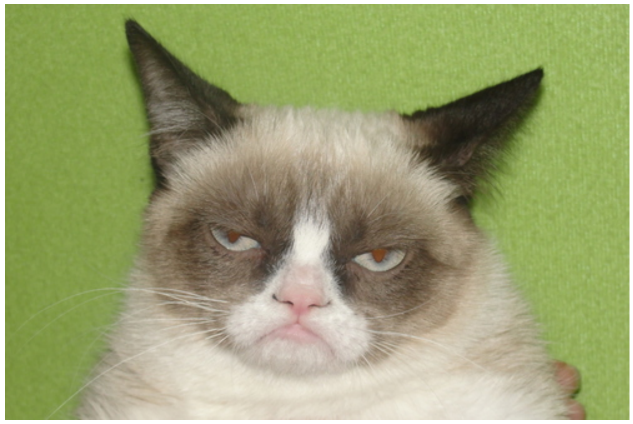
When the cat goes: the paper is ready.



Understanding the effect of selfish behaviour in a series of two queues.

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Abstract

Hierarchical queues in HC; PoA; Simulation model; Heuristics developed to obtain optimal policies;

1 Introduction

- Hierarchy in real life queues;
- Review of papers in BQT;
- PoA;
- Discussion of situation being modelled in this paper.

Queues arise in a variety of settings: road traffic, data transfer and healthcare are just a few examples. A large quantity of literature has investigated strategic decision making (for example: whether or not to join a queue) with regards to queues stemming from []. When leaving one queue; customers (which we will refer to as players) often join a second queue and this is what is referred to as a hierarchical queueing system (as shown in Figure 1).

There is a wide range of literature evidencing the negative effect of selfish actions in queueing systems []. The following is a general conclusion of a majority of the literature:

> Selfish users make busier systems.

- Naor;
- VK+PH;
- RS;
- Bell Stidham;

- Hassani book;
- Rouhgharden;

The effect of selfish behaviour when compared to optimal behaviour can be quantified as the Price of Anarchy (PoA). This is defined as the ratio of the selfish ($\tilde{C}$) and optimal (C^*) costs:

$$\text{PoA} = \frac{\tilde{C}}{C^*}$$

The contribution of this paper is to consider two stations (to avoid confusion we will refer to a queue and service station as a station) in series allowing for two consecutive decisions: whether or not to join the station. Players who join the first station upon completion of service will potentially drop out from the entire system. A potential application of this is a healthcare system where patients must choose to see their general practitioner before obtaining care from a specialist/emergency centre.

In Section 2, the model will be described. A novel aspect of this work is that the cost of a given policy is evaluated through the use of purpose built Agent Based Simulation (ABS). In Section 3 various heuristic approaches to obtaining optimal policies are considered. These range from random search algorithms to a more sophisticated approach that depends on the analytical formula for an $M/M/c$ queue [**?**]. Before concluding in Section 5 a variety of numerical experiments will be shown in Section 4 demonstrating the effect of selfish decision making in queues in series.

2 Model

Let $k\{1,2\}$ be the index for each station in the system. The players arrive into the system at a rate Λ. The players know the rate of service μ_k at each station, the number of servers at each station c_k and also the number of players currently in each station. Upon observing a station, players must select one of two potential strategies: 'join' or 'skip'. The strategy picked at station k is denoted by $d_k \in \{0,1\}$:

$$d_k = \begin{cases} 1, & \text{if player joins queue } k \\ 0, & \text{if player skips queue } k \end{cases} \tag{1}$$

The cost associated with joining system k is denoted by J_k and corresponds to the actual time spent in system k. The cost associated with skipping station k is denoted by β_k. The cumulative cost of going through the entire system is given by $C = C(d) = C_1(d) + C_2(d)$ where:

$$C_k = \begin{cases} \beta_k & \text{if } d_k = 0 \\ J_k & \text{if } d_k = 1 \end{cases} \tag{2}$$

The expected value of J_k when there are m players present upon arrival in system k is given by:

$$E[J_k] = \frac{m+1}{\mu_k c_k} \tag{3}$$

A final parameter of our model is the dropout probability p. This is the probability with which a player who joins the first station will exit the entire system (implying $C_2 = 0$).

