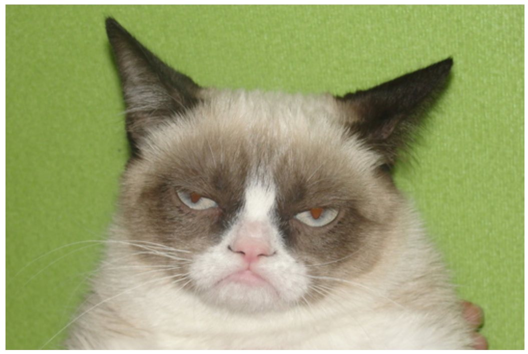
When the cat goes: the paper is ready.



Understanding the effect of selfish behaviour in a series of two queues.

Jason Young, Vincent Knight

December 31, 2013

Abstract

In this paper, we consider a system consisting of $M/M/C$ queues arranged in series. When customers (referred to as players) arrive at a given queue as they move through the system, they make a decision, to either join the queue or skip it at a cost. The system is considered under two types of player behaviour, one where players act selfishly and one where there is a policy dictating the decisions of the players. Players acting selfishly will attempt to reduce their own cost with no thought to the cost of other players in the system. Alternatively a optimal social policy is considered that attempts to reduce the average cost per player. We use the Price of Anarchy as a measure of the inefficiency due to choice to observe the impact that individual behavior has on the system. An Agent Based Simulation is developed that is used in conjunction with various heuristic approaches to obtain the socially optimal and selfish behavior of players. Due to the high computational cost of this model an analytical approximation is obtained and used to identify the effect of the total arrival rate on the system as well as other system parameters. In particular an optimal dropout probability rate is obtained for various scenarios which could have applications such as the optimal referral rate of general practitioners to hierarchical medical services.

1 Introduction

- Hierarchy in real life queues;

- Review of papers in BQT;
- PoA;
- Discussion of situation being modelled in this paper.

Consider a system where players within this system will act for their own good, attempting to reduce their cost or equivalently increase their utility. This work will quantify the impact that this choice has in hierarchical queueing systems. In this paper and without loss of generality, cost is considered to be a unit-less combination of time spent in the system and any utility lost by balking from a given queue. It is well understood that individual behaviour has a negative effect on the efficiency of queueing systems.

To illustrate this effect, consider the classic game theory problem, the Prisoners Dilemma. The game is that two prisoners are being separately questioned and must decide whether to co-operate, or defect, and the cost to each player can be seen in Fig.**??** . Under selfish conditions, both players defect, giving a cost per player of 5. However, if both players co-operate, the average cost per player is reduced to 1.

	Co-operate	Defect
Co-operate	(1,1)	(7,0)
Defect	(0,7)	(5,5)

Figure 1: Payoff matrix for a prisoners dilemma game

In [**?**] Bell and Stidham look into a particular system where the queues were arranged in parrallel. Players knew the service time at each of these queues and could join any of them and recieve the same service. However players were not able to see the length of the queue before joining. This is an example of an unobservable queueing system, where players are not able to see the number of other players ahead of them prior to joining. Some work has been done in showing the differences in player behaviour in unobservable and observable versions of the same queue in [**?**]. In that paper, the effect of the amount of information available to players, including some "intentional vagueness" was studied while the system was under both selfish and social conditions. Boudali [**?**] studied the effect of choice in a system where there was a possibility of a catastrophe occurring and players would have to abandon the system, wasting the time spent waiting.

2 Model

In this paper we will focus on a hierarchical queueing system in which players must progress through a series of queues as seen in Fig.**??**.

