CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

METHODS TO SOLVE ASSET BUBBLE IN FINANCE

A thesis submitted in partial fulfillment of the requirements
For the degree of Master of Science in
Applied Mathematics

by

Jaspreet Kaur

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The thesis of Jaspreet Kaur is approved:

Dr. Stephen Breen	Date
Dr. Vladislav Panferov	Date
Dr. Jorge Balbás, Chair	Date

California State University, Northridge

Dedication

Jas' dedication

Acknowledgements

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ABSTRACT

METHODS TO SOLVE ASSET BUBBLE IN FINANCE

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We will study non parametric estimator Floren Zmirou in local real time on compact domain with stochastic differential equation which has unknown drift and diffusion coeificents. Once we will have volatility from floren zmirou. We will obtain volatility funtion then we will interpolate with cubic spline to see the behavior of the function.

Chapter 1

Numerical Solution, Conclusion and Future Work

Since we have done lot of good work, now it is the time to check the implementation. We will provide examples which will give better understanding toward our problem. Numberical Solutions using implementation

1.1 Examples

1.1.1 EXAMPLE 1

- Ticker: **MWI Veterinary Supply Inc**
- D : 05/16/2014
- T : 60 seconds

Stock Class

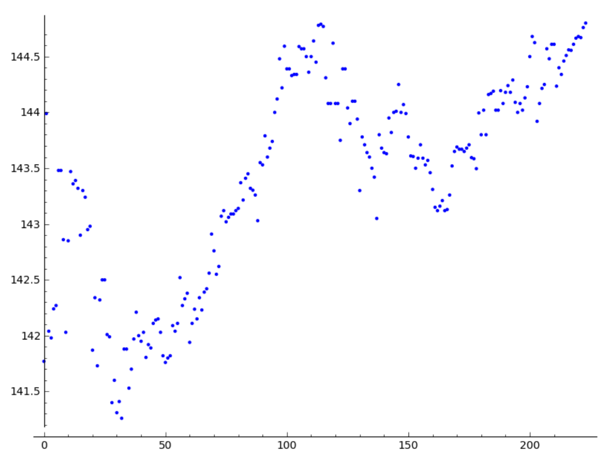

Figure 1.1: Stock Prices vs. Time

Now we are going to check some stocks using Stock class.Information can be downloeded from following Google Finance. We have stock prices for MWI Veterinary. The stock Price graph shows the presence of asset bubble. Next, Floren Zmirou estimator can be used to see the volatility of stock prices.

Floren Zmirou Class

$$S_n(x) = \frac{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}} n(S_{t_i+1} - S_{t_i})^2}{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}}} \tag{1.1}$$

Usable Grid Points	Estimated Sigma Zmirou	Number of Points
141.842890874	1897.69862662	50
144.17445437	290.806107556	108
143.008672622	464.127160557	60

We picked $hn = \frac{1}{n^{(1/3)}}$. By theorem (1.17) $(h_n)_n{}^3$1satisfies $nh_n \to \infty$ and $nh_4n \to 0$, then $S_n(x)$ is a consistent estimator of $\sigma^2(x)$.There are Floren Zmirou's estimated sigma values for usable grid points and number of points in each usable grid point. We deleted those grid points which has less than 1 percent of data. We are using 3 point grid.

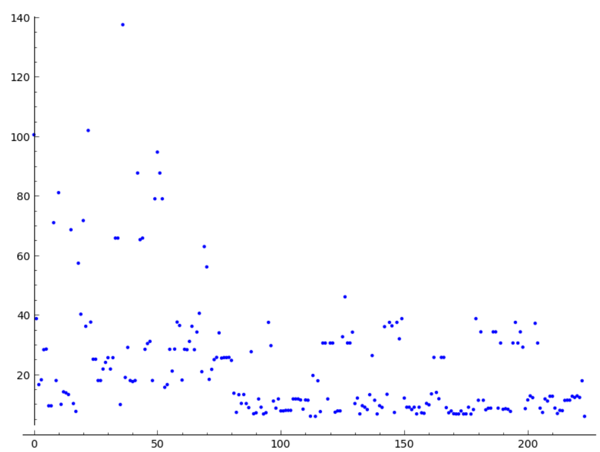

Figure 1.2: Stock Prices vs.Floren Zmirou Standard Deviation Estimation
Next we will need to interpolate $\sigma(x)$. We used Varince of Cubic spline and Standard Deviation of cubic spline. Floren Zmirou's sigma points.

Cublic Spline

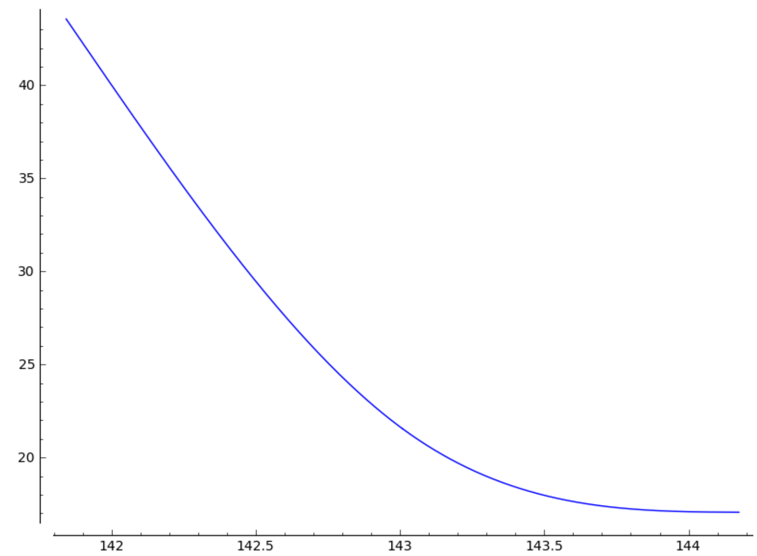

Figure 1.3: Floren Zmirou Standard Deviation Estimation vs. Variance Cubic Spline

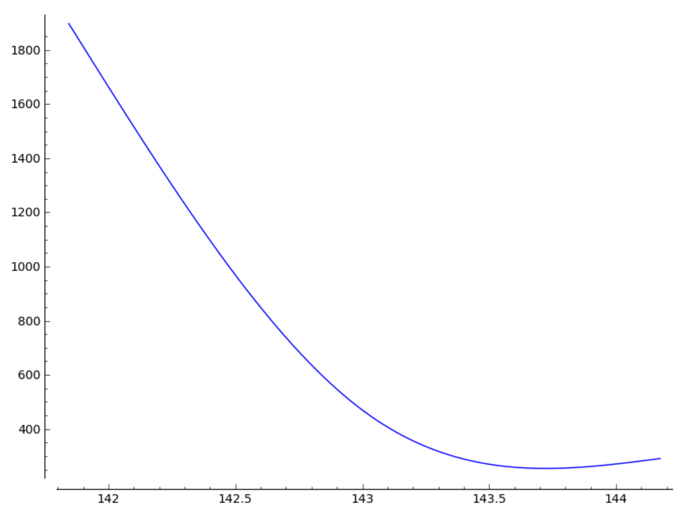

Figure 1.4: Floren Zmirou Standard Deviation Estimation vs.Standard Deviation Cubic Spline

If the graph of the volatility versus the stock price tends to infinity at a faster rate than does the graph $f(x) = x$, then we have bubble.Figure 1.3 and 1.4 shows that volatility function goes to infinity at faster rate so we can conclude that there is a bubble in MWIV stock.The price process under a risk neutral measure is a strick local martingale.

1.1.2 EXAMPLE 2

- Ticker: **GOOGLE Inc.**
- D : 05/16/2014
- T : 60 seconds

Stock Class:

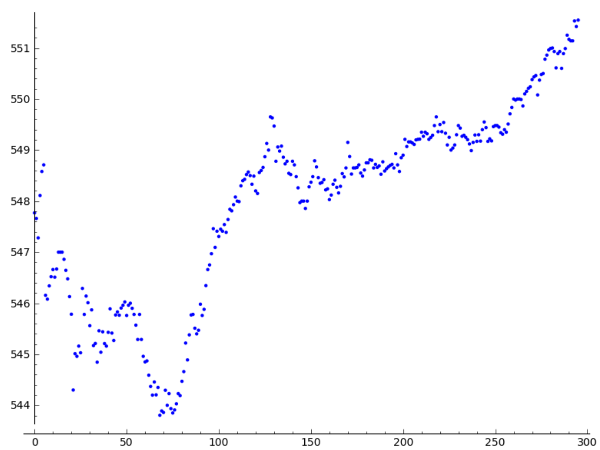

Figure 1.5: Stock Prices vs. Time

Figure 1.5 is showing stock prices for Google Inc. We do not see the existence of an asset bubble. Since the prices are going upward, it is hard to conclude. We will see how the volatility will look for these stock prices.

Floren Zmirou Class

Usable Grid Points	Estimated Sigma Zmirou	Number of Points
547.289611925	267.623605573	64
549.612686541	517.868135963	143
551.935761158	76.0733825073	17
544.966537308	1890.46832	72

We are using 4 points

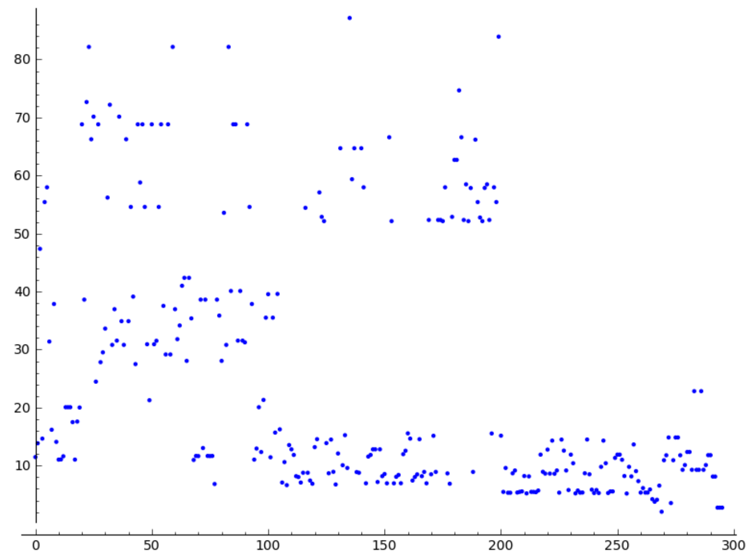

Figure 1.6: Floren Zmirou Standard Deviation Estimation vs. Stock Prices

grid in this example. As we see in figure 1.6, the sigma values are widely distributed so it is very difficult to conclude from $\sigma(x)$ values. We will interpolate by cubic spline to see the behaviour of volatility function.

Cublic Spline

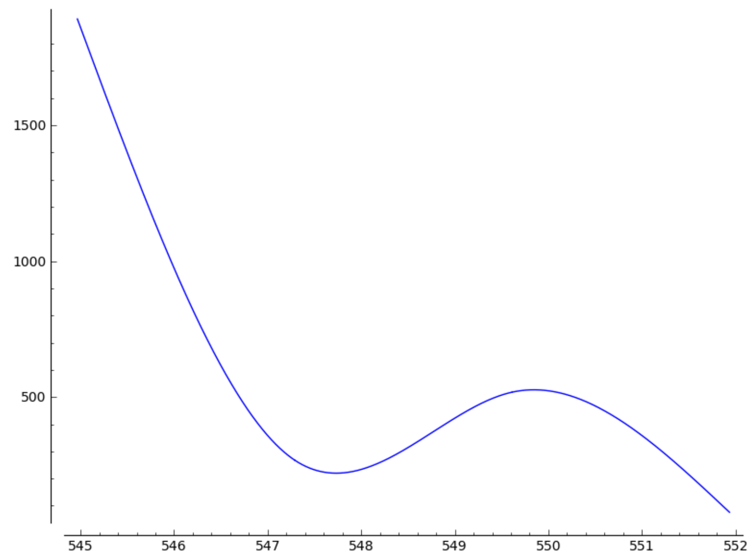

Figure 1.7: Floren Zmirou Standard Deviation Estimation vs. Variance Cubic Spline

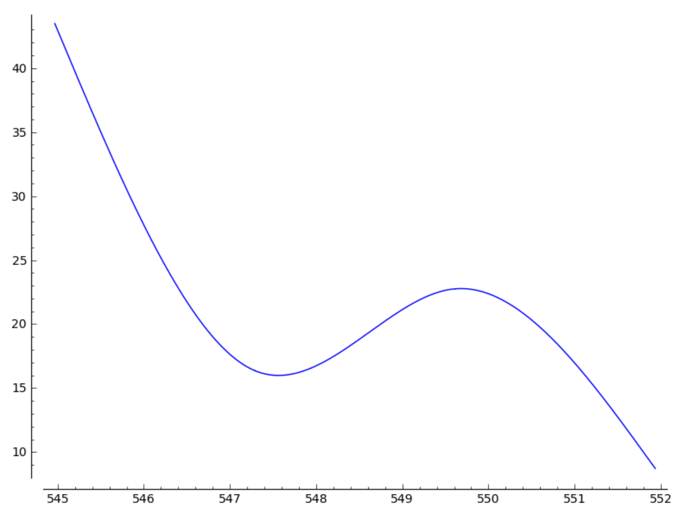

Figure 1.8: Floren Zmirou Standard Deviation Estimation vs.Standard Deviation Cubic Spline

