EE 105 Feedback Control Systems
Department of Electrical and Computer Engineering
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Homework #4

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1 Problem 1

Now use parameter P, I, D instead of B, K, M. Write the transfer function of the system, use Black's formula, C is the transfer function of controller while P is the transfer function of the plant:

$$H(s) = \frac{Y(s)}{R(s)} = \frac{CP}{1+CP} = \frac{Ds^2 + Ps + I}{(m+D)s^2 + (P+b)s + I}$$

Since the input is a unit step function, if we use the final value theorem($D, P, I \neq 0$):

$$y(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s} H(s) = \frac{I}{I} = 1$$

So if we want the system have zero steady-state error, the coefficient I can not be zero, because if $I = 0$:

$$y(\infty) = \lim_{s\to 0} s \cdot \frac{1}{s} H(s) = \frac{P}{P+b} < 1$$

Using the initial value theorem:

$$y(0) = \lim_{s\to \infty} s \cdot \frac{1}{s} H(s) = \frac{D}{m+D}$$

So if we do not want the system have abrupt jump at $t = 0$, D must be zero. Thus, the transfer function of the system becomes:

$$H(s) = \frac{Ps + I}{ms^2 + (P+b)s + I}$$

Poles at:

$$s_p = -\frac{P+b}{2m} \pm \frac{1}{2m}\sqrt{(P+b)^2 - Im}$$

Zeros at:

$$s_z = -\frac{I}{P}$$

Rewrite the system transfer function:

$$H(s) = \frac{P}{m}\frac{s + \frac{I}{P}}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

Where:

$$\omega_n = \sqrt{\frac{I}{m}}, \quad \zeta = \frac{P+b}{2\sqrt{mI}}$$

Ignore the zeros, focus on the poles. We need an underdamped second order system with the rising time no more than 20s and overshoot less than 10%. Damping ratio has to be between 0 and 1. We can solve the overshoot equation:

$$e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 10\%$$

We have a solution $\zeta = 0.59155$. Then solve the rising time equation:

$$\frac{\pi - \arccos(\zeta)}{\omega_n\sqrt{1-\zeta^2}} = 20$$

We have the value of $\omega_n = 0.1366$. Therefore,

$$I = 18.66, \quad P = 111.6$$

As the plots show, the zeros will seriously affect the performance of the system because the zeros is near these poles. Therefor, we have overshoot more than 10% due to the zeros. One way to fix this is to increase the damping ratio ζ. The plot with the notation "no zeros" is the plot with the transfer function of $\omega_n^2/(s^2 + 2\zeta\omega_n + \omega_n^2)$, which can easily see the difference with the practical transfer function $H(s)$.

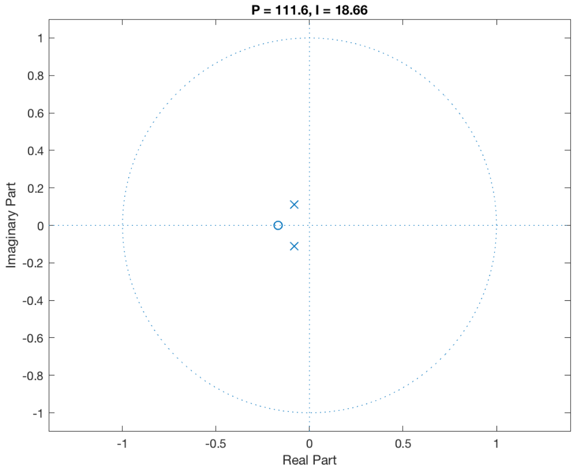

Figure 1: zeros and poles

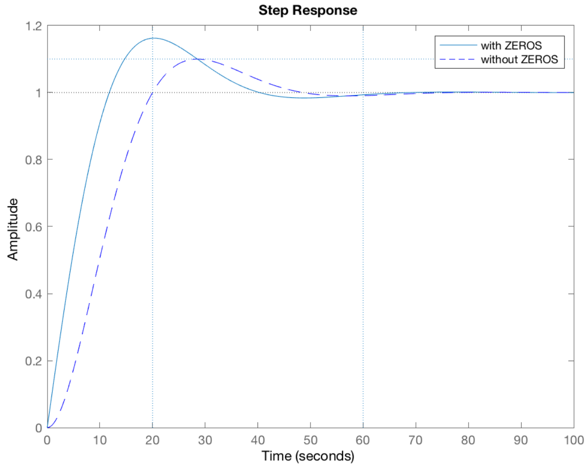

Figure 2: Step response for $P = 111.6$, $I = 18.66$

If we adjust the parameter a bit, like increasing the damping ratio a little by increasing the coefficient P.

$$I = 18.66, \quad P = 160$$

Plots are like these, which have basically satisfied the requiments.

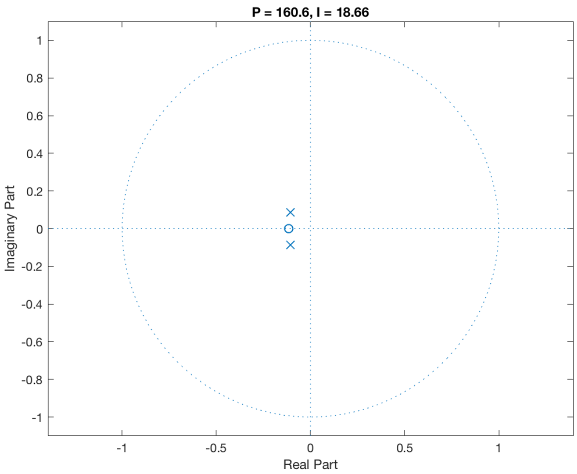

Figure 3: zeros and poles

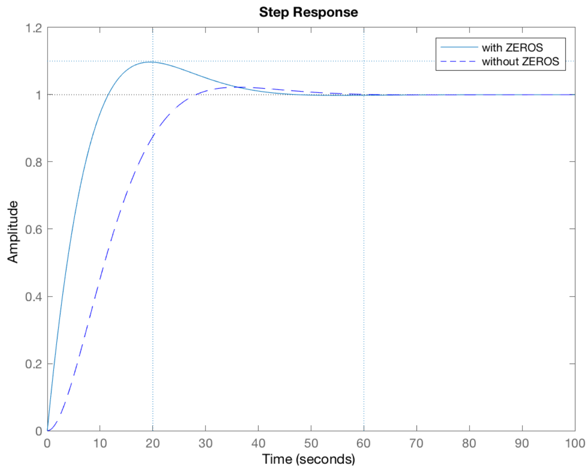

Figure 4: Step response for $P = 160.6$, $I = 18.66$