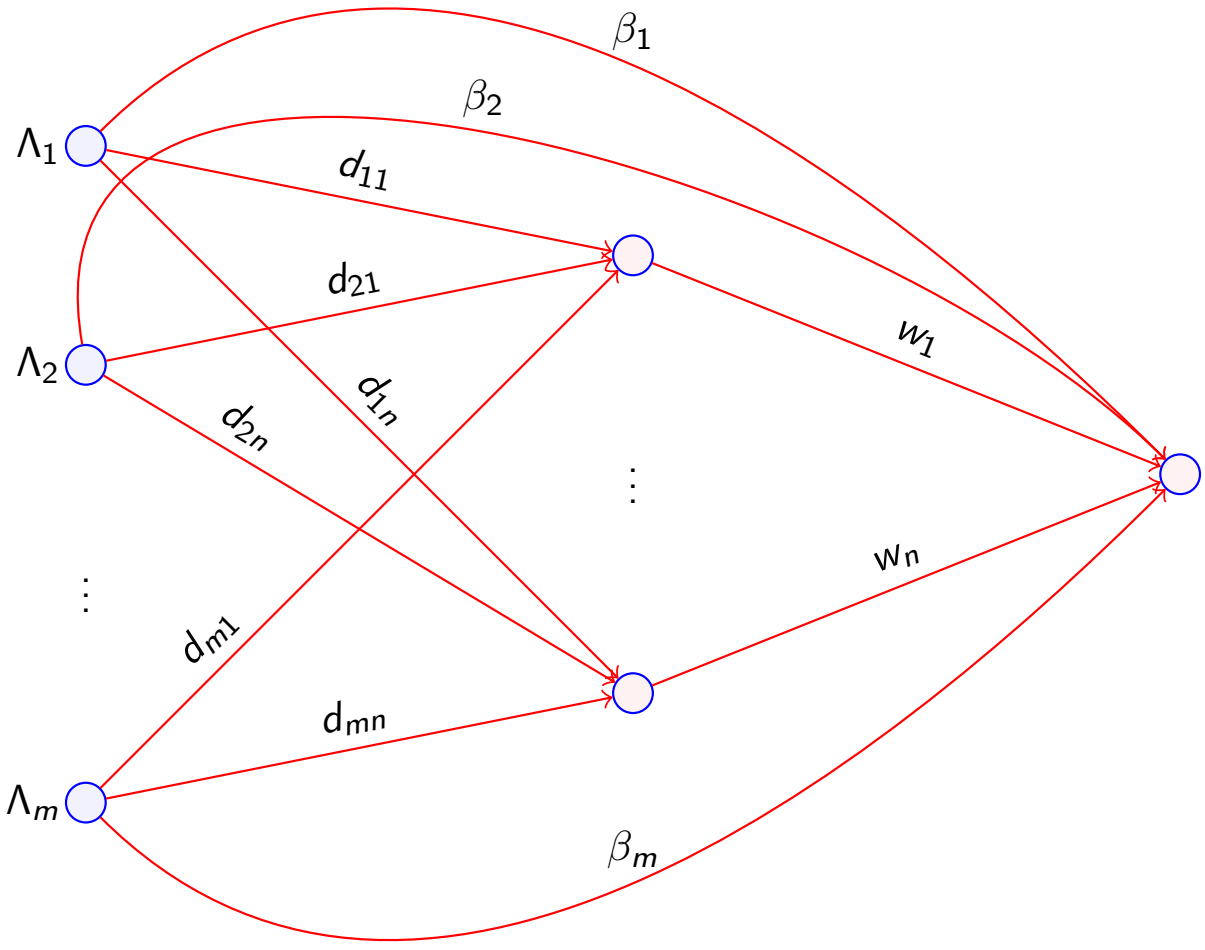
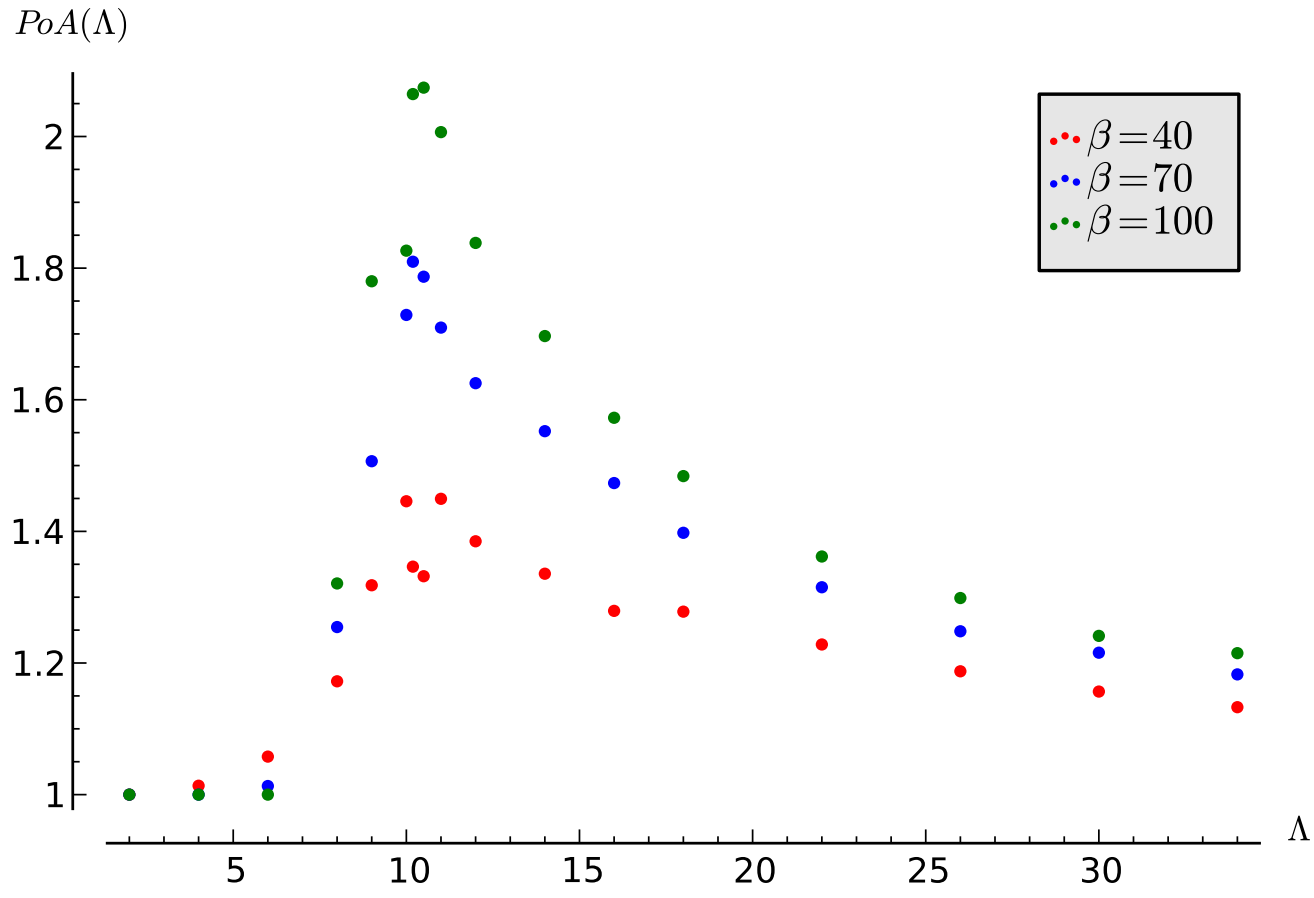
+Vincent.Knight
@drvinceknight
vincent-knight.com/Talks

