ME 180 Digital Control/Dynamic System
Department of Mechanical Engineering
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Problem Set #1

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1 Problem P2.9(b)

Description

For each of the following transfer functions, write the corresponding differential equation.

Section (b)

$$\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)}$$

Solution

Rewrite the transfer function $G(s) = X(s)/F(s)$ as $F(s) = [1/G(s)]X(s)$, which is:

$$F(s) = \frac{1}{15}(s^2 + 21s + 110)X(s)$$

Then:

$$F(s) = \frac{1}{15}s^2X(s) + \frac{7}{5}sX(s) + \frac{22}{3}X(s)$$

Take the inver laplace transform on both sides and ignore the initial value(set to zero):

$$f(t) = \frac{1}{15}\frac{d^2}{dt^2}x(t) + \frac{7}{5}\frac{d}{dt}x(t) + \frac{22}{3}x(t)$$

2 Problem P2.19(a)

Description

Find the transfer function, $G(s) = V_o(s)/V_i(s)$, for each network shown in Figure. Solve the problem using mesh analysis.

Section (a) Circuit

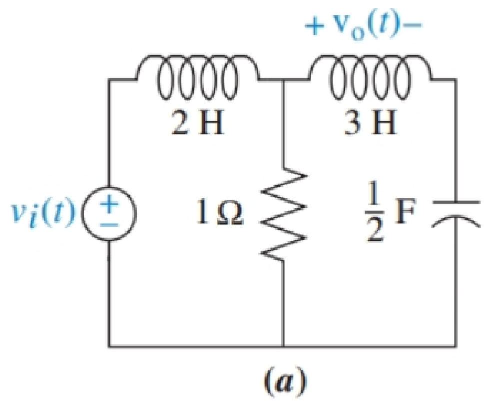
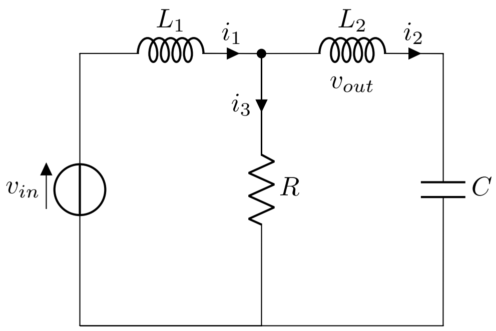


Figure 1: P2.19(a)

Solution

The mesh equations are:

$$\begin{aligned}
I_1(s)(sL_1 + R) \quad & - I_2(s)R & = V_{in} \\
-I_1(s)R \quad & + I_2(s)(R + sL_2 + \frac{1}{sC}) & = 0
\end{aligned}$$



Then solve $I_2(s)$ from the equation:

$$I_2(s) = \begin{bmatrix} sL_1 + R & V_{in} \\ -R & 0 \end{bmatrix} \Big/ \begin{bmatrix} sL_1 + R & -R \\ -R & R + sL_2 + \frac{1}{sC} \end{bmatrix} = \frac{sRC}{s^3(L_1L_2C) + s^2(L_1 + L_2)RC + sL_1 + R} V_{in}(s)$$

Then we can get:

$$V_{out}(s) = I_2(s)(sL_2) = \frac{s^2 L_2 RC}{s^3(L_1L_2C) + s^2(L_1 + L_2)RC + sL_1 + R} V_{in}(s)$$

Substitute the variables with the value above, then the transfer function is:

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{3s^2}{6s^3 + 5s^2 + 4s + 2}$$

3 Problem P2.25

Description

Find the transfer function, $G(s) = X_2(s)/F(s)$, for the translational mechanical network shown in Figure P2.25.

