BAYESIAN NETWORKS

ARTIFICIAL INTELLIGENCE | COMP 131

TODAY ON AI

- Bayesian networks
- Independence
- Automated reasoning
- Optimizations
- Questions?

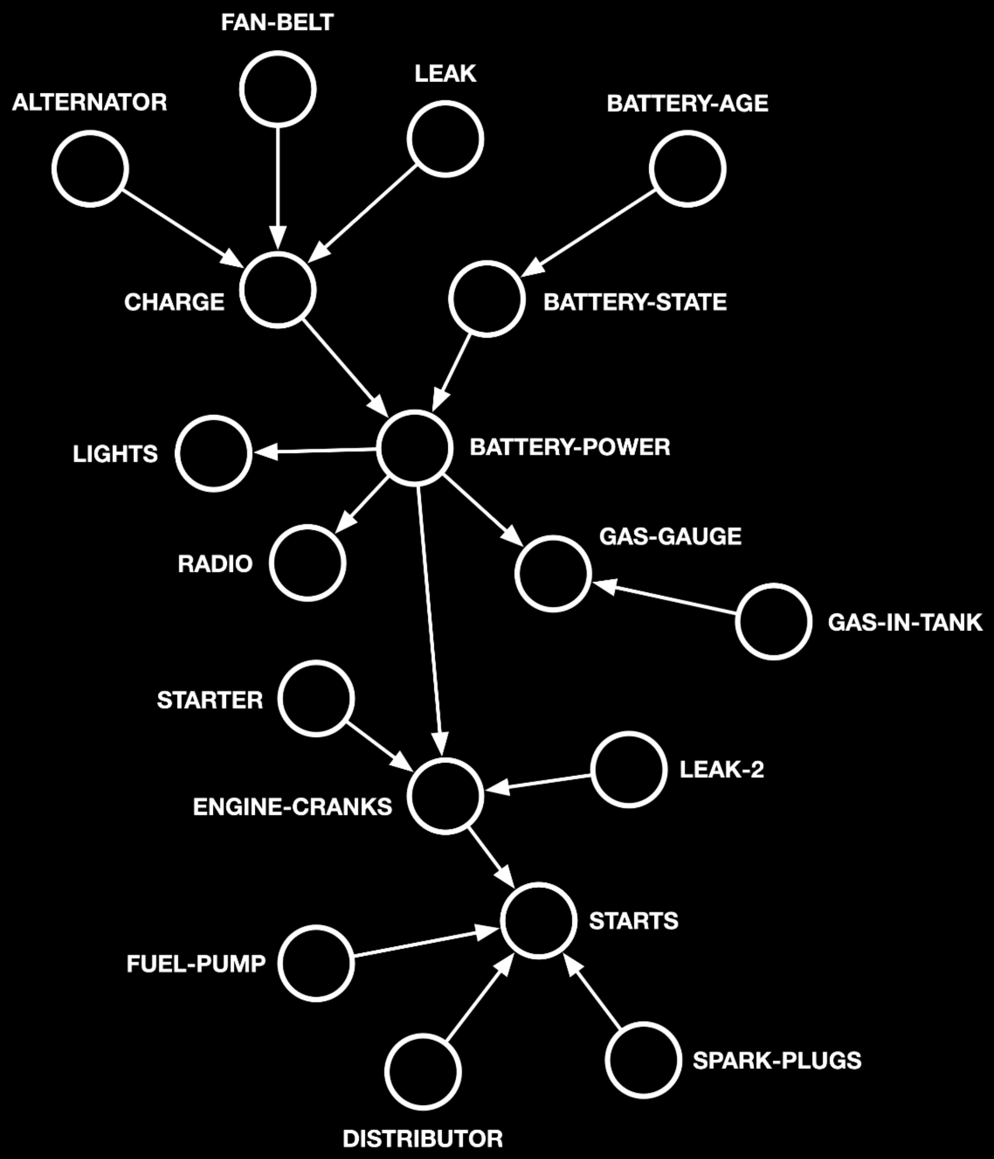
- **Conditional probability**: $P(X|Y) = \frac{P(X,Y)}{P(Y)}$
- **Product rule**: $P(X, Y) = P(X|Y)P(Y)$
- **Chain rule**: $P(X_1, \dots, X_n) = \prod_i P(X_i|X_1, \dots, X_{i-1})$
- ***X* and *Y* are independent** iff $P(X, Y) = P(X)P(Y)$
- **Bayes rule**: $P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$

SECTION 01

Bayesian networks

Bayesian networks, or **Belief networks** or more formally **graphical models**, are a simplified descriptions of how some portion of the world work:

- It is a compact way to describe joint probabilities
- It allows to calculate complex joint distributions using **local conditional probabilities** among random variables
- Local interactions will **chain together** to give global, indirect interactions

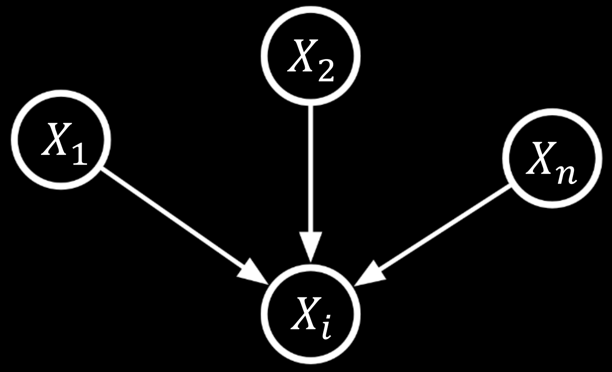


- **Node**: a random variable with its domain
  - Can be assigned (observed) or unassigned (unobserved)
  - There is usually one node per random variable

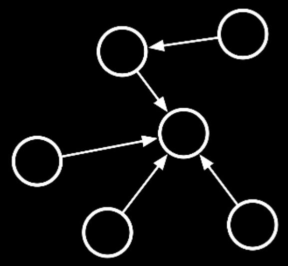
- **Nodes without arcs**: represent independent random variables

- **Arc**: indicates an interaction between variables. It encodes a local conditional probability

$$P(X_i|X_1, \dots, X_n) = P(x_i|parents(X_i))$$



- **Network**: A directed, acyclic graph that encodes conditional independence

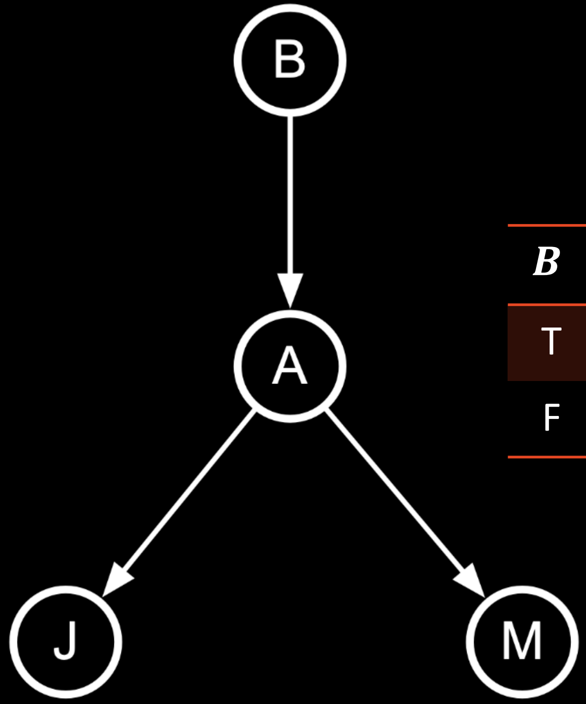


You are at work. You receive a phone call from your neighbors Mary and John who say that they think they hears your alarm going off. Is it possible that you are being burgled?

