

# FIRST-ORDER LOGIC 2

ARTIFICIAL INTELLIGENCE | COMP 131

# TODAY ON AI

- First-order Logic inference
- Resolution-based inference
- Forward chaining
- Backward chaining
- Questions?

The **simplest possible** approach is FOL inference to use what we know about Propositional Logic inference.

- Resolution-based inference
  - Use **Refutation** to confirm or refute a sentence  $p$  (but not to generate all entailed sentences)
  - Requires FOL KB to be reduced to CNF
- Forward chaining
  - Uses **Generalize Modus Ponens** to add new atomic sentences
  - Useful for systems that make inferences as information streams in
  - Requires KB to be in form of first-order definite clauses
- Backward chaining
  - Works backwards from a query to try to construct a proof
  - Can suffer from repeated states and incompleteness
  - Useful for query-driven inference

**Reduction to  
Propositional form**

The concept is based on the theorem stating that a sentence is entailed by a new KB *iff* it is entailed by the original KB.

Every FOL KB can be propositionalized so that entailment is preserved.

Three problems in applying PL algorithms to FOL sentences:

1. The **Existential quantifier**  $\exists$
2. The **Universal quantifier**  $\forall$
3. Ground **terms of functions**

**Get rid of** the existential quantifiers: in an **Existential instantiation** the variable is replaced by a new constant symbol (not in the domain):

### CONSTANTS

{Robot, Sq12, Sq23, Sq21}

### PREDICATES

{ $\exists x$ : radioactive( $x$ )}

### RESULT

radioactive(C1)

**Get rid** of the universal quantifiers: in a **Universal instantiation**  $k$  copies of the universally quantified sentences are added:

### CONSTANTS

{Robot, Sq12, Sq23, Sq21}

### PREDICATES

$\{\forall x: \text{radioactive}(x) \rightarrow \text{unsafe}(x)\}$

### RESULT

radioactive(Robot)  $\rightarrow$  unsafe(Robot)

radioactive(Sq12)  $\rightarrow$  unsafe(Sq12)

radioactive(Sq23)  $\rightarrow$  unsafe(Sq23)

radioactive(Sq21)  $\rightarrow$  unsafe(Sq21)

- An issue arises with function symbols, ground terms are infinitely many, e.g., `father(father(father(John)))`.
- Herbrand (1930) postulated that if a sentence is entailed by an FOL KB, it is entailed by a **finite** subset of the propositional KB.
  - For  $n = 0$  to  $\infty$ , create a propositional KB by instantiating with **depth-n** terms see if is entailed by this KB
  - The problem is that it works if the sentence is **entailed**; it does not work if is **not entailed**



## Resolution-based inference

A **resolution-based inference** uses the same PL refutation method:

- It does not generate all entailed sentences (all the new facts)
- It only confirms or refute the query
- It requires to:
  - Propositionalize the KB from a FOL form to a PL form (with specified depth)
  - The PL form of the KB must be reduced to a CNF form

**Forward chaining**

**Generalized Modus Ponens (GMP)** combines And-Introduction, Universal-Elimination, and Modus Ponens when the Knowledge Base contains only **Implications clauses**:

For  $\{p_i, p'_i, q\}$ , a substitution  $\theta$  such that  
 $\text{Substitution}(\theta, p'_i) = \text{Substitution}(\theta, p_i)$   
for all  $i$ , then:

$$\frac{p'_1, p'_2, \dots, p'_n, (p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q)}{\text{Substitution}(\theta, q)}$$

### CONSTANTS

**{Robot, Sq12, Sq23, Sq21}**

### PREDICATES

**$\{\forall x: \text{radioactive}(x) \wedge \text{empty}(x) \rightarrow \text{unsafe}(x)\}$**

**radioactive(Sq12)**

**empty(Sq12)**

### RESULT

**radioactive(Sq12), empty(Sq12)**

**$(\text{radioactive}(x) \wedge \text{empty}(x) \rightarrow \text{unsafe}(x))$**

---

**unsafe(Sq12)**

There is an implicit assumption that all variables are universally quantified.

**Unification** is a pattern matching procedure that takes two atomic sentences and returns a failure if they do not match and a substitution list if they do.

The substitution list is called the **most general unifier**.

### CONSTANTS

{Robot, Sq12, Sq23, Sq21}

### PREDICATES

$\{\forall x: \text{radioactive}(x) \rightarrow \text{unsafe}(x)\}$

windy(Sq12)

empty(Sq21)

### RESULT

radioactive( $x$ )  $\cup$  radioactive(Sq12)      { $x/\text{Sq12}$ }

radioactive( $x$ )  $\cup$  windy(Sq12)      **FAILURE**

radioactive( $x$ )  $\cup$  empty(Sq21)      **FAILURE**

```

1  function Unify(p, q, θ) return a solution, or FAILURE
2  scan p and q left-to-right to find where p and q are not equal
3  if there is no disagreement
4      return θ
5  let r and s be the different terms
6  if Variable(r) then
7      θ = Union(θ, {r/s})
8      return Unify(Substitution(θ, p), Substitution(θ, q), θ)
9  else if Variable(s) then
10     θ = Union(θ, {s/r})
11     return Unify(Substitution(θ, p), Substitution(θ, q), θ)
12 else return FAILURE

```

## EXAMPLES

adjacent(Sq12, x)  $\cup$  adjacent(Sq12, Sq13)

adjacent(Sq12, x)  $\cup$  adjacent(y, Sq13)

adjacent(Sq12, x)  $\cup$  adjacent(y, free(Sq13))

adjacent(Sq12, x)  $\cup$  adjacent(x, Sq13)

## RESULT

{x/Sq13}

{x/Sq13, y/Sq12}

{y/Sq12, x/free(Sq13)}

**FAILURE**

Like in **Propositional Logic** inference, the algorithm answers queries using the KB to determine new facts until it finds that the query is **true**, or until we've run out of new facts to generate.

Forward chaining works very much like **Breath-first Search**.

```
1 function Forward-Chaining(KB, q) return a substitution, or FAILURE
2   repeat until new is empty
3     new = {}
4     for each  $(p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow c)$  in KB
5       for each  $\theta$  such that  $\text{Substitution}(\theta, p_1 \wedge \dots \wedge p_n) = \text{Substitution}(\theta, p'_1 \wedge \dots \wedge p'_n)$ 
6         for some  $p'_1, \dots, p'_n$  in KB do
7            $c' = \text{Substitution}(\theta, c)$ 
8           if  $c'$  does not unify with any sentence in KB + new then
9             new = new +  $c'$ 
10             $\phi = \text{Unify}(c', q)$ 
11            if  $\phi$  is not failure then
12              return  $\phi$ 
13   KB = KB + new
14   return FAILURE
```

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

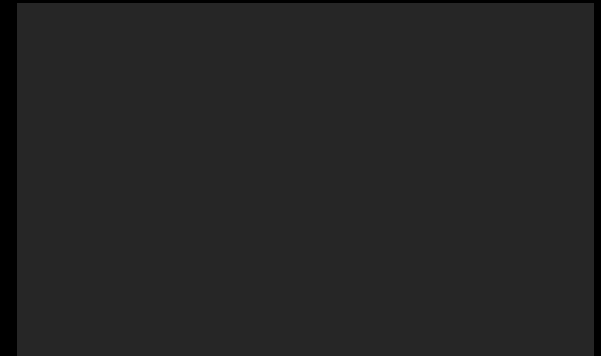
The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE



## NEW FACTS





## NL STORY

The law says **that it is a crime for an American to sell weapons to hostile nations**

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

1  $\forall x \forall y \forall z: \text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x)$

## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

**The country Nono**, an enemy of America, **has some missiles**, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 5  $\exists x: \text{owns}(\text{Nono}, x) \wedge \text{missile}(x)$

## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

**The country Nono**, an enemy of America, **has some missiles**, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$

## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and **all its missiles were sold to it by Colonel West**, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\forall x: \text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono})$
- 3
- 4
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$

## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and **all its missiles were sold to it by Colonel West**, who is American.

