Time Series

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Contents

Purpose

In this paper, we will explore Time-Series Regression. Rather than just creating a variety of time series models and trying to fit them on the bank stocks' data, we will delve into the theory of Time Series - breaking down the models to their mathematical forms and explaining the parameters and their effects to the models.

Examples of Time Series Statistical Models

White Noise

White noise series are a generated series that are a collection of uncorrelated random variables. They are used as a model for noise in engineering applications. We will also add these to other series (like signal) to add noise and make them appear more random.
White noise, it it's most general form, is defined as

$$w_t \sim \mathrm{wn}(0, \sigma_w^2)$$

with mean 0 and finite variance σ_w^2. We can also specify additional restrictions to make the white noise more useful, like Independent and Identically Distributed (IID), and also the probability density. A particularly useful white noise series is Gaussian white noise, which is defined as

$$w_t \overset{\text{IID}}{\sim} \mathrm{N}(0, \sigma_w^2$$

Moving Average and Filtering

This method is used to smooth the series. It helps to take away the large peaks and makes the slower oscillations more apparent. This series is defined by an average of its current value and its immediate neighbours in the past and future.

$$v_t = \frac{1}{3}\left(w_{t-1} + w_t + w_{t+1}\right)$$

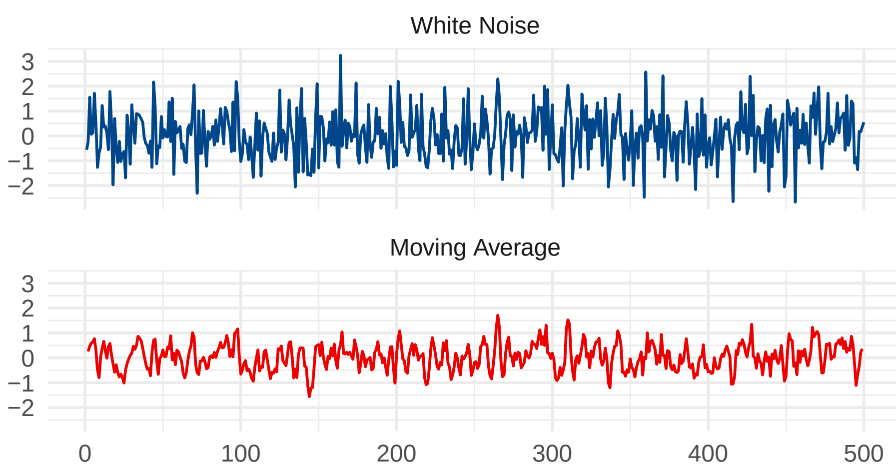

Figure 1: White Noise and Moving Average

Notice how for the moving average subplot in Figure 1, most of the larger peaks are averaging out.

Random Walk with/without Drift

A random walk is an extremely useful time series model, used for purposes like analyzing global temperatures. It can be defined without drift - simply called a Random Walk. The form of those model is

$$\begin{aligned} x_t &= \delta + x_{t-1} + w_t \\ &= \delta t + \sum_{j=1}^{t} w_j \end{aligned}$$

where δ is the drift factor and w_t is the white noise with the initial condition of $x_0 = 0$.

The Random walk, with and without drift, is shown in Figure 2 with a drift value of 0.2. Also, the dashed line represents a line with intercept of 0 and slope of the frist value.

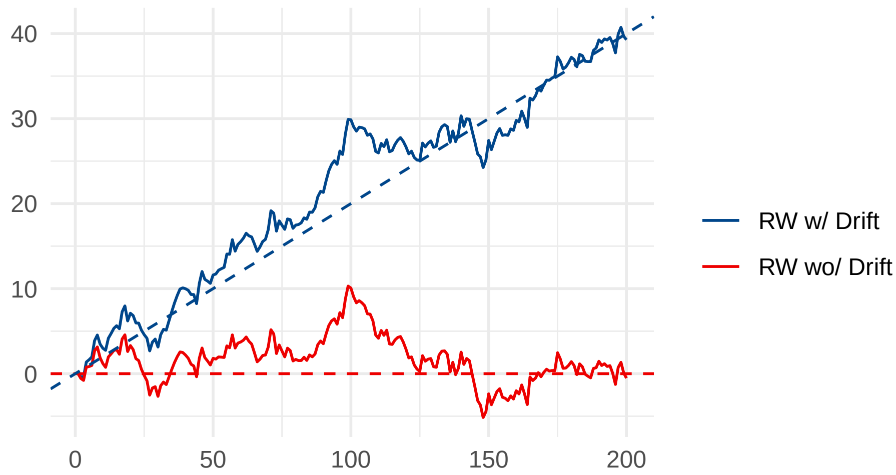

Figure 2: Random Walk with and without Drift

Signal With Noise

Many time series models assume there is an underlying signal with some consistent periodic variation, impaired with some random noise. We will plot a sinusoidal function and contaminate it with white noise, comparing the differences. We note that a sinusoidal function can be written in the following form

$$\mathrm{A}\cos\left(2\pi\omega + \phi\right)$$

We consider the model $x_t = 2\cos\left(\frac{2\pi t}{20}\right)$ and we will add white noise with both $\sigma_w = 1$ & and $\sigma_w = 5$

This is shown in Figure 3, with different noise distributions. Note the difference in the y-axis scale.

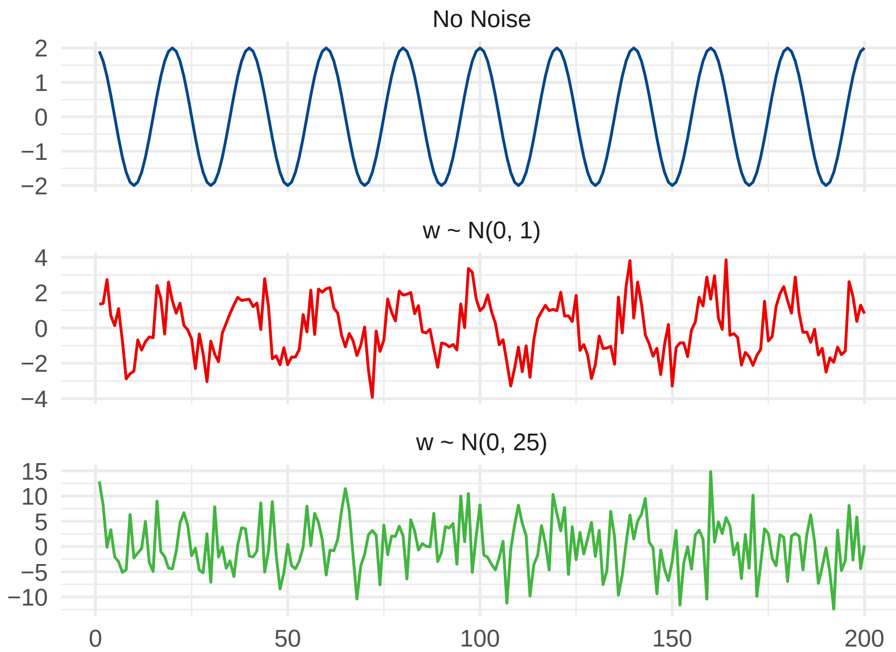

Figure 3: Sinusodial Series with Varying Noise

Acknowledgements

For this entire paper, I used the textbook *Time Series Analysis and Its Applications With R Examples* for a deeper understanding of time series and their applications. It is a wonderful textbook that introduces the concepts in a digestible manner while also using plots to further help with the intuition behind many models.