$$P(B, A, J, M) = P(B) \times P(A|B) \times P(J|A) \times P(M|A)$$

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
B	Burglary	{T, F}

$P(b)$	$P(\neg b)$
0.001	0.999

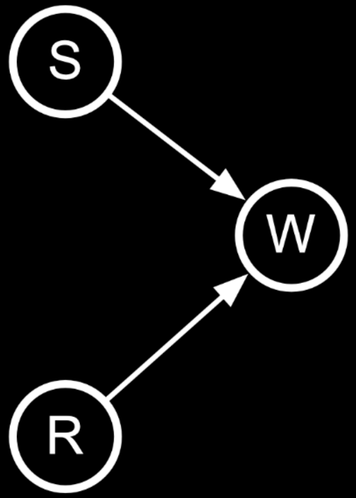


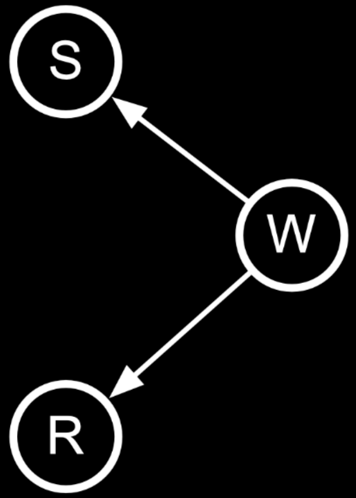
B	$P(a\|B)$	$P(\neg a\|B)$
T	0.95	0.05
F	0.29	0.71

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

The word **causal** is contentious in cases where the model of the data contains no explicit temporal information:





$$P(R, S, W) = P(S)\,P(R)\,P(W|R, S)$$

$$P(R, S, W) = P(W)\,P(R|W)\,P(S|W)$$

- Bayes Networks do not have to be causal. The arcs simple **reflect some correlation**

- When Bayes Networks reflect **a true causal** relationship:
  - They are more intuitive
  - They are simpler to represent from expert knowledge
  - They are topologically simpler

- Remember that the network topology really encodes conditional independence

A useful and fundamental condition that Bayesian network capture is called **conditional independence** and indicated as $\boldsymbol{X} \perp \boldsymbol{Y} \mid \boldsymbol{Z}$:

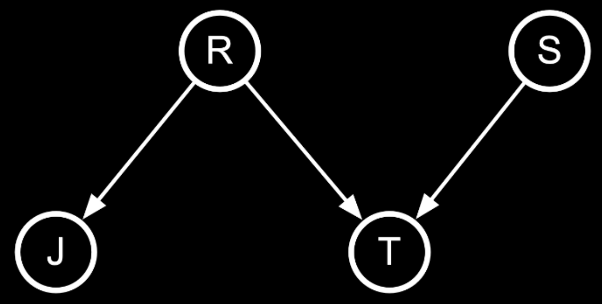
$$\forall x \in X, y \in Y, \forall z \in Z: P(x, y|z) = P(x|z)P(y|z)$$

$$\forall x \in X, y \in Y, z \in Z: P(x|z, y) = P(x|z)$$

$$\forall x \in X, y \in Y, z \in Z: P(y|z, x) = P(y|z)$$

One morning Tracey leaves her house and realizes that her grass is wet. Is it due to overnight rain or did she forget to turn off the sprinkler last night?
Next, she notices that the grass of her neighbor, Jack, is also wet.

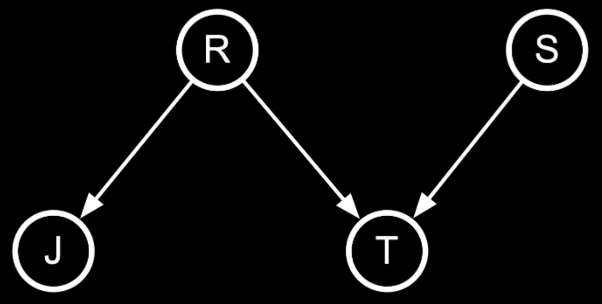
Example:
$R = Rained$
$S = Sprinkler$
$J = Jack's\ grass\ is\ wet$
$T = Tracey's\ grass\ is\ wet$



A **good conditional independence** assumption is: $J \perp T \mid R$

The chain rule says: $P(x_1, \ldots, x_n) = \prod_i P(x_i | x_1, \ldots, x_{i-1})$

Let's look at the previous example:



$$
\begin{aligned}
P(J, T, R, S) &= P(R) \times P(S) \times P(J|R, T, S)P \times (T|J, R, S) \\
&= P(R) \times P(S) \times P(J|R, T) \times P(T|J, R, S)
\end{aligned}
$$

If we assume that $J \perp T \mid R$:

$P(J|R, T) = P(J|R)$ and $P(T|J, R, S) = P(T|R, S)$

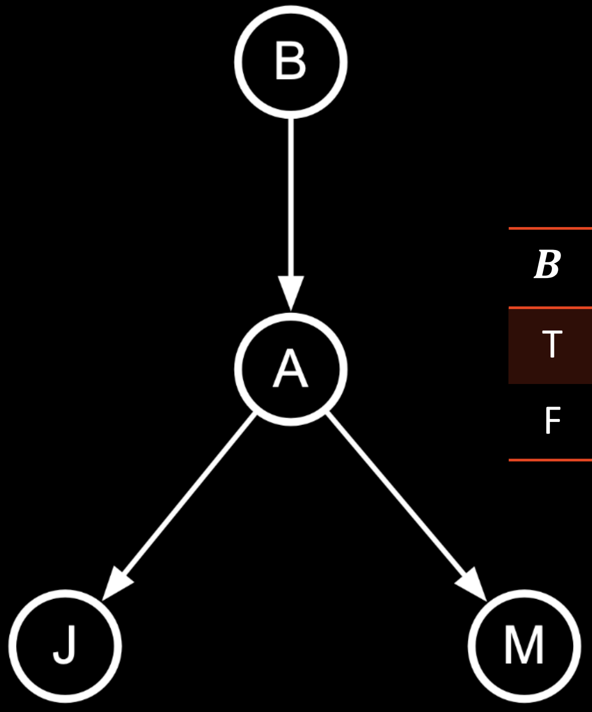
$$P(J, T, R, S) = P(R) \times P(S) \times P(J|R) \times P(T|R, S)$$

SECTION 02

Automated reasoning

You are at work. You receive a phone call from your neighbors Mary and John who say that they think they hears your alarm going off. Is it possible that you are being burgled?