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## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell **weapons** to hostile nations.

The country Nono, an enemy of America, has some **missiles**, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\forall x: \text{missile}(x) \rightarrow \text{weapon}(x)$
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## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell **weapons** to hostile nations.

The country Nono, an enemy of America, has some **missiles**, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

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- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
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## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to **hostile nations**

The country Nono, **an enemy of America**, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

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- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\forall x: \text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x)$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$

## NEW FACTS



## NL STORY

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## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by **Colonel West, who is American**.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$

## NEW FACTS

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

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Is Colonel West a criminal?

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## NEW FACTS

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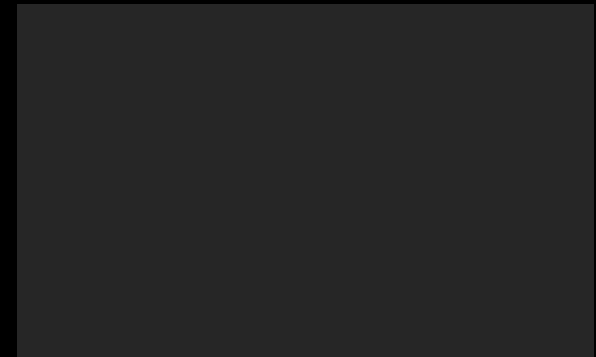
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Is **Colonel West** a **criminal**?

## KNOWLEDGE BASE

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## NEW FACTS



**Criminal(West)?**

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

## NEW FACTS

$\text{weapon}(\text{M1})$

$\text{Criminal}(\text{West})?$

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➡ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x)$
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- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- ➡ 8  $\text{enemy}(\text{Nono}, \text{America})$

## NEW FACTS

$\text{weapon}(\text{M1})$   
 $\text{hostile}(\text{Nono})$

$\text{Criminal}(\text{West})?$

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

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- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

## NEW FACTS

$\text{weapon}(\text{M1})$   
 $\text{hostile}(\text{Nono})$   
 $\text{sell}(\text{West}, \text{M1}, \text{Nono})$

**Criminal(West)?**

## NL STORY

The law says that it is a crime for an American to sell weapons to hostile nations.

The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

Is Colonel West a criminal?

## KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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## NEW FACTS

$\text{weapon}(\text{M1})$   
 $\text{hostile}(\text{Nono})$   
 $\text{sell}(\text{West}, \text{M1}, \text{Nono})$

**Criminal(West): YES!**



## Backward chaining

Like in Propositional Logic, the basic idea behind **Backward chaining** is to work backward from the goal to the facts that must be asserted for the goal to hold.

Backward chaining proceeds in a **Depth-first Search**.

```
1 function Backward-chaining(KB, goals,  $\theta$ ) return substitution
2   if goals is empty then
3     return  $\theta$ 
4   goal = pop from goals
5    $q' = \text{Substitution}(\theta, \text{goal})$ 
6    $\theta'' = \{\}$ 
7   for each r in KB such that  $(p_1 \wedge p_2 \wedge \dots \wedge p_n \rightarrow q)$  do
8      $r' = \text{Standardize}(r)$ 
9      $\theta' = \text{Unify}(q, q')$ 
10    if  $\theta'$  succeeded then
11       $\theta'' = \theta'' + \text{Backward-chaining}(\text{KB}, (p_1 \dots p_n) + \text{goals}, \text{Compose}(\theta, \theta'))$ 
12  return  $\theta''$ 
```

Criminal(**West**)

**QUERY**

Is Colonel West a criminal?

Criminal(**West**)

**KNOWLEDGE BASE**

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
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- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

Criminal(**West**)

**QUERY**

Is Colonel West a criminal?

Criminal(**West**)

**KNOWLEDGE BASE**

- ➔
- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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Criminal(**West**)

{ $x$ /**West**}

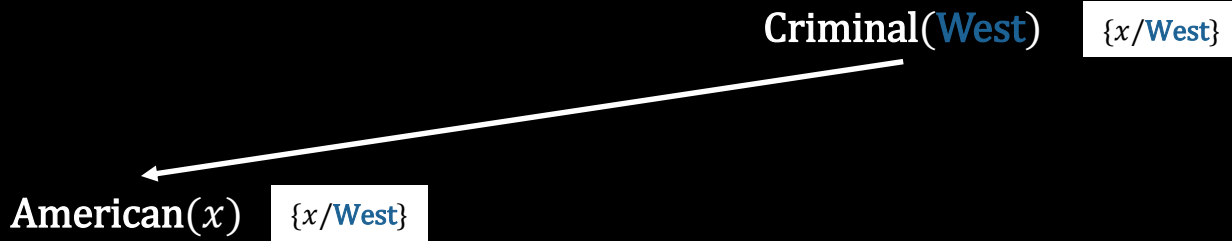
#### QUERY

Is Colonel West a criminal?

Criminal(**West**)

#### KNOWLEDGE BASE

- ➔
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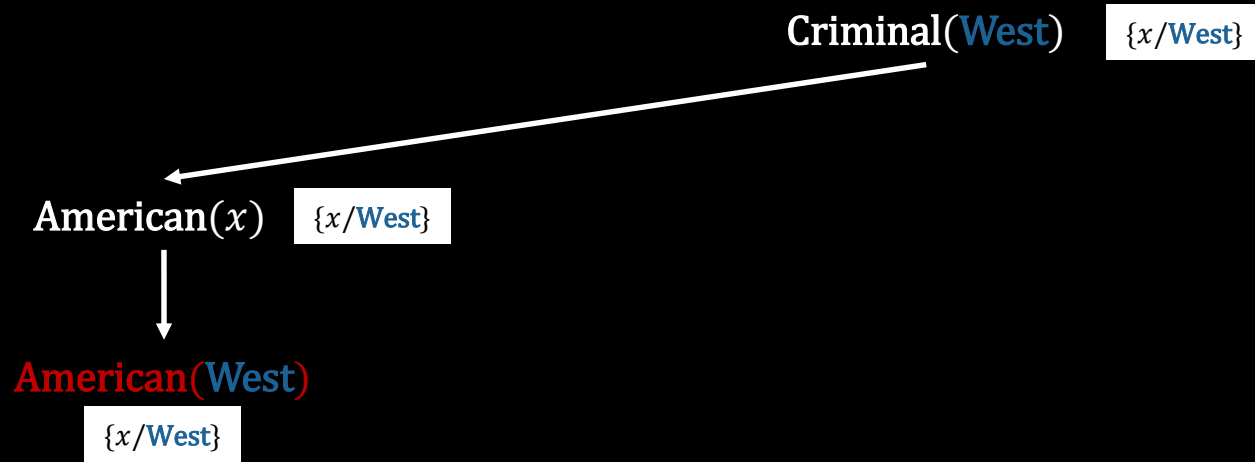


#### QUERY

Is Colonel West a criminal?  
Criminal(West)

#### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

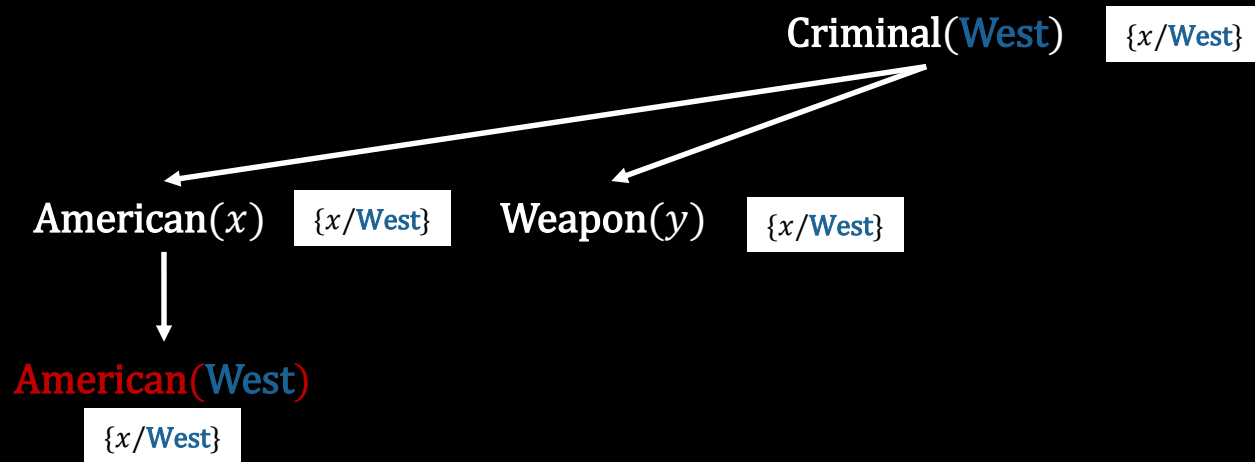


#### QUERY

Is Colonel West a criminal?  
`Criminal(West)`

#### KNOWLEDGE BASE

- ➡ 1 `american( $x$ )  $\wedge$  weapon( $y$ )  $\wedge$  sells( $x, y, z$ )  $\wedge$  hostile( $z$ )  $\rightarrow$  criminal( $x$ ) ...`
- 2 `missile( $x$ )  $\wedge$  owns(Nono,  $x$ )  $\rightarrow$  sells(West,  $x$ , Nono) ...`
- 3 `missile( $x$ )  $\rightarrow$  weapon ( $x$ ) ...`
- 4 `enemy( $x$ , America)  $\rightarrow$  hostile( $x$ ) ...`
- 5 `owns(Nono, M1)`
- 6 `missile(M1)`
- ➡ 7 `american(West)`
- 8 `enemy(Nono, America)`



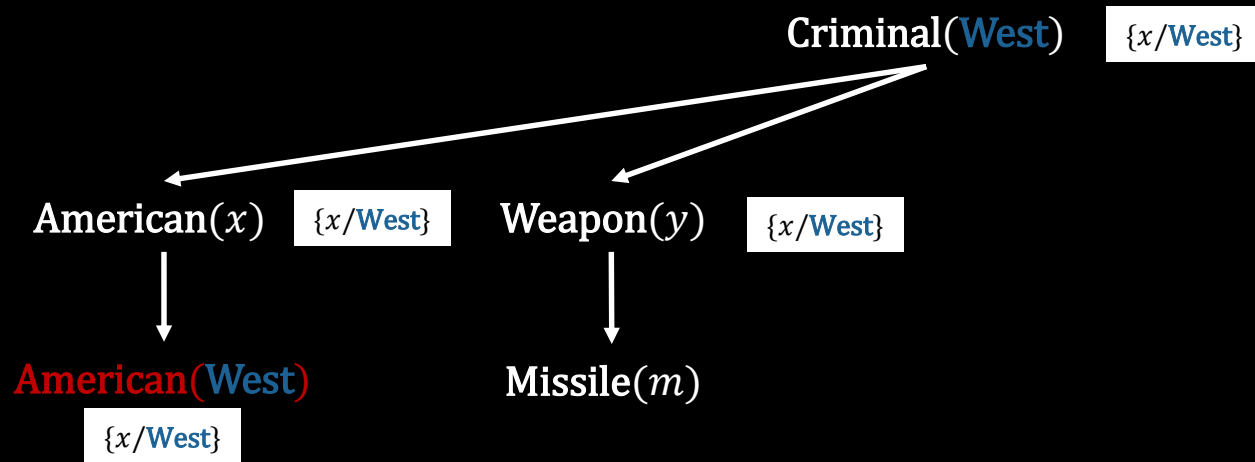
### QUERY

Is Colonel West a criminal?  
**Criminal(West)**

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$



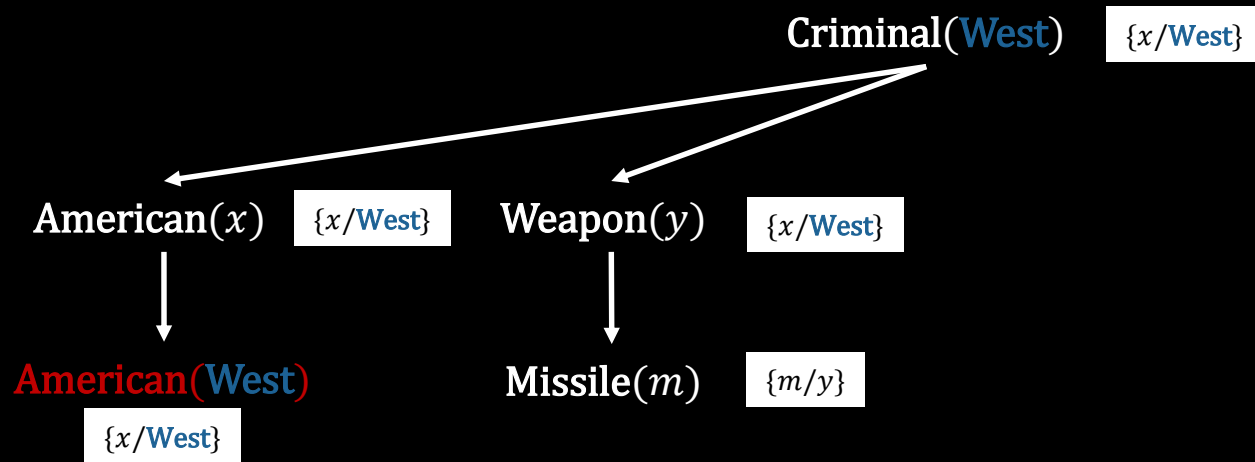


### QUERY

Is Colonel West a criminal?  
**Criminal(West)**

### KNOWLEDGE BASE

- ➡ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- ➡ 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