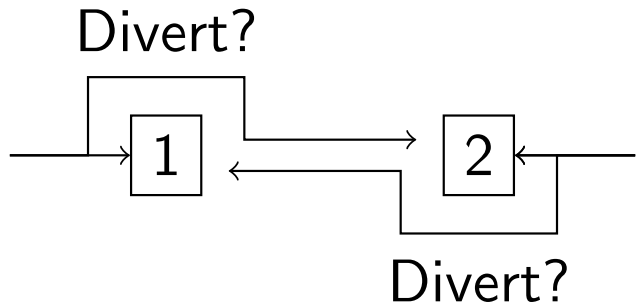
$$\begin{pmatrix} (2,2) & (5,0) \\ (0,5) & (4,4) \end{pmatrix}$$

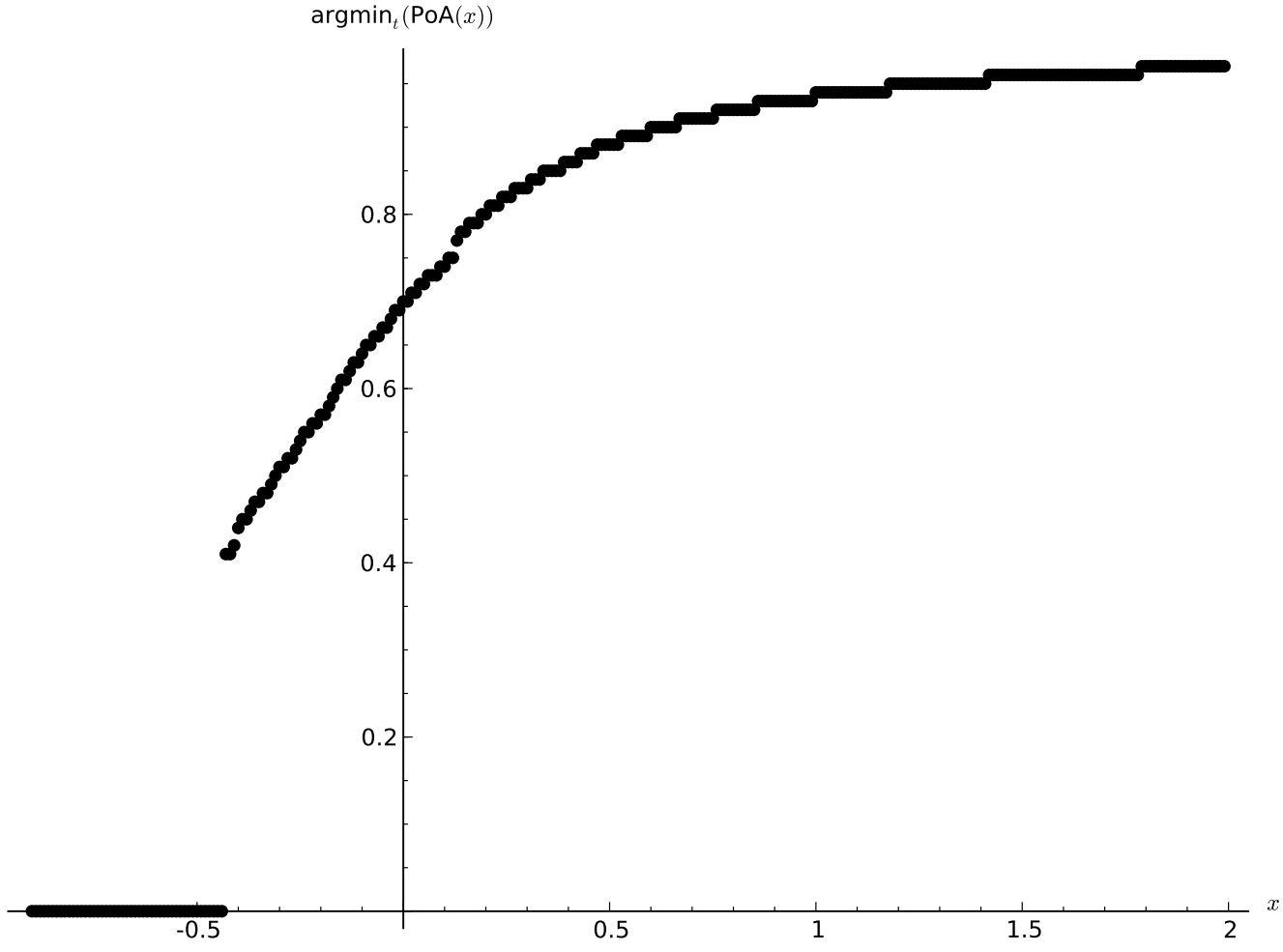

β_1
β_2
Λ_1
d_{11}
d_{21}
Λ_2
w_1
d_{2n}
d_{1n}
w_n
d_{m1}
d_{mn}
Λ_m
β_m


$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

Price of Anarchy in Public Services *EJORS*, 2013.