All of this is summarised in Figure 1.

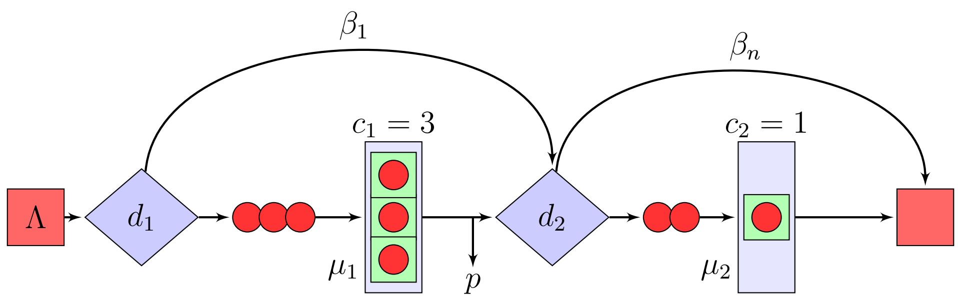

Figure 1: Diagrammatic representation of the model considered.

A player moving through the system with n stations can expect to receive a cost of (4) when making a choice $d_k(1)$ at queue k .

$$E[\xi_n] = \sum_{i=1}^{n} (1 - d_i) \left(\frac{m_i + 1}{\mu_i c_i} \right) + \sum_{i=1}^{n} d_i \beta_i \tag{4}$$

We want to instigate a policy which minimises the average cost per player, which is known as the optimal social policy. In the system, the policy is the length of the queue at which a player arriving at the queue will balk from it. Now consider Fig.**??**, and we can see that a player skips from this queue if there are 3 people in the queue.

By considering policies which benefit the system by reducing the average cost per player, we can see the effect of choice in this system, which is what is implied by a social policy (we also say that the system is under social conditions). Within this paper, we consider the Price of Anarchy (**??**) or PoA, the ratio of average cost per player under selfish conditions (given by $\tilde{\tau}$), to the average cost per player under social conditions (given by τ^*). In the earlier example concerning the prisoners dilemma, the Price of Anarchy is $5/1 = 5$

- Parameters;
- Optimal behaviour;
- Selfish behaviour.
- Cost (how to verify that cost is correct?).