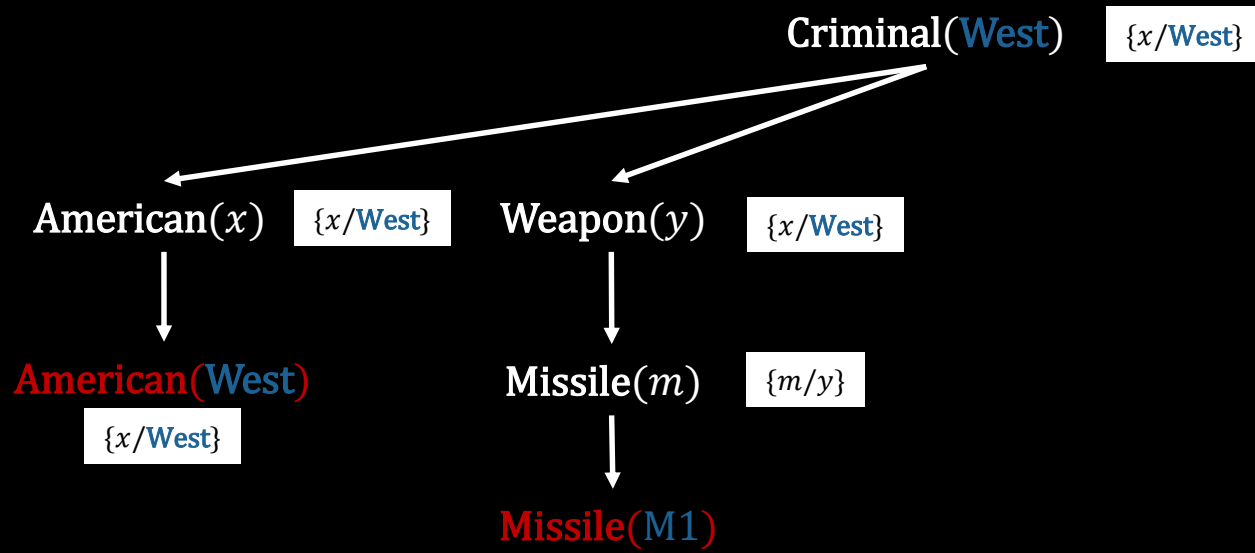


### QUERY

Is Colonel West a criminal?  
Criminal(**West**)

### KNOWLEDGE BASE

- ➔ 1     $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2     $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- ➔ 3     $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4     $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5     $\text{owns}(\text{Nono}, \text{M1})$
- 6     $\text{missile}(\text{M1})$
- 7     $\text{american}(\text{West})$
- 8     $\text{enemy}(\text{Nono}, \text{America})$

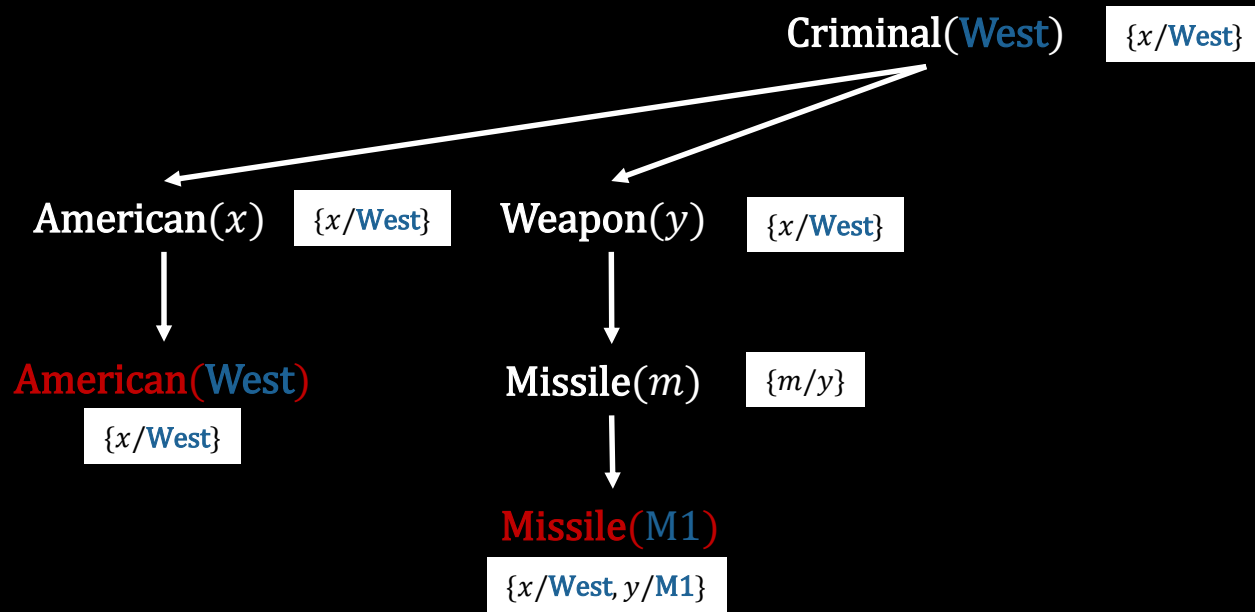


### QUERY

Is Colonel West a criminal?  
**Criminal(West)**

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- ➔ 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- ➔ 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

