EE 105 Feedback Control Systems
Department of Electrical and Computer Engineering
Tufts University Fall 2018
Homework #3

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1 Problem 1

1.1 Section A

The transfer function of this open-loop system , while $W = 0$ is:

$$\frac{Y(s)}{R(s)} = \frac{b}{ms + b}$$

1.2 Section B

The transfer function of the close-loop system, while $W = 0$ is(using Black's formula):

$$\frac{Y(s)}{R(s)} = \frac{\frac{B}{ms+b}}{1 + \frac{B}{ms+b}} = \frac{B}{ms + b + B}$$

1.3 Section C

When $r(t) = \varepsilon(t)(\varepsilon(t)$ is unit step function), $R(s) = 1/s$, $Y(s)$ of open loop system should be:

$$Y(s) = \frac{1}{s}\frac{b}{ms + b} = \frac{b}{m}\frac{1}{(s + b/m)s} = \frac{1}{s} - \frac{1}{s + b/m}$$

So $y(t)$ should be:

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \varepsilon(t)(1 - e^{-\frac{b}{m}t})$$

1.4 Section D

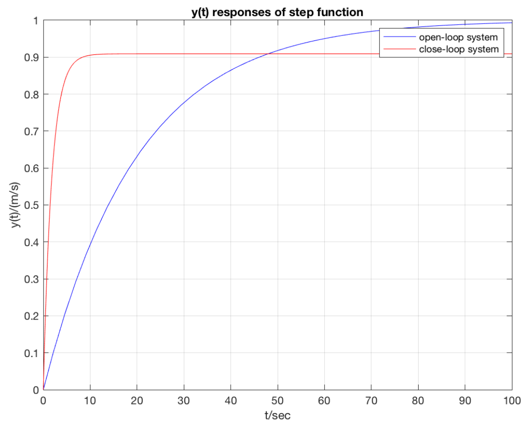
When $r(t) = \varepsilon(t)$, $R(s) = 1/s$, $Y(s)$ of close loop system should be:

$$Y(s) = \frac{1}{s}\frac{B}{ms + b + B} = \frac{B}{m}\frac{1}{s + \frac{B+b}{m}}\frac{1}{s} = \frac{B}{B + b}\left(\frac{1}{s} - \frac{1}{s + \frac{B+b}{m}}\right)$$

So $y(t)$ should be:

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \frac{B}{B + b}\varepsilon(t)(1 - e^{-\frac{B+b}{m}t})$$

1.5 Section E



1.6 Section F

If we let $t \to \infty$ for close-loop system, the transfer function $y(t)$ becomes:

$$\lim_{t\to\infty}(1-y(t)) = 1 - \lim_{t\to\infty}\frac{B}{B+b}u(t)(1-e^{-\frac{B+b}{m}t}) = 1 - \frac{B}{B+b} = \frac{1}{1+B/b}$$

1.7 Section G

The transfer function for the controller effort in open-loop system is:

$$\frac{U(s)}{R(s)} = b$$

1.8 Section H

The transfer function for the controller effort in close-loop system is(using Black's formula):

$$\frac{U(s)}{R(s)} = \frac{B}{1+B/(ms+b)} = \frac{B/(ms+b)}{ms+b+B}$$

1.9 Section I

When $r(t) = \varepsilon(t)$, $R(s) = 1/s$, $U(s)$ of open-loop system should be:

$$U(s) = \frac{b}{s}$$

So we have $u(t)$:

$$u(t) = b\varepsilon(t)$$

1.10 Section J

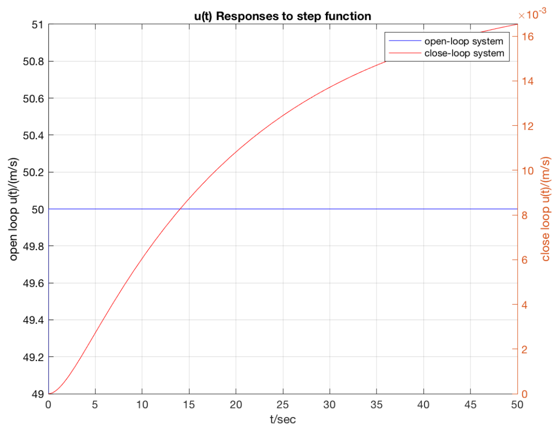
When $r(t) = \varepsilon(t)$, $R(s) = 1/s$, $U(s)$ of close-loop system should be:

$$U(s) = \frac{1}{s}\frac{B/(ms+b)}{ms+b+B} = \frac{B}{b(B+b)}\frac{1}{s} - \frac{1}{b}\frac{1}{s+b/m} + \frac{1}{B+b}\frac{1}{s+(B+b)/m}$$

So we have $u(t)$:

$$u(t) = \frac{B}{b(B+b)}\varepsilon(t) - \frac{1}{b}\varepsilon(t)e^{-\frac{b}{m}t} + \frac{1}{B+b}\varepsilon(t)e^{-\frac{B+b}{m}t}$$

1.11 Section K



1.12 Section L

The transfer function for the disturbance effort in open-loop system is:

$$\frac{Y(s)}{W(s)} = \frac{1}{ms+b}$$

1.13 Section M

The transfer function for the disturbance effort in close-loop system is(Using Black's formula):

$$\frac{Y(s)}{W(s)} = \frac{1/(ms+b)}{1+B/(ms+b)} = \frac{1}{ms+B+b}$$

1.14 Section N

For the open-loop system, if $W(s) = 1/s$ then:

$$Y(s) = \frac{1}{s}\frac{1}{ms+B+b} = \frac{1}{b}\Big(\frac{1}{s} - \frac{1}{s+b/m}\Big)$$

So $y(t)$ is:

$$y(t) = \frac{1}{b}\varepsilon(t)(1-e^{-\frac{b}{m}t})$$

For the close-loop system, we have:

$$Y(s) = \frac{1}{s}\frac{1}{ms+B+b} = \frac{1}{B+b}\Big(\frac{1}{s} - \frac{1}{s+(B+b)/m}\Big)$$

So $y(t)$ of close-loop system is:

$$y(t) = \frac{1}{B+b}\varepsilon(t)(1-e^{-\frac{B+b}{m}t})$$

Here is the plot: