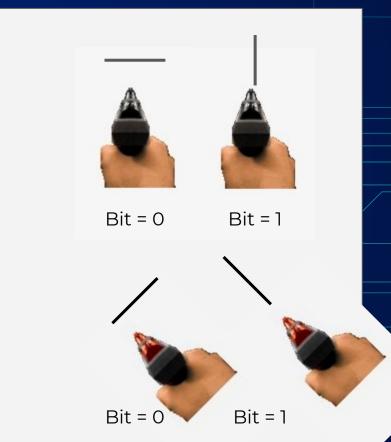
- Alice randomly chooses whether to shoot it:
 - upright
 - o at a 45 deg angle

Fired upright

 She loads her bullet (bit) and shoots it

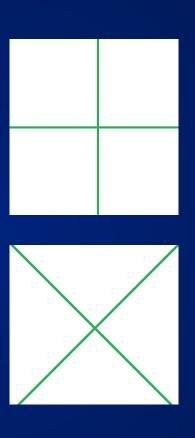
Fired at 45 deg angle

It is a flat bullet



Eve's / Bob's Detector

 Eve and Bob have a detector that can only detect in one basis at any given point in time



Detectors

 Eve and Bob have a detector that can only detect in one basis at any given point in time



Detectors

They can only rotate the detector



 Bob can only detect in one basis (either like this, or 45 degrees like an x)

> If he chooses the same basis that Alice sent the bit in, he will correctly measure the bit value that Alice sent







































Bob's Detector

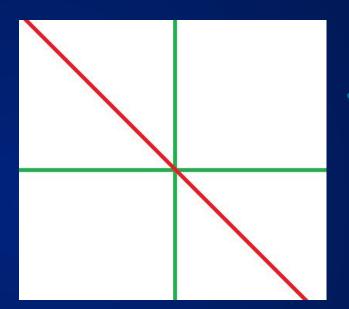
 Bob can only detect in one basis (either like this, or 45 degrees like an x)





Bob's Detector

 Bob can only detect in one basis (either like this, or 45 degrees like an x)



If he chooses the wrong basis, he will get a random value (can either go left or right, thus '0' or '1')

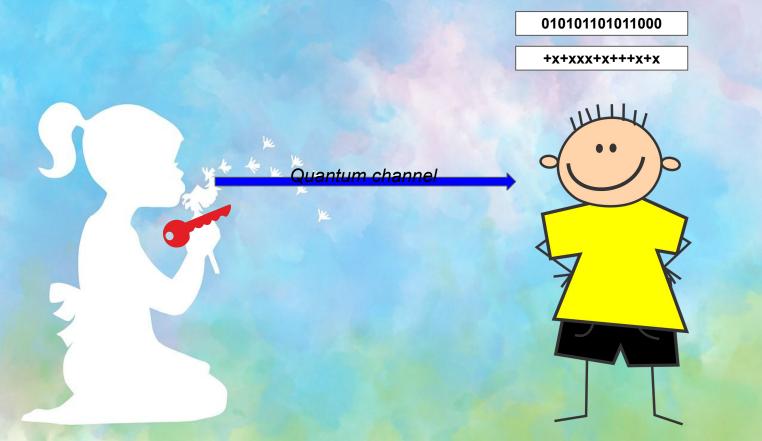
"Is a diagonal line horizontal or vertical?"

In the detection phase, Bob doesn't know which bits are sent in what basis.

He just measures the bits in random bases and stores the bases he measured them in.

QKD

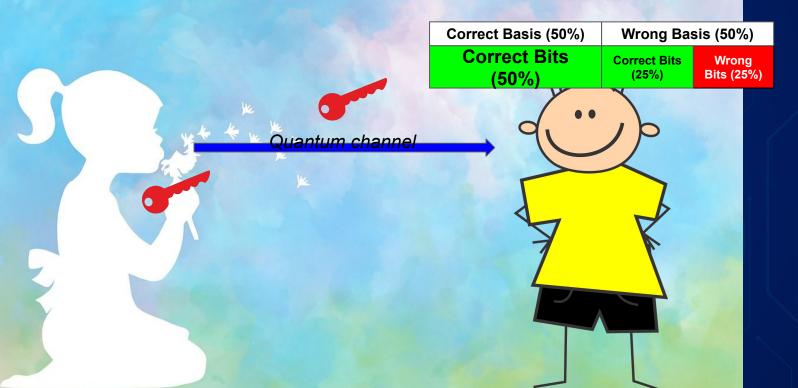
Detection



QKD

Detection

In other words, 50% of his bits will be the same as Alice's, 50% will be random.



At this point Alice and Bob ditch the quantum channel and use the classical channel.

Alice sends Bob the sequence of bases she sent the bits in. (+,x,+,+...)

Correct Basis (50%) Wrong Basis (50%)

Correct Bits
(50%) Correct Bits
(25%) Wrong
Bits (25%)

QKD

Sifting



Bob discards the bits that he measured in the wrong basis. (because they are now random bits)

He tells Alice which bits he discarded and she also discards those bits.

We call the result the raw sifted key.

Correct Basis (50%)

Correct Bits

(50%)

In a perfect scenario, this has no errors.



QKD

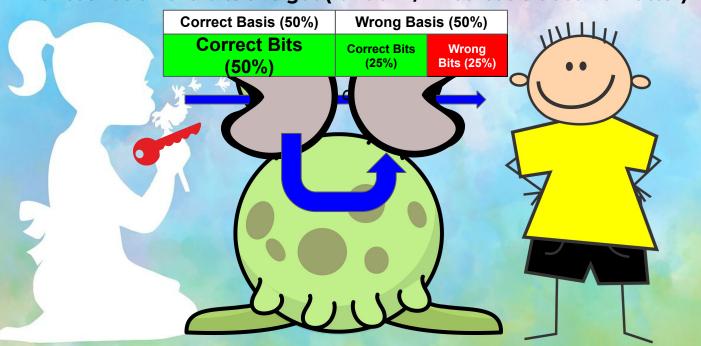
Sifting

Why does it have to be so complicated?

