Data-driven Power Flow Linearization: Simulation
Supplementary Material

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I. Introduction

The supplementary material includes

- an review of existing experiments in DPFL studies,
- the thorough illustration of the settings for each evaluated method, including their predictors, responses, hyperparameters,
- additional ranking results (36 in total) with the mean and maximal relative errors of all the evaluated methods,

II. Review of Existing Experiments

We here aim to offer a detailed review of existing DPFL experiments in the literature, as depicted in Table I. This review intends to present the experimental accomplishments of previous DPFL studies, while simultaneously revealing the importance and need for an extensive numerical comparison of all DPFL methods. Below are the further discussions of Table I.

Firstly, Table I indicates that both transmission and distribution grids were used as test cases to verify DPFL methods. While distribution grids differ from transmission grids in terms of symmetry and topology, DPFL methods are generally applicable to both types of systems, such as the methods in [1]–[5]. The reasons are twofold. First, even if the three phases in a distribution systems are unbalanced, DPFL methods can still be implemented by either training one DPFL model for each phase [6], or training an overall DPFL model for all variables in three phases [2], [5], [7], [8]. Note that the latter can generate a DPFL model reflecting the mutual influences between phases. Second, from a DPFL perspective, radial topologies do not pose any unique challenges compared to mesh topologies, as the difference is only in the number of dependent and independent variables. In summary, while distribution grids may have unbalanced characteristics and radial topologies, these attributes do not bring special difficulties to DPFL studies.

Secondly, Table I reveals that many evaluations solely depended on artificial data for training and testing, without considering the effects of noise and outliers on the data. This ideal testing environment is rarely found in real life, and the resulting conclusions may not hold in practice. To address this issue, it is recommended that synthetic data be injected with noise and outliers to mimic real-world scenarios.

Thirdly, as indicated in Table I, only a few studies report the load fluctuation range used in their simulations. This is worth mentioning because the accuracy of the DPFL model is highly dependent on the simulated fluctuations. E.g., a narrow fluctuation range typically leads to higher accuracy for the DPFL model. Without this information being made public, it is difficult to determine the reason for the high accuracy of the evaluated DPFL model.

Finally, Table I indicates which of the DPFL approaches have been evaluated in existing DPFL studies. These evaluations aimed to implement a comparative analysis between established and novel DPFL methods at that time. However, the scope of these comparisons is quite narrow, with only a few DPFL studies undertaking evaluations against a limited number of existing DPFL approaches (some works only conducted comparisons with PPFL methods). Such constrained comparisons fail to provide an in-depth understanding of the overall performance of DPFL methods. Consequently, a more exhaustive and inclusive comparison across all the DPFL approaches is clearly needed, in order to demonstrate their relative merits and limitations comprehensively.