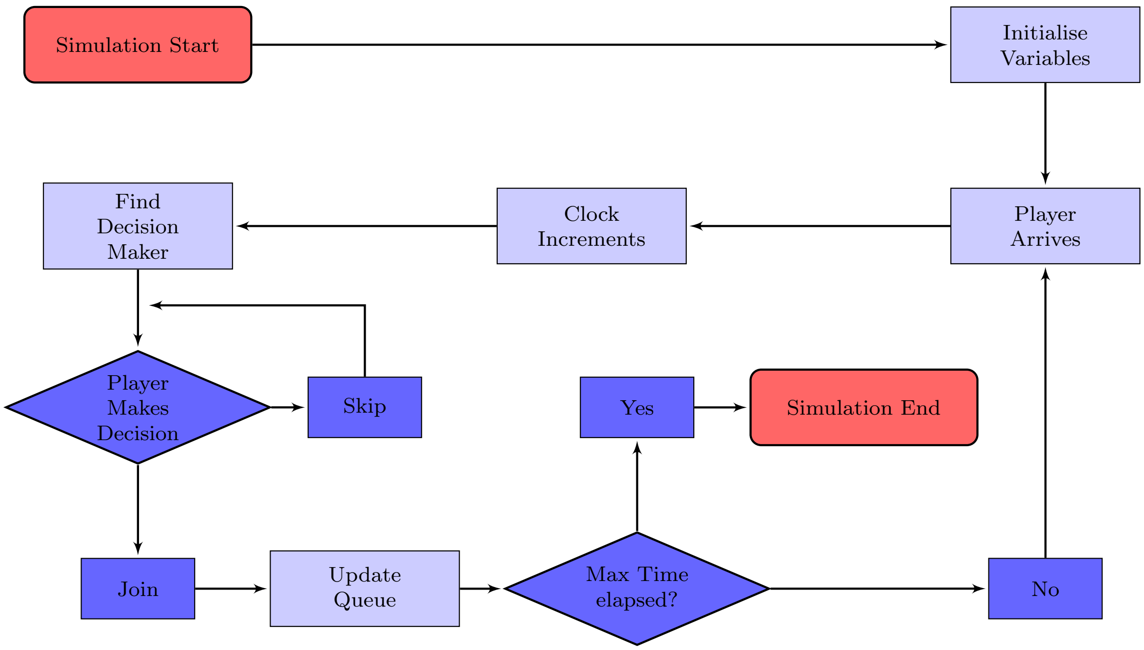

Figure 2: Flow chart describing the simulation model

3 Heuristic Optimal Policies

- Heuristic 1: basic search;
- Heuristic 2: based on assumption of Markovian arrival rate at second queue;
- Heuristic 3: Based on Naor which applies only for single server systems.

4 Results

- Scenarios...