Because Eve doesn't know the basis either, so if she's intercepting and resending, she also has to measure each bit with a random basis.

50% of Eve's bits will be the same as Alice's, 50% will be random.

Eve resends all the bits she got (random / fixed basis doesn't matter)



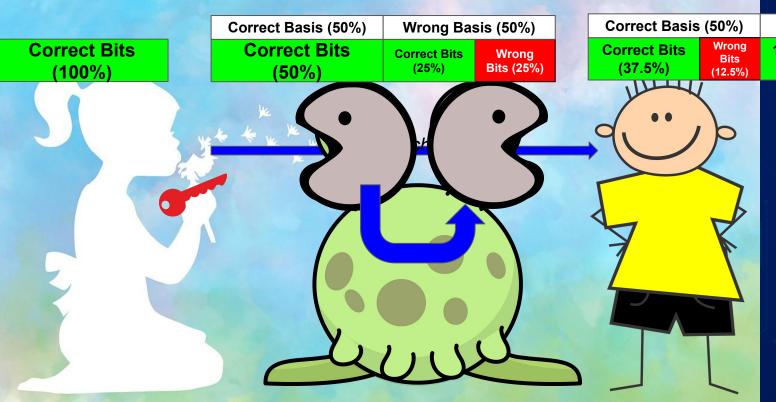
QKD

Detection

Thus with Eve, Bob will have some errors.

QKD





Wrong Basis (50%)

18.75 % 18.75 6.5 %

6.2 6. 5 25 % %

Total Error rate without Eve: **25%**

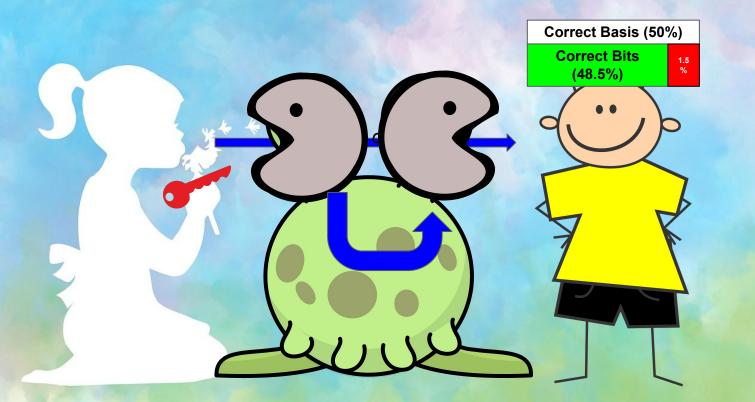
Total Error rate with Eve:

(12.5 + 18.75 + 6.25) = **37.5%**

After basis reconciliation: **¼ of raw sifted key is wrong**

Eve, or other measurement errors (e.g. accidental detections) introduce errors, so it's never perfect.

For simplicity, let's just assume the errors were introduced by Eve who eavesdropped a small percentage of the bits.



QKD

How do you derive a key from this? Let's discuss a simplified version of Cascade, an error correction algorithm.

QKD



Imagine this scenario: Alice is a teacher and she's sick, so she's at home

Bob is the remedial teacher

QKD





Imagine the key as a class of students, with each bit representing a chair

We assume the students sometimes play truant or go to the wrong class ("errors")



QKD

Alice can tell which chair is supposed to be occupied because she has the attendance sheet

Bob can only see the students and the empty chairs



QKD

Alice and Bob split the class into groups (e.g. groups of 4)

Alice then tells Bob "okay, the first group should have an even number of students. The second group should have an odd number of students..."

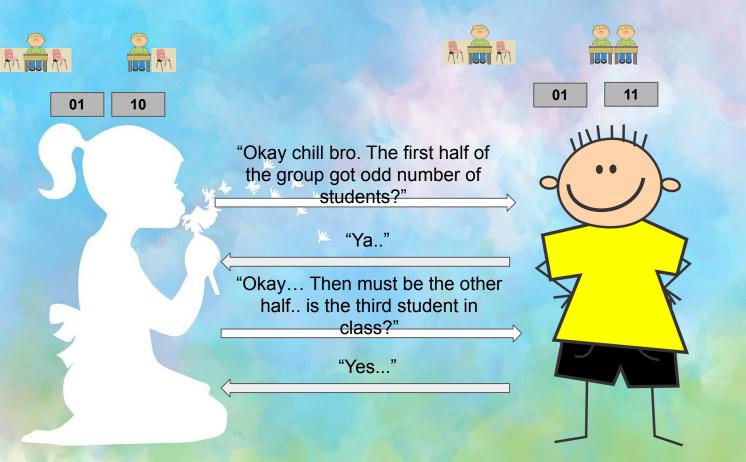


QKD

If Bob realizes a parity doesn't tally, he'll tell Alice. 0111 1011 0000 1011 0000 0110 "The first group only has an odd number of students..."

QKD

Once they identify a problem group, they will repeatedly split the group into two, find the chair that is causing the issue, then fix it.



QKD

Once they identify a problem group, they will repeatedly split the group into two, find the chair that is causing the issue, then fix it.



QKD

Once they identify a problem group, they will repeatedly split the group into two, find the chair that is causing the issue, then fix it. "What are you doing in my 0111 class??" 0110 "Aha! Then the 4th student not from our class!"

QKD

Once they identify a problem group, they will repeatedly split the group into two, find the chair that is causing the issue, then fix it.



QKD

They do this multiple times with multiple block sizes until they are confident that their bits are the same.

They also keep a history of what they sent and received so when they correct a student, they can check back and see if there's any more bits they can correct (thus the name "Cascade" from the cascading effect)



QKD

How do you derive a key from this? Let's discuss a simplified version of Cascade, an error correction algorithm.

