Q11 → Weak Induction →

Identity → $1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

↓ Re-write as sum

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Base Case → $n=1$

$$\sum_{i=1}^{1} i^3 = \left[\frac{1(1+1)}{2}\right]^2$$

$$= 1^3 = \left[\frac{1(2)}{2}\right]^2$$

$$= 1 = \left[\frac{2}{2}\right]^2$$

$$= 1 = 1^2$$

$$= 1 \overset{\checkmark}{=} 1$$ Base Case holds!

Inductive Step →

Inductive Hypothesis → Assume that $\sum_{i=1}^{k} i^3 = \left[\frac{k(k+1)}{2}\right]^2$ for some $k \geq 1$

$$\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 + (k+1)^3$$ Properties of sums

$$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$$ By our inductive hypothesis

$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$ Distribute the square

$$= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4}$$ Multiply by 4 (to make sure denominators match)

$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$ Simplify

$$= \frac{(k+1)^2(k^2 + 4(k+1))}{4}$$ Factor out $(k+1)^2$

$$= \frac{(k+1)^2(k^2 + 4k + 4)}{4}$$ Distribute the 4

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$$= \frac{(k+1)^2(k+2)^2}{4}$$ Factor

$$= \left[\frac{(k+1)(k+2)}{2}\right]^2$$ Pull out the Square

Conclusion $\Rightarrow$ Therefore, By Weak induction, we've shown that

$$\boxed{\forall n \geq 1,\ 1^3+2^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2}$$