5 Conclusions and Further work

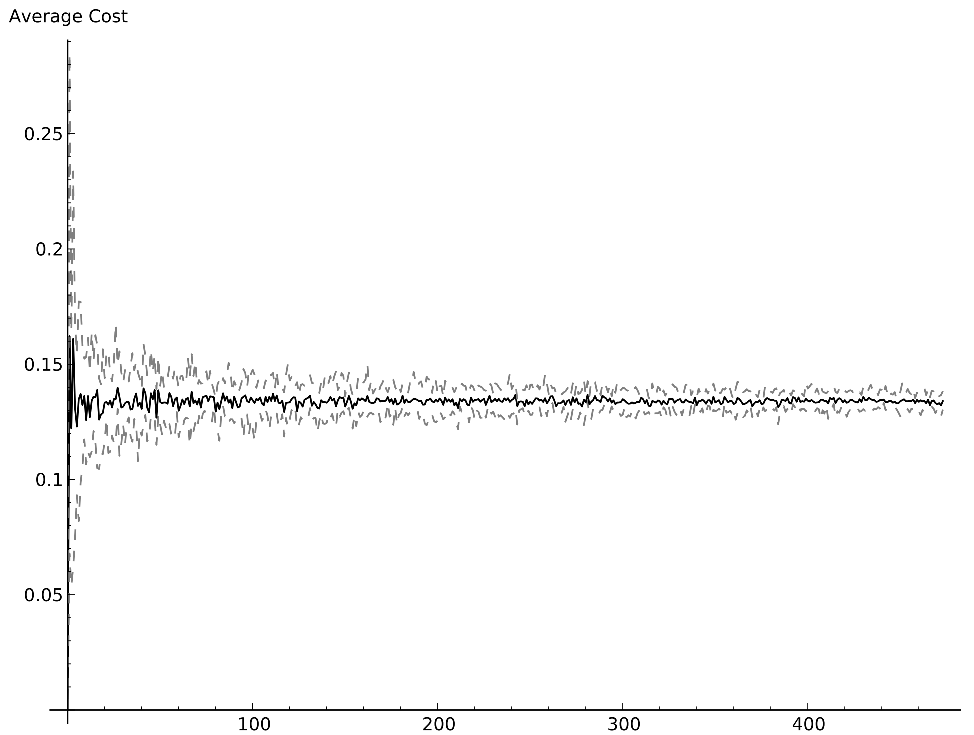

Figure 3: Run time

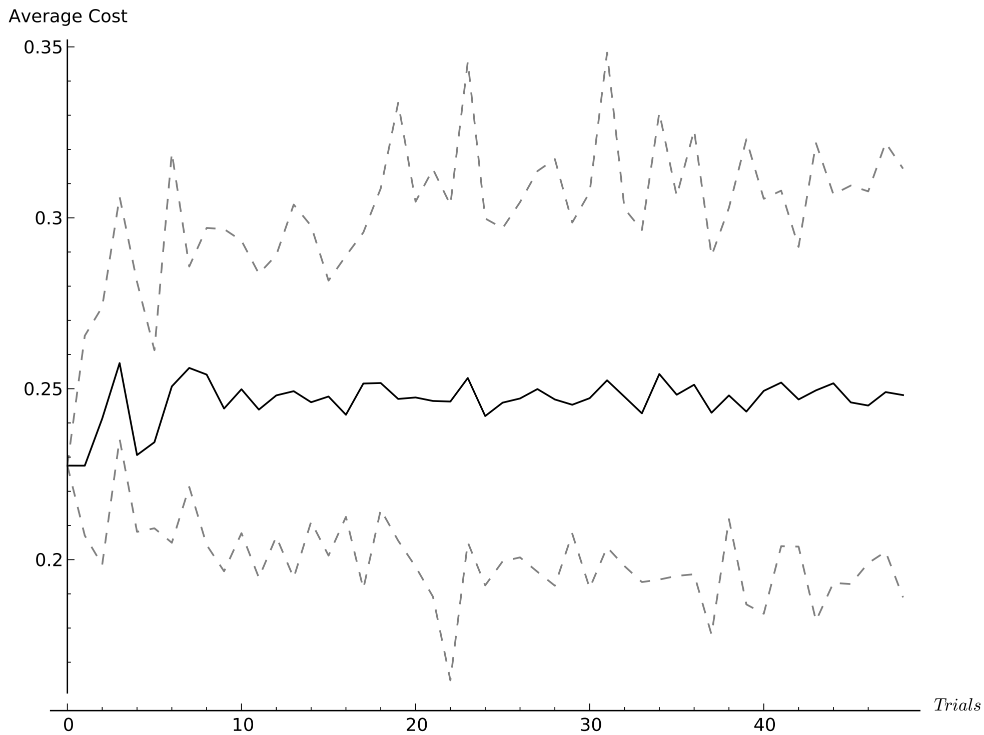