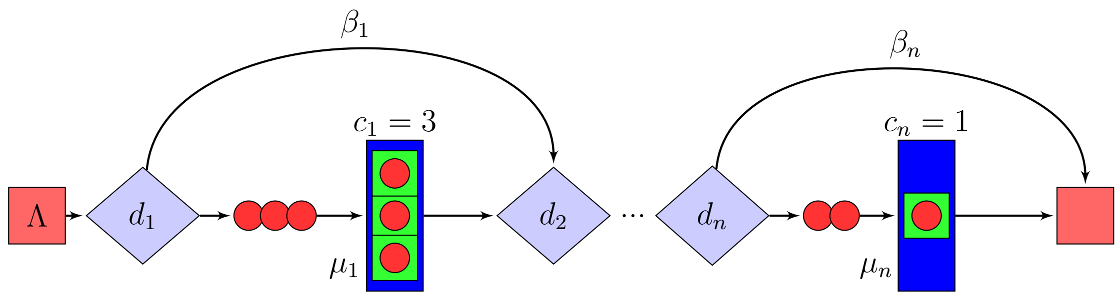


Figure 2: Diagram of n stations in series

The players arrive into the system at a rate Λ and make a decision on whether or not to join the first queue. The players know the rate of service μ_k at each queue , the number of servers at each queue c_k and also the number of players currently in each queue. Once they observe a queue, they must either join the queue or skip the queue. There is a cost associated with skipping a particular queue give by β_k . There is also a chance that after completing service at a queue, the player would leave the system completely, known as the dropout probability p_k. Players make their decision d_k by comparing the skipping cost to the expected cost of joining the queue .

$$d_k = \begin{cases} 1, & \text{if player joins queue } k \\ 0, & \text{if player skips queue } k \end{cases} \tag{1}$$

A player arriving at queue k and observing m agents ahead of them will expect to receive a cost given by equation (**??**).

$$E[\sigma_k] = \frac{m+1}{\mu_k c_k} \tag{2}$$

where σ_k denotes the actual cost to a player at station k.

In addition, we let ξ denote the cumulative cost incurred by a player moving through the system. T_k in eq **??** denotes the actual time spent in station k by a player. It is the sum of the time spent waiting to be served (T_k^w) and the time spent being served (T_k^s). Thus we have the cumulative cost to a player in station

$$\xi_k = \begin{cases} 0, \text{ if } i = 0 \\ \xi_{k-1} + \begin{cases} T_k, & \text{if player joins queue k} \\ \beta_k, & \text{if player skips queue k} \end{cases} \end{cases} \tag{3}$$

Upon arriving at queue k a player will make a decision that will benefit them by comparing (**??**) to β_k. For example, a player arriving at a queue with the parameters listed in Fig.**??** will expect to add .5 to their cost if they receive service at that station. As the balking cost for this queue is lower than the expected cost ($\beta = 0.4$), this player would skip under selfish conditions.

Parameter	Value
μ	2
c	4
m	3
β	0.4

Figure 3: Example parameters for a station

A player moving through the system with n stations can expect to receive a cost of (**??**) when making a choice d_k(**??**) at queue k .

$$E[\xi_n] = \sum_{i=1}^{n} (1 - d_i) \left(\frac{m_i + 1}{\mu_i c_i} \right) + \sum_{i=1}^{n} d_i \beta_i \tag{4}$$

We want to instigate a policy which minimises the average cost per player, which is known as the optimal social policy. In the system, the policy is the length of the queue at which a player arriving at the queue will balk from it. Now consider Fig.**??**, and we can see that a player skips from this queue if there are 3 people in the queue.

By considering policies which benefit the system by reducing the average cost per player, we can see the effect of choice in this system, which is what is implied by a social policy (we also say that the system is under social conditions). Within this paper, we consider the Price of Anarchy (**??**) or PoA, the ratio of average cost per player under selfish conditions (given by $\tilde{\tau}$), to the average cost per player under social conditions (given by τ^*). In the earlier example concerning the prisoners dilemma, the Price of Anarchy is $5/1 = 5$

$$PoA = \frac{\tilde{\tau}}{\tau^*} \tag{5}$$

This paper is structured as follows: Section 2 gives an outline of how a simulation model was used to study the queueing system, Section 3 explains the heuristic methods involved in finding the optimal social policy, Section 4 describes an analytical approximation of the model used to study the system at a much lower computational cost. This approximation was validated with the simulation model. Section 5 will discuss the results we gained from the analytical approximation version of the model and then it will conclude with some further work.

This paper initially proposes a bespoke simulation model [**??**] written in Python [**?**] to investigate the system. The model itself is a combination of Discrete Event Simulation[**?**] and an Agent Based Model [**?**]. The flow of the simulation model can be seen in Fig.**??**.