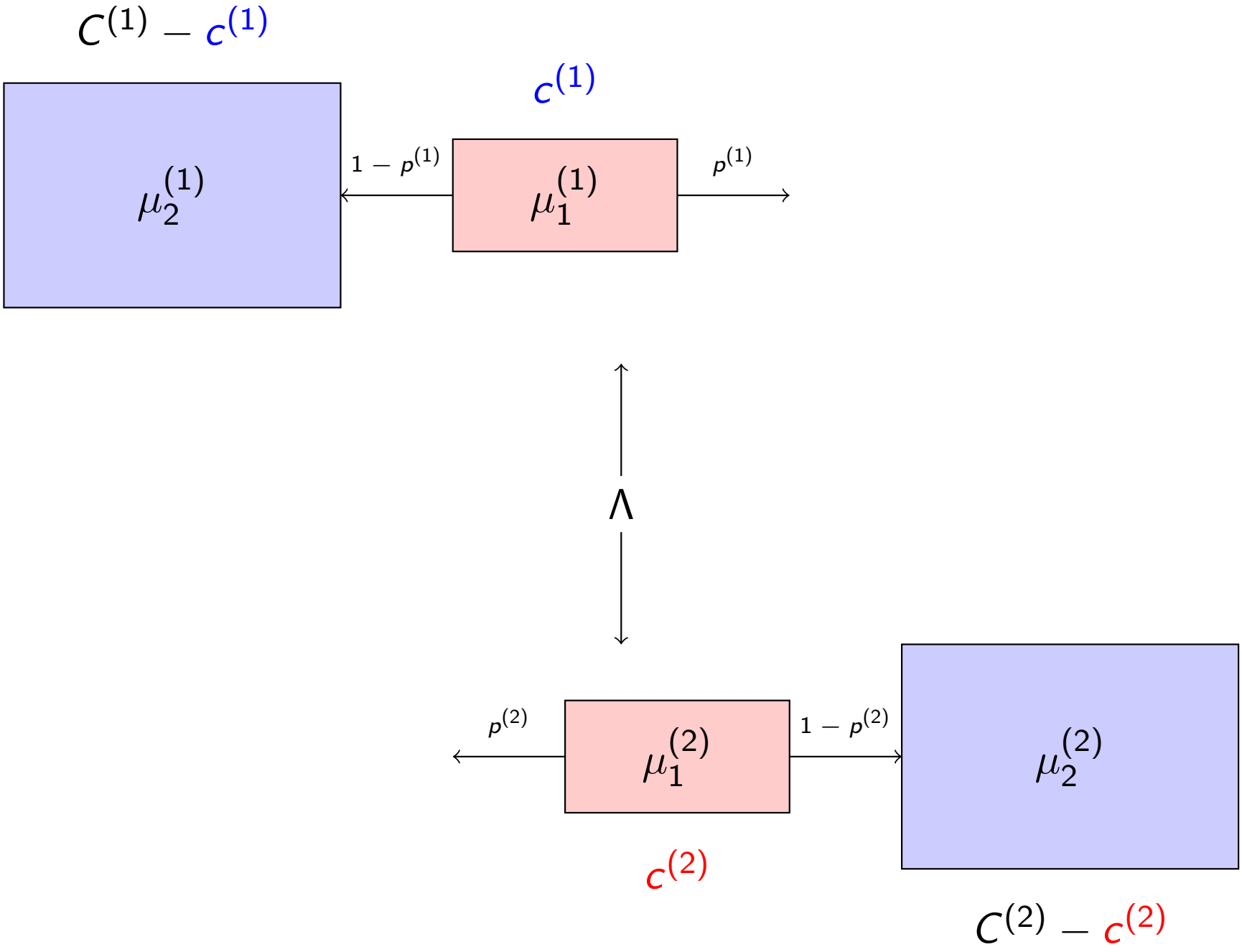
What about the controllers?

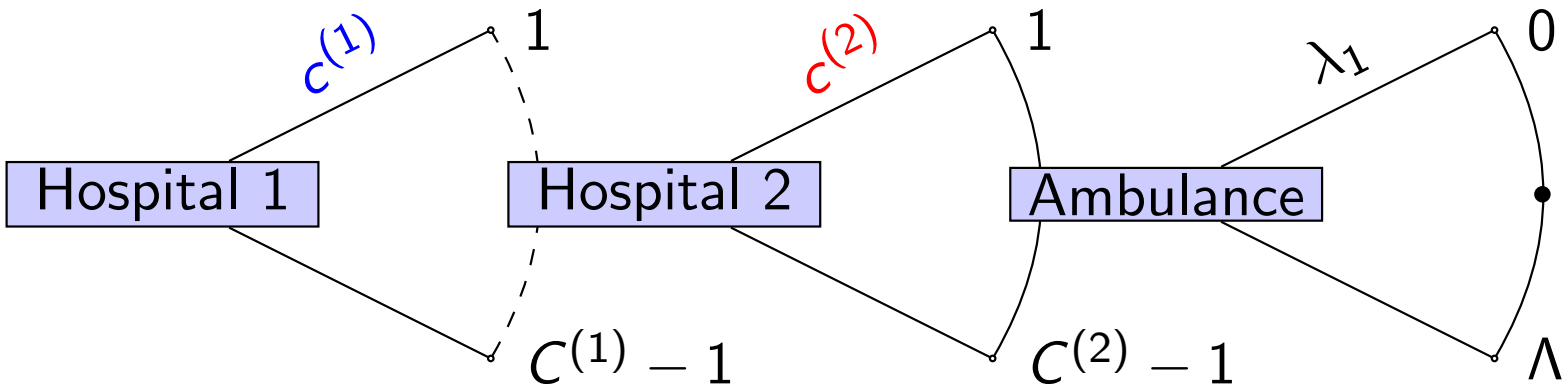

Divert?
1
2
Divert?

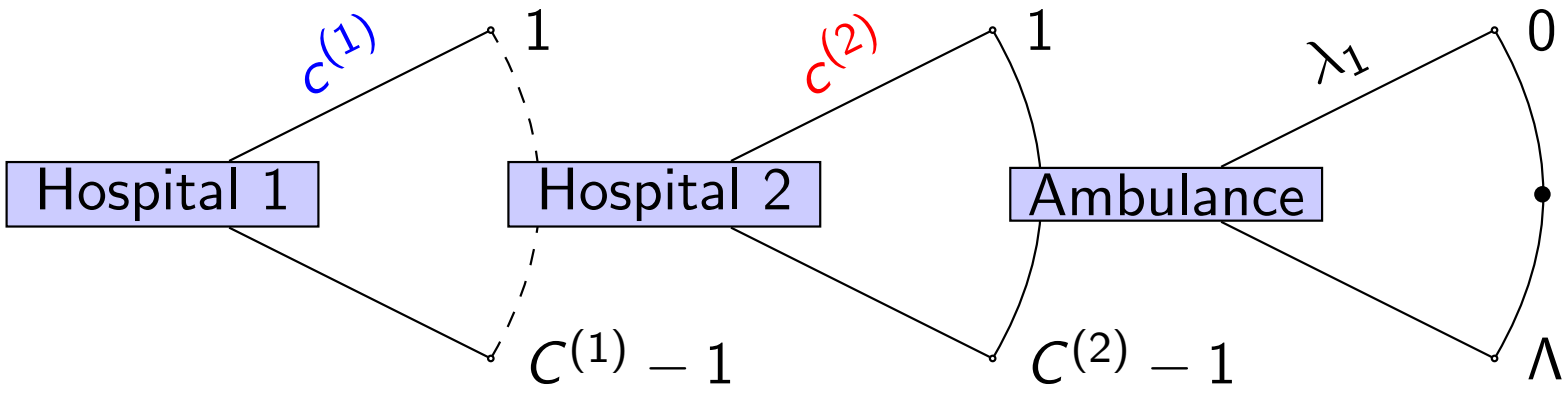

argmin$_t$(PoA(x))
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

Measuring the Price of Anarchy in Critical Care Unit Interactions, *Submitted to OMEGA*

