EE 105 Feedback Control Systems
Department of Electrical and Computer Engineering
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Homework #2

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1 Problem 1

Answer:
(a) According to the problem statement.

$$i_L + \frac{L}{R}\frac{\mathrm{d}i_L}{\mathrm{d}t} = I$$

So Laplace transform:

$$I_L(s) = I(\frac{1}{1+\frac{R}{L}s})$$

Since $I = u(t)$ and $\mathcal{L}(u(t)) = 1/s$, I_L becomes:

$$I_L = \frac{1}{s}(\frac{1}{1+\frac{R}{L}s}) = \frac{L/R}{s^2+(L/S)s}$$

And the $i_L(t)$ is:

$$\mathcal{L}^{-1}(I_L(s)) = u(t)(1-e^{-\frac{L}{R}t})$$

(b) Using mathematica to draw the plot when $L = 75\text{nH}$ and $R = 50\Omega$. We can see that the time constant

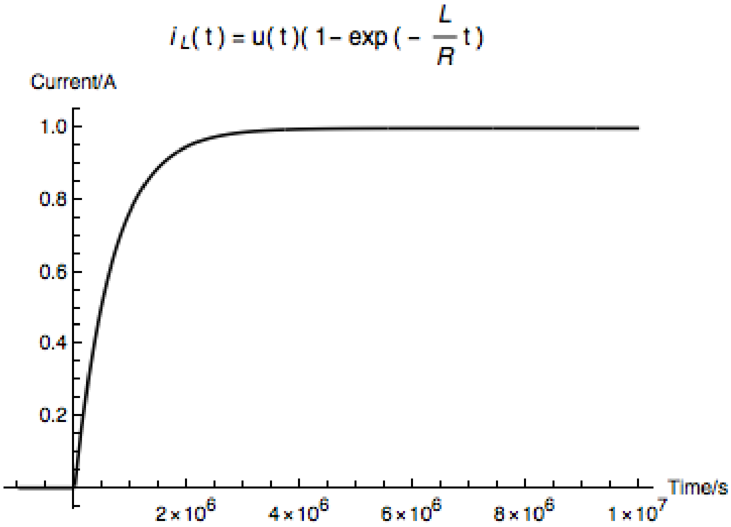

Figure 1: Plot of $i_L(t)$

$\tau = R/F = 6.7 \times 10^5$, so it will take hours to reach the stable state.

2 Problem 2

Answer:
(a) $e^{-t} - e^{-2t}$ We have:

$$F(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

(b) t^2 We have:

$$F(s) = \frac{2}{s^3}$$

(c) $\cosh(3t)$ We have:

$$F(s) = \frac{s}{s^2 - 9}$$

(d) $t\cosh(3t)$ We have:

$$F(s) = \frac{\mathrm{d}}{\mathrm{d}s}\mathcal{L}^{-1}(\cosh(3t)) = -\frac{s^2+9}{(s^2-9)^2}$$

(e)$e{-}2t\cosh(3t)$ We have:

$$F(s) = \frac{s+2}{(s+2)^2-9} = \frac{s+2}{s^2+4s-5}$$

(f)$(2t)^2$ We have:

$$F(s) = \frac{1}{2}\frac{2}{(s/2)^3} = \frac{8}{s^3}$$

(g)$(t-5)^2u(t-5)$ We have:

$$F(s) = \frac{2e^{-5s}}{s^3}$$

3 Problem 3

Answer:
(a) According to the problem statement:

$$F(s) = \frac{s}{(s+1)(s+4)} = \frac{1}{3}\Big(\frac{4}{s+4} - \frac{1}{s+1}\Big)$$

which means:

$$f(t) = \frac{1}{3}u(t)(4e^{-4t} - e^{-t})$$

(b) According to the problem statement:

$$F(s) = \frac{s^3+6s^2+6s}{s^2+6s^+8} = s - \frac{2s}{s^2+6s+8} = s - 2(\frac{2}{s+4} - \frac{1}{s+2})$$

So the inverse Laplace transform is:

$$f(t) = \delta'(t) - 4e^{-4t} + 2e^{-2t}$$

(c) According to the problem statement:

$$F(s) = \frac{s+1}{s^2+2s} = \frac{s+1}{s(s+2)} = \frac{1}{2}(\frac{1}{s} + \frac{1}{s+2})$$

So the inverse Laplace transform is:

$$f(t) = \frac{1}{2}u(t)(1+e^{-2t})$$

(d) According to the problem statement:

$$F(s) = \frac{e^{-2s}}{(s+1)(s+2)^2} = e^{-2s}\Big(\frac{1}{s+1} - \frac{1}{s+2} - \frac{1}{(s+2)^2}\Big)$$

Assume:

$$F(s) = e^{-2s}F_1(s)$$

We have:

$$\mathcal{L}(F_1(s)) = u(t)(e^{-t} + e^{-2t} + te^{-2t})$$

Then we get:

$$f(t) = \mathcal{L}(F(s)) = \mathcal{L}(e^{-2s}F_1(s)) = u(t-2)(e^{-(t-2)} + e^{-2(t-2)} + te^{-2(t-2)})$$

4 Problem 4

Answer:
This block diagram is :

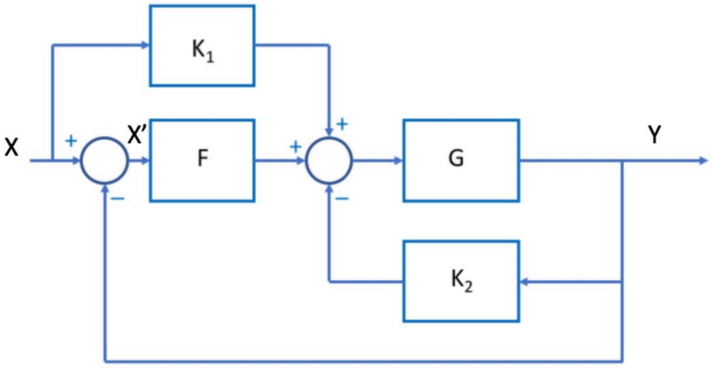

Figure 2: The block diagram

Where X is the input of the system, and Y is the output of the system.
Here we can use the Black's Formula to combine the G and K_2.

$$P = \frac{G}{1 + GK_2}$$

And we can list the following equations:

$$X' = X - Y$$

$$Y = P(FX' + K_1X)$$

Then we can figure out the relationship between Y and X:

$$\frac{Y}{X} = \frac{PF + PK_1}{1 + PF} = \frac{G(F + K_1)}{1 + G(F + K_2)}$$

So the transfer function of the single block should be that.