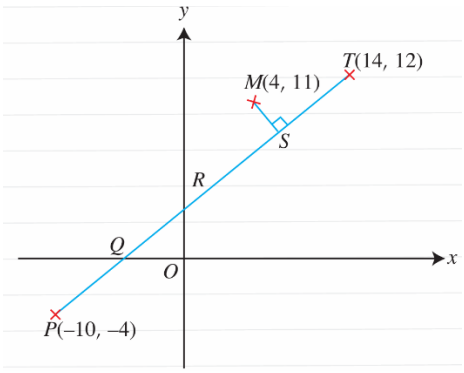
Coordinate Geometry & Trigonometric Identities

1. In the figure, $PQRST$ is a straight line cutting the x-axis at Q and the y-axis at R. The coordinates of P, T and M are $(-10, -4)$, $(14, 12)$ and $(4, 11)$ respectively, where MS is the perpendicular from M to PT.



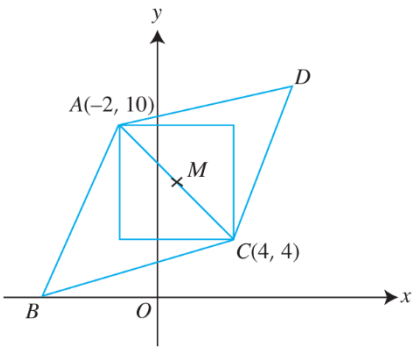
Find

(i) the equation of PQ and of MS,

(ii) the coordinates of Q, R and S,

(iii) the area of ΔPMS.

2. The figure shows a rhombus $ABCD$, where $A(-2, 10)$ and $C(4, 4)$ are opposite corners. The midpoint of AC is M and B lies on the x-axis.



(a) Find

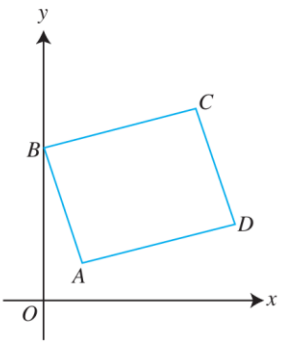
(i) the coordinates of M and of B,

(ii) the equation of BD,

(iii) the area of $ABCD$.

(b) Prove that $\angle ABC$ is not a right angle.

3. The figure shows a parallelogram $ABCD$. The coordinates of A, B and C are $(2, 2)$, $(0, 8)$ and $(8, 10)$ respectively.



(a) Find

(i) the coordinates of the point of intersection of the diagonals AC and BD,

(ii) the coordinates of D,

(iii) the equation of the diagonal BD,

(iv) the area of the parallelogram $ABCD$.

(b) Explain why the diagonals AC and BD are not perpendicular to each other.

4. Prove each of these identities.

a $\dfrac{\cos^2 x - \sin^2 x}{\cos x - \sin x} = \cos x + \sin x$

b $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = \dfrac{2}{\sin x}$

c $\dfrac{\cos^4 x - \sin^4 x}{\cos^2 x} = 1 - \tan^2 x$

d $\dfrac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x\,(1 - \sin^2 x)} = \tan^4 x$

5. Prove each of these identities.

a $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \sec x \operatorname{cosec} x$

b $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} = 2\tan x \sec x$

c $\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x} = 2\operatorname{cosec}^2 x$

d $\dfrac{\cos x}{1 - \tan x} + \dfrac{\sin x}{1 - \cot x} = \sin x + \cos x$

e $\dfrac{\cos x}{1 + \sin x} + \dfrac{\cos x}{1 - \sin x} = 2\sec x$

f $\dfrac{\cos x}{\operatorname{cosec} x + 1} + \dfrac{\cos x}{\operatorname{cosec} x - 1} = 2\tan x$

Answers:

[1] **(i)** $3y - 2x - 8 = 0,\ 2y + 3x = 34$

(ii) $Q(-4, 0), R\left(0, 2\frac{2}{3}\right), S\left(6\frac{8}{13}, 7\frac{1}{13}\right)$

(iii) $47\frac{1}{13}$ units^2

[2] **(a)** **(i)** $(1, 7), (-6, 0)$ **(ii)** $y = x + 6$

(iii) 84 units^2

[3] **(a)** **(i)** $(5, 6)$ **(ii)** $(10, 4)$

(iii) $5y + 2x = 40$ **(iv)** 52