TABLE I
REVIEW OF THE NUMERICAL EXPERIMENTS IN THE EXISTING DPFL LITERATURE

Ref.	Approach	Transmission Case	Distribution Case	Fluctuation$^{\diamond}$	Data†	Noise	Outlier	DPFL Comparison‡
[9]	LS	-	Modified IEEE 123-bus Single-phase Case [10]	-	Real Measured Data from [11]	Gaussian Noise	-	-
[12], [13]	LS_COD	IEEE 5-,24-,30-,57-,118-,and 300-bus Cases; PEGASE 1354-bus Case; RTE 1888-bus Case; Polish 2383-bus Case	-	40%–100%	Generated by MATLAB	-	-	LS_TOL [14], PLS_BDL [15]
[2], [5]	LS_HBLD LS_HBLE	22-, 85-, and 141-bus Cases	IEEE 13-, 123-, 8500-bus Three-phase Cases	50%–150%	Generated by MATLAB and OpenDSS	-	Generated from [0 p.u., 3 p.u.]	-
[14]	LS_TOL	IEEE 9-, 14-, 33-, and 57-bus Cases; NREL 118-bus Case; Case of Henan province, China	-	80%-120%	Partially Generated by MATLAB; Partially from [16]	Gaussian Noise	-	PLS_BDL [15]
[17]	LS_CLS	-	IEEE 123-bus Single-phase Case	-	Generated by GridLAB-D	-	-	-
[1], [18]	LS_LIFX LS_LIFXi	IEEE 5, 24, 30, 57, and 118-bus Cases; NREL 118-bus Case [16]	IEEE 33-bus Single-phase Case; Modified IEEE 123-bus Single-phase Case [19]	50%–400%	Generated by Undisclosed Software	Gaussian Noise	-	PLS_BDL [15]
[20]	LS_REC	IEEE 118-bus [21] and Polish 2383-bus Cases [22]	-	80%–120%	Generated by MATPOWER and OpenDSS	Gaussian Noise	Claimed to Consider but Details Undisclosed	LS [9], LS_HBLD, LS_HBLE [2], [5], LS_COD [12], [13]
[23]	PLS_SIMRX	IEEE 30-, 57-, 118-, and 300-bus Cases	-	-	Generated by Undisclosed Software	-	-	PLS_BDL [15]
[15]	PLS_BDL PLS_BDLY2	IEEE 5-, 57-, 118-, and 300-bus Cases	IEEE 33-bus Single-phase Case [24]; Modified 123-bus Single-phase Case [25]	80%–120%	Generated by MATLAB	Gaussian Noise	-	-
[26]	PLS_REC PLS_RECW	-	IEEE 33-bus Single-phase Case	-	Generated by MATLAB	-	-	-
[4]	RR RR_VCS	IEEE 14-, 30-, 47-, 118-, and 300-bus Cases	IEEE 33- and 123-bus Single-phase Cases	-	Generated by MATLAB	-	-	PLS_BDL [15], SVR_POL [27], [28]
[7]	RR_KPC	-	Modified IEEE 37-bus, [29], IEEE 123-bus [30], and 606-bus [31] Three-phase Cases	-	Generated by MATLAB	Gaussian Noise	-	PLS_BDL [15]
[32]	RR_WEI	8-generator 36-bus Case of PJM, the U.S.	-	-	Partially from PJM and Partially from [33]	Gaussian Noise	-	-
[8]	SVR	-	Modified IEEE 33-bus Three-phase Case [8]	50%–150%	Generated by MATLAB	-	Generated from [0.8 p.u., 1.2 p.u.]	PLS_BDL [15]
[34]	SVR_CCP	6-bus Case [35]; 118-bus Case [36]; Texas 2000-bus Case [37]	-	70%–130%	Generated by MATLAB	-	-	LS§
[27], [28]	SVR_POL	-	IEEE 123-bus Single-phase Case; Southern California Edison Single-phase Case, the U S.	-	Partially Generated by MATLAB; Partially Real Measured Data	Noise from Undis-closed Model	Generated from Gaussian Model	LS§
[38]	SVR_RR	-	IEEE 123-bus Three-phase Case; IEEE 33-bus Case	50%–150%	Generated by MATLAB	-	Generated from [0 p.u., 1 p.u.]	PLS_BDL [15], RR§
[39]	LCP_BOX LCP_COU	IEEE 118-bus Case	-	-	Generated by MATLAB	-	-	SVR§
[3]	DRC_XM DRC_XYM DRC_XYD	IEEE 118-bus Case; Polish 2383-bus Case	IEEE 123-, 8500-bus Three-phase Case	60%–140%	Generated by MATLAB	-	-	LS_HBLD, LS_HBLE [2], [5], LS_COD [12], [13]
[40], [41]	DC_LS	Tennessee Valley Authority (TVA) Case of the U.S. [42]	-	-	Real Measured Data from [42]	-	-	-
[43]	DLPF_C	IEEE 14-bus, New England 39-bus, European 89-bus, IEEE 118-bus, Illinois 200-bus, and European 1354-bus Cases [44]	-	Public Data	From U.S. National Renewable Energy Laboratory [45]	-	-	-

$^{\diamond}$: Refers to the fluctuation range for nodal power injections within the training and testing dataset.
‡ : Refers to the numerical comparison with DPFL approaches **ONLY**.
§ : Refers to the evaluated methods that the corresponding reference was not explicitly given.

III. Experiment Settings

The reader is referred to Table II for a thorough illustration of the settings for each method, including their predictors, responses, hyperparameters (or the cross-validation ranges[1]), datasets, solvers (when applicable), and additional relevant information.

TABLE II
Settings for the Evaluated Methods

Approach	Predictor$^{\diamond}$	Response$^{\diamond}$	Miscellaneous	Solver
LS	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized¶	-
LS_SVD	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized	-
LS_COD	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized	-
LS_HBLD	P, Q, V^2, θ_{ref}	V, PF	$\delta^{HUB} \in \{0.01, 0.02, \cdots, 0.05\}$; $N_{cv} = 5^{\dagger}$; Data: Normalized	FMINUNC
LS_HBLE	P, Q, V^2, θ_{ref}	V, PF	$\delta^{HUB} \in \{0.01, 0.02, \cdots, 0.05\}$; $N_{cv} = 5$; Data: Normalized	MOSEK
LS_TOL	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized	-
LS_CLS	P, Q, V^2, θ_{ref}	V, PF	$k \in \{2, 3, \cdots, 10\}$; $N_{cv} = 5$; Data: Normalized	-
LS_LIFX	P, Q, V^2, θ_{ref}	V, PF	f_{lift}: Gauss; Data: Normalized	-
LS_LIFXi	P, Q, V^2, θ_{ref}	V, PF	f_{lift}: Gauss; Data: Normalized	-
LS_WEI	P, Q, V^2, θ_{ref}	V, PF	ϖ: 0.6 [9]; Data: Normalized	MOSEK
LS_REC	P, Q, V^2, θ_{ref}	V, PF	New Data Share = 40%‡; $\kappa = 0.99$; Data: Normalized	-
PLS_SIMRX	P, Q, V^2, θ_{ref}	V, PF	$N_p = \text{Rank}(\boldsymbol{X})$; Data: Normalized	-
PLS_BDL	P, Q, V^2	V, PF	$N_p = \text{Rank}(\boldsymbol{X})$; Data: Normalized	-
PLS_BDLY2	P, Q, V^2	V, PF	$N_p = \text{Rank}(\boldsymbol{X})$; Data: Normalized	-
PLS_REC	P, Q, V^2, θ_{ref}	V, PF	New Data Share = 40%; Data: Normalized	-
PLS_RECW	P, Q, V^2, θ_{ref}	V, PF	New Data Share = 40%; $\varpi = 0.6$ [26]; Data: Normalized	-
RR	P, Q, V^2, θ_{ref}	V, PF	$\lambda = 10^{-10}$; Data: Normalized	-
RR_VCS	P, Q, V^2	V^2, R_{ij}, C_{ij}	$\lambda = 10^{-10}$; Data: Original¶	-
RR_KPC	P, Q, V^2, θ_{ref}	V, PF	$\lambda = 10^{-10}$; $k \in \{2, 3, \cdots, 10\}$; $\eta \in \{10^2, 10^3, \cdots, 10^5\}$; $N_{cv} = 5$; Data: Normalized	-
RR_WEI	P, Q, V^2, θ_{ref}	V, PF	$\lambda = 10^{-10}$; $\tau \in \{0.1, 0.11, \cdots, 0.35\}$; $N_{cv} = 5$; Data: Normalized	-
SVR	P, Q, V^2, θ_{ref}	V, PF	$\epsilon = 10^{-4}$; $\omega = 10$; Data: Normalized	GUROBI
SVR_CCP	P, Q, V^2, θ_{ref}	V, PF	$\epsilon = 10^{-4}$; $\omega = 10$; $M = 10^6$; $\varsigma_j^{CCP} = 95\%$; Data: Normalized	GUROBI
SVR_POL	P, Q, V^2, θ_{ref}	V, PF	$\epsilon = 10^{-4}$; Kernel Model: 3rd-order Polynomial; Data: Normalized	FITRSVM
SVR_RR	P, Q, V^2, θ_{ref}	V, PF	$\epsilon = 10^{-4}$; $\omega = 10$; $\lambda = 10^{-4}$; Data: Normalized	GUROBI
LCP_BOX	P, Q	V, θ	Bound: min/max coefficients of the 1st-order Taylor Approximation Model§; Data: Original	MOSEK
LCP_BOXN	P, Q	V, θ	Data: Original	MOSEK
LCP_COU	V, θ	PF	$\delta^{LIN} = 10^{-2}$; Data: Original	MOSEK
LCP_COUN	V, θ	PF	Data: Original	MOSEK
LCP_JGD	P, Q	V, θ	Structure: 1st-order Taylor Approximation Model; Data: Original	SDPT3
LCP_JGDN	P, Q	V, θ	Data: Original	SDPT3
DRC_XM	P, Q, V^2, θ_{ref}	V, PF	$\epsilon_j = 10^{-4}$; $\varsigma_j^{DRC} = 95\%$; Data: Normalized	MOSEK
DRC_XYM	P, Q, V^2, θ_{ref}	V, PF	$\epsilon_j = 10^{-4}$; $\varsigma_j^{DRC} = 95\%$; Data: Normalized	MOSEK
DRC_XYD	P, Q, V^2, θ_{ref}	V, PF	$\epsilon_j = 10^{-4}$; $\varsigma_j^{DRC} = 95\%$; Data: Normalized	GUROBI
DC_LS	P	θ	Data: Original	-
DLPF_C	P, Q, V, θ_{ref}	V, PF	Data: Original	-
LS_PIN	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized	-
LS_PCA	P, Q, V^2, θ_{ref}	V, PF	$N_p \in \{40, 50, \cdots, 80\}$; $N_{cv} = 5$; Data: Normalized	-
LS_GEN	P, Q, V^2, θ_{ref}	V, PF	Covariance Estimation Model: ARIMA; Data: Normalized	FGLS$^{\&}$
PLS_NIP	P, Q, V^2, θ_{ref}	V, PF	Data: Normalized	-
PLS_CLS	P, Q, V^2, θ_{ref}	V, PF	$k \in \{2, 3, \cdots, 10\}$; $N_{cv} = 5$; Data: Normalized	-
DC	P	θ	Data: Original	-
PTDF	P	PF	Data: Original	-
TAY	P, Q	V, θ	Data: Original	-
DLPF	P, Q, V, θ_{ref}	V, PF	Data: Original	-