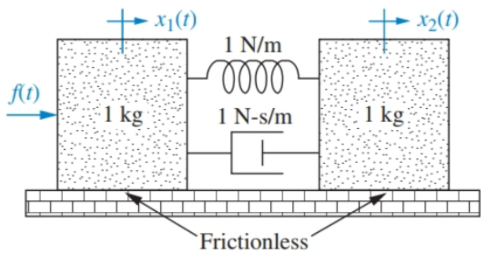


Figure 2: P2.25

Solution

According to the text, we could obtain the dynamics of this mechanical system:

$$\begin{aligned} X_1(s)(m_1 s^2 + \zeta s + k) \quad & - X_2(s)(\zeta s + k) & = F(s) \\ -X_1(s)(\zeta s + k) \quad & + X_2(s)(m_2 s^2 + \zeta s + k) & = 0 \end{aligned}$$

Substitute with the value above

$$\begin{aligned} X_1(s)(s^2 + s + 1) \quad & - X_2(s)(s + 1) & = F(s) \\ -X_1(s)(s + 1) \quad & + X_2(s)(s^2 + s + 1) & = 0 \end{aligned}$$

which could be solved by Cramer's Rule,

$$X_2(s) = \begin{bmatrix} s^2 + s + 1 & F(s) \\ -(s + 1) & 0 \end{bmatrix} \Big/ \begin{bmatrix} s^2 + s + 1 & s + 1 \\ -(s + 1) & s^2 + s + 1 \end{bmatrix} = \frac{s + 1}{(s^2 + 2s + 2)s^2} F(s)$$

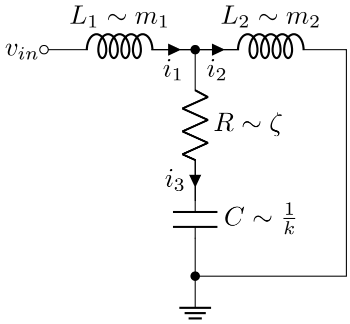
Thus the transfer function is:

$$\frac{X(s)}{F(s)} = \frac{s + 1}{(s^2 + 2s + 2)s^2}$$

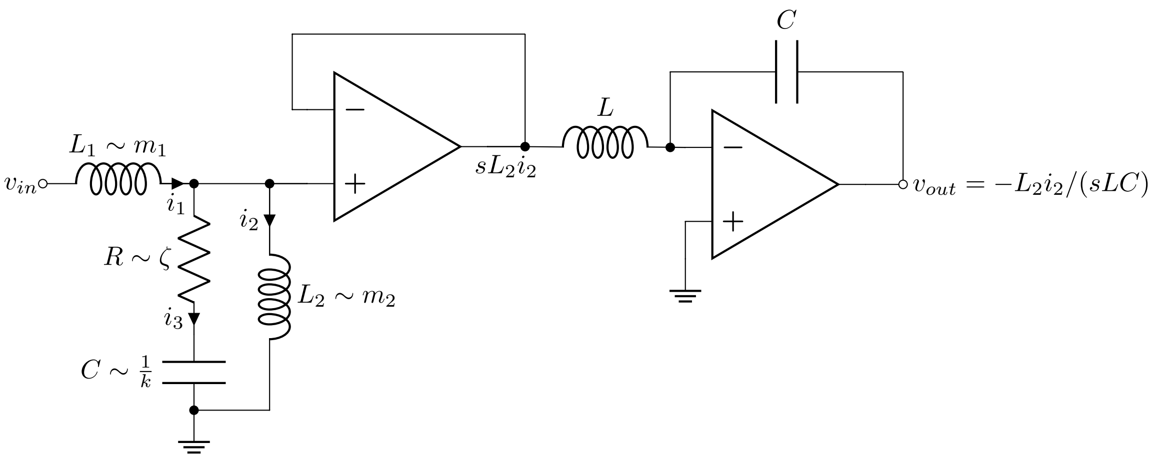
Bonus: the equavalant circuit dynamics should be like:

$$\begin{aligned}
\frac{1}{s}I_1(s)(L_1s^2+Rs+\frac{1}{C}) \quad & -\frac{1}{s}I_2(s)(Rs+\frac{1}{C}) && = V_{in}(s) \\
-\frac{1}{s}I_1(s)(Rs+\frac{1}{C}) \quad & +\frac{1}{s}I_2(s)(L_2s^2+Rs+\frac{1}{C}) && = 0
\end{aligned}$$

Thus the schematic of the equavaent circuit is:



while the input should be $V_{in}(s)$, the output should be $\frac{1}{s}I_2(s)$. To obtain the output, consider using a AMP intergrator:



Then pick $LC = L_2$, we can obtain the $I(s)/s$ presented in the form of voltage.