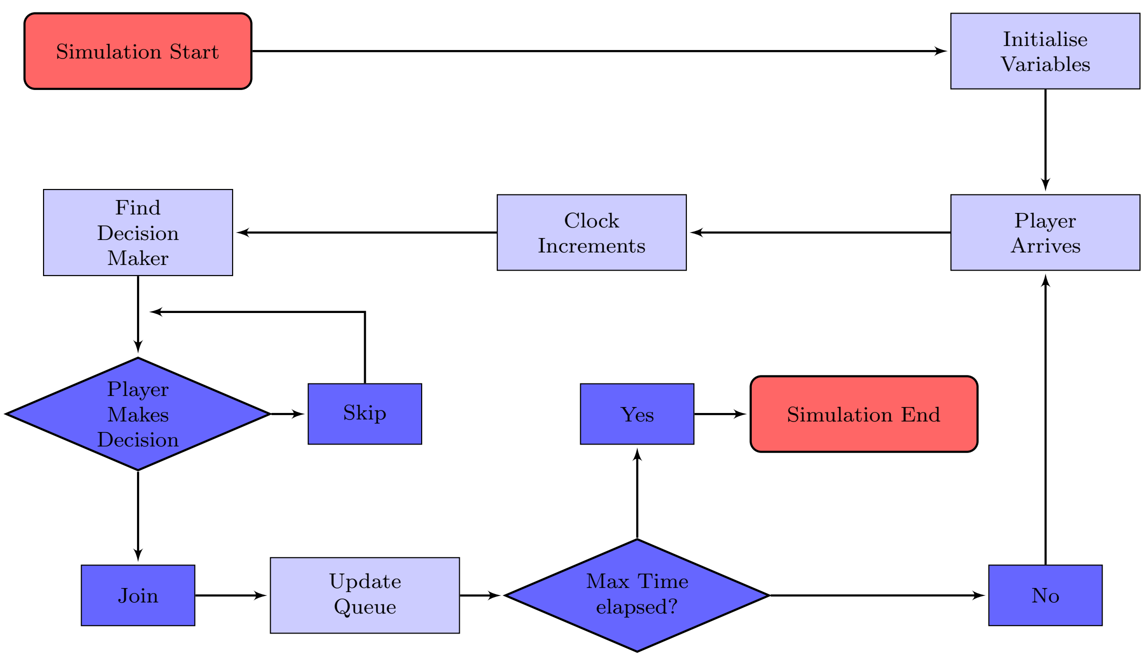


Figure 4: Flow chart describing the simulation model

Our simulation model will initialise with the system empty, however this would not represent the system it is attempting to represent. This is due to the fact that most systems do not begin with an empty queue (e.g. a GP surgery would already have appointments scheduled, which would represent players in a queue in the context of our model.) and the potential impact of this is discussed in [?]. The behaviour we are investigating occurs when the system has reached steady state. To ensure that we are getting results from the steady state of a model, we must first determine the length of time it takes before the results from the model become invariant with respect to time. In [?], a method is discussed for finding the correct length for a model to run before collecting results. The algorithm involves the simulation being run for an increasing amount of time, and the mean utilisation recorded along with the standard deviation. By plotting the mean utilisation, along with a 95% confidence interval, we can see when the model reaches steady state conditions by seeing when the graph stabilises. In Fig.??, the mean and 95% confidence interval of the results was plotted to find the parameters necessary to ensure the accuracy of our results. The three parameters which needed to be determined were the length of the simulation, the warm-up period and the number of times each instance would be repeated. . Using the method suggested previously, we concluded the we needed to run the model for 250 time units and a warm up period of 50 time units was required. The number of trials did not cause a large variation even for small number of repeats and we decided to go with 16.

To be able to effectively investigate larger scenarios, we started to use the Cardiff University supercomputer, Merlin [?].The model is an example of embarrassingly parallel problem[?], due to the number of trials which enabled us to take advantage of the large number of cpu's available on Merlin[?].

3 Optimisation

The motivation for this paper is to study the effect that choice has on these particular systems, by using the Price of Anarchy as a measure of the inefficiency. To understand the impact of choice, we do not need the optimal social policy, but merely a good approximation to the optimal social policy. In [?] a variety of heuristic methods are discussed, along with criteria for when a heuristic is applicable and what makes a good heuristic. We decided to use heuristic methods based on the conditions

- No exact method was known to find the optimal solution