Figure 4: Trials

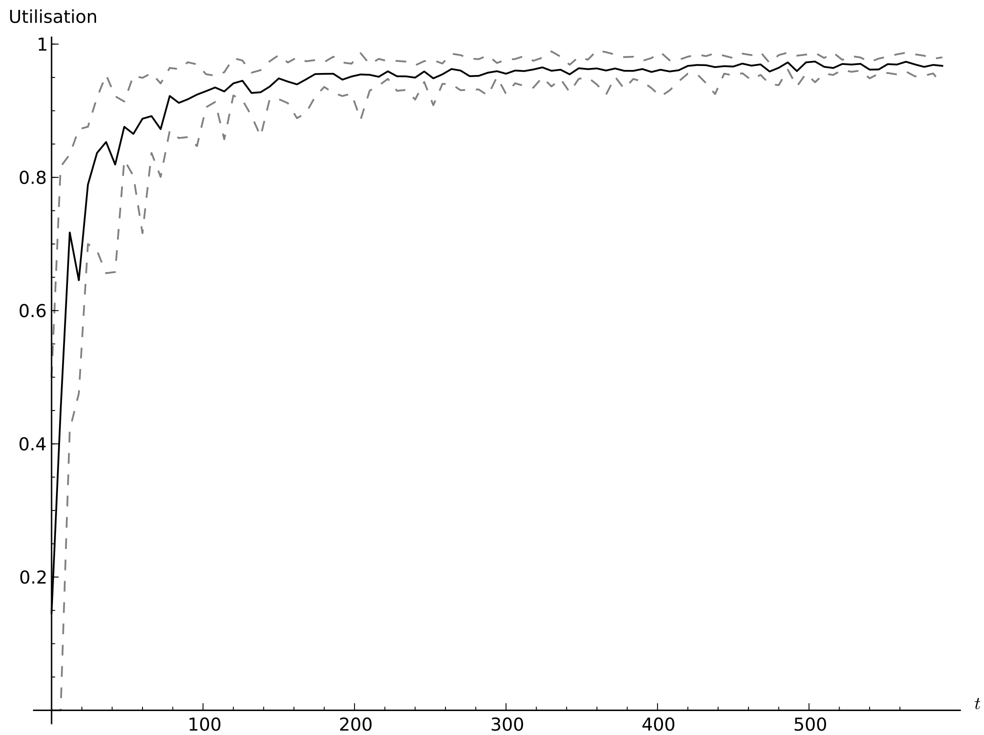

Figure 5: Warm up

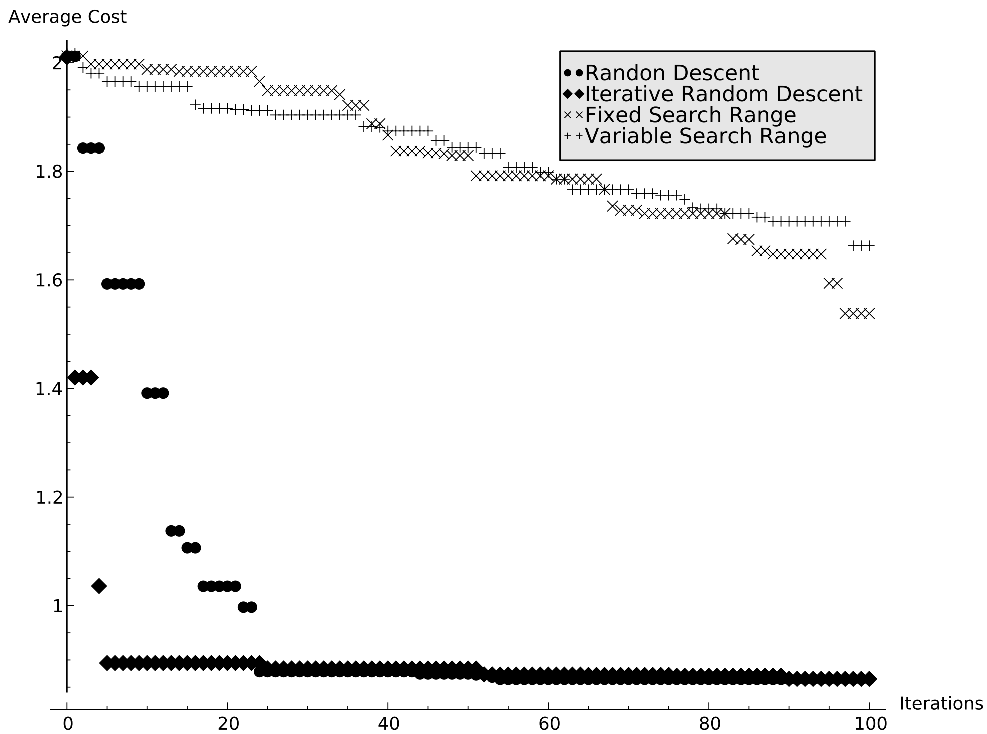