So Alice will split her key into blocks.

She calculates the parity of each block.

Parity = 0 if sum of the bits in a block is even
Parity = 1 if sum of the bits in a block is odd



QKD

Bob checks the parities of each block. If he finds there is a difference, he knows that block has an error and he will inform Alice.



QKD

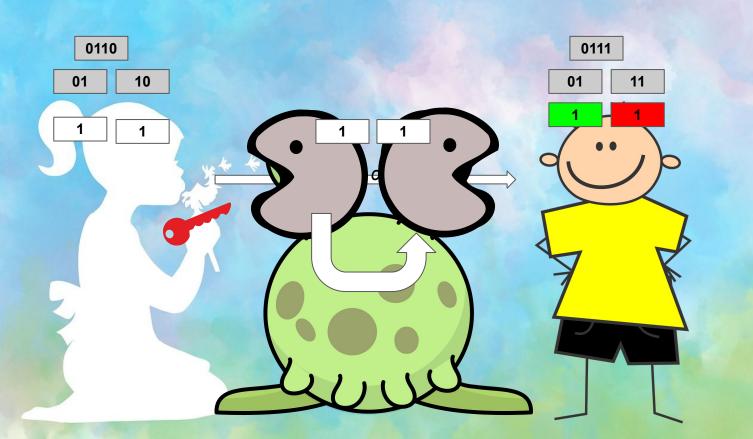
Bob checks the parities of each block. If he finds there is a difference, he will inform Alice.

Alice will then recursively split the block and calculate parities for each sub-block until the error is in a block of size 1, then they correct the error by



QKD

They do this multiple times with multiple block sizes until they are confident that their bits are correct and their sifted keys have been corrected.



QKD

At this point Alice & Bob have a fraction of their raw sifted key. However, since communications were done through classical channel, Eve also knows:

- 1. What bases Alice used and the bases Bob accepted
- 2. Every parity bit that was exchanged between Alice and Bob

Eve can use these two pieces of information to derive parts of the key that Alice and Bob have now.



QKD

Privacy Amplification

So the question is: Given that Eve knows t bits, can they still use the key, or do they have to throw it away and choose a new key?

QKD

Privacy Amplification



The answer is... they can still use it!

The leftover hash lemma tells us that they can produce a final key of length (n - t) from a raw sifted key of length n, over which Eve has negligible knowledge using a universal hash function. This is called privacy amplification, as it "amplifies" the privacy of the key.

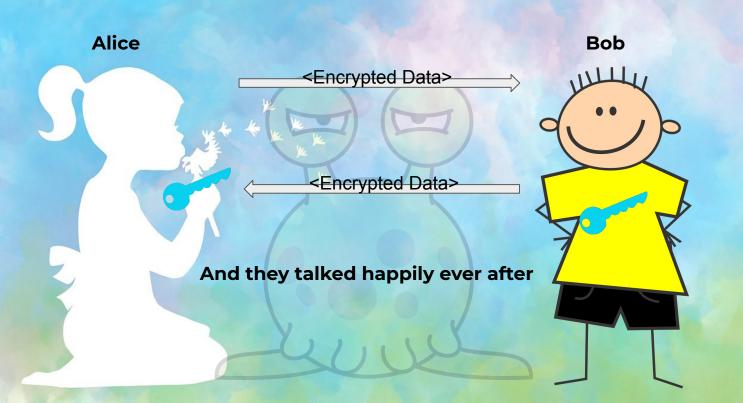


QKD

PrivacyAmplification



Post-QKD



There's more...

How do they store their key?

QKD

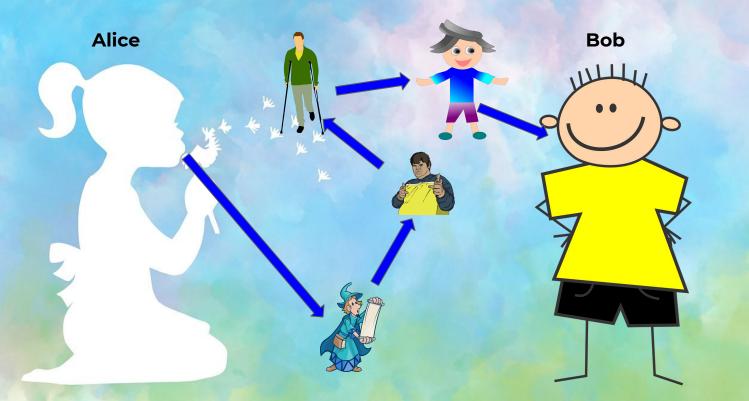
Post-QKD







What if Alice and Bob only have an indirect quantum channel connection?



QKD

Post-QKD

O2Open Source Solutions

Implementing QKD at a larger scale

Open Source Solutions

Questions to ask

- Can it be used with SpeQtral software stack?
- How well developed is it?
- How well maintained is it?

I've looked at 2 stacks:

- OpenQKDNetwork
- CQPToolkit

Open Source Solutions Review

OpenQKDNetwork

Reviewed on June 8

- A suite of Java Maven Projects to support QKD at a Wide Area Network (WAN) level
- Pros:
 - Good ideas (I am probably ill-equipped to evaluate their concepts)
- Cons:
 - Unoptimized implementation & naive algorithms used (appears to be proof of concept at best)
 - Incomplete
 - Zero documentation
 - Zero tests