EMERGENCY UNIT
AMBULANCE
AMBULANCE
AMBULANCE

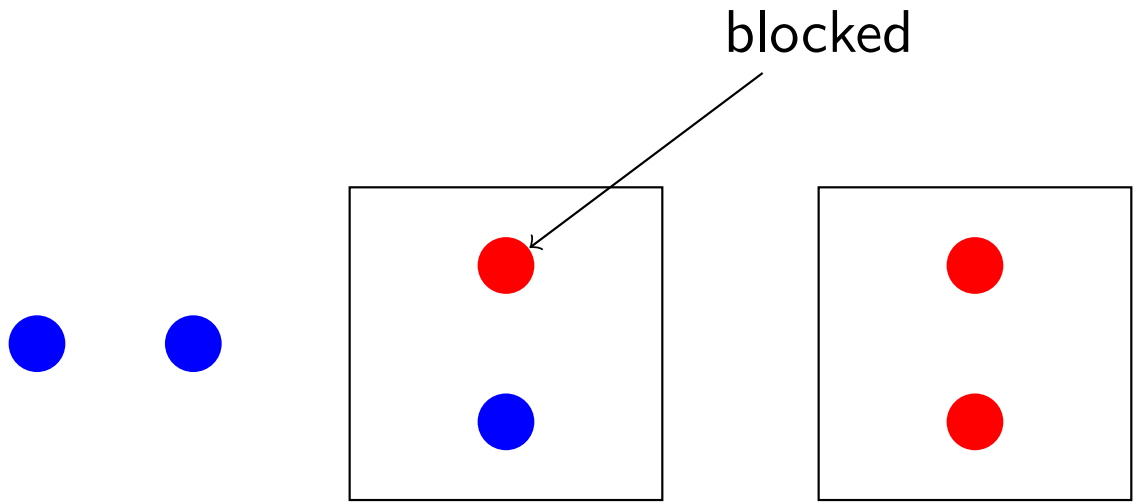

$C^{(1)} - c^{(1)}$
$c^{(1)}$
$\mu_2^{(1)}$
$1 - p^{(1)}$
$\mu_1^{(1)}$
$p^{(1)}$
Λ
$p^{(2)}$
$\mu_1^{(2)}$
$1 - p^{(2)}$
$\mu_2^{(2)}$
$c^{(2)}$
$C^{(2)} - c^{(2)}$


$c^{(1)}$
1
$c^{(2)}$
1
λ_1
0
Hospital 1
Hospital 2
Ambulance
$C^{(1)} - 1$
$C^{(2)} - 1$
Λ

