

Q13 →

$$T(n) = \begin{cases} T(n-3) + 4 & n \geq 2 \\ 7 & n < 2 \end{cases}$$

$$T(n) = T(n-3) + 4$$

$$= T(n-6) + 4 + 4 = T(n-6) + 8$$

$$= T(n-9) + 4 + 4 + 4 = T(n-9) + 12$$

$$\vdots$$
$$= T(n-3k) + 4k$$

Sanity check with  $k=3$   
 $T(n-3(3)) + 4(3) = 12 \checkmark$

The recursion terminates at  $n-3k=1$ .

$$n-3k=1$$
$$\underline{-3k = 1-n}$$
$$\underline{-1}$$

$$\underline{3k = -1+n}$$
$$3$$

$$k = \frac{-1+n}{3}$$

number of unrollings to get to base case

at which point  $T(n-3k)=7$ . Thus,

$$T\left(n-3\left(\frac{-1+n}{3}\right)\right) + 4\left(\frac{-1+n}{3}\right)$$

$$= T(1) + \left(\frac{-4+n}{3}\right)$$

$$= T(1) + \frac{(-4)+4n}{3}$$

$$= 7 + \left(-\frac{4}{3}\right) + \frac{4}{3}n$$

Therefore, for  $n \geq 2$ , this is bounded above and below by products of constants with  $n$  (Example  $\Rightarrow 2$  and  $1$ ). So

$$T(n) \text{ is } \Theta(n)$$