$^{\diamond}$: The default predictors and responses are $\{P, Q, V^2, \theta_{ref}\}$ and $\{V, \text{PF}\}$, respectively, unless they do not apply to the approach (see Table **??**).
¶: The unit energy normalization is used for training and testing datasets by default, unless the approach is not applicable (see Table **??**).
†: N_{cv} is the fold number for cross-validation, used for auto-tuning the parameter based on the given parameter set and the training dataset.
‡: "New Data Share" is the fraction of new data in the training set, used for recursive DPFL methods to update models in a point-wise manner.
§: Every training data point has been used as a tangent point to find the min/max coefficients among all data points.
$^{\&}$: FGLS in MATLAB has been modified by replacing the ordinary least squares with the least squares with Pseudoinverse, as previously mentioned.

[1]In this study, cross-validation is employed for hyperparameter tuning only in cases where the literature does not specify the hyperparameter values.

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9-bus-L: Relative Error of PF (log-scale)

9-bus-L: Relative Error of Vm (log-scale)

9-bus-S: Relative Error of PF (log-scale)

9-bus-S: Relative Error of Vm (log-scale)

14-bus-L: Relative Error of PF (log-scale)

14-bus-L: Relative Error of Vm (log-scale)

14-bus-S: Relative Error of PF (log-scale)

14-bus-S: Relative Error of Vm (log-scale)

33-bus-L-NJ: Relative Error of PF (log-scale)

33-bus-L-NJ: Relative Error of Vm (log-scale)

33-bus-L-OJ: Relative Error of PF (log-scale)

33-bus-L-OJ: Relative Error of Vm (log-scale)

33-bus-L: Relative Error of PF (log-scale)

33-bus-L: Relative Error of Vm (log-scale)

33-bus-S-NI: Relative Error of PF (log-scale)

33-bus-S-NI: Relative Error of Vm (log-scale)

33-bus-S-OI: Relative Error of PF (log-scale)

33-bus-S-OI: Relative Error of Vm (log-scale)

33-bus-S: Relative Error of PF (log-scale)

33-bus-S: Relative Error of Vm (log-scale)

39-bus-L: Relative Error of PF (log-scale)

39-bus-L: Relative Error of Vm (log-scale)

39-bus-S: Relative Error of PF (log-scale)
RR_KPC mean: 3.6x10^-6 max: 5.7x10^-4
LS_LIFX mean: 4.6x10^-6 max: 1.6x10^-4
PLS_RECW mean: 8.0x10^-5 max: 2.9x10^-3
LS_LIFXi mean: 9.3x10^-5 max: 2.8x10^-3
RR_WEI mean: 1.1x10^-4 max: 3.5x10^-3
DLPF_C mean: 1.1x10^-4 max: 3.5x10^-3
PLS_NIP mean: 1.1x10^-4 max: 3.5x10^-3
PLS_REC mean: 1.1x10^-4 max: 3.5x10^-3
LS_PIN mean: 1.1x10^-4 max: 3.5x10^-3
LS_COD mean: 1.1x10^-4 max: 3.5x10^-3
LS_PCA mean: 1.1x10^-4 max: 3.5x10^-3
RR_VCS mean: 1.1x10^-4 max: 3.5x10^-3
RR mean: 1.1x10^-4 max: 3.5x10^-3
LS_HBLE mean: 1.1x10^-4 max: 3.5x10^-3
LS_GEN mean: 1.1x10^-4 max: 3.5x10^-3
LS_HBLD mean: 1.1x10^-4 max: 3.5x10^-3
DRC_XYM mean: 1.2x10^-4 max: 3.5x10^-3
DRC_XYD mean: 1.9x10^-4 max: 3.4x10^-3
SVR_RR mean: 9.0x10^-4 max: 3.5x10^-3
SVR mean: 9.0x10^-4 max: 3.5x10^-3
SVR_POL mean: 1.9x10^-3 max: 6.9x10^-3
PLS_CLS mean: 2.7x10^-3 max: 2.1x10^-2
PLS_SIMRX mean: 3.6x10^-3 max: 1.5x10^-2
LS_WEI mean: 8.8x10^-3 max: 3.9x10^-1
LCP_COUN mean: 1.0x10^-2 max: 1.6x10^-1
LCP_COU mean: 2.0x10^-2 max: 1.7x10^-1
DRC_XM mean: 2.8x10^-2 max: 1.1x10^-1
SVR_CCP mean: 4.9x10^-2 max: 1.4x10^-1
DC mean: 6.3x10^-2 max: 4.9x10^-1
PTDF mean: 6.3x10^-2 max: 4.9x10^-1
DC_LS mean: 8.5x10^-2 max: 4.0x10^-1
DLPF mean: 8.7x10^-2 max: 1.1x10^0
LS_SVD mean: 1.8x10^-1 max: 4.0x10^-1
LS_TOL mean: 6.7x10^0 max: 3.5x10^1
PLS_BDL mean: 8.9x10^0 max: 3.2x10^1
PLS_BDLY2 mean: 9.0x10^1 max: 2.8x10^2
TAY failure
LCP_JGDN failure
LCP_JGD failure
LCP_BOXN failure
LCP_BOX failure
LS_REC failure
LS_CLS failure
LS failure

39-bus-S: Relative Error of Vm (log-scale)

118-bus-L-NI: Relative Error of PF (log-scale)

118-bus-L-NI: Relative Error of Vm (log-scale)

118-bus-L-OI: Relative Error of PF (log-scale)

118-bus-L-OI: Relative Error of Vm (log-scale)

118-bus-L: Relative Error of PF (log-scale)