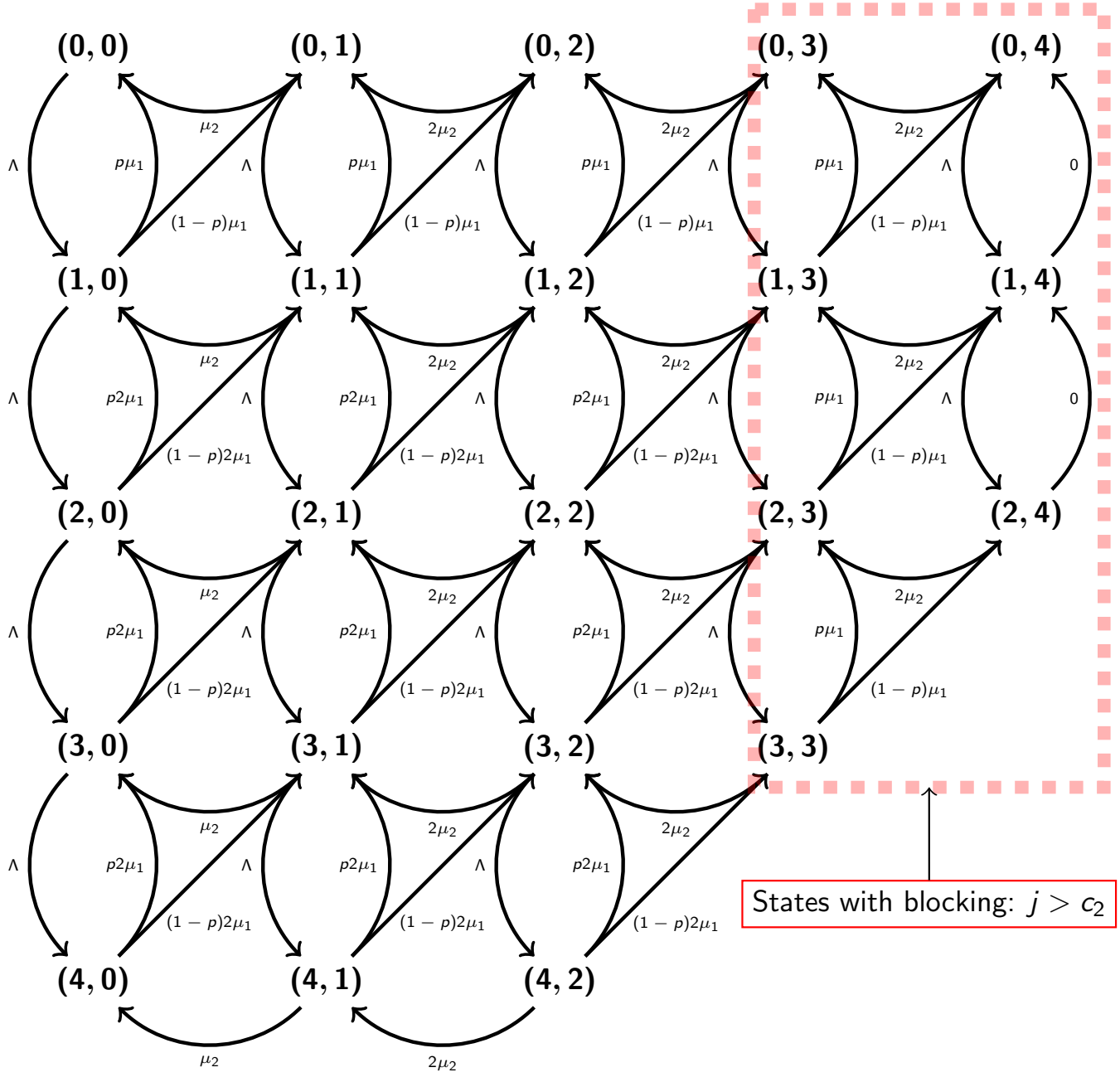


$$(|u_1^{(1)} - u_2^{(1)}|, |u_1^{(2)} - u_2^{(2)}|, |w^{(1)} - w^{(2)}|)$$

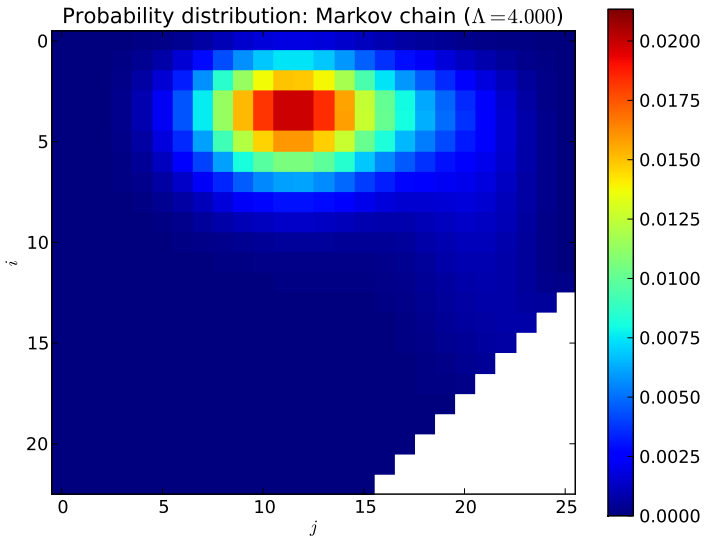
$$S = \left\{(i,j) \in \mathbb{Z}^2_{\geq 0} \mid 0 \leq j \leq c_1 + c_2,\ 0 \leq i \leq c_1 + N - \max(j - c_2, 0)\right\}$$

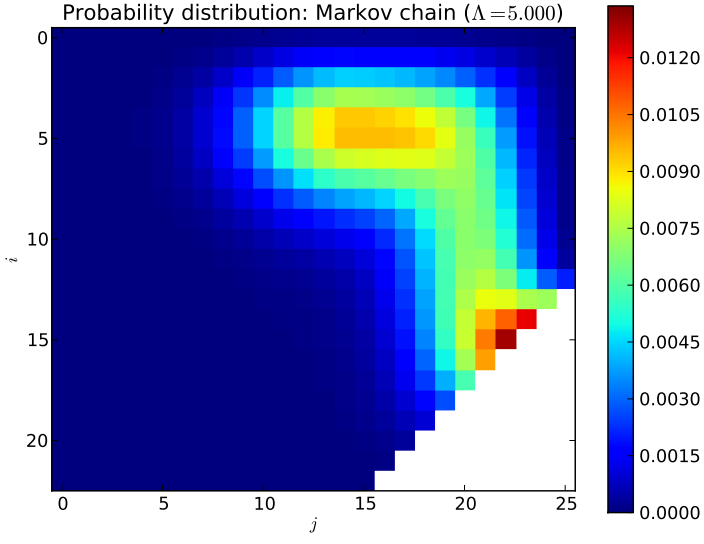


$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \Lambda, & \text{if } \delta = (1,0) \\ \min(c_1 - \max(j_1 - c_2, 0), i_1)(1-p)\mu_1, & \text{if } \delta = (-1,1) \\ \min(c_1 - \max(j_1 - c_2, 0), i_1)p\mu_1, & \text{if } \delta = (-1,0) \\ \min(c_2, j_1)\mu_2, & \text{if } \delta = (0,-1) \end{cases}$$

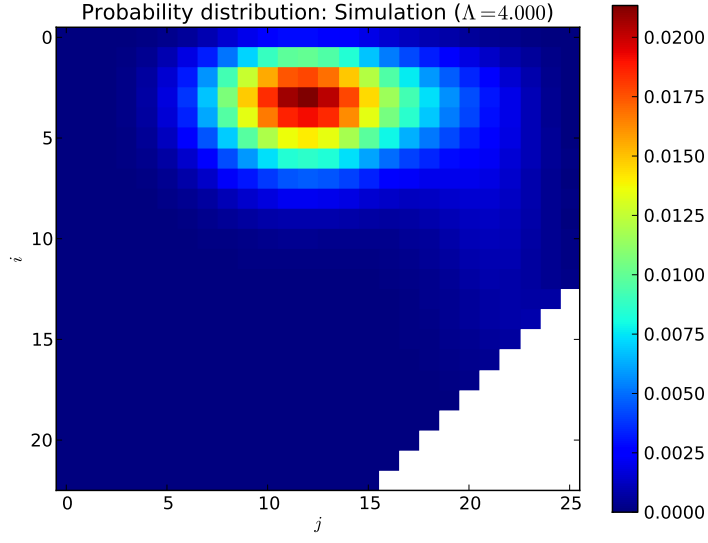

(0, 0)
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(2, 4)
(3, 0)
(3, 1)
(3, 2)
(3, 3)
(4, 0)
(4, 1)
(4, 2)
Λ
μ_2
$p\mu_1$
$(1-p)\mu_1$
$2\mu_2$
$p2\mu_1$
$(1-p)2\mu_1$
0
States with blocking: $j > c_2$

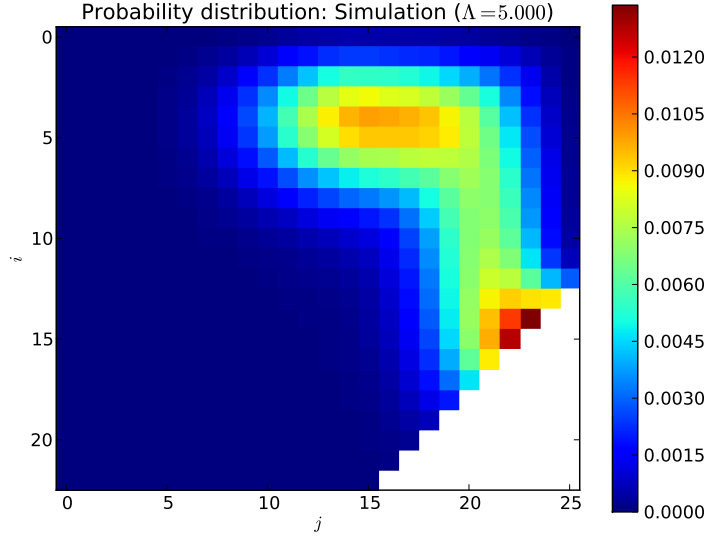
Analytical





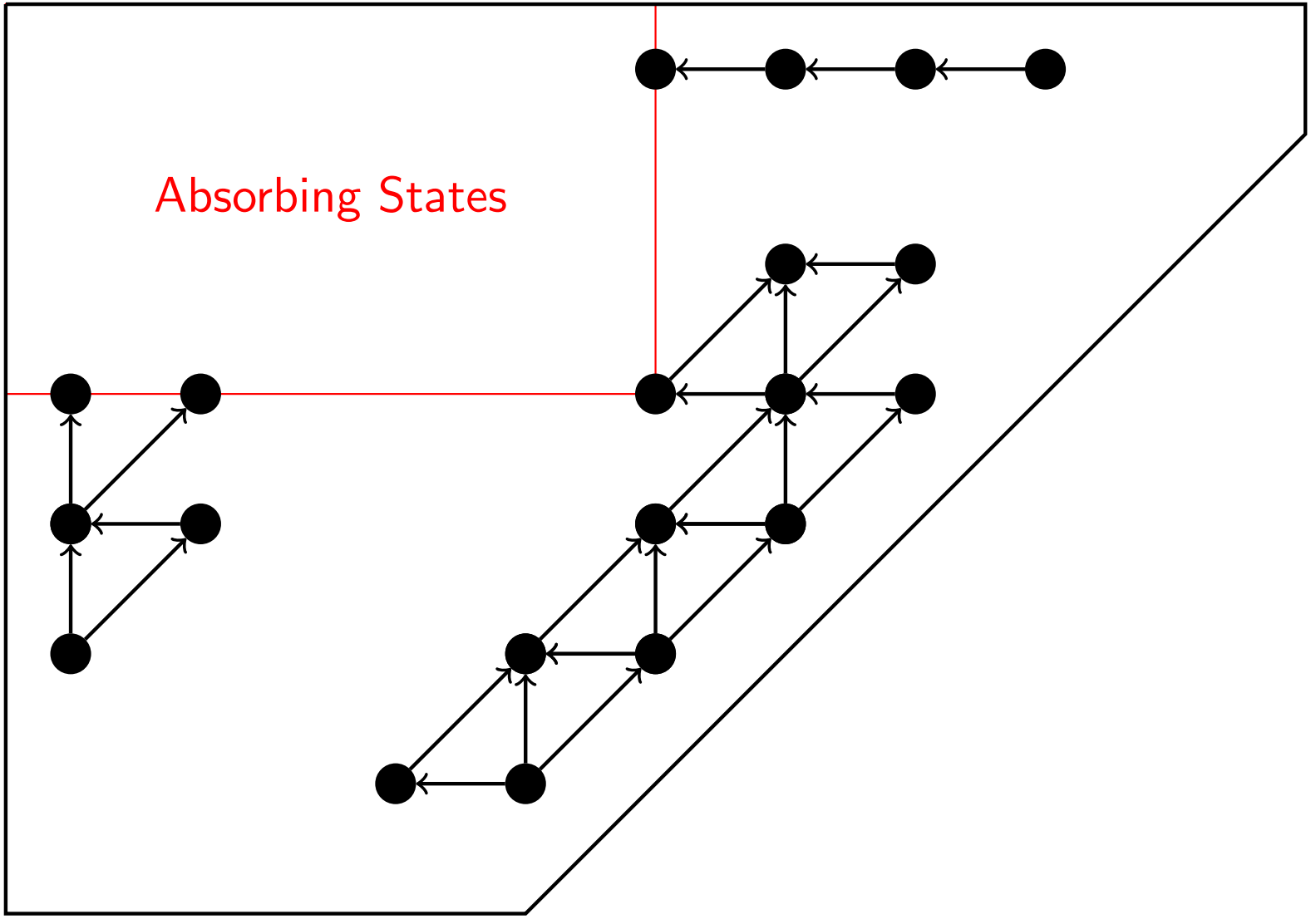
Simulation





Expected wait:

$$w = \frac{\sum_{(i,j)\in S_A} c(i,j)\pi_{(i,j)}}{\sum_{(i,j)\in S_A} \pi_{(i,j)}}$$

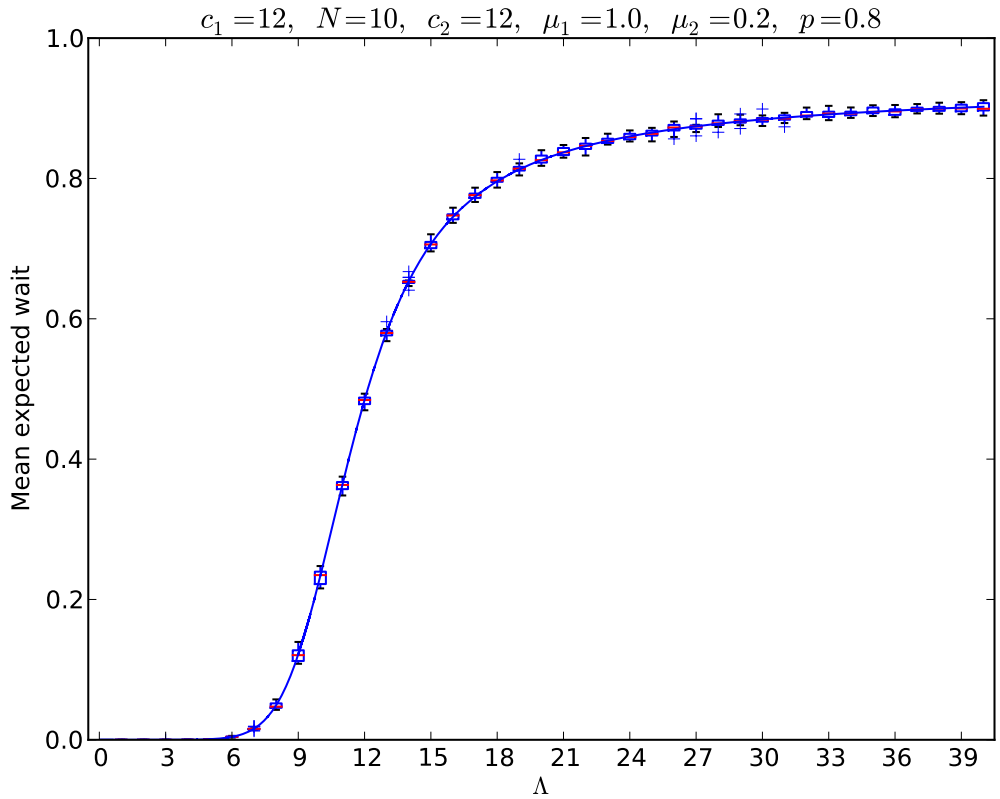


Sojourn time in state (i, j):