Figure 6: Comparing basic search

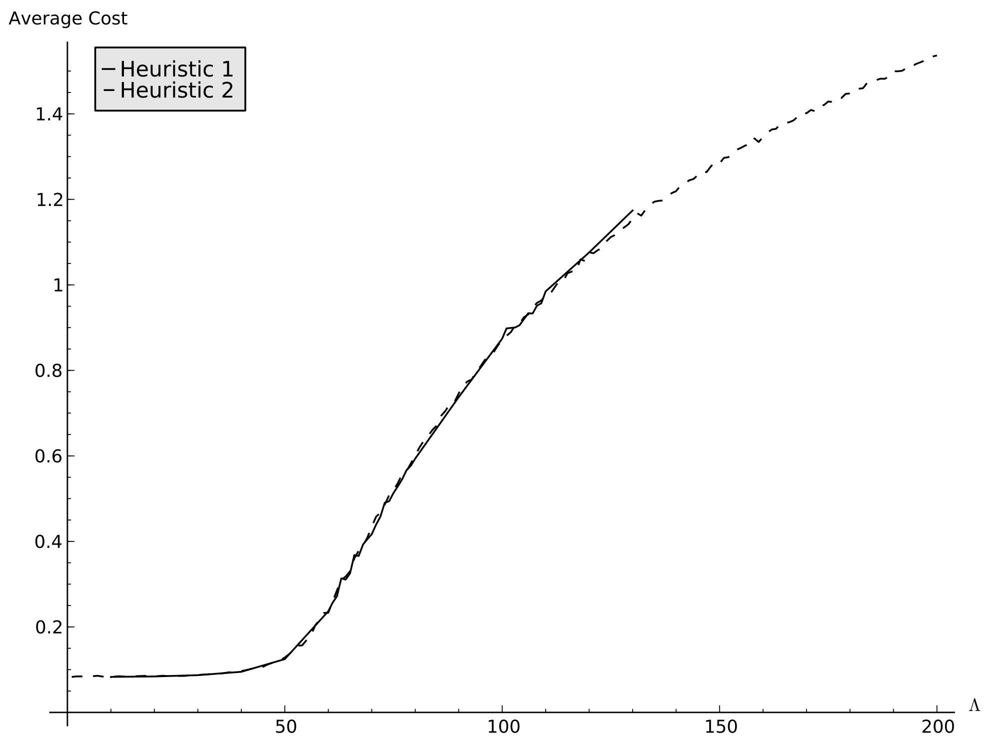

Figure 7: Comparing Heuristic 1 and 2

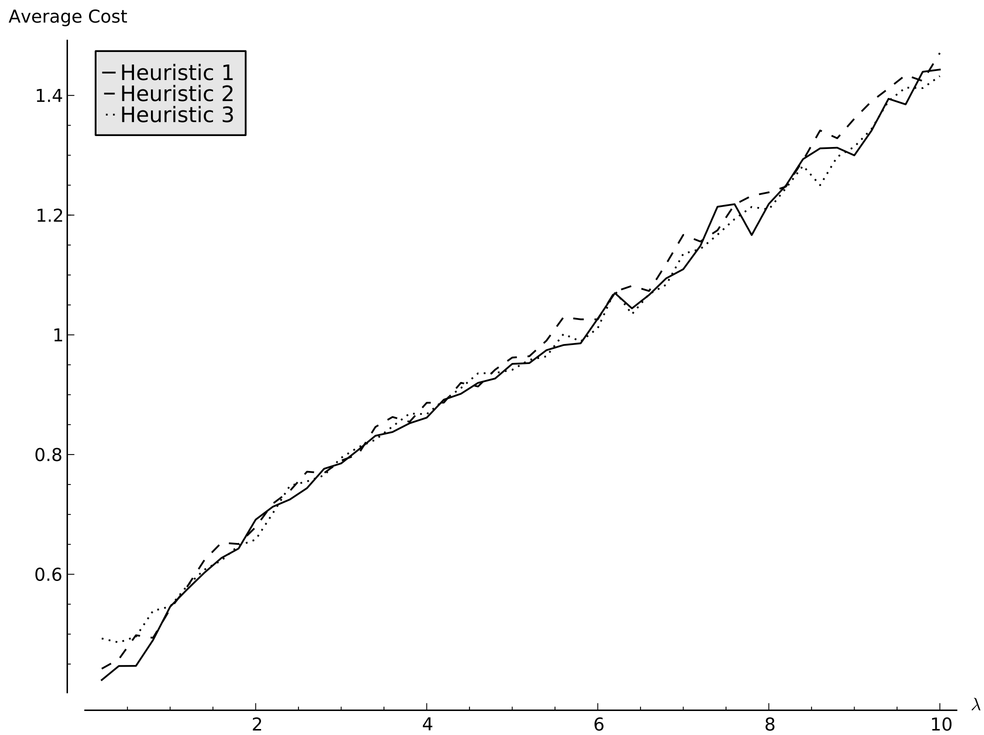