Figure 1.7 and 1.8 shows that the $\sigma(x)$ values goes to 0. The graphs are not going to infinity, so we can conclude that there is no bubble in asset prices. Therefore it is martingale.

1.1.3 EXAMPLE 3

- Ticker: **APPLE Inc.**
- D : 05/21/2014
- T : 60 seconds

Stock Class

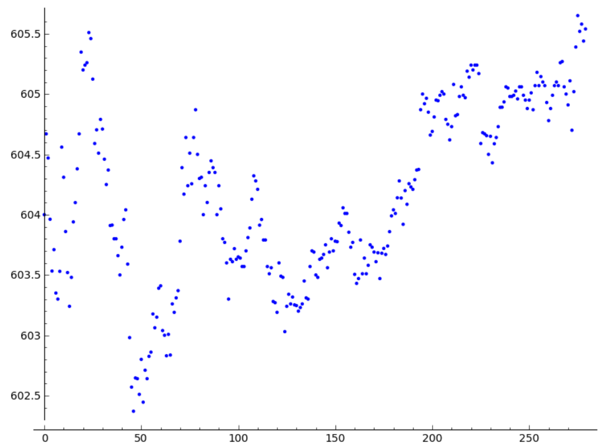

Figure 1.9: Stock Prices vs. Time

Here we have stock prices for Apple for one day in seconds. We tested the existence of bubble. Results are inconclusive but we still can not see bubble in price. We will have better understanding about volatility function in Floren Zmirou's estimation graph.

Floren Zmirou Estimation

$$S_n(x) = \frac{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}} n(S_{t_i+1} - S_{t_i})^2}{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}}} \tag{1.2}$$

Usable Grid Points	Estimated Sigma Zmirou	Number of Points
602.871457276	138.351149247	42
603.874371827	245.251175157	125
604.877286378	102.97102087	104

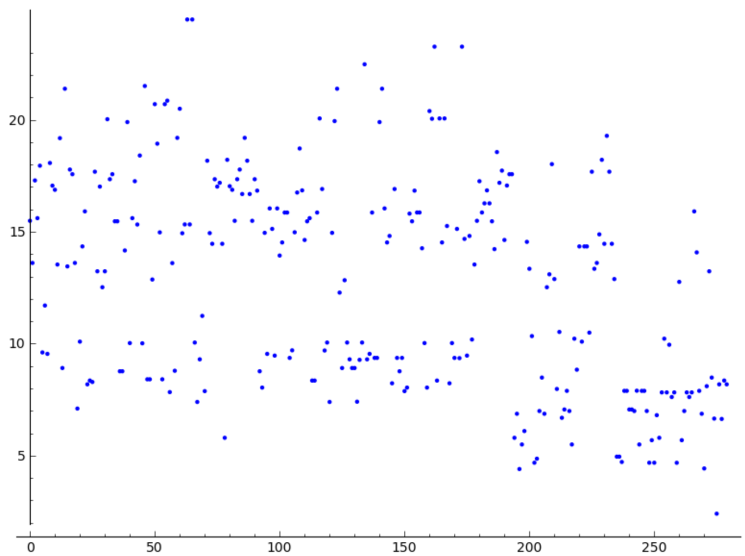

Figure 1.10: Floren Zmirou Standard Deviation Estimation vs. Stock Prices

We are using 3 points grid. We can not conclude if the volatility funcion $\sigma(x)$ is going to infinity or not in figure 1.10. We will interpolate $\sigma(x)$ to see the behaviour of the volatility function.

Cublic Spline

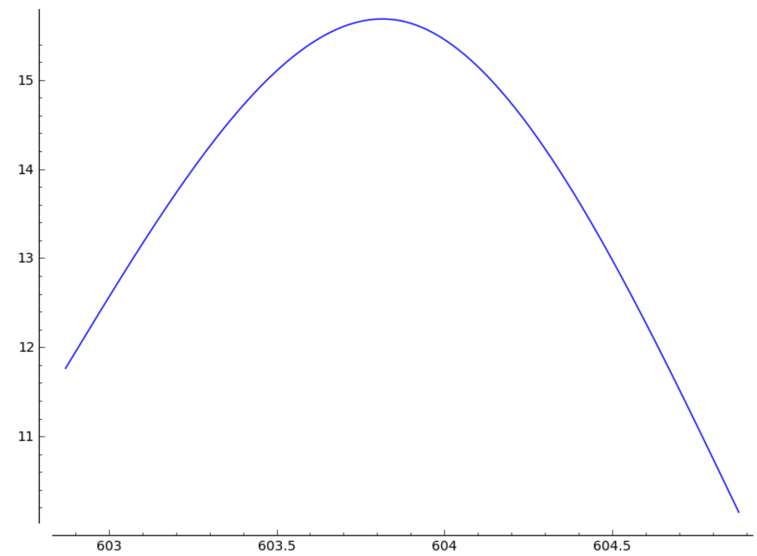

Figure 1.11: Floren Zmirou Standard Deviation Estimation vs. Variance Cubic Spline

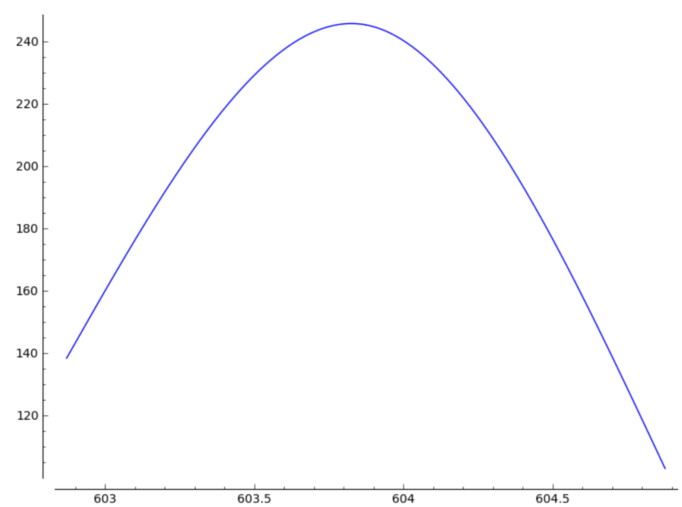

Figure 1.12: Floren Zmirou Standard Deviation Estimation vs.Standard Deviation Cubic Spline

Figure 1.11 and 1.12 shows that the $\sigma(x)$ goes to 0 as x goes to infinity so under the risk neutral probabilities, Apple asset price is a martingale therefore there is no bubble in the price.

1.1.4 EXAMPLE 4

- Ticker: **GROUPON Inc.**
- D : 05/21/2014
- T : 60 seconds

Stock Class

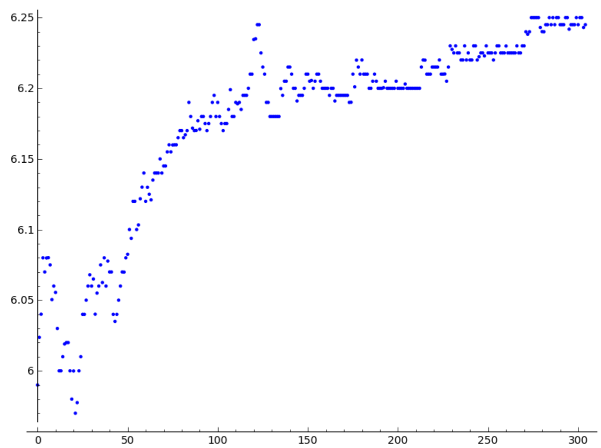

Figure 1.13: Stock Prices vs. Time

Figure 1.13 shows that stock prices of Groupon are going to infinity as time increases. We can not determine determine the exsitence of bubble.

Floren Zmirou Estimation

$$S_n(x) = \frac{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}} n(S_{t_i+1} - S_{t_i})^2}{\Sigma_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}}} \tag{1.3}$$

Usable Grid Points	Estimated Sigma Zmirou	Number of Points
6.01159662403	0.0106001479673	25
6.0947898721	0.00331796023881	38
6.17798312017	0.000229293586847	155
6.26117636824	7.51642424675e-05	86

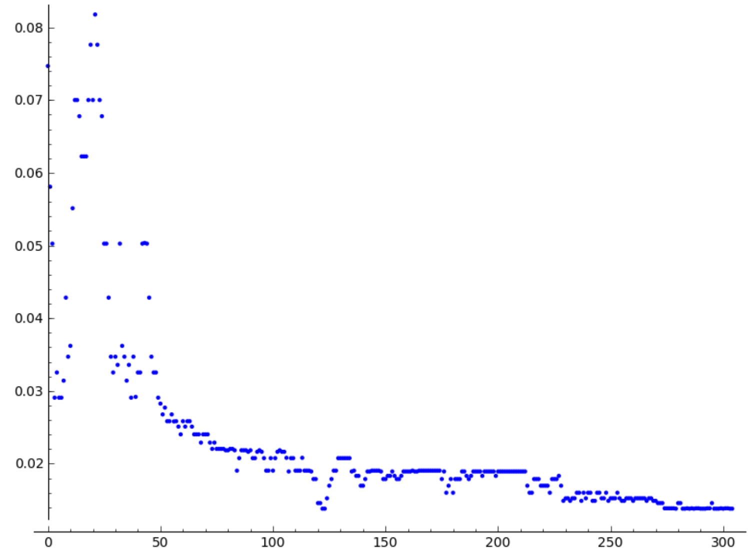

Figure 1.14: Floren Zmirou Standard Deviation Estimation vs. Stock Prices

There are 4 points grid which we are using in this example. Floren Zmirou Estimated Sigma values are very small. In Figure 1.14, $\sigma(x)$ values are going to ∞ as x goes to ∞. We will interpolate for better results.

Cublic Spline

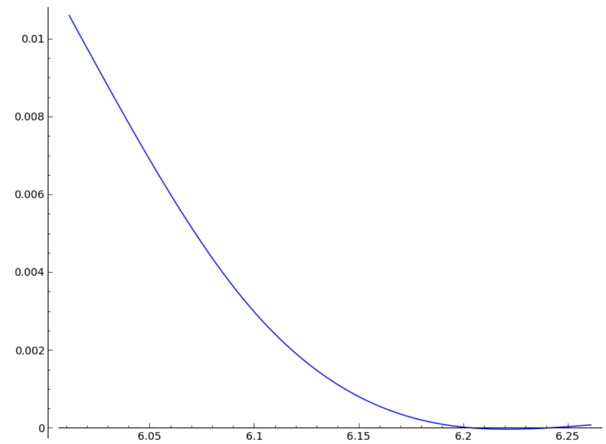

Figure 1.15: Floren Zmirou Standard Deviation Estimation vs. Variance Cubic Spline

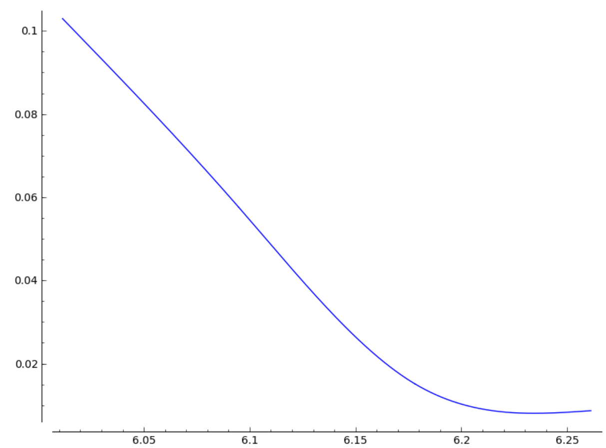

Figure 1.16: Floren Zmirou Standard Deviation Estimation vs.Standard Deviation Cubic Spline

Figure 1.15 and 1.16 shows that volatility function is going to infinity as x values are going to infinity so groupon asset price is a supermartingale therefore we can conclude that there is a bubble in asset price.

1.2 Future Work

- Still need to know tail of the volatility function.
- Need to extrapolate the volatility with either Comparison Theorem Method or Reproducing Kernal Hilbert Spaces.
- Need to know themorem 0.1.12 equation(3)

$$\int_{\alpha}^{\infty} \frac{x}{\sigma^2(x)} dx < \infty \tag{1.4}$$

  for all $\alpha > 0$ is finite or infinite.
- Determine from intergral if there is bubble or not.