4 Problem P2.35

Description

For the rotational mechanical system with gears shown in Figure P2.20,find the transfer function, $G(s) = \theta_3(s)/T(s)$. The gears have inertia and bearing friction as shown. [**notes of instructor:** Do not solve the TF. Instead, determine apparent inertia at θ_3]

Solution

The transfer function of the this gear system should have a form of:

$$(\gamma_1\gamma_2)T(s) = (J_{eq}s^2 + D_{eq}s + K_{eq})\Theta_3(s)$$

where $\gamma_1 = \frac{N_1}{N_2}$, $\gamma_2 = \frac{N_3}{N_4}$, are the gear ratio. And J_{eq}, D_{eq}, K_{eq} represent the equavalant impedance. Thus,

$$\begin{aligned}
J_{eq} &= J_5 + J_4 + \gamma_2^2[J_3 + J_2 + \gamma_1^2(J_1)] \\
&= J_5 + J_4 + \gamma_2^2(J_3 + J_2) + \gamma_2^2\gamma_1^2 J_1 \\
&= J_5 + J_4 + \left(\frac{N_3}{N_4}\right)^2(J_3 + J_2) + \left(\frac{N_3}{N_4}\right)^2\left(\frac{N_1}{N_2}\right)^2 J_1
\end{aligned}$$

5 Problem P2.54

Description

Consider the differential equation:

$$\frac{d^3x}{dt^3} + 10\frac{d^2x}{dt} + 20\frac{dx}{dt} + 15x = f(x)$$

where is the input and is a function of the output, x.If $f(x) = 3e^{-5x}$, linearize the differential equation for x near 0.[**notes instructor:** linearize at $x = 1$ instead of $x = 0$]

Solution

the component $f(x) = 3e^{-5x}$ is a nonlinear component in terms of $x(t)$, thus

1. determine the operating point, which corresponds to $x_0 = 1$:

$$f_0 = f(1) = 3e^{-5}$$

2. determine the slop in terms of x at the operating point :

$$m = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} = \left.(-15)e^{-5x}\right|_{x=x_0} = -15e^{-5}$$

3. change of variables, using the local linearized function $\delta f = f_0 + m\delta x$ to approximate the original nonlinear function f,

$$\delta f = 3e^{-5} + (-15)e^{-5}\delta x, \quad \delta x = x - x_0$$

The local linearized differential equation in terms of δx at operating point $x_0 = 1$ is:

$$\frac{d^3}{dt^3}\delta x + 10\frac{d^2}{dt}\delta x + 20\frac{d}{dt}\delta x + 15\delta x = 3e^{-5} + (-15)e^{-5}\delta x$$

6 Problem P5.13

Description

For the system shown in Figure P5.13, find the poles of the closed-loop transfer function, $T(s) = C(s)/R(s)$. [**notes of instructor:** obtain the transfer function]

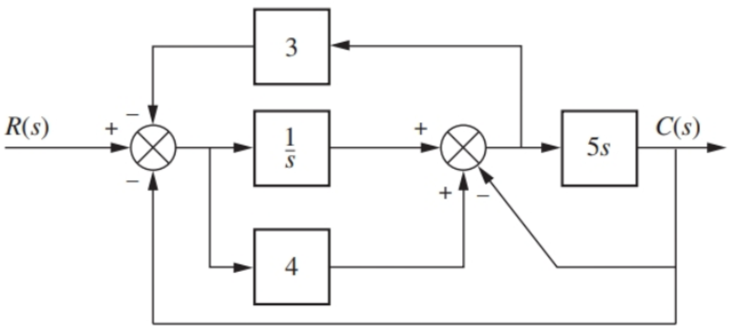


Figure 3: P5.13

Solution

First, get the input $X(s)$ of the $1/s$ block:

$$X(s) = R(s) - \frac{3C(s)}{5s} - C(s)$$

Then calculate the equation of the second nodes:

$$\frac{C(s)}{5s} = (\frac{1}{s} + 4)X(s) - C(s)$$

Substitute the $X(s)$ in the second equation by the first equation:

$$\frac{C(s)}{5s} = \left(\frac{1}{s} + 4\right)\left(R(s) - \frac{3C(s)}{5s} - C(s)\right) - C(s)$$

Then

$$(25s^2 + 22s + 3)C(s) = (20s^2 + 5s)R(s)$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{20s^2 + 5s}{25s^2 + 22s + 3}$$