Figure 8: Comparing all heuristics

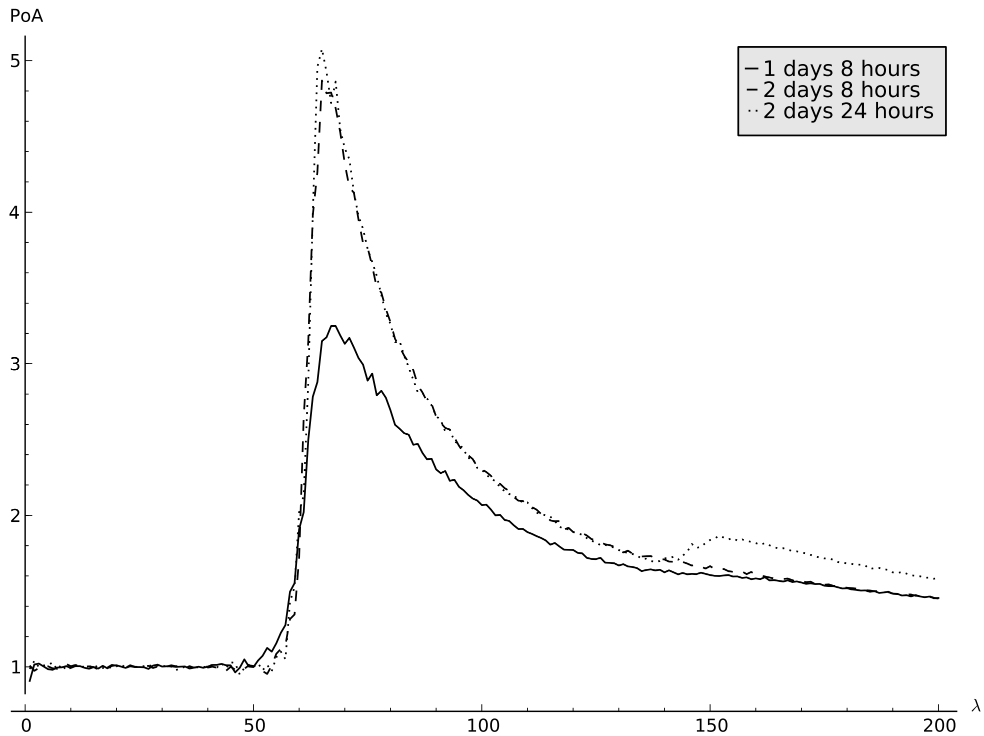

Figure 9: PoA for varying lambda