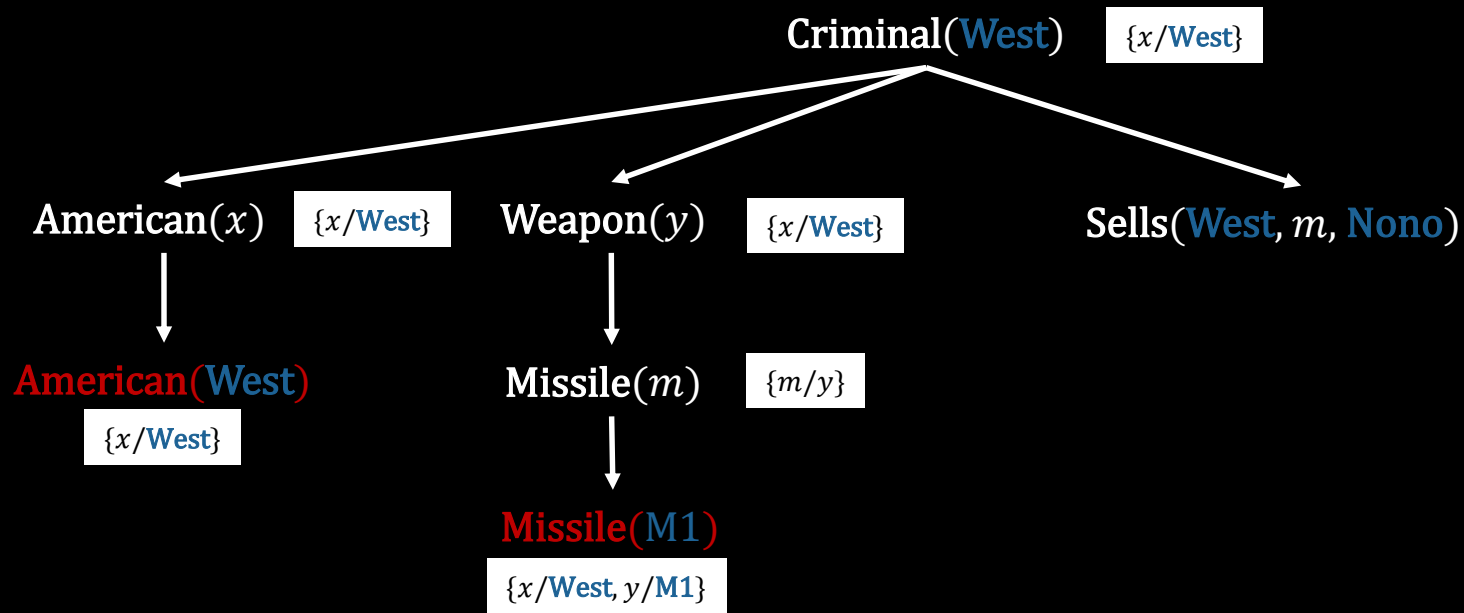


### QUERY

Is Colonel West a criminal?  
**Criminal(West)**

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- ➔ 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
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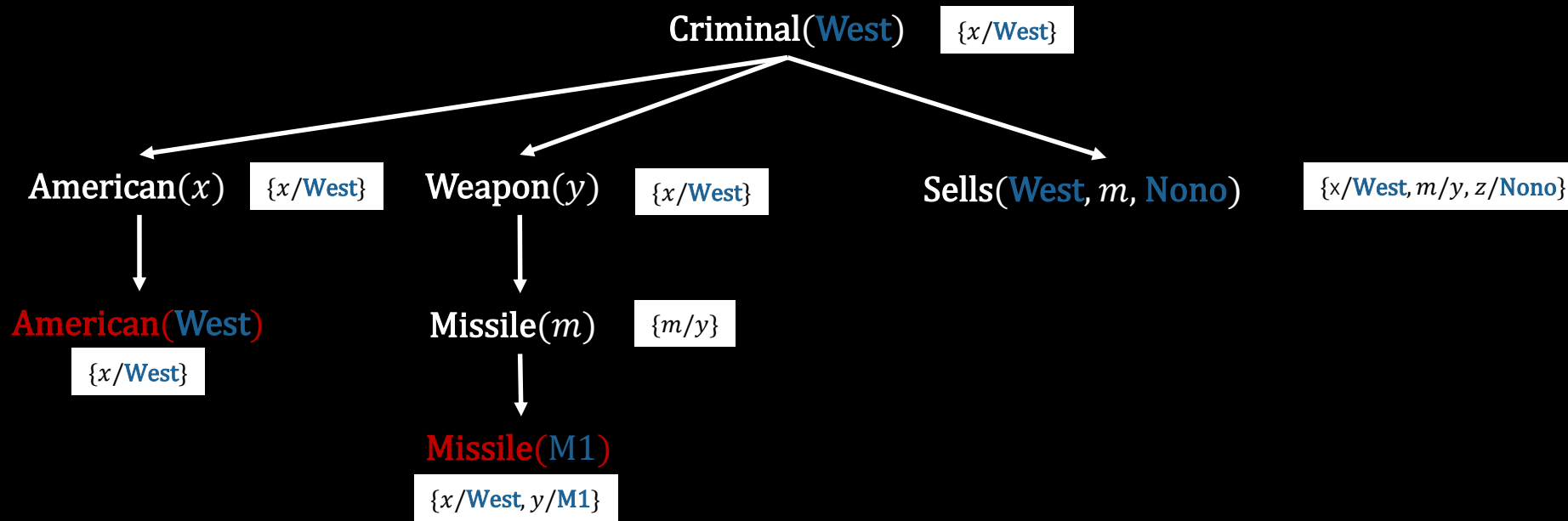


### QUERY

Is Colonel West a criminal?  
 Criminal(**West**)

### KNOWLEDGE BASE

- ➡ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- ➡ 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

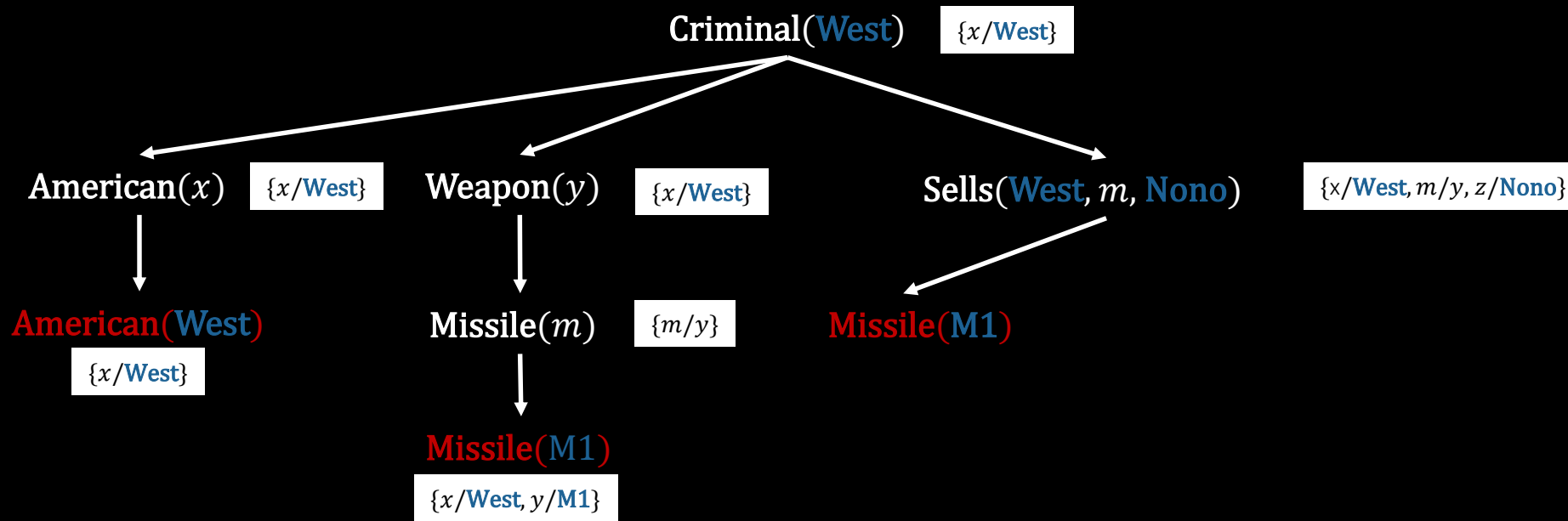


### QUERY

Is Colonel West a criminal?  
 Criminal(West)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- ➔ 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

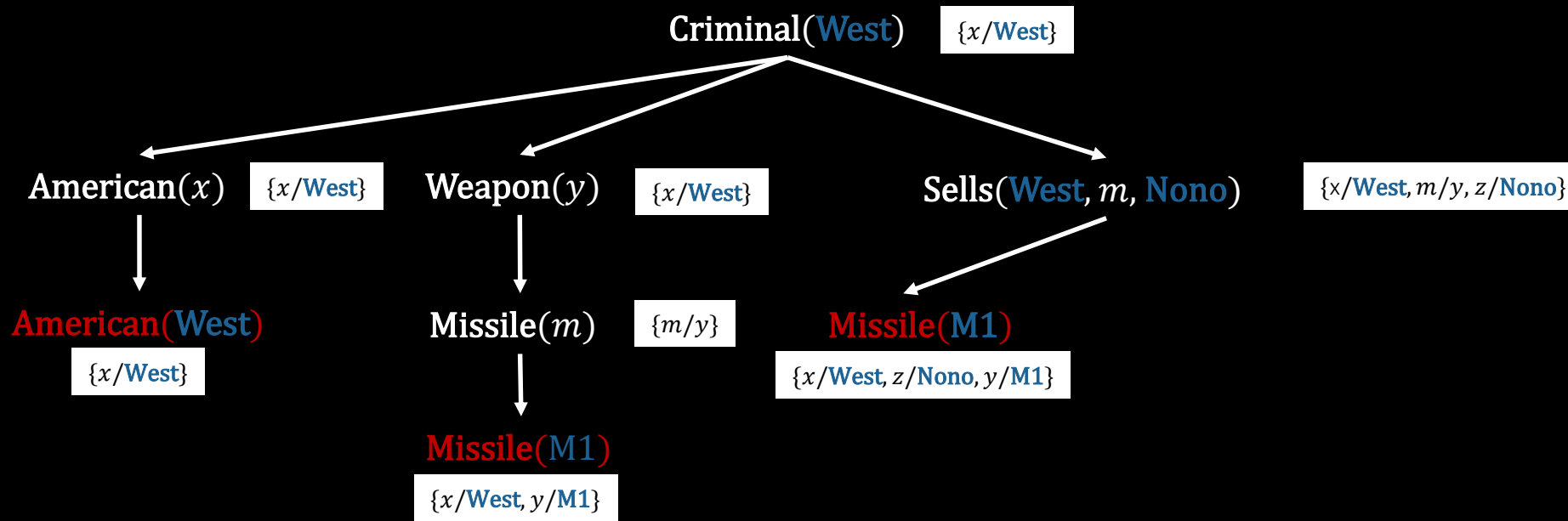


### QUERY

Is Colonel West a criminal?  
**Criminal(West)**

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- ➔ 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
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- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- ➔ 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$