$P(b)$	$P(\neg b)$
0.001	0.999



B	$P(a \mid B)$	$P(\neg a \mid B)$
T	0.95	0.05
F	0.29	0.71

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
B	Burglary	{T, F}

A	$P(j \mid A)$	$P(\neg j \mid A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m \mid A)$	$P(\neg m \mid A)$
T	0.70	0.30
F	0.01	0.99

Alarm network

AN EXAMPLE

You are at work. You receive a phone call from your neighbors Mary and John who say that they think they hears your alarm going off. Is it possible that you are being burgled?

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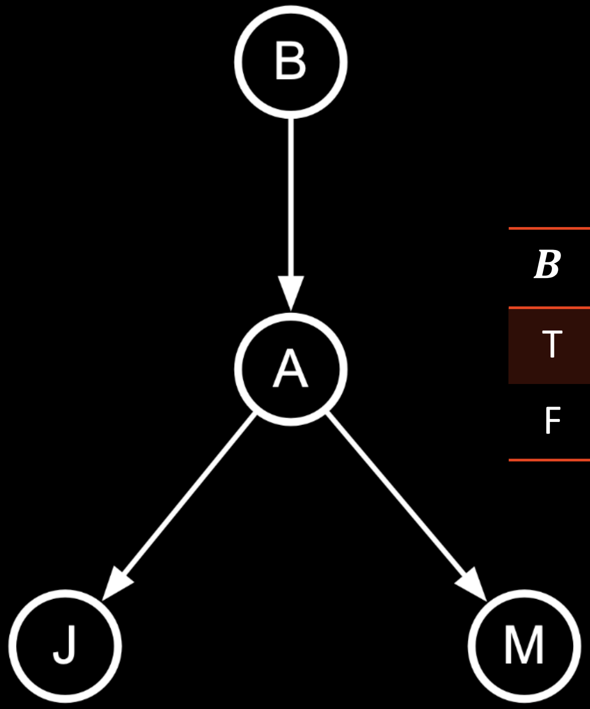
0.001	0.999



B	$P(a\|B)$	$P(\neg a\|B)$
T	0.95	0.05
F	0.29	0.71

$$P(B, A, J, M) = P(B) \times P(A|B) \times P(J|A) \times P(M|A)$$

$$P(b, a, \neg j, m) = ?$$

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
B	Burglary	{T, F}

A	$P(j\|A)$	$P(\neg j\|A)$
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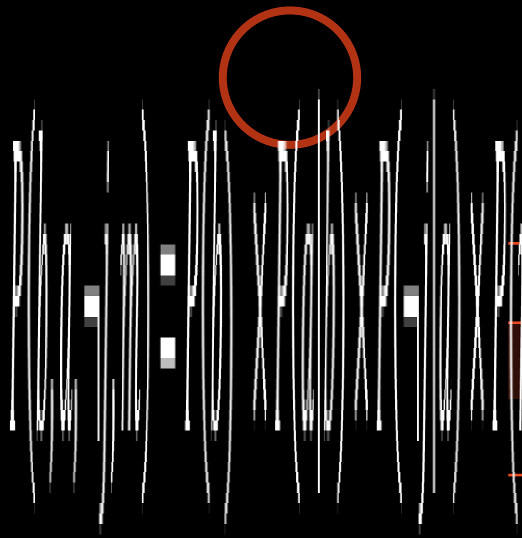

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$$P(B, A, J, M) = P(B) \times P(A|B) \times P(J|A) \times P(M|A)$$

$$P(b, a, \neg j, m) = P(b) \times P(a|b) \times P(\neg j|a) \times P(m|a)$$
$$P(b, a, \neg j, m) = \mathbf{0.001}$$

$$P(b, a, \neg j, m) = \mathbf{0.001}$$

J	John calls	{T, F}
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B	Burglary	{T, F}

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$$P(B, A, J, M) = P(B) \times P(A|B) \times P(J|A) \times P(M|A)$$

$$P(b, a, \neg j, m) = P(b) \times P(a|b) \times P(\neg j|a) \times P(m|a)$$
$$P(b, a, \neg j, m) = 0.001 \times \mathbf{0.95}$$

$$P(b, a, \neg j, m) = \mathbf{0.00095}$$

B	$P(a|B)$	$P(\neg a|B)$
T	0.95	0.05
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$$P(b, a, \neg j, m) = P(b) \times P(a|b) \times P(\neg j|a) \times P(m|a)$$

$$P(b, a, \neg j, m) = 0.001 \times 0.95 \times \mathbf{0.10}$$

$$P(b, a, \neg j, m) = \mathbf{0.000095}$$

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
B	Burglary	{T, F}

A	$P(j\|A)$	$P(\neg j\|A)$
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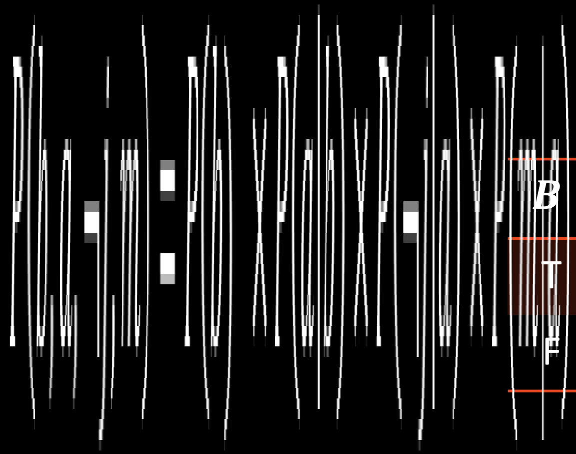

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T	0.95	0.05
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$$P(b, a, \neg j, m) = P(b) \times P(a|b) \times P(\neg j|a) \times P(m|a)$$
$$P(b, a, \neg j, m) = 0.001 \times 0.95 \times 0.10 \times \mathbf{0.70}$$

$$P(b, a, \neg j, m) = \mathbf{0.0000665}$$

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
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A	$P(j\|A)$	$P(\neg j\|A)$
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T	0.70	0.30
F	0.01	0.99

Given a **joint distribution query**: $P(q_1, \dots, q_k)$

- **Evidence variables**: $None$
- **Query variable(s)**: $Q_1, \dots, Q_k$
- **Hidden variables**: $H_1, \dots, H_r$