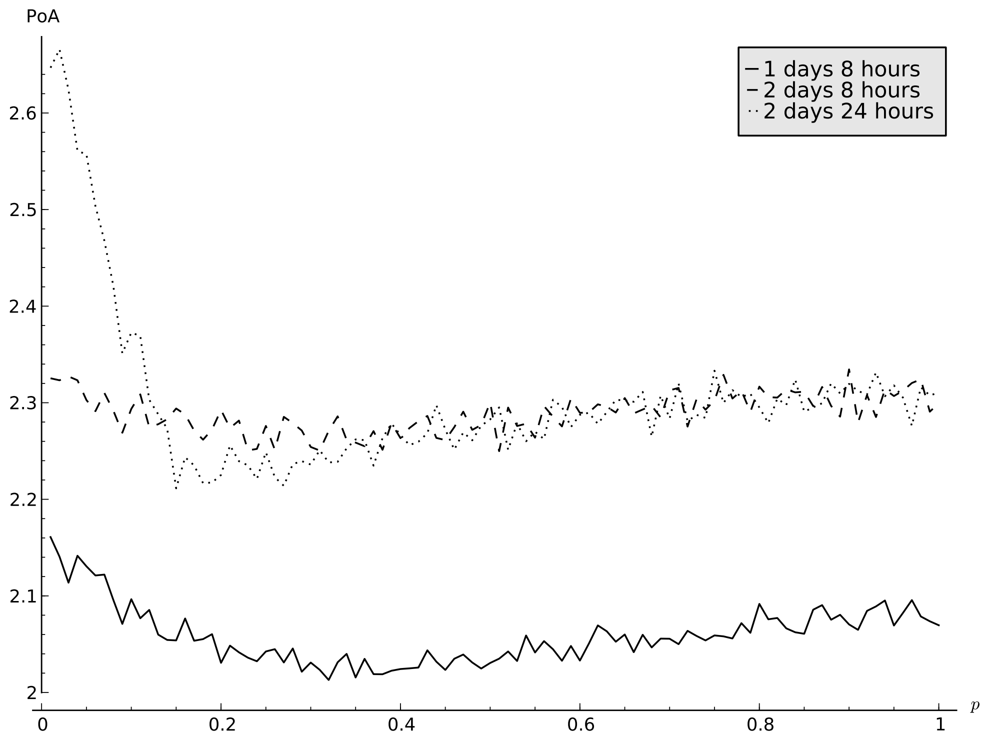

Figure 10: PoA for varying exit prob

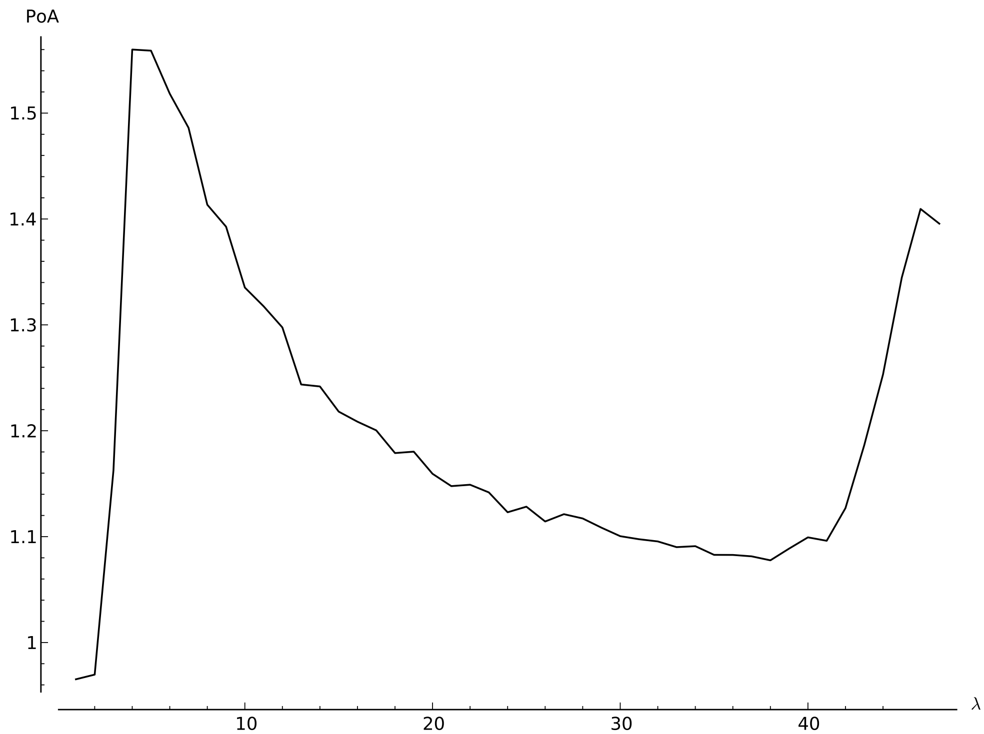

Figure 11: Another set of scenarios