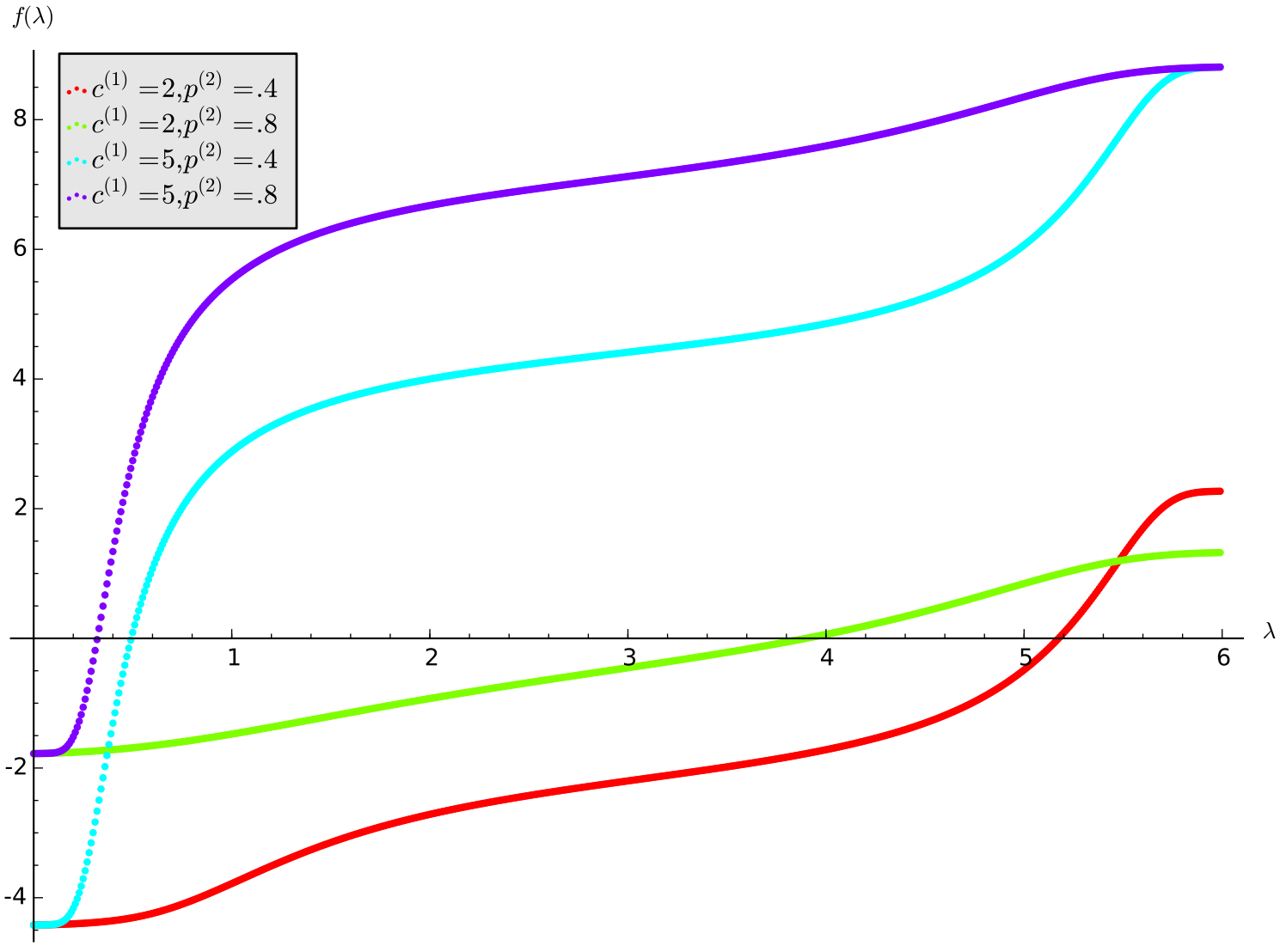
$$w(i,j) = \frac{1}{\min(c_2, j)\mu_2 + \min(c_1 - \max(j - c_2, 0), i)\mu_1}$$

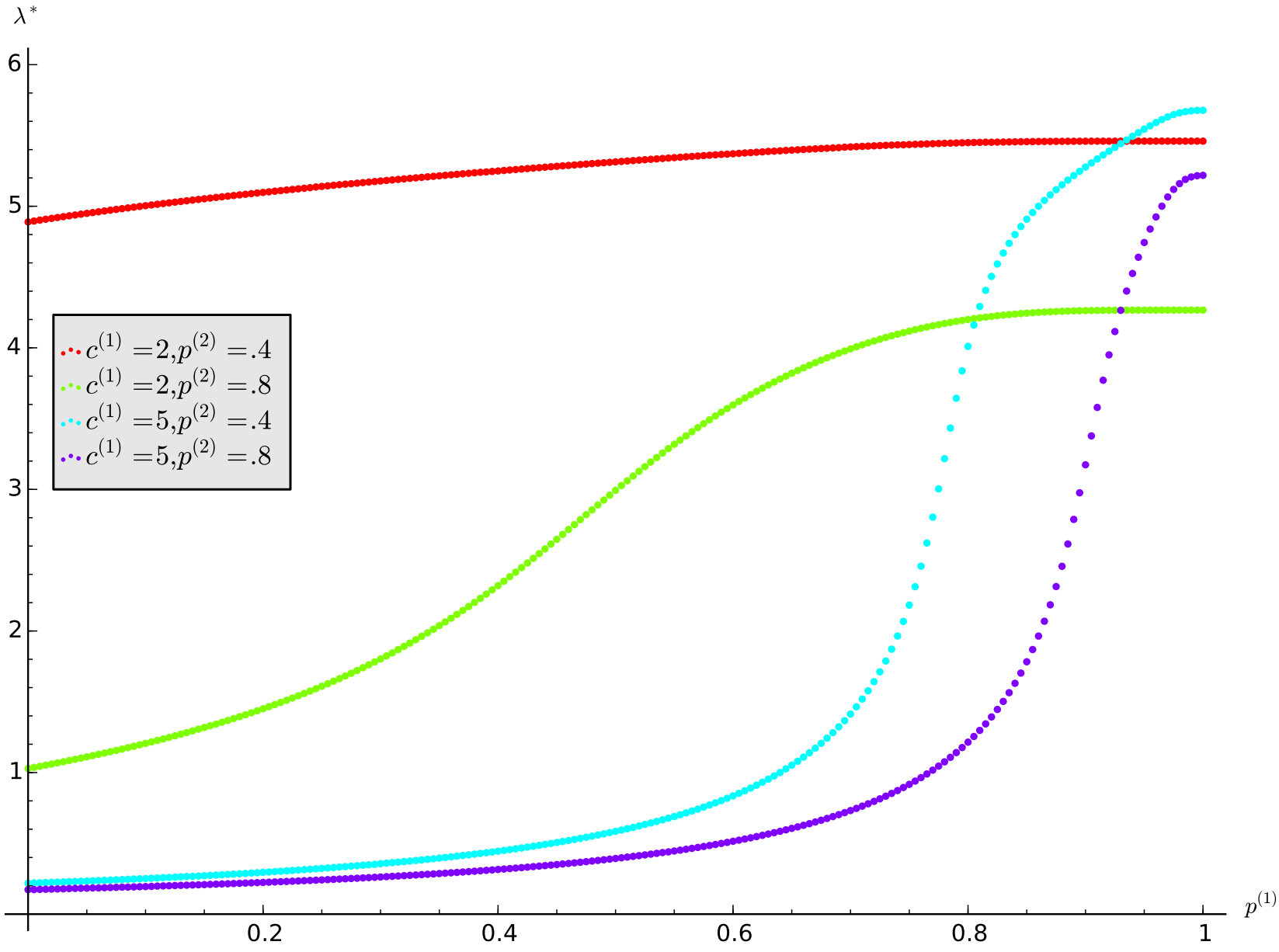
Cost of state (i, j):