Open Source Solutions Review

2. CQPToolkit

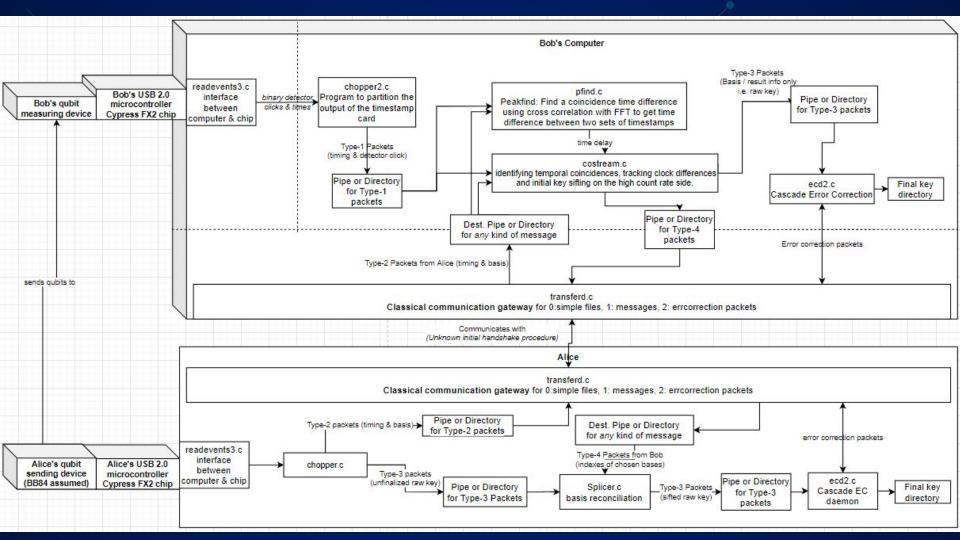
Reviewed on June 8

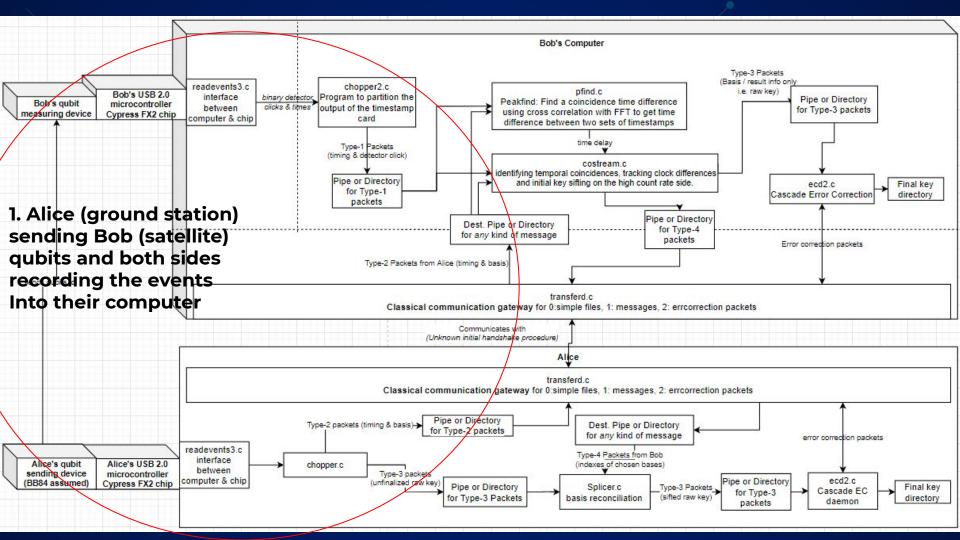
- A C++ framework to manage QKD devices and keys (can support QKD at a WAN level as well)
- Pros:
 - Comprehensive implementation
 - Decent documentation
 - Feels complete
 - Has test cases
- Cons:
 - Can be useful but will require technical expertise
 - Large and complicated internal code
 - Documentation not in-line with code
 - QKD portion will require our own implementation

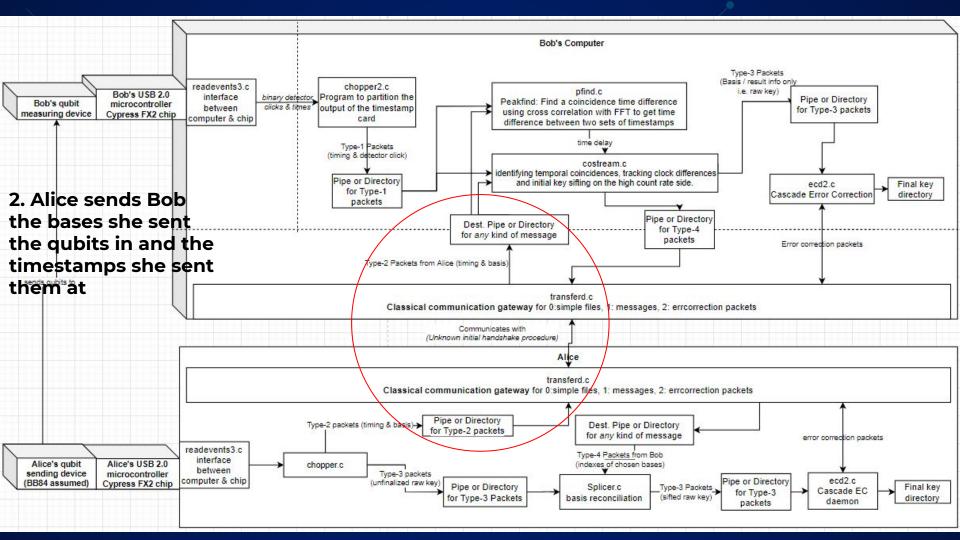
03

qCrypto

Brief architectural overview of our software stack, which is a suite of programs for QKD

