Q14 ⇒

functions ⇒ $f(n) = 3^{2n+5} + n^4$

$g(n) = 4^n$

Limit comparison test ⇒

$$\lim_{n\to\infty} \frac{3^{2n+5} + n^4}{4^n}$$

$$= \lim_{n\to\infty} \frac{3^{2n+5}}{4^n} + \frac{n^4}{4^n}$$ Properties of Fractions

$$= \lim_{n\to\infty} \frac{3^{n+5}}{4^n} + \lim_{n\to\infty} \frac{n^4}{4^n}$$ Properties of limits

↓ Simplify exponent ↓ Apply L'H

(Apply L'H means Applied L'Hopital's rule)

$$= \lim_{n\to\infty} \frac{3^5 \cdot 3^n}{4^n} + \lim_{n\to\infty} \frac{4n^3}{\ln(4)\, 4^n}$$

↓ pull out constant ↓ Apply L'H

$$= 3^5 \lim_{n\to\infty} \frac{3^n}{4^n} + \lim_{n\to\infty} \frac{12n^2}{\ln^2(4)\, 4^n}$$

↓ Simplify ↓ Apply L'H

$$= 3^5 \lim_{n\to\infty} \left(\frac{3}{4}\right)^n + \lim_{n\to\infty} \frac{24n}{\ln^3(4)\, 4^n}$$

↓ Apply L'H

$$\lim_{n\to\infty} \frac{24}{\ln^4(4)\, 4^n}$$

Solve limit at infinity ↓ Solve limit at infinity

$$= \infty + 0$$

$$= \infty$$

That's an omega :)

Therefore, by limit Comparison Test, $\boxed{3^{2n+5} + 4^n \text{ is } \Omega(4^n)}$