Building a lock-in amplifier PHY 312

In this lab you will learn some basic electronics techniques. The goal of the lab is to build a "lock-in amplifier"- a device that was invented by Princeton professor Robert Dicke and to use for measurement of the speed of sound. Lock-in amplifier in general is useful in detecting very small signals at a specific frequency in the presence of a large background noise. First, consider several methods for selecting a small signal at a specific frequency in the presence of noise.

1. Filter

The simplest way to detect a signal at a specific frequency is to use a frequency filter. Frequency filters for electrical signals can be easily constructed using simple electronic components, like resistors and capacitors. They can also be implemented in the software at the data analysis stage.

Filters can be divided into low-pass, high-pass, band-pass and band-stop types. As the names imply, low pass filters allow all frequencies below a certain cut-off frequency to pass through, high-pass filters pass all frequencies above a cut-off frequency. Band-pass filters pass signals in a certain frequency band and band-stop filters reject signals in a particular band, allowing all others to pass.

Filters are never perfect. That is, they can not pass signals in one frequency region undistorted and completely reject signals at other frequencies. The transition from the pass to the reject region is usually gradual. Filters can also introduce other undesirable effects, such as a delay and distortion of the signals.

Filters are useful for rejecting noise that is far away in frequency from the signal of interest. Filters are easy to construct and they work in real time, giving a filtered signal without the need for numerical analysis.

2. Fourier Transform

Fourier transform is a mathematical operation that separates signals into different frequency components. If S(t) is the signal as a function of time, then the Fourier transform is given by $F(\omega) = \int S(t)e^{-i\omega t}dt$. The Fourier transform can also be performed on a discrete set of data points by replacing the integral with a sum. If the number of data points is equal to an integer power of 2, the transform is numerically efficient and is called a Fast Fourier Transform (FFT). Since the Fourier transform is a complex number, one often looks at the Fourier Transform Power given by $F(\omega)F^*(\omega)$, which is proportional to the power of an electrical signal S as a function of frequency.

Fourier transforms are convenient for looking at multiple frequency components contained in a signal. They also allow one to measure the strength of the signal at a particular frequency and the noise in the signal as a function of frequency. One can also implement a filter by multiplying the Fourier transform by a certain function of frequency and then performing an inverse Fourier transforms. Fourier filters have better properties than real-time filters, but they require numerical analysis and cannot be implemented in real time.

The conversion of analog signals to digital form is usually performed by integrated circuits called A/D converters. The A/D conversion is characterized by the sampling rate and the digital resolution in number of bits. If the signal is sampled at a rate equal to R samples/sec, one can

show that the digital form will accurately represent signals with frequencies below $f_c = R/2$. Signals with a frequency f above f_c will appear at a frequency $2f_c-f$. This phenomenon is called aliasing. To avoid this problem, A/D conversion is usually preceded by an analog low-pass filter with a cut-off frequency below f_c .

3. Lock-in Amplifier

Lock-in amplifier is a device that "locks-in" on a signal of a particular frequency and amplifies it, rejecting all other frequencies. To illustrate its operation, lets assume that the signal contains two frequencies, $S(t) = A_1 \sin(\omega_1 t + \phi) + A_2 \sin(\omega_2 t)$. In the lock-in amplifier the signal is multiplied by a reference function, $R(t) = \sin(\omega_r t)$ and the result is filtered using a low-pass filter. With simple trigonometric identities one can show that

$$X(t) = S(t)R(t) = A_1(\cos[(\omega_1 - \omega_r)t + \phi] - \cos[(\omega_1 + \omega_r)t + \phi])/2 +$$

$$A_2(\cos[(\omega_2 - \omega_r)t + \phi] - \cos[(\omega_2 + \omega_r)t + \phi])/2$$

If the reference frequency ω_r is close to ω_1 , then $\omega_1-\omega_r$ is a low frequency that will pass through the low-pass filter, while the rest of the terms will be rejected by the filter. In this way the lock-in can select the frequencies close to the reference frequency. If $\omega_1=\omega_r$ and $\phi=0$, then the lock-in simply measures the amplitude of the signal at ω_1 . If the two frequencies are slightly different, it measures the beats between them.

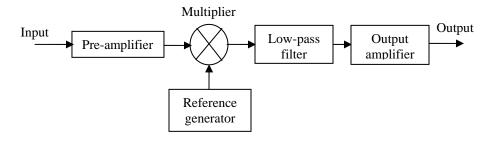
One can also determine the phase of the signal relative to the reference. If we multiply the signal by a reference function $R_1(t) = \cos(\omega_r t)$ then we obtain $Y(t) = A_1 \sin[(\omega_1 - \omega_r)t + \phi]/2 + (\text{high frequency terms})$. If $\omega_1 = \omega_r$, then $Y/X = \tan(\phi)$. Sometimes it's also convenient to look at $R = \sqrt{X^2 + Y^2}$, which does not depend on the phase of the signal.

The cut-off frequency of the low-pass filter determines the selectivity of the amplifier. It's more common to refer to the time constant of the lock-in amplifier $\tau = 1/(2\pi f_c)$. A long time constant (a small cut-off frequency) is better for rejecting the noise but slows down the response of the lock-in to changes in the signal. A small time constant (high cut-off frequency) increases the speed of the response but also makes the signal noisier.

Lock-in amplifiers are usually used in combination with modulation techniques to detect a very weak signal. The idea is to modulate a small signal on and off at a known frequency and then detect the resulting response with a lock-in referenced to the same frequency.

4. Components of a lock-in amplifier

A block diagram of a lock-in amplifier is shown in the figure. Each of the elements can be constructed using a simple integrated circuit.



a) Multiplier.

The heart of the lock-in is a multiplier that performs multiplication of the input signal and reference signal. Analog multipliers use non-linear properties of transistors, with quite a bit more circuitry to make an accurate transfer function $V_{out}=V_1\times V_2$. An example of such chip is AD633.

It is also possible to implement a lock-in amplifier using a switch instead of an analog multiplier. With a switch one can get a signal that alternates between V and -V, which is equivalent to multiplying the signal V by a square reference wave. The preceding discussion of lock-in properties will mostly work for a square reference wave (can you think of any differences?). An example of a fast electronic switch useful here is ADG419.

b) Reference generator

To make a sine reference wave we need an oscillator with sine output. It can be generated using any amplifier with positive feedback (think of high-pitch audio feedback). However, the gain of the oscillator needs to be controlled to obtain high-quality sine wave without distortion. For example, you can use an TLE4121 operational amplifier and a Wien-bridge oscillator described in Analog application note AN-580.

If we use a switch to multiply a signal by a square wave, it is sufficient to build a square-wave oscillator. A standard way of doing that is to use LM555 timer chip.

In some cases the reference frequency is not arbitrary, but is determined by an external reference frequency. In this case the term "lock-in" is perhaps most appropriate, since the device is supposed to lock-in to the specific frequency and keep track of it even if the frequency is changing. A standard way of doing so is to use what is known as a "Phase-Locked-Loop"- an oscillator whose frequency is locked to external reference. It can be implemented using LM565 PLL chip.

It is important to think about the phase between your signal and the reference. If they happen to be 90° out of phase you will get no output! Hence it is good to incorporate some possibility of phase adjustment between the signal and the reference. This can be accomplished, for example, with an RC delay circuit.

c) Pre-amplifier

The purpose of the pre-amplifier is to increase the size of a small input signal. Depending on the input type, different types of pre-amplifiers are used. For example, one often uses an instrumentation amplifier to make differential measurements that are not affected by noise on the ground lines. If the input signal is a current (from a photo-diode, for example) then a current-to-voltage amplifier is used, usually implemented with an operational amplifier. TLE2141 op-amp or AD622 instrumentation amplifier can be used here.

d) Low-pass filter

The low-pass filter determines the output bandwidth of the lock-in and eliminates the first and second harmonic of the reference signal generated in the multiplication process. The cut-off frequency of the filter must be much lower than the lock-in operating frequency. Low-pass filter can be implemented with passive elements, such as an RC-filter, or R and C elements can be used together with an op-amp. Several stages of filtering improve the sharpness of frequency cut-off.

e) Output amplifier

After signals at the reference frequency and above are eliminated by the filter, the remaining voltage is proportional to the signal we are trying to measure and hence will be fairly small. Now one can amplify this signal without risking overloading the amplifier with a large reference signal. A high-gain amplifier is appropriate here, which can be implemented using 1-2 stages of op-amp amplification.

Testing lock-in amplifier

After building the lock-in amplifier and verifying that it works with large signals, you want to test its utility for detecting very weak signals. For example, you can see if you can a very weak sound tone at a specific frequency. A good application of the phase-sensitive nature of the lock-in amplifier is to perform a measurement of the speed of sound. The delay in the propagation of the sound from speakers to the microphone will cause a phase shift of the detected signal. Make a measurement of the speed of sound using this idea and see how it depends on the sound frequency.

Design choices

Before building anything, read the datasheets for the chips you will use and understand how they work. The handout on electronic circuits provides some general background. Decide on the ultimate goal for signal detection and pick the operating frequency of the lock-in. Decide if you want to use a sine or square reference waveform, external or internal reference. Remember that you need some way to ensure that the lock-in "knows" the frequency of the weak signal you are detecting. The lock-in can be build using the push-in breadboard.