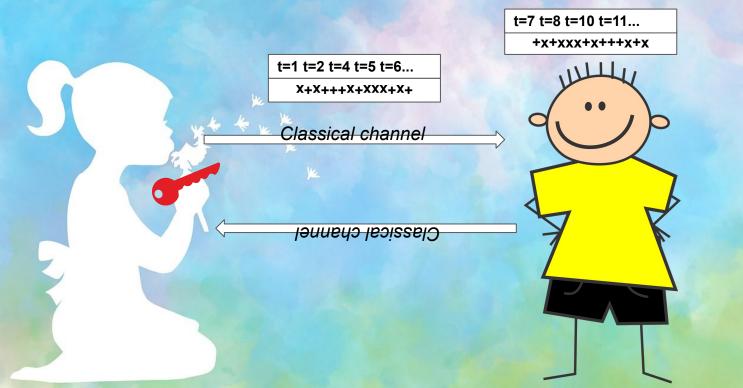






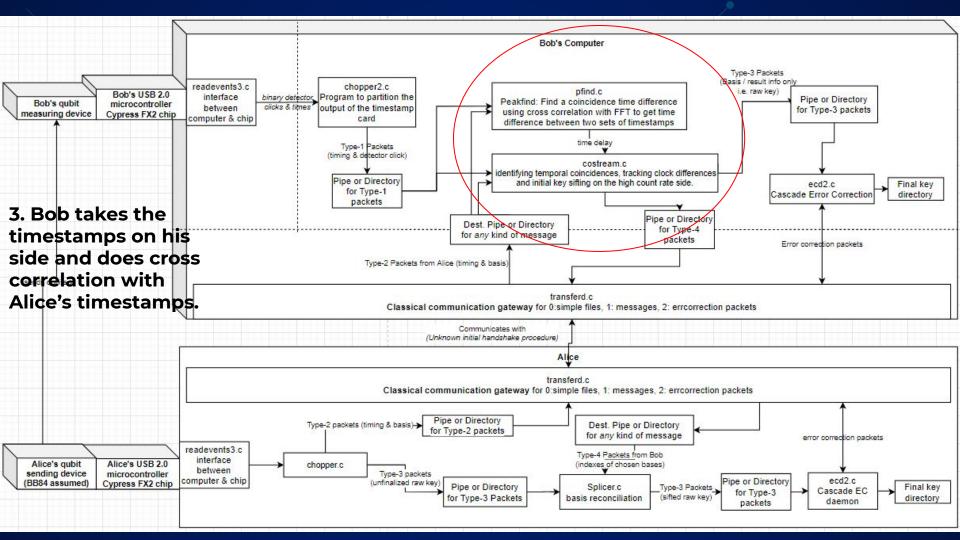
Since the quantum channel is just Alice shooting photons (light) at Bob, some photons may go missing or Bob may measure random light thinking it was from Alice.

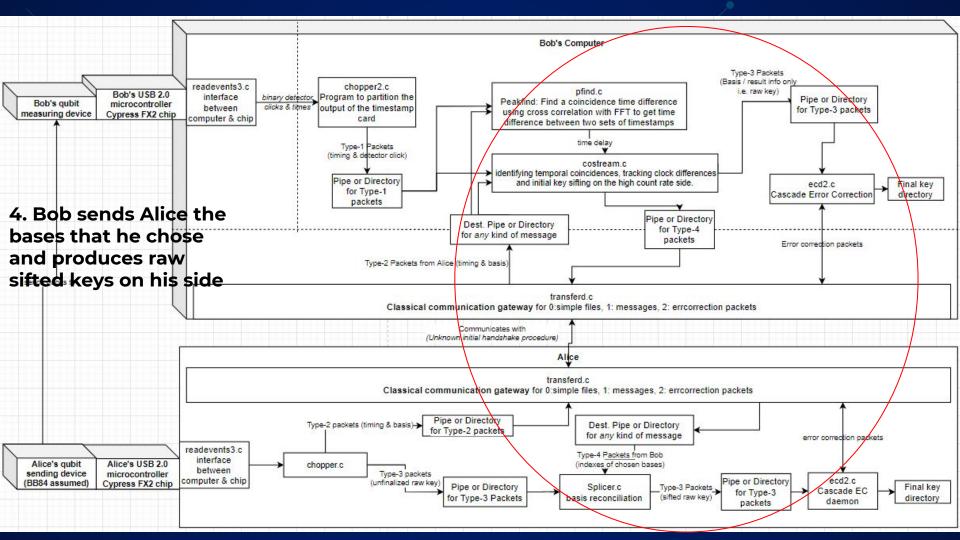
So on top of sending the bases, Alice also sends the timestamps, which Bob can cross correlate with his set of timestamps to determine which bases match up.

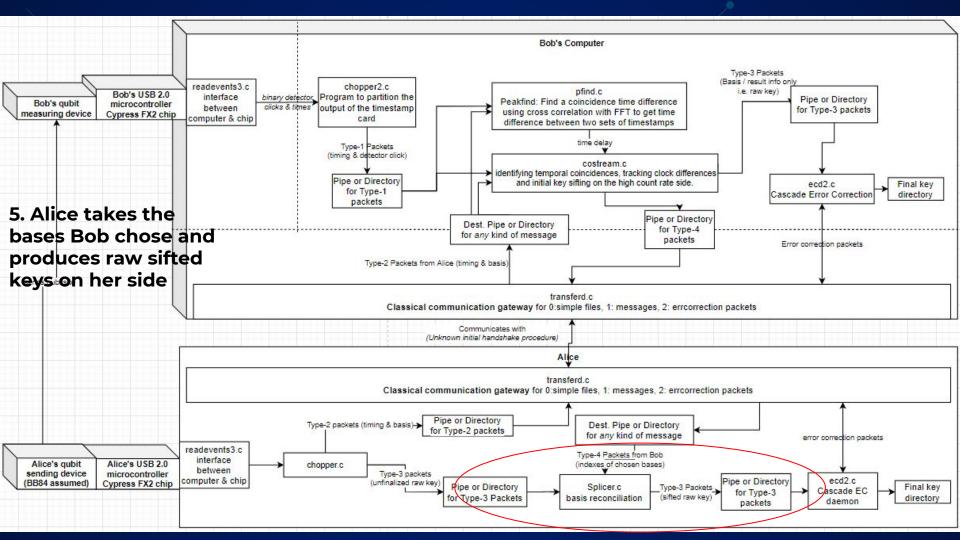


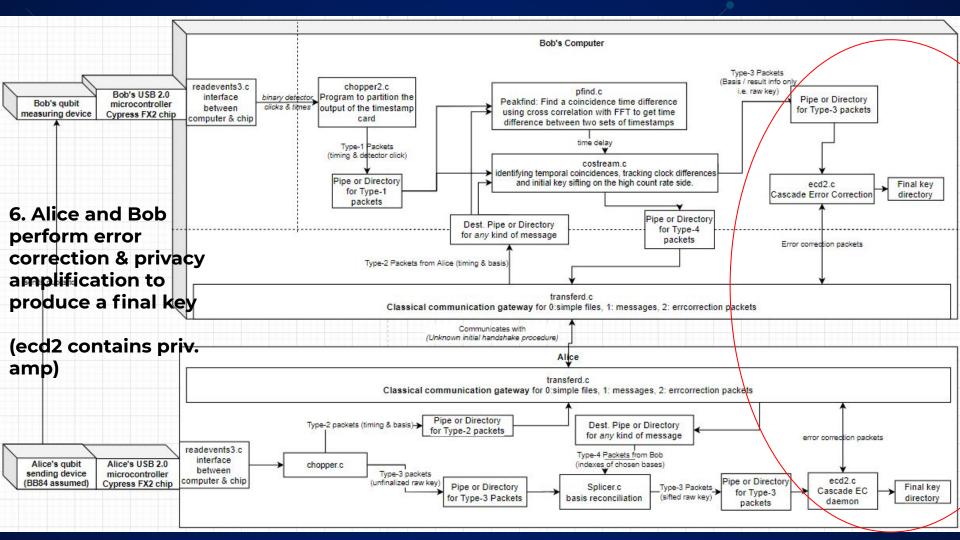
QKD

Sifting









qCrypto Stack on Google Collab

- Jupyter Notebook stored on Google Drive which can be run on Google's virtual machines
 - Used because it is easier for less technical people to follow
 - Repurposed to run Bash, C and C++ code
- Comprehensive documentation on entire qCrypto stack, as well as points of consideration for future developers
- Shows you how to setup and run the stack, and then literally runs the code to set it up and execute on a linux virtual machine
- Other documentation (including differences from Dr Kurtsiefer's code) on the document itself

Main differences from Dr. Kurtsiefer's qCrypto

- Various (minor) changes to various components in the stack
 - Mainly refactoring (improving readability without affecting original functionality)
 - Added features which were useful in setting the stack up (more debug logs, ordering of bases in timestamp events on Bob's side, more input params on transferd.c to configure local port number etc)
 - Added documentation generation
- Error correction module refactored and restructured to support other algorithms
- More detailed version on the Google Collab document for those who are interested



Error Correction Procedure

- Estimate Quantum Bit Error Rate (QBER) by taking a sample from the raw sifted key (that sample is later discarded)
- Recall that the raw sifted key comprises of a sequence of binary states (now represented as bits)
- 3. Cascade algorithm: Divide raw sifted key into blocks, exchange parities for each block, and recursively binary search for flipped bits if the parities do not match
 - Binary search as in if a parity for a block doesn't match, recursively split the block into two and repeat the parity checking procedure until the erroneous bit is found
 - b. Iterate this a few times until satisfied that there should be no more errors
 - c. Used in qCrypto's ecd2

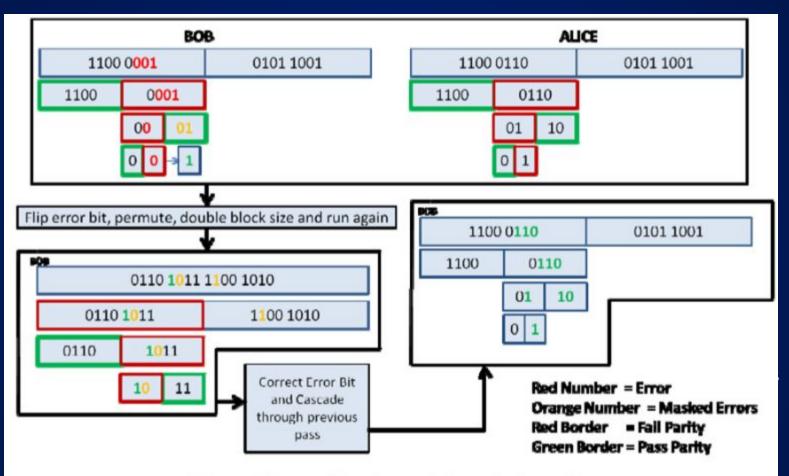


Figure 1. Binary Bisection and Cascade Operation

Privacy Amplification Procedure

- Perform privacy amplification (matrix multiplication of the corrected key with a universal hash family: a random binary matrix will do)
 - a. QBER is used as a parameter for binary entropy function which is then used to calculate length of final key (L).
 - Revealed bits in Cascade are either discarded during the algorithm or factored into the final key length calculations.
 - c. Matrix multiplication: 1 x n corrected key times n x L random binary matrix





04

LDPC

Low Density Parity Check

Codes (if the name sounds complicated means protocol is also complicated)

Imagine that you compare the parity of a block in the Cascade algorithm.

How did you do it?

Alice adds every bit (student) in a block (group), and checks whether it is even or odd.

Alice tells Bob, who then checks if it's the same parity on his side.



LDPC

Now let's do the same thing differently.

Alice now tells Bob:

"I'll give you this number, 0 or 1. If you add it to the number of students in the group, that group will always have an even number."

"If it is not even, then there is an error in that group."



