EE 105 Feedback Control Systems
Department of Electrical and Computer Engineering
Tufts University Fall 2018
Homework #11

Name: *Shang Wang* Student ID: *1277417* E-mail: *shang.wang@tufts.edu*

1 Problem 1

1.1 Part A

The state equation and the output equation:

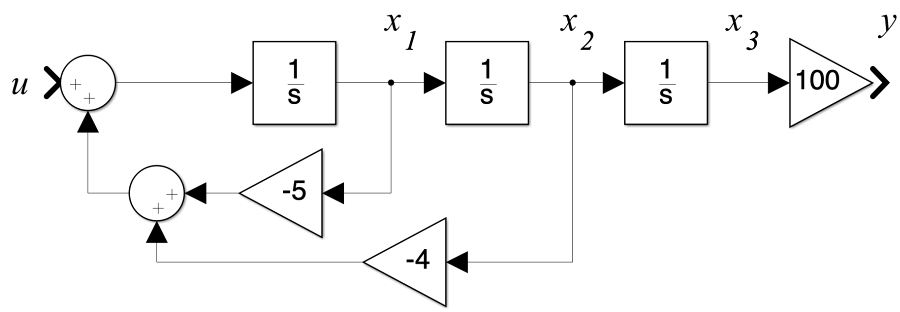
$$\begin{aligned}
\dot{x}_1 &= -5x_1 - 4x_3 + u \\
\dot{x}_2 &= x_1 \\
\dot{x}_3 &= x_2 \\
y &= 100x_3
\end{aligned}$$

So the control matrices are:

$$A = \begin{bmatrix} -5 & -4 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = 0$$

1.2 Part B

The block diagram is:



1.3 Part C

To move the poles to $p = -1, -2, -4$, we need to calculate the characteristic polynomial:

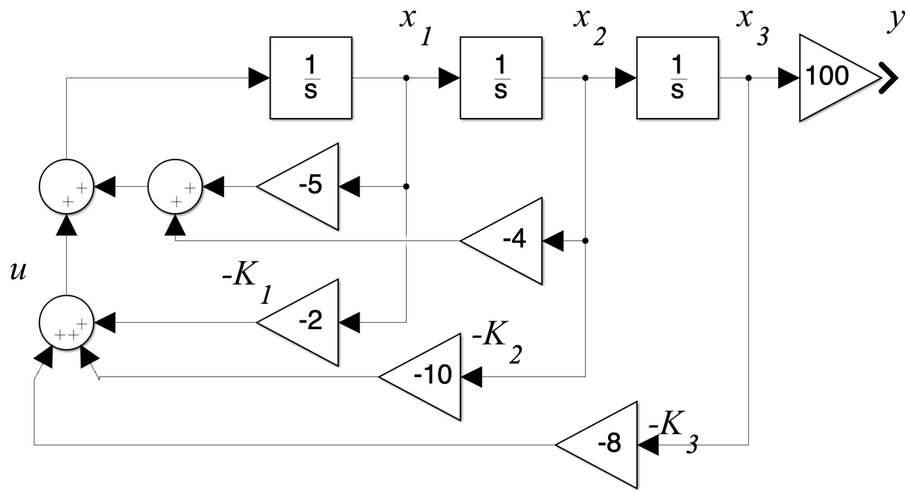
$$(p+1)(p+2)(p+4) = p^3 + 7p^2 + 14p + 8$$

Then subtract it by characteristic polynomial of the dynamic system. then we have:

$$K = [2,\ 10,\ 8]$$

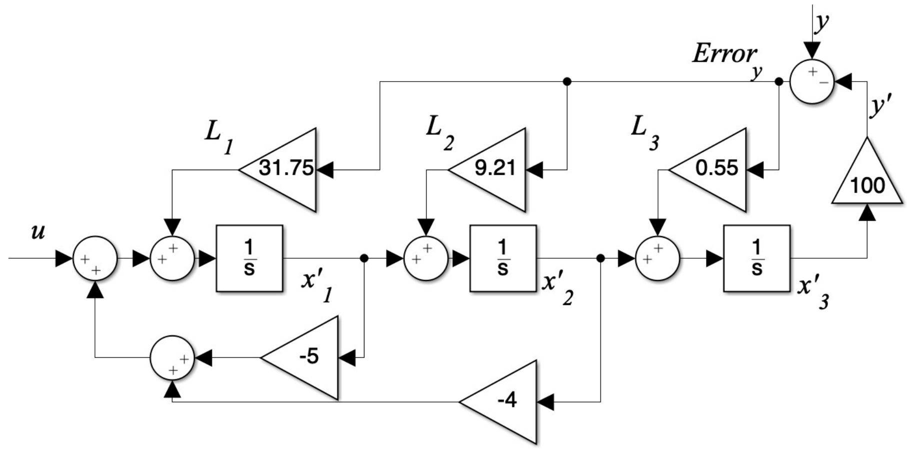
1.4 Part D

Here is the system with feadback path:



1.5 Part E

Here is the estimator:



1.6 Part F

To calculate the L, we need the characteristic equation of the estimator:

$$(p+20)^3 = p^3 + 60p^2 + 1200p + 8000 = 0$$

And the same characteristic equation could be derived in matrices form by using matrices A, C, L:

$$\det[pI - (A - LC)] = 0$$

subtitute by the numetrical value:

$$\det\left\{ \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} - \left(\begin{bmatrix} -5 & -4 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 100 \end{bmatrix} \right) \right\}$$

Then we have:

$$\det\left\{ \begin{bmatrix} p+5 & 4 & 100L_1 \\ -1 & p & 100L_2 \\ 0 & -1 & p+100L_3 \end{bmatrix} \right\} = 0$$

The characteristic equation is

$$p^3 + (100L_3 + 5)p^2 + (4 + 100L_2 + 500L_3)p + (100L_1 + 500L_2 + 400L_3) = 0$$

So we get

$$100L_3 + 5 = 60, \quad 4 + 100L_2 + 500L_3 = 1200, \quad 100L_1 + 500L_2 + 400L_3 = 8000,$$

Thus:

$$L_1 = 31.75, \quad L_2 = 9.21, \quad L_3 = 0.55$$

1.7 Part G

Here is the whole design: