Applied Mathematics
Note

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

ASSET BUBBLE DETECTION USING THE FLOREN-ZMIROU ESTIMATOR

A thesis submitted in partial fulfillment of the requirements
For the degree of Master of Science in
Applied Mathematics

by

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August 2013

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Dedication

Jas' dedication

Acknowledgements

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ABSTRACT

ASSET BUBBLE DETECTION USING THE FLOREN-ZMIROU ESTIMATOR

By

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Master of Science in Applied Mathematics

We will study non parametric estimator Floren Zmirou in local real time on compact domain with stochastic differential equation which has unknown drift and diffusion coeificents. Once we will have volatility from floren zmirou. We will obtain volatility funtion then we will interpolate with cubic spline to see the behavior of the function.

Chapter 1

Introduction

1. Market forces exhibit stochastic behavior.
    - Stochastic behavior operates in a probabilistic sense[18].
    - Uncertainties in market value [6].
    - Intrinsic value type 1, 2 and 3 bubbles[12].
    - Extrinsic value(bubble)[17, 16].
    - Uncertainties(risks) in financial portfolios, and sectors of markets(e.g .com bubble, housing bubble)JAPBIIM
2. To minimize risk in portfolio against bubble, one would need to detect when a bubble exists in an asset[8].
3. In this paper, we use a non-parametric estimator Floren Zmirou to detect or determine type 3 bubbles in an asset.
4. Further testing is needed to determine if this method is valuable in minimizing portfolio risk against type 3 bubble.

1.1 Previous Work on Asset Bubble Detection

- Identify other means to detect or determine asset bubble.

[7] paper covers variance bounds testing and malformed time series as a method of detection and detection difficulty respectively. [21] panel data test and cointegration are used to detect bubbles in housing prices.[24] RKHS theory is used to detect asset bubbles.[13] used a bootrapping methodology to indicate type 3 bubbles in the Nasdaq price index and Case-Shiller house price index. [27] use a Kalman filter to estimate a bubble and then use a state space model with Markov switching to model the survivability of a bubble, switching between collapsing and surving states.

Chapter 2

Background on Asset Bubble Detection Using the Floren-Zmirou Estimator

2.1 Background of Portfolio Theory

2.2 Mathematical Background of Portfolio Theory

$$S_n(x) = \frac{\sum_{i=1}^{n} 1_{|S_{t_i}-x|<h_n\}} n \left(S_{t_{i+1}} - S_{t_i}\right)^2}{\sum_{i=1}^{n} 1_{\{|S_{t_i}-x|<h_n\}}} \tag{2.1}$$

Theorem 1. *A stochastic process S is a strict local martingale if and only if*

$$\int_{\alpha}^{\infty} \frac{x}{\sigma^2(x)} dx < \infty \tag{2.2}$$

for all $\alpha > 0$.

The *market price* is the current price of an asset.

Definition 1 (Probability Space)**.** $(\Omega, \mathcal{F}, P)$ where Ω is a set, $\mathcal{F}$ is a sigma algebra of Ω, and P is a probability measure

CHAPTER 2 OUTLINE

1. Background of Portfolio Theory
   - (A) Objective of portfolio theory: to minimize risk given an expected return on assets[20].
     - (i) Explain in words to the reader the key definitions:
       - (a) Portfolio has set of asset in manners which minimize the risks.
       - (b) Primary way to manage portfolio with the aim of distributing risk.
       - (c) Market Price- current price
       - (d) Fundamental Price- Intrisic price.
       - (e) The bubble is the difference between market and fundamental price.
       - (f) The fluctuation of asset price is volatillity od asset.
     - (ii) Our thesis' problem consists detecting type 3 bubble within market price[19]
2. Mathematical Background of Portfolio Theory
   - (A) Random Variable [1]
     - (i) moment Generator function of random variable.[3]
     - (ii) Expected value of an random variable[3].
     - (iii) Variance of a random variable[3]
   - (B) Sigma Albegra[5]
     - (i) Probability space[10]
     - (ii) Stochastic Process[10]
     - (iii) Weiner process[10]
     - (iv) Martingale[10]

(C) Stochastic Process
  (i) Fundamential pricesa stochastic process[14, 22]
  (ii) Market Price as a stochastic process[14, 22]
(D) Asset Bubble[7]
(E) Volatility[7]
(F) SDE[9]
(G) Euler Marayama[14]
(H) Floren Zmirou[8]
(I) Theorem strict local martingale iff [23]
(J) Test: using FZ tp estimate sigma and then testing integral.

1. Background of Portfolio Theory

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Modern Portfolio theory provides enough information to investors with portfolio which has set of assets in manners to minimize the risk given on expected return on asset.It attemptes to give explanation for risk and expected return. Investors can create portfolio to obtain maximum possible return from portfolio management. The risk is the possibility return on an investment will be different from it's expected value. Expected value is the average of a probability distribution of possible returns. There is no guarantee that an asset will be profitable for an investor.In order to reduce risk in a portfolio, one will be interested to know if an asset has a bubble. A bubble occurs when prices for an asset rise for above their actual value. A famous example of a bubble is the dot-com bubble of the 1990s. Bubble is realted to price and volatility.Measuring the variation and stablility of price is called volatility. [16]Asset bubbles can be three types: type 1 is the uniformly integrable martingale, type 2 is a martingale that is not a uniformly integrable martingale and type 3 is a strict local martingale that is not martingale. Type 3 bubbles occurs in asset with finite maturities. Our thesis problem consists detecting type 3 bubbles within market price.

Chapter 3

Implementation of Asset Bubble Detection Using the Floren-Zmirou Estimator

In equation (**??**)...
in theorem 1
See the definition 1

3.1 Strategy

3.2 Imlementation

3.3 Algorithms in Pseudocode

CHAPTER 3 OUTLINE

(a) Strategy
  (A) We use Minute to minute stock price data
  (B) Bubble testing on FZ estimation- explain
    (i) MVT on floren zmirou derivative[25, 8]
    (ii) Intro natural cubic spline [4]
      (a) Spline on Floren Zmirou
      (b) MVT on spline interpolation of FZ [25, 4]

(b) Implementation

(c) Algorithms in Pseudocode
  (A) verbal explanantion sage python- floren zmirou, spline, bubble testing.
  (B) verbal explanantion google- minute to minute

Chapter 4
Examples of Asset Bubble Detection

(a) 1 example of asset bubble detection

- stock
- day of data
- table of grid points
- identify usable grid points.
- floren zmirou graph
- spline graphs together
- bubble test.
- Conclusion

Chapter 5

Future Work on Asset Bubble Detection and Conclusion

5.1 Future Work

(a) Identify this only detects bubbble when sigma square is certain to make equation 3 explode.

(b) when uncertainty arises, extrapolation is need to identify bubbles e.g rkhs, etc[24, 27]

(c) Further exammination of testing historical examples of type 3 bubb;es is needed to determine assurance of tests and methodology.

(d) testing on longer periods of time(e.g 1 week instead of a day)and faster rates of data(e.g.seconds to seconds) stock price data instead of minute to minute [24]

5.2 Conclusion

(a) cegurgitate

(b) what you did

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