118-bus-L: Relative Error of Vm (log-scale)
RR_KPC mean: 3.5x10^-7 max: 4.4x10^-5
PLS_CLS mean: 3.9x10^-7 max: 4.4x10^-5
LS_LIFX mean: 7.2x10^-7 max: 2.6x10^-5
LS_LIFXi mean: 6.1x10^-6 max: 1.6x10^-4
RR_VCS mean: 2.3x10^-5 max: 5.7x10^-4
RR_WEI mean: 2.5x10^-5 max: 5.8x10^-4
LS_HBLE mean: 2.5x10^-5 max: 5.7x10^-4
LS_GEN mean: 2.5x10^-5 max: 5.8x10^-4
LS_PCA mean: 2.5x10^-5 max: 5.8x10^-4
LS_PIN mean: 2.5x10^-5 max: 5.8x10^-4
DLPF_C mean: 2.5x10^-5 max: 5.8x10^-4
PLS_NIP mean: 2.5x10^-5 max: 5.8x10^-4
PLS_REC mean: 2.5x10^-5 max: 5.8x10^-4
LS_COD mean: 2.5x10^-5 max: 5.8x10^-4
LCP_BOXN mean: 2.5x10^-5 max: 5.8x10^-4
RR mean: 2.5x10^-5 max: 5.8x10^-4
LS_HBLD mean: 2.5x10^-5 max: 5.8x10^-4
LCP_BOX mean: 2.5x10^-5 max: 5.8x10^-4
LS_WEI mean: 2.7x10^-5 max: 5.4x10^-4
PLS_RECW mean: 2.7x10^-5 max: 5.4x10^-4
TAY mean: 3.4x10^-5 max: 6.1x10^-4
PLS_SIMRX mean: 7.4x10^-5 max: 7.3x10^-4
SVR_POL mean: 1.4x10^-4 max: 8.9x10^-4
DRC_XYD mean: 7.4x10^-4 max: 2.6x10^-3
SVR_RR mean: 1.0x10^-3 max: 2.6x10^-3
SVR mean: 1.0x10^-3 max: 2.6x10^-3
DLPF mean: 2.0x10^-3 max: 1.4x10^-2
DC mean: 2.3x10^-2 max: 6.4x10^-2
DC_LS mean: 2.3x10^-2 max: 6.4x10^-2
LS_TOL mean: 3.1x10^-2 max: 1.9x10^-1
SVR_CCP mean: 5.0x10^-2 max: 1.2x10^-1
LS_SVD mean: 1.2x10^-1 max: 3.5x10^-1
PLS_BDLY2 mean: 1.7x10^0 max: 1.4x10^1
PLS_BDL mean: 1.9x10^0 max: 1.4x10^1
PTDF failure
LCP_COUN failure
LCP_COU failure
LS_REC failure
LS_CLS failure
LS failure

118-bus-S: Relative Error of PF (log-scale)
LS_LIFX mean: 6.1x10^-6 max: 1.7x10^-3
RR_KPC mean: 8.3x10^-6 max: 6.4x10^-3
PLS_CLS mean: 3.7x10^-5 max: 1.7x10^-2
LS_LIFXi mean: 1.6x10^-4 max: 2.3x10^-2
LS_HBLE mean: 3.2x10^-4 max: 6.0x10^-2
RR_VCS mean: 3.2x10^-4 max: 6.0x10^-2
DLPF_C mean: 3.2x10^-4 max: 6.0x10^-2
PLS_NIP mean: 3.2x10^-4 max: 6.0x10^-2
PLS_REC mean: 3.2x10^-4 max: 6.0x10^-2
LS_COD mean: 3.2x10^-4 max: 6.0x10^-2
LS_PIN mean: 3.2x10^-4 max: 6.0x10^-2
LS_PCA mean: 3.2x10^-4 max: 6.0x10^-2
RR mean: 3.2x10^-4 max: 6.0x10^-2
RR_WEI mean: 3.2x10^-4 max: 6.0x10^-2
LS_GEN mean: 3.2x10^-4 max: 6.0x10^-2
LS_HBLD mean: 3.2x10^-4 max: 6.0x10^-2
PLS_RECW mean: 3.5x10^-4 max: 6.4x10^-2
LS_WEI mean: 3.5x10^-4 max: 6.4x10^-2
SVR mean: 1.1x10^-3 max: 6.4x10^-2
SVR_RR mean: 1.1x10^-3 max: 6.4x10^-2
PLS_SIMRX mean: 1.5x10^-3 max: 5.5x10^-2
SVR_POL mean: 2.2x10^-3 max: 4.5x10^-2
DRC_XYD mean: 1.7x10^-2 max: 4.6x10^-1
SVR_CCP mean: 5.0x10^-2 max: 4.6x10^-1
LS_SVD mean: 9.8x10^-2 max: 3.5x10^-1
LCP_COU mean: 2.2x10^-1 max: 1.2x10^1
PTDF mean: 2.6x10^-1 max: 6.7x10^0
DC mean: 2.6x10^-1 max: 6.7x10^0
DC_LS mean: 3.3x10^-1 max: 2.8x10^1
DLPF mean: 5.0x10^-1 max: 1.8x10^1
LS_TOL mean: 7.4x10^-1 max: 2.1x10^0
LCP_COUN mean: 10.0x10^-1 max: 1.0x10^0
PLS_BDL mean: 8.6x10^1 max: 1.9x10^3
PLS_BDLY2 mean: 1.6x10^2 max: 2.8x10^3
LS_REC mean: 3.0x10^32 max: 1.9x10^34
TAY failure
LCP_BOXN failure
LCP_BOX failure
LS_CLS failure
LS failure

