

**EE 105 Feedback Control Systems**  
**Department of Electrical and Computer Engineering**  
**Tufts University Fall 2018**  
**Homework #9**

Name: *Shang Wang*

Student ID: *1277417*

E-mail: *shang.wang@tufts.edu*

## 1 Problem 1

### 1.1 Part A

For ideal Op-Amp,

$$\frac{v_{out}}{v_{in}} = -\frac{Z_{out}}{Z_{in}}$$

where

$$Z_{in} = (\frac{1}{sC} + R_2) // R_1; \quad Z_{out} = R_1$$

So

$$D(s) = -\frac{v_{out}}{v_{in}} = \frac{(R_1 + R_2)s + 1}{R_2Cs + 1}$$

We have

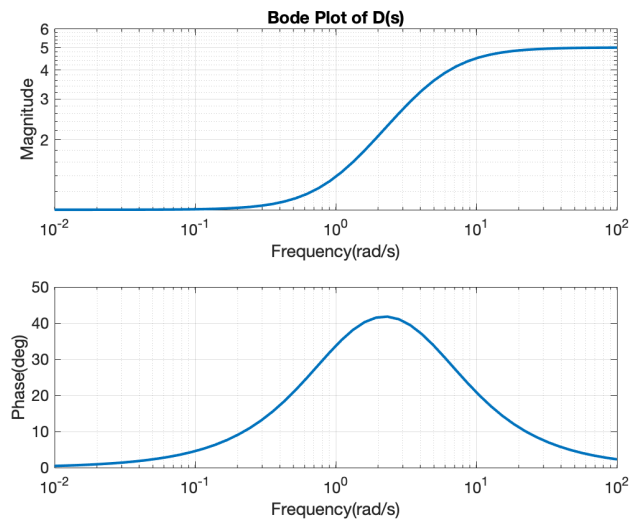
$$(R_1 + R_2)C = \frac{1}{0.2} R_2C$$

So I choose parameters

$$R_1 = 80 \text{ k}\Omega, \quad R_2 = 20 \text{ k}\Omega, \quad C = 10 \text{ }\mu\text{F}$$

### 1.2 Part B

The bode plot of  $D(s)$ (after applying the unit inverter)



We can see the turning point at about  $\omega = 1$  and 5.

### 1.3 Part C

Say  $V_m$  is the votage of the inverting input.

$$\begin{cases} V_{out} = -V_m \frac{A_0}{\tau s + 1} \\ V_m = V_{in} + \frac{Z_{in}}{Z_{in} + Z_{out}} (V_{out} - V_{in}) \end{cases}$$

eliminate  $V_m$ ,

$$\frac{V_{out}}{V_{in}} = -\frac{Z_{out}}{Z_{in}} \frac{A_0}{(\tau s + 1)(1 + \frac{Z_{out}}{Z_{in}}) + A_0} = -D(s) \frac{L(s)}{1 + L(s)}$$

where

$$L(s) = \frac{A_0}{(\tau s + 1)(D(s) + 1)} = \frac{A_0}{2} \frac{\alpha s + 1}{(\tau s + 1)(\beta s + 1)}$$

In which,

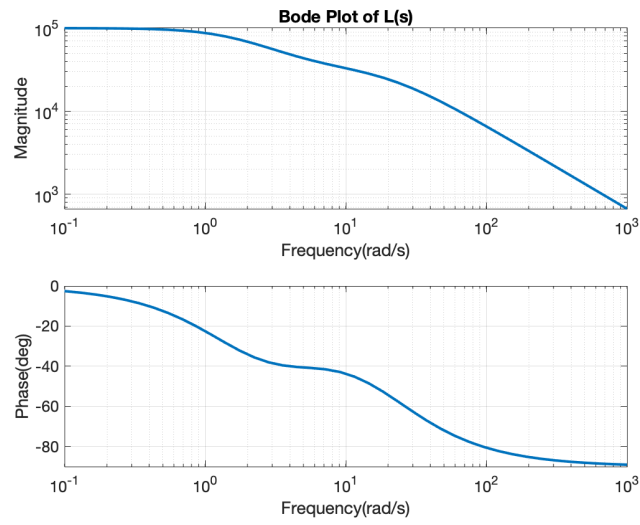
$$\alpha = R_2 C, \quad \beta = (\frac{R_1}{2} + R_2) C$$

## 1.4 Part D

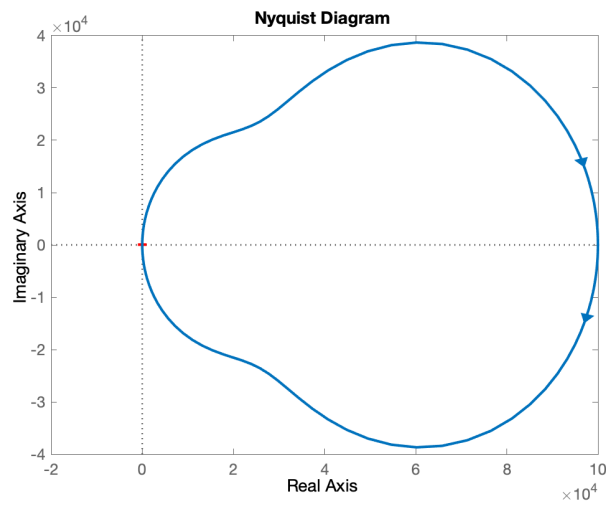
Substitute these quantity into  $L(s)$ ,

$$L(s) = \frac{20000s + 100000}{0.03s^2 + 0.65s + 1}$$

The bode plot,



And the Nyquist plot,



## 1.5 Part E

According to the Bode plot, the phase of  $L(s)$  will never be lower than 90 degree. So the close loop system is stable.