EE 105 Feedback Control Systems
Department of Electrical and Computer Engineering
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Homework #5

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1 Problem 1

1.1 Part A

$$L(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$$

Answer:
Poles at ($p = 0, -1, -5, -10$), while zeros at ($z = -2, -6$). All the poles and zeros are in the real axis. So the pole(p = -10) will finally find ($z = -6$), likewise, ($p = -5$) will reach the point ($z = -2$). The other two poles ($p = 0$) and ($p = -1$) will join together, then seperat with departure angle $\pm\theta_d$, finally go to infinity with angle $\pm\pi/2$. The remaining parameters depature angle θ_d, Real-Axis intercept α and Break Away point σ still need to be calculated.

The "Real-Axis intercept" α:

$$\alpha = \frac{\sum_1^n p_i - \sum_1^m z_j}{n-m} = -4$$

The "Break Away point" σ (which is a real number) satisfies the formular:

$$\frac{\mathrm{d}}{\mathrm{d}\sigma} K = \frac{\mathrm{d}}{\mathrm{d}\sigma} \frac{-1}{L(\sigma)} = 0$$

Meaning :

$$\frac{\mathrm{d}}{\mathrm{d}\sigma} \frac{-\sigma(\sigma+1)(\sigma+5)(\sigma+10)}{(\sigma+2)(\sigma+6)} = 0$$

We have σ:

$$\sigma = -0.5676$$

Then we can calculate the departure angle θ_d. Assume $s_0 = b + xi$ is on the root locus near break away point. The angle is really small so we have $\tan(\theta) = \theta$. Using Master Criterian.

$$\sum_{i=1}^{n} \frac{x}{b - z_i} - \sum_{j=1}^{m} \frac{x}{b - p_j} = \pm\pi$$

$$x \cdot f(b) = \pm\pi \rightarrow \frac{x}{b-\sigma} = \tan(\theta_d) = \pm\frac{\pi}{f(b)(b-\sigma)}$$

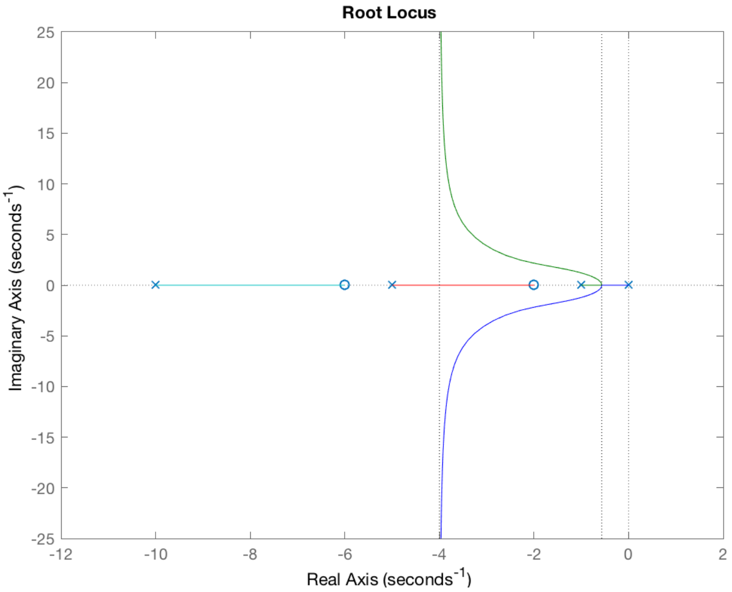
Where $f(b) = \sum_{i=1}^{n} \frac{1}{b-z_i} - \sum_{j=1}^{m} \frac{1}{b-p_j}$, clearly $\sigma \neq z_i, p_j$. Thus we have:

$$\lim_{b\to\sigma} f(b) = \text{Const}$$

So,

$$\lim_{b\to\sigma} \tan(\theta_d) = \pm\infty$$

We have $\theta_d = \pm\pi/2$.
The plot is like:



1.2 Part B

$$L(s) = \frac{s^2 + 2s + 12}{s(s^2 + 2s + 10)}$$

Answer:
Poles at $(p = 0, -1 \pm 3i)$, zeros at $(z = -1 \pm i\sqrt{11})$. Poles $(p = -1 \pm 3i)$ will finally find zeros $z = -1 \pm i\sqrt{11}$. Pole $p = 0$ will go to minus infinity on the real axis.

The Real-Axis intercept α:

$$\alpha = \frac{\sum_1^n p_i - \sum_1^m z_j}{n - m} = 0$$

The departure angle of pole$(p = -1 + 3i)$ is θ_d

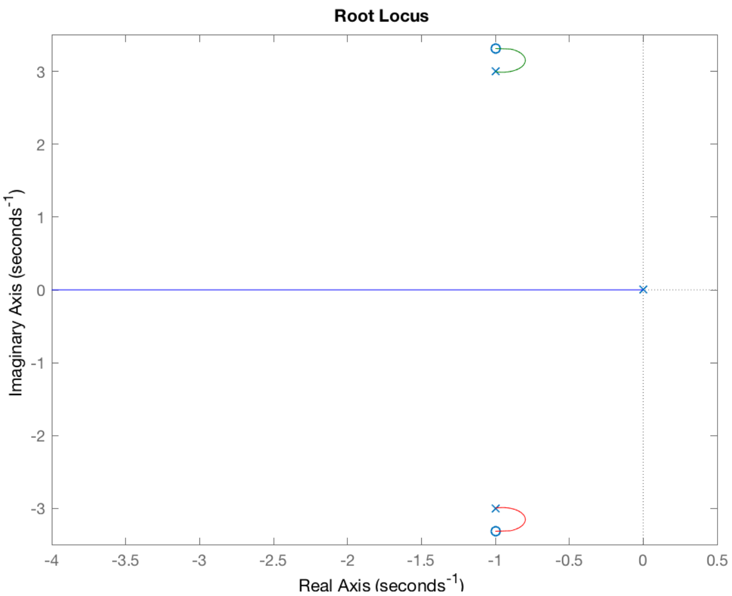
$$-\theta_d + \sum \psi_i - \sum \phi_j = -\pi$$

We have:

$$-\theta_d - \pi/2 - (\pi - \arctan(\frac{3}{1})) = -\pi$$

$$\theta_d = \arctan(3) - \pi/2 = -18.44^\circ$$

The departure angle of pole$(p = -1 - 3i)$ is $\theta_d = 18.44^\circ$.
So the plot is:



1.3 Part C

$$L(s) = \frac{s+3}{s^3(s+4)}$$

Answer:
Poles at $(p = -4)$, triple poles at $p = 0$, zeros at $z = -3$. One of the triple poles will find the zero$(z = -3)$, the other two will flee to infinity. Pole$(p = -4)$ will go to minus infinity. The departure angle of two poles at $(p = 0)$ θ are:

$$-3\theta_d + \sum \psi_i - \sum \phi_j = \pm\pi$$

We have $\theta_d = \pm\pi/3$.
The real axis intercept α:

$$\alpha = \frac{\sum_1^n p_i - \sum_1^m z_j}{n-m} = -\frac{1}{4}$$

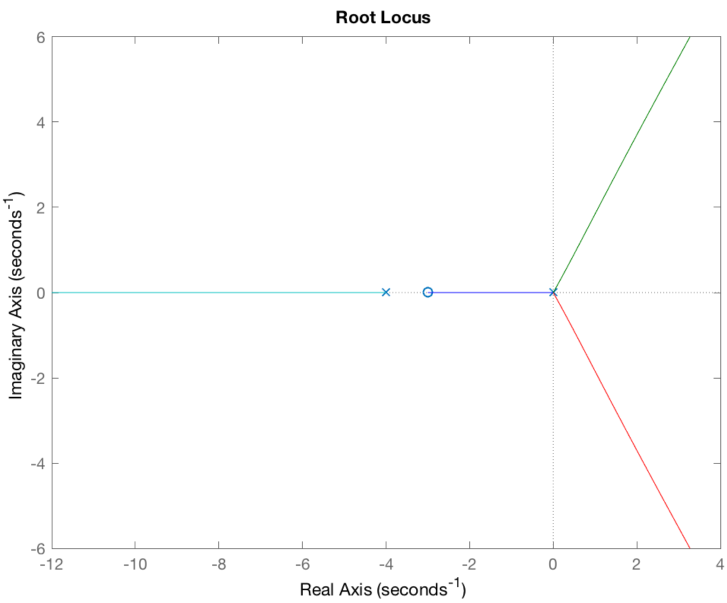
The Asymptote Angles at Large s, we have:

$$\angle\frac{1}{s^3} = (2n+1)\pi, \; n \in \mathbb{Z}$$

So the asymptote angle θ_a:

$$\theta_a = \pm\frac{\pi}{3}, \pi$$

The plot is like this:

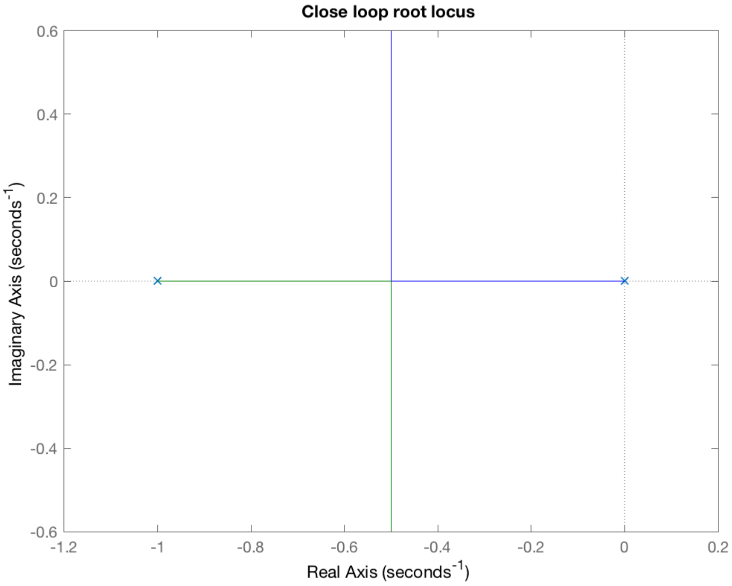


2 Problem 2

2.1 Part A

Answer:
$G(s)$ has poles at $(p = 0, 1)$, no zeros. So the two poles will join together and break away to infinity at $\sigma = 1/2$ on the real axis. Their departure angle and asymptote angle are both $\pm\pi/2$. Thus, it's impossible for the root locus to have interception with imaginary axis $Re(s) = -1$ at $s = -1 \pm j\sqrt{3}$.

The root locus is like this:



2.2 Part B

Answer:
Write the close loop transfer function:

$$H(s) = \frac{D_c(s)G(s)}{1 + D_c(s)G(s)} = \frac{K(s+z)}{s^3 + (1+p)s^2 + (p+K)s + z}$$

According to the requirement, we need close loop poles at $p = -1 \pm i\sqrt{3}$. So, we can write a denominator:

$$(s^2 + 2s + 4)(s + 2) = s^3 + 4s^2 + 8s + 8$$

Parameter K, p, m can be calculated accordingly.

$$p = 3, \;\; K = 5, \;\; z = \frac{8}{5}$$

Thus,

$$D_c(s) = \frac{4(s + 8/5)}{s + 3}$$

The root locus plot is like: