Tufts University
EE105 Feedback Control Systems, Fall 2018
Prof. Brian Aull
Homework #10   Due before you leave for Thanksgiving.

**(Bear in mind that the ECE Office may be closed on Wednesday.   If you need that time and can't email the homework, let me know.**)

In class we recalled the tightrope walker example from Lecture 6 and began to recast that problem into a modern control theory framework.   In Lecture 6 we derived the differential equation for the tilt angle $\theta$ of the tightrope walker.

$$I\frac{d^2\theta}{dt^2} - \tau_g\theta = \tau_p$$

$\tau_p$ is the external torque, $\tau_g$ is the gravitational torque, and I is the moment of inertia.  The transfer function of the system is

$$\frac{\theta(s)}{\tau_p(s)} = \frac{1}{Is^2 - \tau_g}$$

(a)  Choose a state vector **x** and write a pair of equations describing the system in control canonical form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-a_1 & -a_2\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u$$

You should specify $x_1$, $x_2$, u, and the matrix elements $-a_1$ and $-a_2$ in terms of the physical parameters of the system.

Assume that $\tau_g/I = 1$ and that the desired output $y = \theta$

(b) Draw the control canonical form block diagram (see slide 15 of lecture 20 as an example of what this should look like.)
(c) The transfer function has poles at -1 and +1, so it is unstable.   Design a control law that will move these poles to -2 and -3.  Show the block diagram with the control law feedback included.
(d) Generate a root locus plot for the system with an equivalent PD controller and verify that when the gain is unity, you indeed get closed-loop poles at -2 and -3.