LDPC

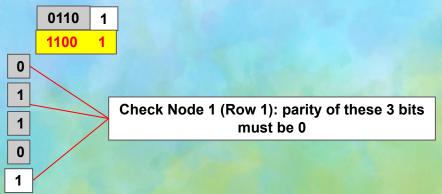
If this is represented as a parity-check matrix, this is how it (in yellow) will look like:

0110 0 1111 1

This matrix can also be represented as a Tanner Graph (bipartite graph): (the values substituted in to make it easier to understand). Each edge is a '1'

Check Node 1 (Row 1): parity of these 5 bits must be 0

If we split the block in half and calculated the parity for the left sub-block:



LDPC

If this action is represented as a parity-check matrix, this is how the parity-check matrix (in yellow) will look like:

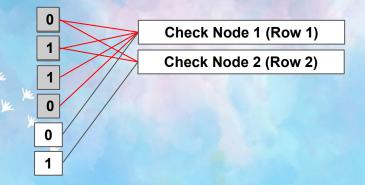
The idea is that your raw sifted key, with the parity bits concatenated behind, should give [0 0 0 0 ... 0] when multiplied with the parity check matrix:

It's just a slightly more complicated way of checking if the parity is correct.

LDPC

If we combine both rows together:

0110	0	1
1111	1	0
1100	0	1



"Low-density": Few number of '1's

"Parity Check": Uses parities to "check" the message

"Code": Something you send that contains the message (raw sifted key in this scenario) and the parity bits

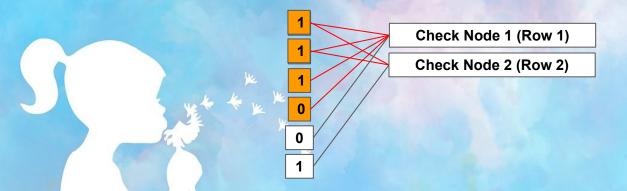
* for QKD you only send the parity bits

Alice & Bob establish the parity check matrix to use beforehand

LDPC

Alice will send Bob the parity bits only "01" and Bob will substitute the message portion with his own raw sifted key.

1110	0	1
1111	1	0
1100	0	1



Bob iteratively corrects the message bits using the constraint that the parity of the bits that each check node is connected to must be zero

May fail if too few parity bits are used, too few check nodes, check nodes have too high a degree etc

LDPC

Let's do a simple decoding (Gallager) example with the same matrix. Alice computed parity bits "01" and sends the parity bits to Bob:

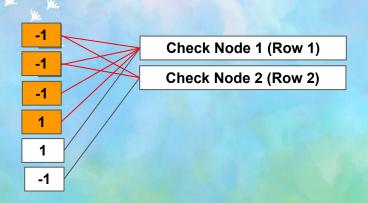
 0110
 0
 1

 1111
 1
 0

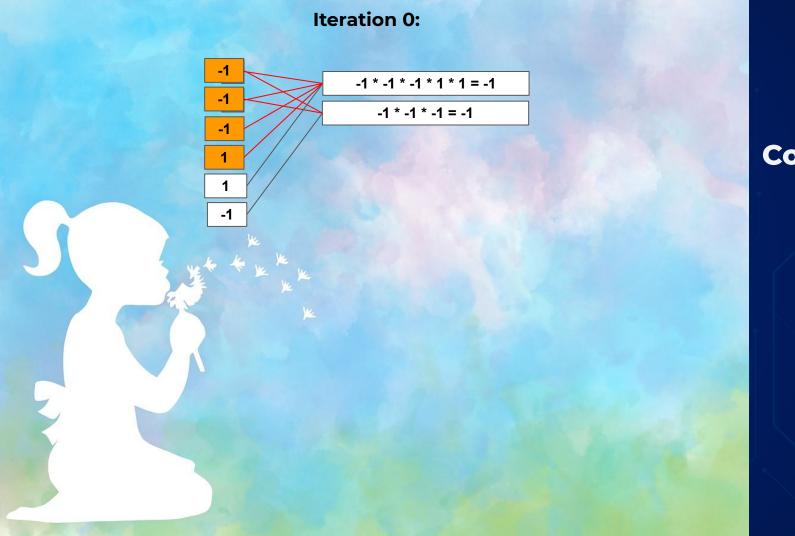
 1100
 0
 1

Let's say Bob's raw sifted key bits are 1110

Bob passes the raw sifted key bits and the parity bits into the Tanner graph: For decoding purposes, '1' is converted to -1, '0' is converted to 1



LDPC



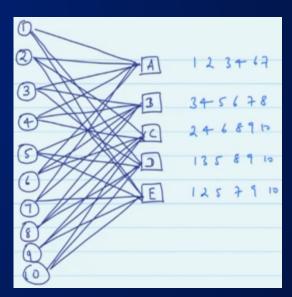
LDPC

LDPC

- 1. Error correcting code (basically in qCrypto's ecd2 you swap Cascade for LDPC)
- 2. Both Alice and Bob establish a parity check matrix H
- 3. Alice calculates a block of parity bits p with H, such that when p is appended to her sifted key m_{Δ} , $[m_{\Delta} p] \times H^{T} = 0$ (**encoding**)
 - a. Alice sends p to Bob
 - b. Bob appends p to his own sifted key m_B and using the relationship that $[m_A p] \times H^T = 0$, iteratively corrects his key using a decoding algorithm (**decoding**)
 - Decoding can fail; often referred to as Frame Error Rate (FER).
 Usually due to insufficient parity bits (this is due to the code rate of the LDPC code used, which is the ratio between message bits and parity bits)
 - ii. Final decoded value checked with a CRC (checksum) sent by Alice to verify correctness

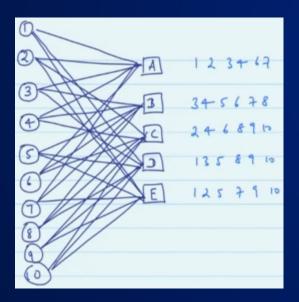
LDPC (Encoding)

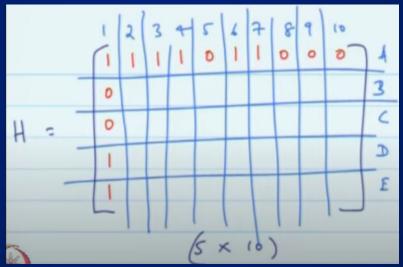
- 1. Error correcting code (basically in qCrypto's ecd2 you swap Cascade for LDPC)
- 2. Alice calculates a block of parity bits p using a parity check matrix H, such that when appended to her sifted key m_{Δ} , $[m_{\Delta} p] \times H^{T} = 0$ (encoding)
- 3. H is a sparse binary matrix and can be represented as a Tanner Graph (bipartite graph):
- 4. Left nodes: "Variable Nodes". Each VN is a column in H
- 5. Right Nodes: "Check Nodes". Each CN is a row in H
- 6. An edge represents a '1' in H_{variable_node_i, check_node_j}



LDPC (Parity Check Matrix)

- 1. Left nodes: "Variable Nodes". Each VN is a column in H
- 2. Right Nodes: "Check Nodes". Each CN is a row in H
- 3. An edge represents a 'l' in H_{variable_node_i, check_node_j}





LDPC (Decoding)

- 1. Alice sends that block of parity bits to Bob
- 2. Bob appends p to his own sifted key m_B and using the relationship that $[m_A p] \times H^T = 0$, iteratively corrects his key using a decoding algorithm (**decoding**)
 - a. Variables and check nodes are either directly or indirectly linked to each other, allowing the decoding algorithm to guess what is the most likely value for a particular variable node etc
 - b. Decoding can fail; often referred to as **Frame Error Rate (FER)**. Usually due to insufficient parity bits (which are not sent to improve the **code rate**, which is the ratio between message bits and parity bits)
 - Final decoded value checked with a CRC (checksum) sent by Alice to verify correctness

LDPC Privacy Amplification

- 1. Almost identical to Cascade (since this is LDPC operating on a binary symmetric channel (BSC))
- 2. Only difference is all the parities sent in one go
- 3. Privacy amplification algorithm factors the number of parity bits sent into the final key length
- 4. For LDPC, there is an additional trade-off: sending less parity bits improves reconciliation efficiency (leading to higher final key length), but can lead to higher frame error rate (e.g. more decoding failures)

Implementation

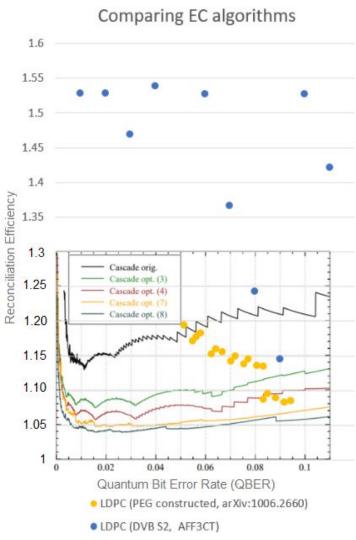
- 1. AFF3CT C++ library
 - a. LDPC is complicated, implementation by myself was prone to error, likely not so efficient and lengthy in development time
 - b. Use a library to do it instead
 - c. Downside: Initially designed to be a simulator (since LDPC usually done on a very low level, even semiconductor level), so not really optimized (e.g. represents each bit as an 32-bit integer when encoding)
 - d. No documentation on how to use it as a library, but I've already developed code on how to use it as a library so that is no longer an issue

Implementation

- 1. Main considerations:
 - a. What matrix to use (and for what QBER)
 - i. Rate-variable LDPC codes, where you can modify the parity bits sent, can reduce the number of matrices needed
 - ii. Construct your own / use others?
 - b. What decoding algorithm to use?
 - i. Already tested and sorted out (now using belief propagation flooding sum-product algorithm), but may want to test other decoders if using good matrices
 - QKD LDPC for the binary symmetric channel is different from QKD LDPC for the continuous channel (already noted in README_LDPC.md & factored into code)

LDPC Parity Check Matrix

- 1. Parity Check Matrices tested, ordered by secret key rate (ascending order):
 - a. From the recently released 5G standard
 - i. Code rate can be varied by not sending some parity bits (known as *puncturing*). In the 5G standard, parity bits are punctured starting from the last parity bit
 - ii. Frame error rate too high, resulting in bad performance
 - b. A few sample matrices from the Sparse Matrix Encyclopedia
 - Matrices weren't able to support QBERs as well as DVB S2. Probably because they were quite dated
 - From the DVB S2 standard (decent, but does not measure up to efficiencies quoted by papers on LDPC that constructed their own matrices)



Comparing Cascade with LDPC

- X-axis: QBER, Y-axis: Reconciliation efficiency (the closer to 1, the better).
- 2. **Blue dots**: LDPC implementation using DVB S2 matrices. Not rate matched, so there is room for improvement. All at 0% FER
- 3. **Yellow dots**: LDPC with matrices constructed by improved PEG algorithm, arXiv:1006.2660v1 by David et al.. Data from the paper, all at presumably 0% FER
 - a. Construction algorithm not released
- 4. **Jagged lines**: Cascade reconciliation efficiencies based on arXiv:1407.3257v2 by Jesus et al. Diagram from the paper.
- 5. **Omitted**: LDPC with matrices constructed by PSD-PEG algorithm from doi:10.1109/ACCESS.2017.2688701



Future Steps

- 1. Integrating LDPC into qCrypto
 - a. Matrix used needs to be optimized (constructed using whatever algo.)
 - i. Investigate matrix construction algorithms
 - 1. Optimizing degree distribution & girth of matrices
 - ii. Look into varying the rate of DVB S2 matrices?
 - b. Aff3ct library that I used will need to be optimized for use
 - Will require some copying pasting, re-implementation of some sections
 - c. LDPC protocol needs to be added into ecd2
 - i. C++ can be integrated with C quite easily
 - ii. Should be easy as ecd2 is better structured to support multiple algorithms



Thanks!

Do you have any questions?

Pictures from Pixabay NES Zapper by Mikel2

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