118-bus-S: Relative Error of Vm (log-scale)

1354-bus-L: Relative Error of PF (log-scale)
PLS_CLS mean: 3.0x10^-5 max: 3.8x10^0
RR_KPC mean: 3.0x10^-5 max: 3.8x10^0
RR mean: 2.6x10^-3 max: 3.6x10^2
PLS_NIP mean: 2.6x10^-3 max: 3.6x10^2
LS_COD mean: 2.6x10^-3 max: 3.6x10^2
DLPF_C mean: 2.6x10^-3 max: 3.6x10^2
PLS_REC mean: 2.6x10^-3 max: 3.6x10^2
LS_PIN mean: 2.6x10^-3 max: 3.6x10^2
LS_PCA mean: 2.6x10^-3 max: 3.6x10^2
RR_WEI mean: 2.6x10^-3 max: 3.7x10^2
RR_VCS mean: 2.8x10^-3 max: 3.6x10^2
PLS_RECW mean: 3.4x10^-3 max: 2.2x10^2
PLS_SIMRX mean: 4.9x10^-2 max: 1.7x10^3
LS_SVD mean: 5.9x10^-2 max: 9.1x10^2
DC mean: 1.6x10^-1 max: 1.5x10^4
PTDF mean: 1.6x10^-1 max: 1.5x10^4
DLPF mean: 1.7x10^-1 max: 1.6x10^4
LS_TOL mean: 8.1x10^-1 max: 9.4x10^2
DC_LS mean: 9.9x10^-1 max: 4.8x10^3
PLS_BDL mean: 1.4x10^2 max: 4.3x10^6
PLS_BDLY2 mean: 2.9x10^3 max: 1.1x10^8
TAY failure
LS_REC failure
LS_CLS failure
LS failure

1354-bus-L: Relative Error of Vm (log-scale)
RR_KPC
mean: 2.2x10^-6
max: 1.2x10^-4
PLS_CLS
mean: 2.2x10^-6
max: 1.2x10^-4
RR_VCS
mean: 1.6x10^-4
max: 5.4x10^-3
RR_WEI
mean: 1.8x10^-4
max: 6.2x10^-3
PLS_REC
mean: 1.8x10^-4
max: 6.1x10^-3
DLPF_C
mean: 1.8x10^-4
max: 6.1x10^-3
PLS_NIP
mean: 1.8x10^-4
max: 6.1x10^-3
LS_PIN
mean: 1.8x10^-4
max: 6.1x10^-3
LS_COD
mean: 1.8x10^-4
max: 6.1x10^-3
RR
mean: 1.8x10^-4
max: 6.1x10^-3
LS_PCA
mean: 1.8x10^-4
max: 6.1x10^-3
TAY
mean: 2.4x10^-4
max: 8.7x10^-3
PLS_RECW
mean: 2.7x10^-4
max: 1.3x10^-2
PLS_SIMRX
mean: 1.7x10^-3
max: 3.1x10^-2
DLPF
mean: 9.3x10^-3
max: 1.0x10^-1
DC
mean: 4.0x10^-2
max: 1.0x10^-1
DC_LS
mean: 4.0x10^-2
max: 1.0x10^-1
LS_SVD
mean: 6.0x10^-2
max: 2.8x10^-1
LS_TOL
mean: 5.3x10^-1
max: 1.9x10^0
PLS_BDL
mean: 5.5x10^0
max: 1.3x10^2
PLS_BDLY2
mean: 1.2x10^2
max: 3.3x10^3
PTDF
failure
LS_REC
failure
LS_CLS
failure
LS
failure

1354-bus-S: Relative Error of PF (log-scale)
RR_KPC
mean: 2.5x10^-6
max: 9.9x10^-2
PLS_CLS
mean: 2.5x10^-6
max: 9.9x10^-2
RR
mean: 1.9x10^-4
max: 2.8x10^1
LS_PIN
mean: 1.9x10^-4
max: 2.8x10^1
PLS_NIP
mean: 1.9x10^-4
max: 2.8x10^1
DLPF_C
mean: 1.9x10^-4
max: 2.8x10^1
PLS_REC
mean: 1.9x10^-4
max: 2.8x10^1
LS_COD
mean: 1.9x10^-4
max: 2.8x10^1
RR_WEI
mean: 1.9x10^-4
max: 2.8x10^1
RR_VCS
mean: 3.3x10^-4
max: 2.8x10^1
PLS_RECW
mean: 3.4x10^-4
max: 5.7x10^1
LS_PCA
mean: 3.8x10^-4
max: 7.5x10^1
LS_SVD
mean: 8.3x10^-2
max: 7.8x10^1
DC
mean: 1.7x10^-1
max: 2.4x10^4
PTDF
mean: 1.7x10^-1
max: 2.4x10^4
DLPF
mean: 1.8x10^-1
max: 2.7x10^4
LS_TOL
mean: 7.9x10^-1
max: 5.9x10^1
DC_LS
mean: 1.0x10^0
max: 7.3x10^3
PLS_SIMRX
mean: 1.1x10^0
max: 4.9x10^4
PLS_BDLY2
mean: 6.3x10^0
max: 2.3x10^5
PLS_BDL
mean: 8.7x10^0
max: 2.5x10^5
TAY
failure
LS_REC
failure
LS_CLS
failure
LS
failure

1354-bus-S: Relative Error of Vm (log-scale)
PLS_CLS mean: 2.3x10^-7 max: 9.4x10^-6
RR_KPC mean: 2.3x10^-7 max: 9.4x10^-6
RR_VCS mean: 1.2x10^-5 max: 3.2x10^-4
RR_WEI mean: 1.3x10^-5 max: 3.6x10^-4
RR mean: 1.3x10^-5 max: 3.6x10^-4
PLS_REC mean: 1.3x10^-5 max: 3.6x10^-4
DLPF_C mean: 1.3x10^-5 max: 3.6x10^-4
PLS_NIP mean: 1.3x10^-5 max: 3.6x10^-4
LS_COD mean: 1.3x10^-5 max: 3.6x10^-4
LS_PIN mean: 1.3x10^-5 max: 3.6x10^-4
LS_PCA mean: 2.2x10^-5 max: 5.4x10^-4
PLS_RECW mean: 2.3x10^-5 max: 6.6x10^-4
TAY mean: 9.4x10^-5 max: 2.3x10^-3
DLPF mean: 8.9x10^-3 max: 6.9x10^-2
DC mean: 4.0x10^-2 max: 8.7x10^-2
DC_LS mean: 4.0x10^-2 max: 8.7x10^-2
PLS_SIMRX mean: 5.3x10^-2 max: 5.8x10^-1
LS_SVD mean: 8.2x10^-2 max: 3.1x10^-1
PLS_BDLY2 mean: 2.4x10^-1 max: 6.1x10^0
PLS_BDL mean: 3.4x10^-1 max: 8.4x10^0
LS_TOL mean: 5.1x10^-1 max: 2.2x10^0
PTDF failure
LS_REC failure
LS_CLS failure
LS failure