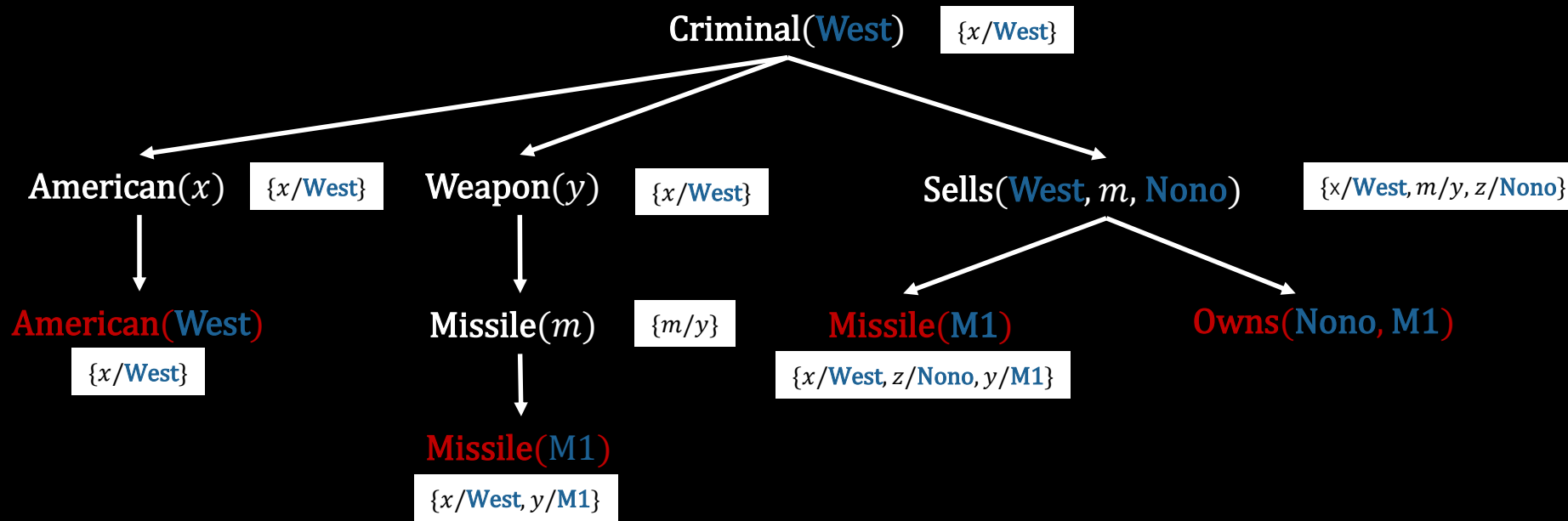
### QUERY

Is Colonel West a criminal?  
 Criminal(West)

### KNOWLEDGE BASE

- ➡ 1 `american(x) ∧ weapon(y) ∧ sells(x, y, z) ∧ hostile(z) → criminal(x) ...`
- ➡ 2 `missile(x) ∧ owns(Nono, x) → sells(West, x, Nono) ...`
- 3 `missile(x) → weapon(x) ...`
- 4 `enemy(x, America) → hostile(x) ...`
- 5 `owns(Nono, M1)`
- ➡ 6 `missile(M1)`
- 7 `american(West)`
- 8 `enemy(Nono, America)`



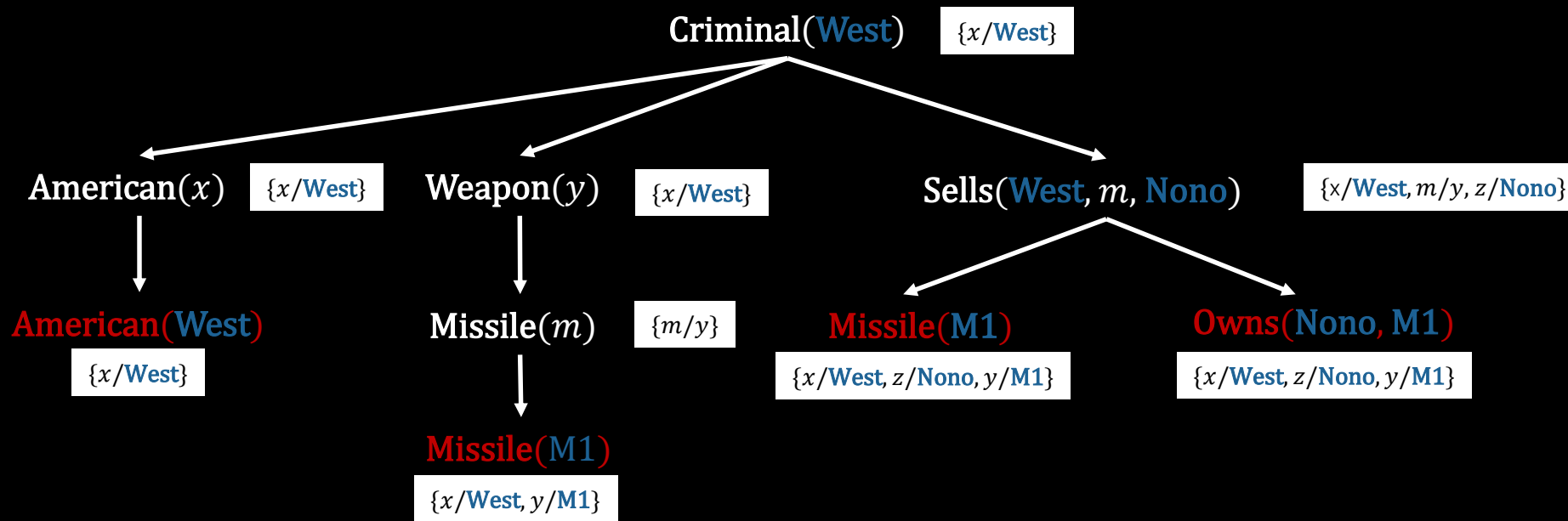


### QUERY

Is Colonel West a criminal?  
 Criminal(**West**)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- ➔ 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- ➔ 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

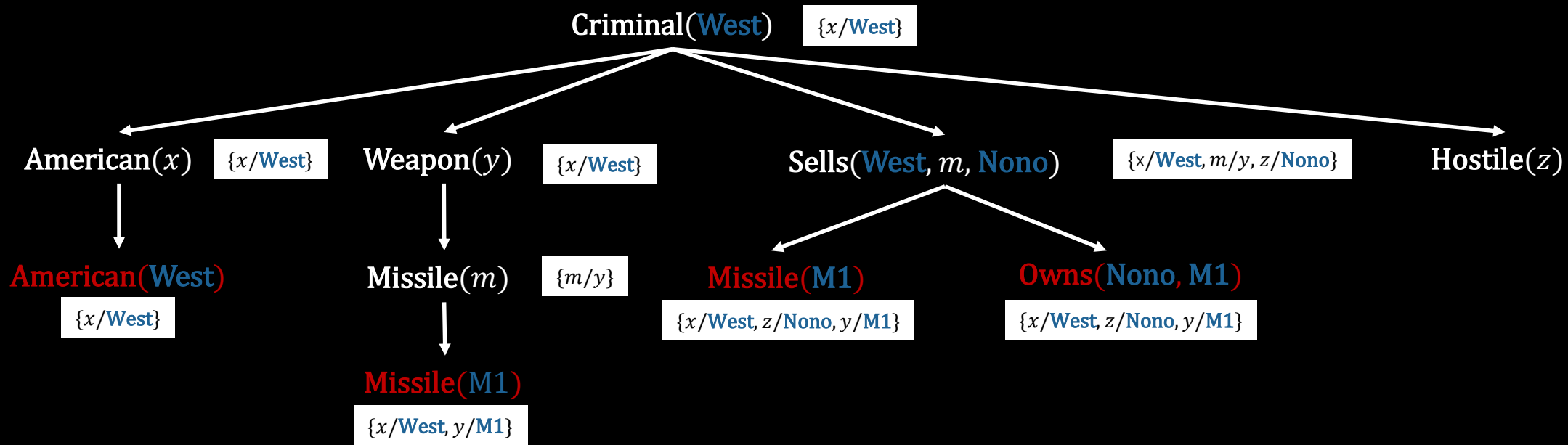


### QUERY

Is Colonel West a criminal?  
 Criminal(**West**)

### KNOWLEDGE BASE

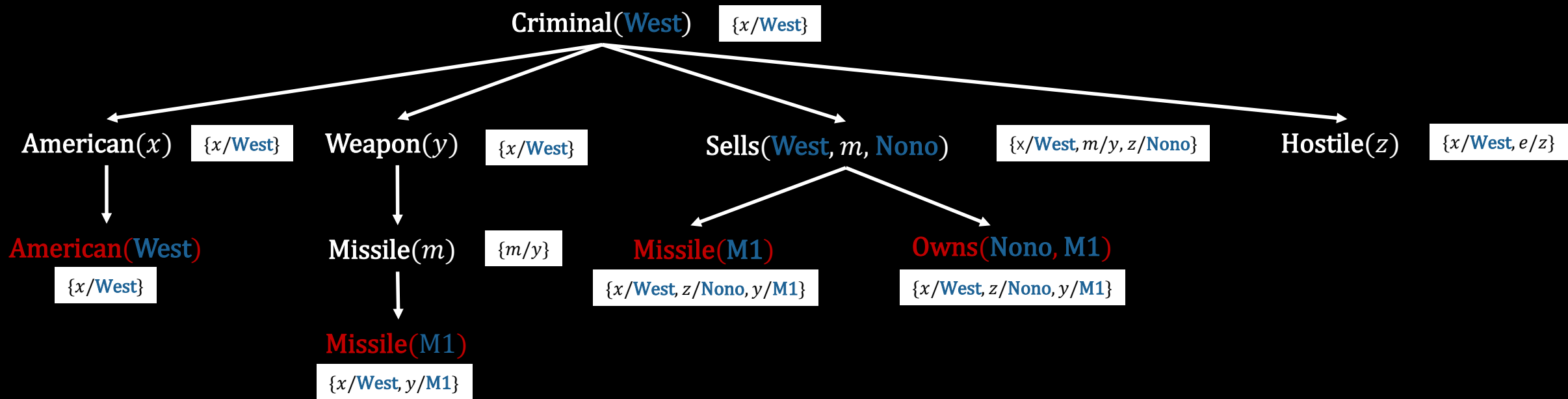
- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- ➔ 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
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- ➔ 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$



Is Colonel West a criminal?  
Criminal(**West**)

### KNOWLEDGE BASE

- ➡ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
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- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➡ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
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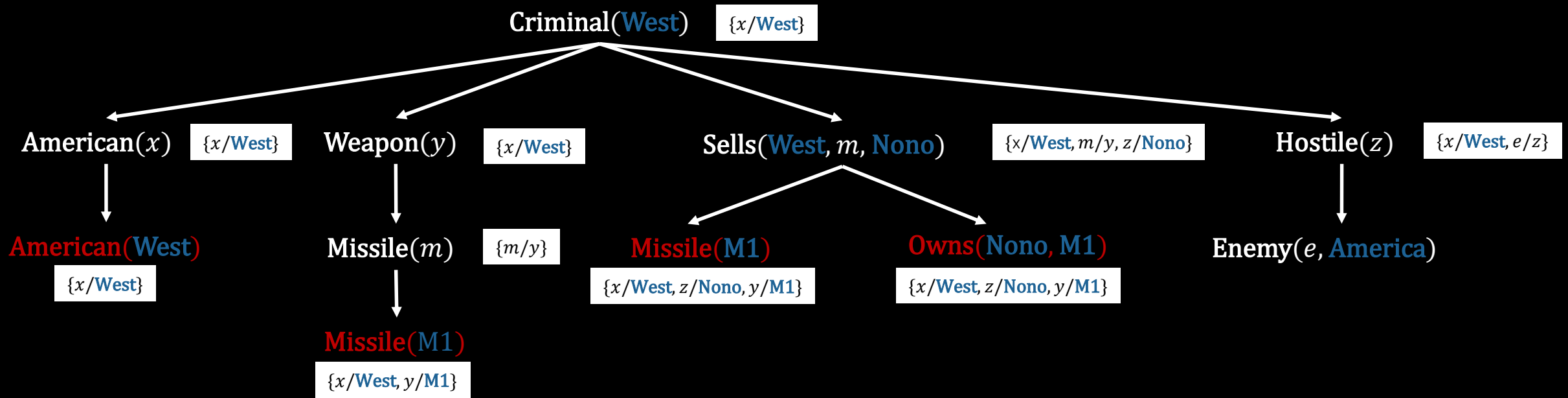


### QUERY

Is Colonel West a criminal?  
 Criminal(**West**)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➔ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$