All the variables of the model $X_1, \dots, X_n$

- **Step 1**: Calculate the joint distribution from the network using the Bayes Network rule:

$$P(h_1, \dots, h_r, q_1, \dots, q_k) = \prod_i P(x_i | parents(X_i))$$

- **Step 2**: Sum out to get the joint probability of query and evidence:

$$P(q_1, \dots, q_k) = \sum_{h_1, \dots, h_r} P(h_1, \dots, h_r, e_1, \dots, e_k)$$

$$P(\neg j) = ?$$

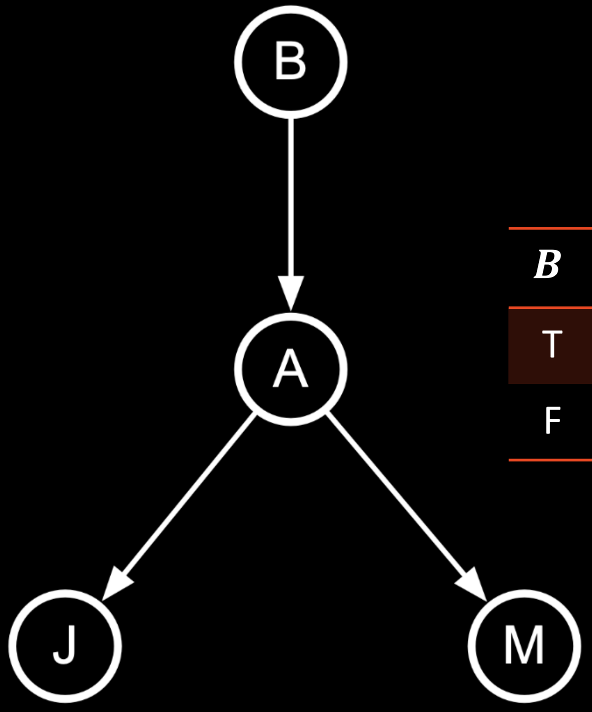
$$P(\neg j) = \sum_{B,A,M} P(B, A, \neg j, M)$$

$$P(\neg j) = \sum_{B,A,M} P(B)\, P(A|B)\, P(\neg j|A)\, P(M|A)$$

$$\begin{aligned} P(\neg j) = {} & P(b)P(a|b)P(\neg j|a)P(m|a) + \\ & P(b)P(a|b)P(\neg j|a)P(\neg m|a) + \\ & P(b)P(\neg a|b)P(\neg j|\neg a)P(m|\neg a) + \\ & P(b)P(\neg a|b)P(\neg j|\neg a)P(\neg m|\neg a) + \\ & P(\neg b)P(a|\neg b)P(\neg j|a)P(m|a) + \\ & P(\neg b)P(a|\neg b)P(\neg j|a)P(\neg m|a) + \\ & P(\neg b)P(\neg a|\neg b)P(\neg j|\neg a)P(m|\neg a) + \\ & P(\neg b)P(\neg a|\neg b)P(\neg j|\neg a)P(\neg m|\neg a) \end{aligned}$$

$$\begin{aligned} P(\neg j) = {} & 0.001 \times 0.95 \times 0.1 \times 0.7 + \\ & 0.001 \times 0.95 \times 0.1 \times 0.3 + \\ & 0.001 \times 0.05 \times 0.95 \times 0.01 + \\ & 0.001 \times 0.05 \times 0.95 \times 0.99 + \\ & 0.999 \times 0.29 \times 0.1 \times 0.7 + \\ & 0.999 \times 0.29 \times 0.1 \times 0.3 + \\ & 0.999 \times 0.71 \times 0.95 \times 0.01 + \\ & 0.999 \times 0.71 \times 0.95 \times 0.99 \qquad = \mathbf{0.9775} \end{aligned}$$

$P(b)$	$P(\neg b)$
0.001	0.999



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A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Given the joint distribution, we have a **conditional query**:
$P(Q_1, \dots, Q_l | e_1, \dots, e_k)$

- **Evidence variables**: $E_1 = e_1, \dots, E_k = e_k$
- **Query variable(s)**: $Q_1, \dots, Q_l$
- **Hidden variables**: $H_1, \dots, H_r$

All the variables of the model $X_1, \dots, X_n$

- **Step 1:** Using the Product rule, if we calculate the joint probability instead of the conditional:

$$P(Q_1, \dots, Q_l | e_1, \dots, e_k) = \frac{\mathrm{P}(\mathrm{Q}_1, \dots, \mathrm{Q}_l, e_1, \dots, e_k)}{\mathrm{P}(e_1, \dots, e_k)}$$

- **Step 2:** Calculate the joint distribution from the network using the Bayes Network rule:

$$P(Q_1, \dots, Q_l, h_1, \dots, h_r, e_1, \dots, e_k) = \prod_i P(x_i | parents(X_i))$$

- **Step 3:** Sum out to get joint of query and evidence :

$$P(Q_1, \dots, Q_l, e_1, \dots, e_k) = \sum_{h_1, \dots, h_r} P(Q_1, \dots, Q_l, h_1, \dots, h_r, e_1, \dots, e_k)$$

- **Step 4:** Recursively, using the same algorithm, calculate:

$$Z = P(e_1, \dots, e_{k_i})$$

- **Step 5:** Normalize:

$$P(Q_1, \dots, Q_l | e_1, \dots, e_k) = \frac{1}{\mathrm{Z}} \mathrm{P}(\mathrm{Q}_1, \dots, \mathrm{Q}_l, e_1, \dots, e_k)$$

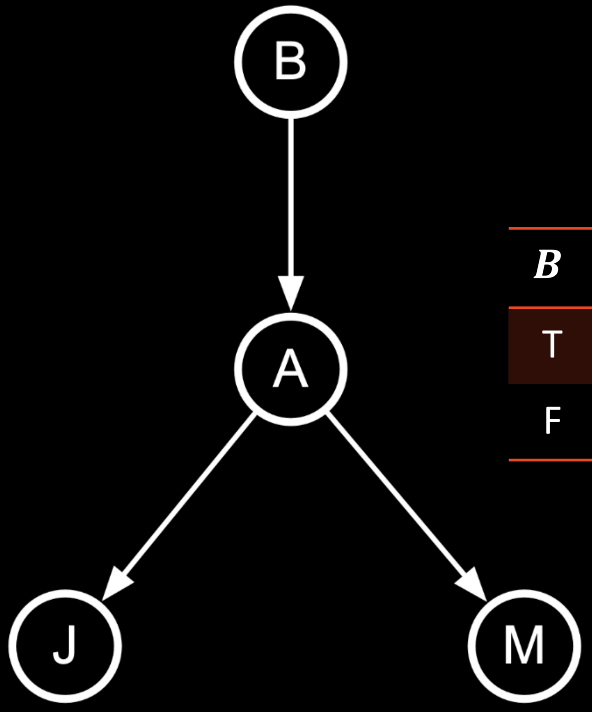
$$P(a|\neg j) = \frac{P(a,\neg j)}{P(\neg j)}$$

$$P(a,\neg j) = \sum_{B,M} P(B)\, P(a|B)\, P(\neg j|a)\, P(M|a)$$

$$\begin{aligned} P(a,\neg j) = {} & P(b)P(a|b)P(\neg j|a)P(m|a) + \\ & P(b)P(a|b)P(\neg j|a)P(\neg m|a) + \\ & P(\neg b)P(a|\neg b)P(\neg j|a)P(m|a) + \\ & P(\neg b)P(a|\neg b)P(\neg j|a)P(\neg m|a) \end{aligned}$$

$$\begin{aligned} P(a,\neg j) = {} & 0.001 \times 0.95 \times 0.1 \times 0.3 + \\ & 0.001 \times 0.95 \times 0.1 \times 0.7 + \\ & 0.999 \times 0.29 \times 0.1 \times 0.3 + \\ & 0.999 \times 0.29 \times 0.1 \times 0.7 = \quad \mathbf{0.0291} \end{aligned}$$

$P(b)$	$P(\neg b)$
0.001	0.999



B	$P(a\|B)$	$P(\neg a\|B)$
T	0.95	0.05
F	0.29	0.71

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Alarm network

AN EXAMPLE: CONDITIONAL QUERIES

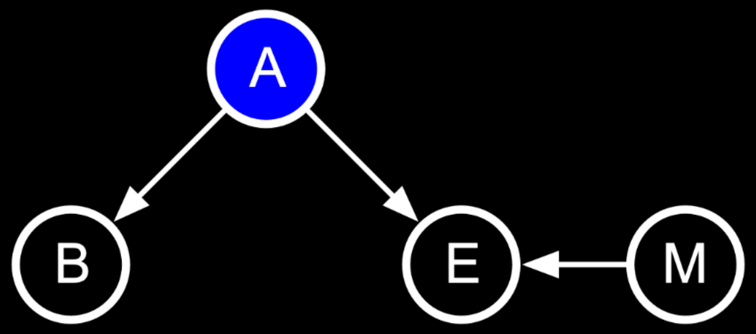
Given the joint distribution, we have a **conditional independence query**:

$$\{Q_1, \dots, Q_m\} \perp \{Q'_1, \dots, Q'_j\} \mid \{E_1, \dots, E_k\}$$

$$\left.\begin{array}{l} \bullet\ \textbf{Evidence variables}: E_1 = e_1, \dots, E_k = e_k \\ \bullet\ \textbf{Query variable(s)}: Q_1, \dots, Q_l \\ \bullet\ \textbf{Hidden variables}: H_1, \dots, H_r \end{array}\right\} \text{All the variables of the model } X_1, \dots, X_n$$

$$P(B,E|A) = \frac{1}{P(A)} \sum_M P(B,E,M,A)$$
$$= \frac{1}{P(A)} \sum_M P(A)P(M)P(E|M,A)P(B|A)$$
$$= P(B|A) \sum_M P(E|M,A)P(M)$$

$$P(E\ |A) = \frac{1}{P(A)} \sum_{B,A,M} P(B,E,M,A)$$
$$= \frac{1}{P(A)} \sum_{B,A,M} P(A)P(M)P(E|M,A)P(B|A)$$
$$= \sum_B P(B|A) \sum_M P(E|M,A)P(M)$$
$$= \sum_M P(E|M,A)P(M)$$



$$P(B,E|A) = \mathrm{P}(B|A)\mathrm{P}(E|A)$$

So, **yes**, B and E are conditionally independent given A.

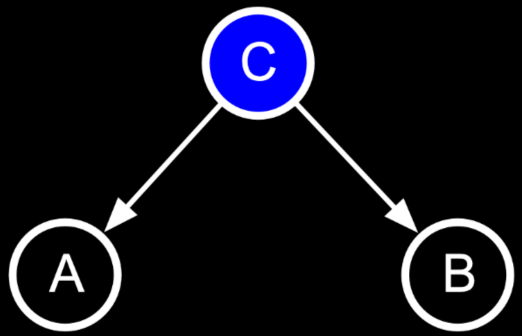
The **D-separation algorithm**, proposed by Pearl in the 1980s, can automatically discover, with some limitations, if variables are conditionally independent.

Tail-Tail rule (or Common cause)

It's possible to demonstrate that in this simple case: $A \perp B \mid C$:

$$P(A,B,C) = P(C)P(A|C)P(B|C)$$

$$P(A,B|C) = \frac{P(A,B,C)}{P(C)} = \mathbf{P(A|C)P(B|C)}$$

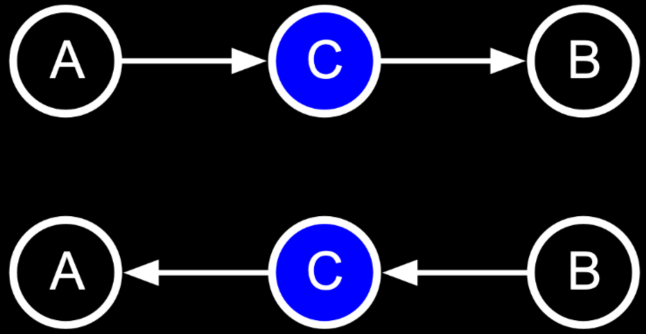


Head-Tail rule (or Chain)

It’s possible to demonstrate that in this simple case: $A \perp B \mid C$:

$$
\begin{aligned}
P(A,B,C) &= P(A)P(C|A)P(B|C) \\
&= P(A,C)P(B|C) \\
&= P(A|C)P(C)P(B|C)
\end{aligned}
$$

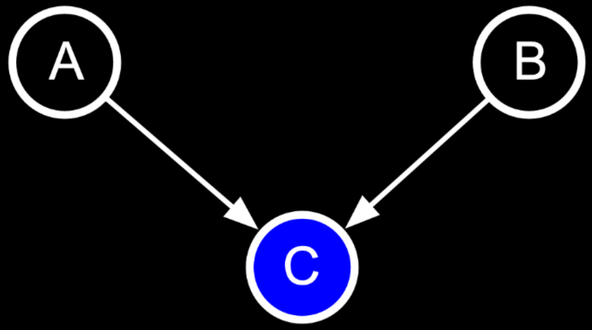
$$P(A,B|C) = \frac{P(A,B,C)}{P(C)} = \boldsymbol{P(A|C)P(B|C)}$$



Head-Head rule (or Collider)

It’s possible to demonstrate that in this simple case: $A \perp B \mid C$:

$$P(A, B, C) = P(A)P(B)P(C|A, B)$$



$$P(A, B|C) = \frac{P(A, B, C)}{P(C)}$$

We don’t know

It’s not always the case: it depends how C is behaves. For example, knowing A and C also gives you information about B:

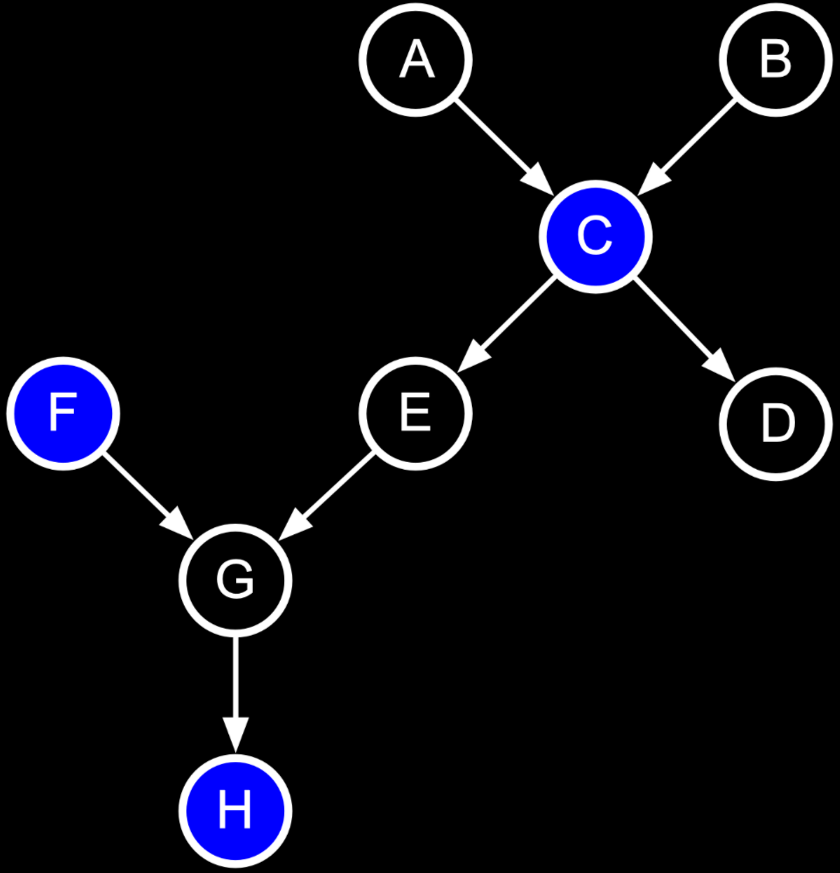
$$C = \begin{cases} 1 & if\ A = B \\ 0 & otherwise \end{cases}$$

- Given the query $\{A_1, \ldots, A_m\} \perp \{B_1, \ldots, B_l\} \mid \{C_1, \ldots, C_k\}$

- We define a path between vertices of A and B as **blocked** if it passes through a vertex c is one of these two conditions happen:

  - the edges are **head-tail** or **tail-tail** and $c \in C$
  - the edges are **head-head** and $c \notin C$ and none of the descendants belong to C

- If all such paths are blocked, then A and B are **D-separated** by C and therefore **conditionally independent** with respect to C



Are the random variables D-separable?

1. $A \perp E \mid C$ YES
2. $\mathrm{E} \perp G \mid C$ NO
3. $B \perp F \mid C, H$ YES
4. $E \perp D \mid H$ YES
5. $F \perp E \mid H$ NO
6. $A \perp B \mid H$ NO
7. $A \perp B \mid F$ YES

SECTION 03

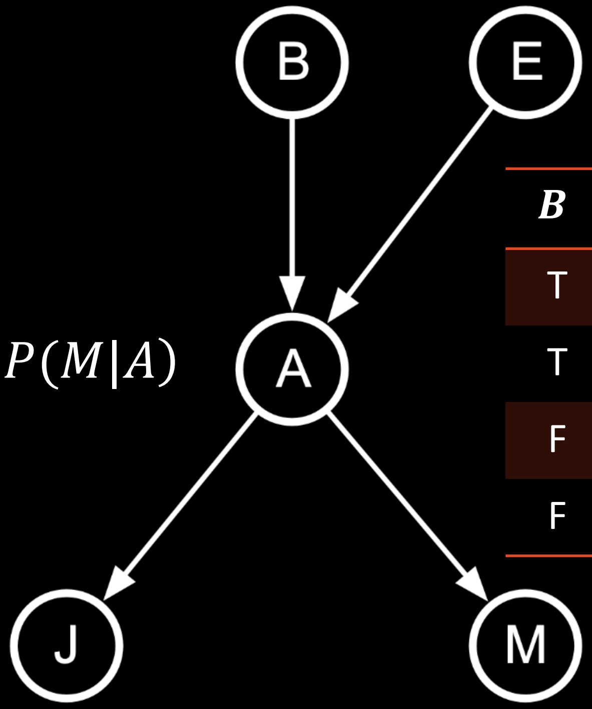
Bayes network optimizations

You are at work. You receive a phone call from your neighbors Mary and John who say that they think they hears your alarm going off. Is it possible that you are being burgled?

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998

$$P(B,E,A,J,M) = P(B) \times P(E) \times P(A|B,E) \times P(J|A) \times P(M|A)$$



B	E	$P(a\|B,E)$	$P(\neg a\|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

J	John calls	{T, F}
M	Mary calls	{T, F}
A	Alarm	{T, F}
B	Burglary	{T, F}

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Variable elimination gradually simplifies the original network:

- It removes hidden variables summing them out
- The target network is only one node that represents the joint probability $P(Q_1, \dots, Q_l, e_1, \dots, e_k)$

- While there are hidden variables:
  - **Step 1:** Select a hidden variable H_i
  - **Step 2:** Join all factors that mention H_i
  - **Step 3:** Eliminate H_i
  - Step 4: Return the P(Q|E)

$$P(Q_1, \dots, Q_l | e_1, \dots, e_k) = \frac{P(Q_1, \dots, Q_l, e_1, \dots, e_k)}{\sum_{Q_1,\dots,Q_l} P(Q_1, \dots, Q_l, e_1, \dots, e_k)}$$

```
function VariableElimination(B, Q, E) return P(Q|E)
𝒯 = set of Conditional Probability Tables of B
Remove rows inconsistent with E from all the tables in T
for i = 1 to n
  𝒯' = all the tables in 𝒯 that involve X_i
  T = the point-wise product of the tables in 𝒯' with X_i marginalized
  Remove T' from 𝒯 and add T
T = the product of the tables in 𝒯
P(Q|E) = normalize T
return P(Q|E)
```

$$f_1(X_1 \ldots X_i) \star f_2(Y_1 \ldots Y_j) = f_{12}(X_1 \ldots X_i \cup Y_1 \ldots Y_j)$$

B	E	$f_1(B,E)$
T	T	0.30
T	F	0.70
F	T	0.90
F	F	0.10

$\star$

A	E	$f_2(B,E)$
T	T	0.20
T	F	0.80
F	T	0.60
F	F	0.40

$=$

A	B	E	$f_{12}(A,B,E)$
T	T	T	$0.20 \times 0.30 = 0.06$
T	T	F	$0.80 \times 0.70 = 0.56$
T	F	T	$0.20 \times 0.90 = 0.18$
T	F	F	$0.80 \times 0.10 = 0.08$
F	T	T	$0.60 \times 0.30 = 0.18$
F	T	F	$0.40 \times 0.70 = 0.28$
F	F	T	$0.60 \times 0.90 = 0.54$
F	F	F	$0.10 \times 0.40 = 0.04$

B	E	$f_1(B,E)$
T	T	0.30
T	F	0.70
F	T	0.90
F	F	0.10

$\star$

E	$f_2(E)$
T	0.10
F	0.20

$=$

B	E	$f_{12}(B,E)$
T	T	0.03
T	F	0.14
F	T	0.09
F	F	0.02

Alarm network and earthquakes

POINT-WISE PRODUCT

$P(B|j,m) = ?$ $\quad P(B|j,m) = \dfrac{P(B,E,A,J,M)}{P(J,M)}$ $\quad P(B|j,m) = \dfrac{P(B,E,A,j,m)}{P(J,M)}$ $\quad P(B|j,m) = \alpha\, P(B,E,A,j,m)$

$$P(B,E,A,J,M) = P(B) \times P(E) \times P(A|B,E) \times P(J|A) \times P(M|A)$$

$$P(B,E,A,j,m) = P(B) \times P(E) \times P(A|B,E) \times P(j|A) \times P(m|A)$$

Order of elimination: **A**, **E**, **B**

$$P(B|j,m) = \alpha \sum_{A,B,E} P(B)\, P(E)\, P(A|B,E)\, P(j|A)\, P(m|A) \qquad P(B|j,m) = \alpha\, P(B) \sum_{E} P(E) \sum_{A} P(A|B,E)\, P(j|A)\, P(m|A)$$

$$f_5(A) = P(m|A) \quad f_4(A) = P(j|A) \quad f_3(A,B,E) = P(A|B,E) \quad P(B|j,m) = \alpha\, P(B) \sum_{E} P(E) \underbrace{\sum_{A} f_3(A,B,E) \star f_4(A) \star f_5(A)}_{f_2(B,E)}$$

$$P(B|j,m) = \alpha\, P(B) \underbrace{\sum_{E} P(E) \star f_3(B,E)}_{f_1(B)} \qquad P(B|j,m) = \alpha\, P(B) \star f_1(B)$$

$$f_5(A) = P(m|A) \qquad f_5(A) =$$

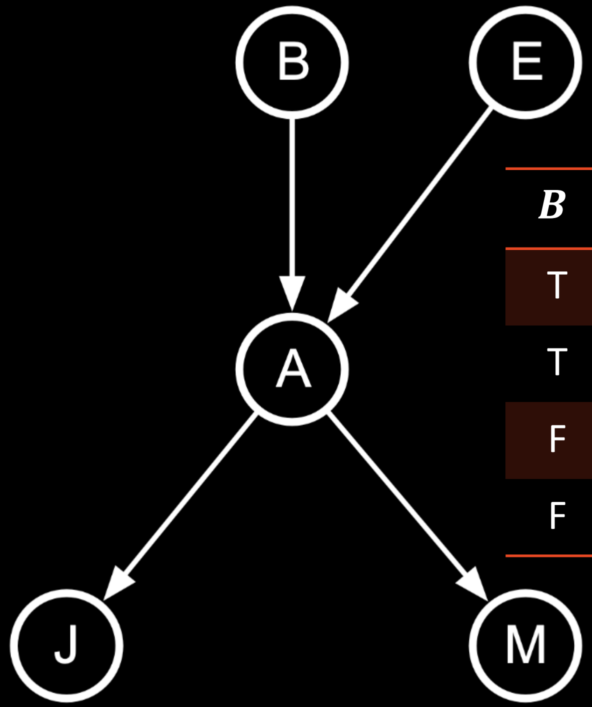
A	$f_5(A)$
T	0.70
F	0.01

$$f_4(A) = P(j|A) \qquad f_4(A) =$$

A	$f_4(A)$
T	0.90
F	0.05

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998



B	E	$P(a\|B,E)$	$P(\neg a\|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Alarm network and earthquakes

AN EXAMPLE

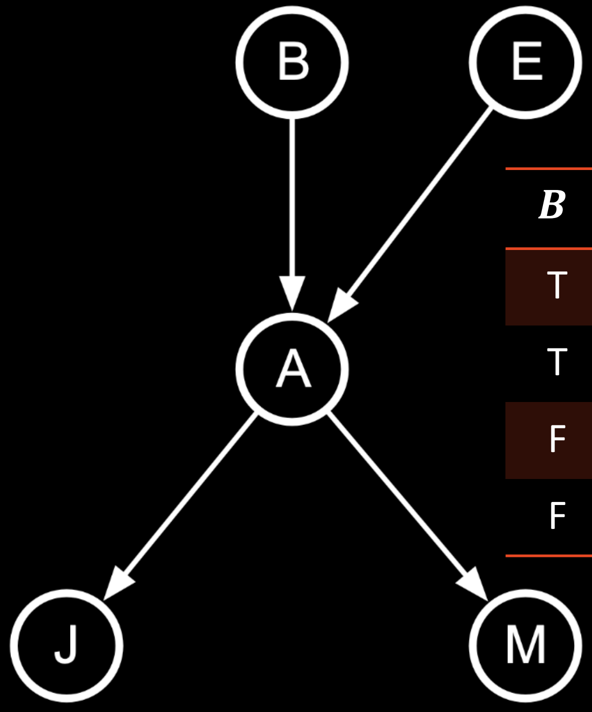
$$f_2(B,E) = \sum_A f_3(A,B,E) \star f_4(A) \star f_5\,(A)$$

$f_3(A,B,E) =$

A	B	E	$f_3(A,B,E)$
T	T	T	0.95
T	T	F	0.94
T	F	T	0.29
T	F	F	0.001
F	T	T	0.05
F	T	F	0.06
F	F	T	0.71
F	F	F	0.999

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998



B	E	$P(a\|B,E)$	$P(\neg a\|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

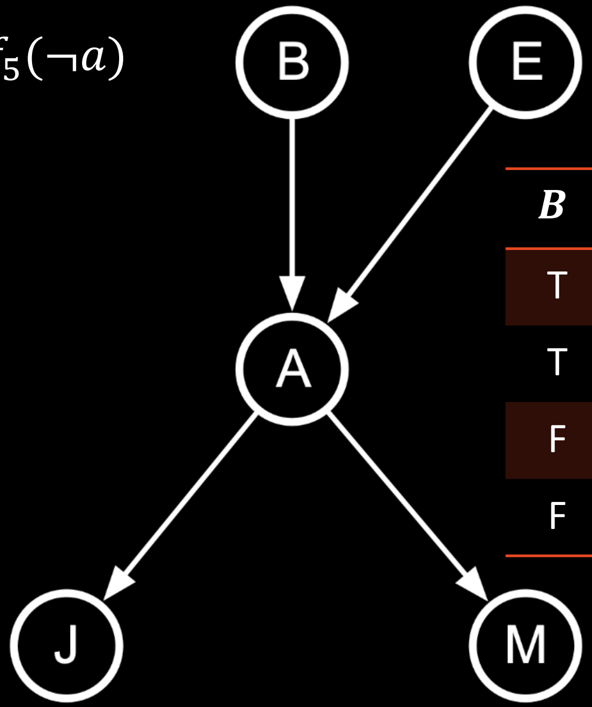
A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

$$f_2(B,E) = \sum_A f_3(A,B,E) \star f_4(A) \star f_5\,(A)$$

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998

$$f_2(B,E) = f_3(a,B,E) \star f_4(a) \star f_5(a) + f_3(\neg a,B,E) \star f_4(\neg a) \star f_5(\neg a)$$



$f_2(B,E) =$

B	E	$f_2(B,E)$
T	T	0.5985
T	F	0.5922
F	T	0.1831
F	F	0.0011

B	E	$P(a\|B,E)$	$P(\neg a\|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Alarm network and earthquakes

AN EXAMPLE

$$f_E(E) = P(E)$$

$f_E(E) =$

E	$f_E(E)$
T	0.002
F	0.998

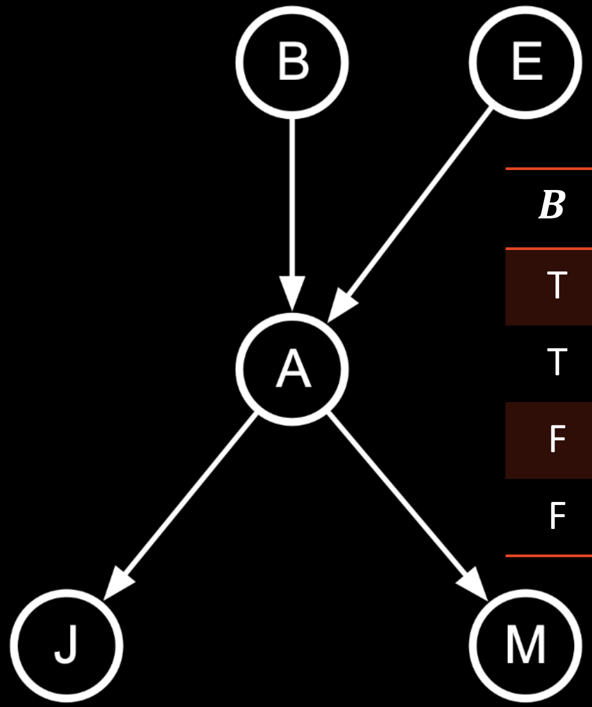
$$f_1(B) = \sum_E f_E(E) \star f_2(B,E)$$

$f_1(E) =$

B	$f_1(B)$
T	0.5922
F	0.0015

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998



B	E	$P(a\|B,E)$	$P(\neg a\|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j\|A)$	$P(\neg j\|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m\|A)$	$P(\neg m\|A)$
T	0.70	0.30
F	0.01	0.99

Alarm network and earthquakes

AN EXAMPLE

$$f_B(B) = P(B) \qquad f_B(B) = $$

B	$f_B(B)$
T	0.001
F	0.999

$$P(B|j,m) = \alpha\, f_B(B) \star f_1(B)$$

$$P(B|j,m) = \alpha$$

B	$\alpha P(B|j,m)$
T	0.0006
F	0.0016

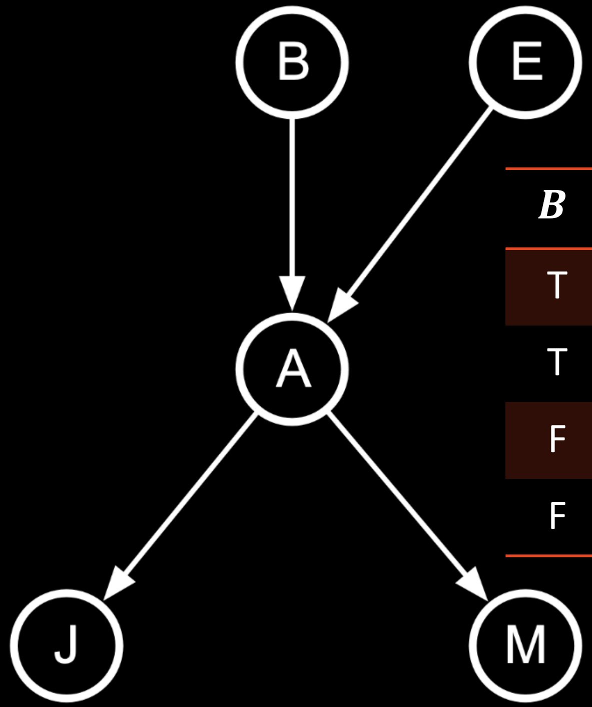
$$1 = \alpha \times P(b|j,m) + \alpha \times P(\neg b|j,m)$$

$$1 = \alpha[P(b|j,m) + P(\neg b|j,m)]$$

$$\alpha = \frac{1}{P(b|j,m)+P(\neg b|j,m)}$$

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998



B	E	$P(a|B,E)$	$P(\neg a|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j|A)$	$P(\neg j|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m|A)$	$P(\neg m|A)$
T	0.70	0.30
F	0.01	0.99

Alarm network and earthquakes

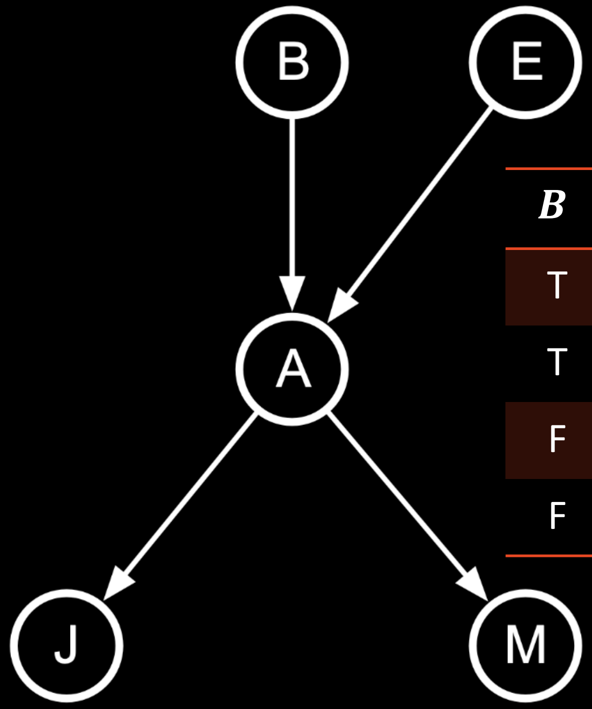
AN EXAMPLE

$P(B|j,m) =$

B	$P(B|j,m)$
T	0.2727
F	0.7272

$P(b)$	$P(\neg b)$
0.001	0.999

$P(e)$	$P(\neg e)$
0.002	0.998



B	E	$P(a|B,E)$	$P(\neg a|B,E)$
T	T	0.95	0.05
T	F	0.94	0.06
F	T	0.29	0.71
F	F	0.001	0.999

A	$P(j|A)$	$P(\neg j|A)$
T	0.90	0.10
F	0.05	0.95

A	$P(m|A)$	$P(\neg m|A)$
T	0.70	0.30
F	0.01	0.99

READINGS

Chapter 14

QUESTIONS ?

ARTIFICIAL INTELLIGENCE

COMP 131

FABRIZIO SANTINI

VERSION 3.0