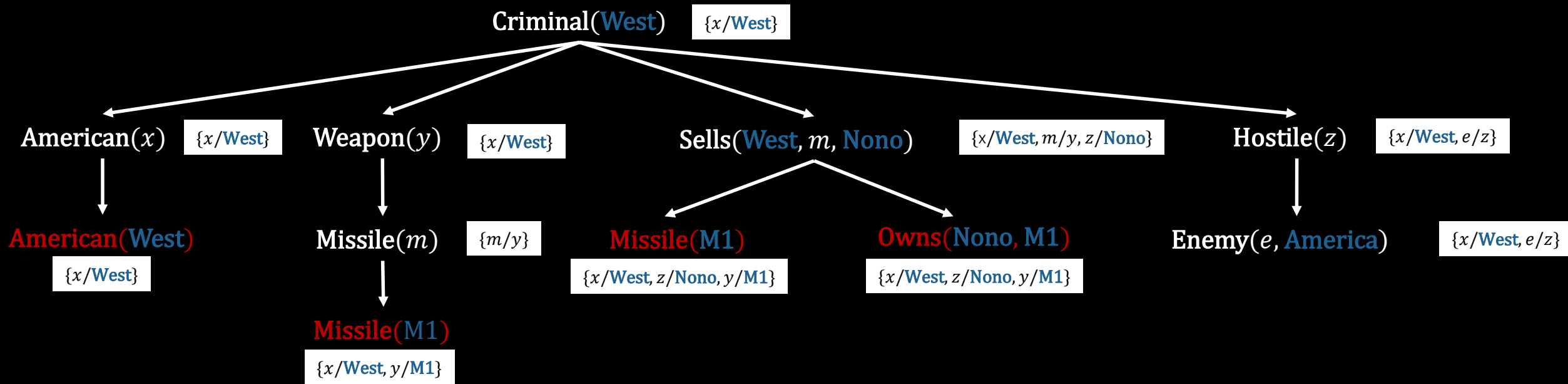


### QUERY

Is Colonel West a criminal?  
 Criminal(**West**)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➔ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
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- 6  $\text{missile}(\text{M1})$
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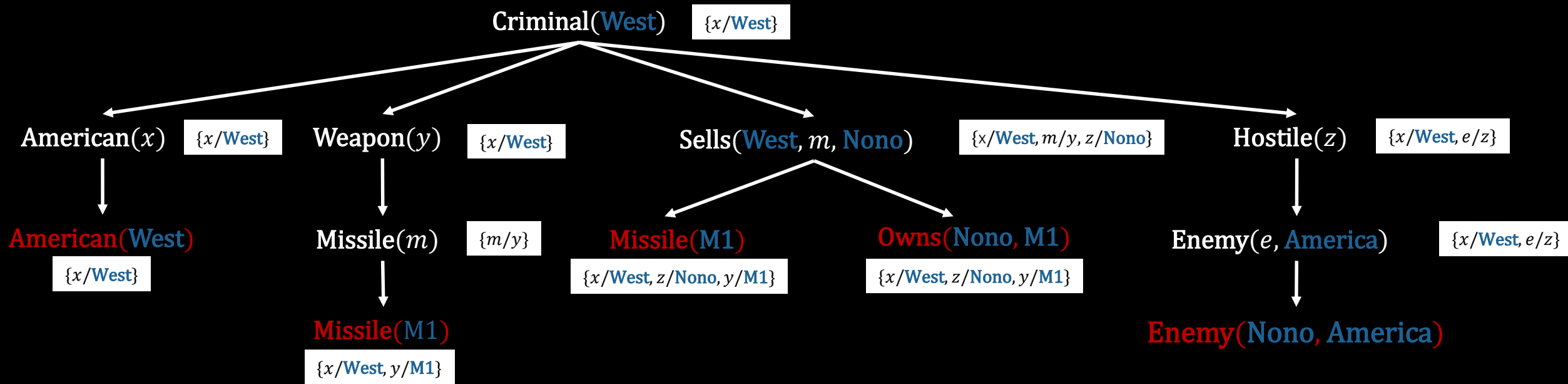


### QUERY

Is Colonel West a criminal?  
Criminal(West)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➔ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
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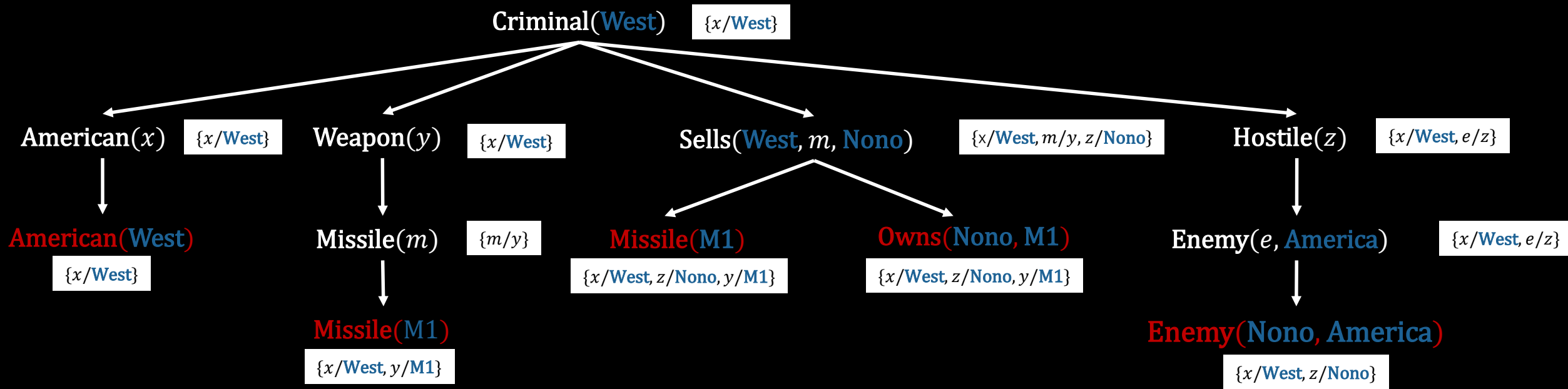


### QUERY

Is Colonel West a criminal?  
Criminal(West)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➔ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- ➔ 8  $\text{enemy}(\text{Nono}, \text{America})$



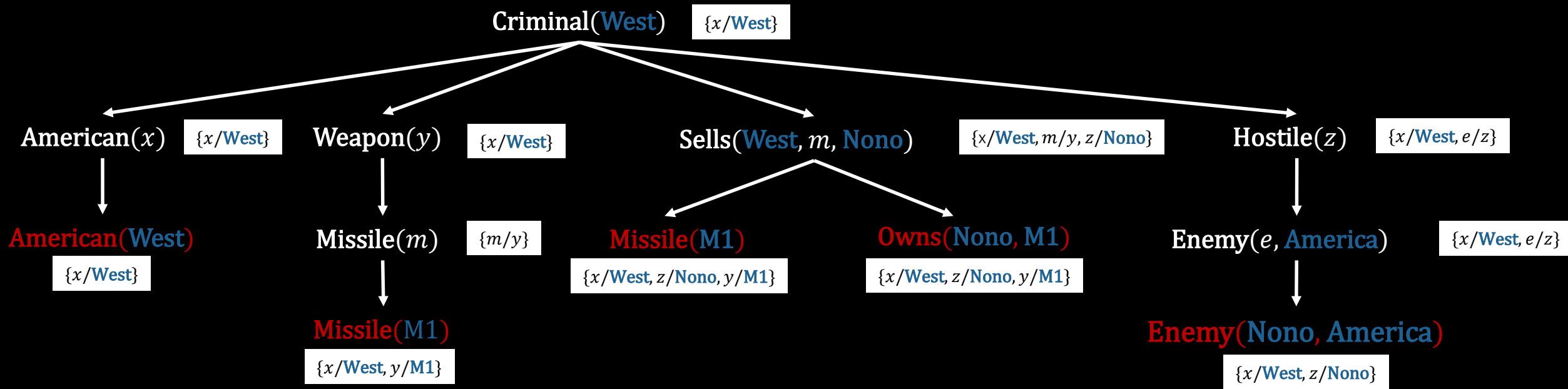
### QUERY

Is Colonel West a criminal?  
Criminal(West)

### KNOWLEDGE BASE

- ➔ 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- ➔ 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- ➔ 8  $\text{enemy}(\text{Nono}, \text{America})$





### QUERY

Is Colonel West a criminal?  
 Criminal(West)

{x/West, y/M1, z/Nono}

### KNOWLEDGE BASE

- 1  $\text{american}(x) \wedge \text{weapon}(y) \wedge \text{sells}(x, y, z) \wedge \text{hostile}(z) \rightarrow \text{criminal}(x) \dots$
- 2  $\text{missile}(x) \wedge \text{owns}(\text{Nono}, x) \rightarrow \text{sells}(\text{West}, x, \text{Nono}) \dots$
- 3  $\text{missile}(x) \rightarrow \text{weapon}(x) \dots$
- 4  $\text{enemy}(x, \text{America}) \rightarrow \text{hostile}(x) \dots$
- 5  $\text{owns}(\text{Nono}, \text{M1})$
- 6  $\text{missile}(\text{M1})$
- 7  $\text{american}(\text{West})$
- 8  $\text{enemy}(\text{Nono}, \text{America})$



**QUESTIONS ?**

# ARTIFICIAL INTELLIGENCE COMP 131

FABRIZIO SANTINI