$$c(i,j) = \begin{cases} 0, & \text{if } (i,j) \in A \\ w(i,j) + p_{s_2}\, c(i, j-1) + p_{s_1}\,(pc(i-1,j) + (1-p)c(i-1,j+1)), & \text{otherwise} \end{cases}$$



$$f(\lambda) = w^{(1)}(\lambda) - w^{(2)}(\Lambda - \lambda)$$




λ^*
6
5
4
3
2
1
0.2
0.4
0.6
0.8
1
$p^{(1)}$
$c^{(1)}=2, p^{(2)}=.4$
$c^{(1)}=2, p^{(2)}=.8$
$c^{(1)}=5, p^{(2)}=.4$
$c^{(1)}=5, p^{(2)}=.8$

$$\Lambda = 6,\ C^{(1)} = 6,\ C^{(2)} = 4,\ N^{(1)} = N^{(2)} = 3$$

$$A = \begin{pmatrix} 0.795 & 0.688 & 0.792 \\ 0.506 & 0.488 & 0.503 \\ 0.183 & 0.159 & 0.178 \\ 0.0104 & 0.0193 & 0.00523 \\ 0.0121 & 0.108 & 0.0159 \end{pmatrix} \quad B = \begin{pmatrix} 0.667 & 0.243 & 0.00105 \\ 0.480 & 0.154 & 0.196 \\ 0.396 & 0.0774 & 0.253 \\ 0.470 & 0.140 & 0.205 \\ 0.664 & 0.239 & 0.00837 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} 3.17 & 1.18 & 2.88 \\ 5.18 & 3.90 & 4.87 \\ 5.37 & 4.39 & 5.07 \\ 5.21 & 4.01 & 4.90 \\ 3.46 & 1.67 & 3.18 \end{pmatrix} \quad S = \begin{pmatrix} 0.672 & 0.481 & 0.672 \\ 0.381 & 0.427 & 0.429 \\ 0.315 & 0.341 & 0.352 \\ 0.376 & 0.418 & 0.423 \\ 0.666 & 0.535 & 0.671 \end{pmatrix}$$

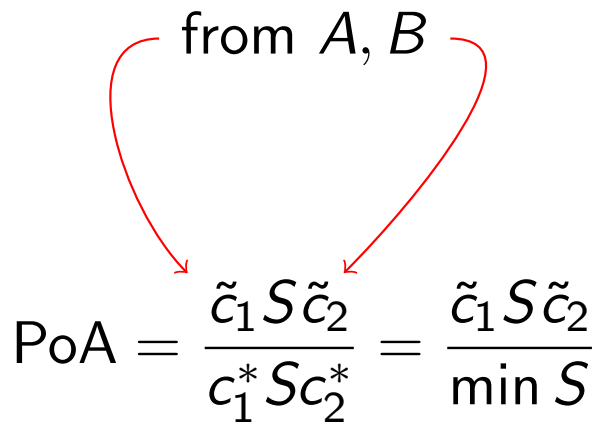
$$\Lambda = 6,\ C^{(1)} = 6,\ C^{(2)} = 4,\ N^{(1)} = N^{(2)} = 3$$

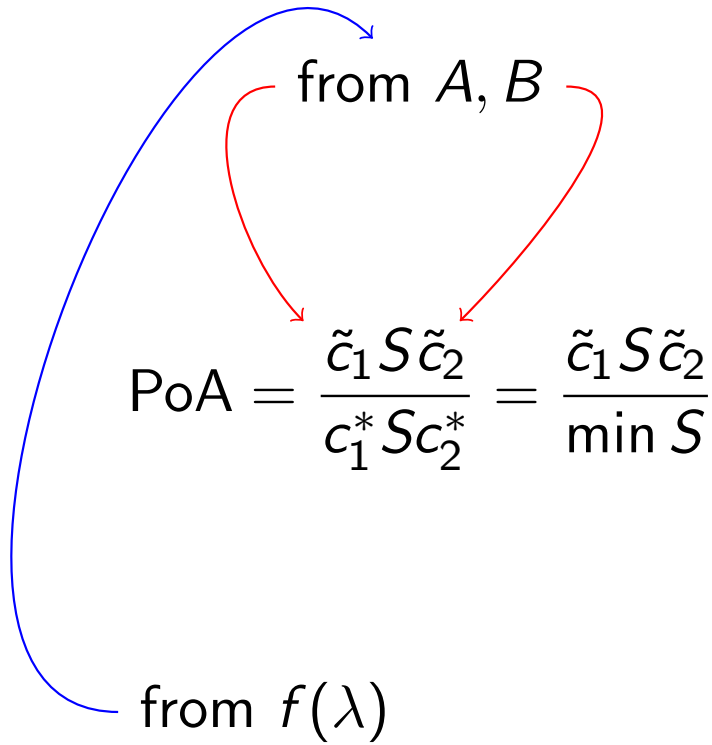
$$A = \begin{pmatrix} 0.795 & 0.688 & 0.792 \\ 0.506 & 0.488 & 0.503 \\ 0.183 & 0.159 & 0.178 \\ 0.0104 & 0.0193 & 0.00523 \\ 0.0121 & 0.108 & 0.0159 \end{pmatrix} \quad B = \begin{pmatrix} 0.667 & 0.243 & 0.00105 \\ 0.480 & 0.154 & 0.196 \\ 0.396 & 0.0774 & 0.253 \\ 0.470 & 0.140 & 0.205 \\ 0.664 & 0.239 & 0.00837 \end{pmatrix}$$

$$\lambda_1 = \begin{pmatrix} 3.17 & 1.18 & 2.88 \\ 5.18 & 3.90 & 4.87 \\ 5.37 & 4.39 & 5.07 \\ 5.21 & 4.01 & 4.90 \\ 3.46 & 1.67 & 3.18 \end{pmatrix} \quad S = \begin{pmatrix} 0.672 & 0.481 & 0.672 \\ 0.381 & 0.427 & 0.429 \\ 0.315 & 0.341 & 0.352 \\ 0.376 & 0.418 & 0.423 \\ 0.666 & 0.535 & 0.671 \end{pmatrix}$$

$\tilde{c}_1 = 4, \tilde{c}_2 = 2$ and $c_1^* = 3, c_2^* = 1$ for PoA $= 1.330$.

$$\mathrm{PoA} = \frac{\tilde{c}_1 S \tilde{c}_2}{c_1^* S c_2^*} = \frac{\tilde{c}_1 S \tilde{c}_2}{\min S}$$


from A, B
$$\mathrm{PoA} = \frac{\tilde{c}_1 S \tilde{c}_2}{c_1^* S c_2^*} = \frac{\tilde{c}_1 S \tilde{c}_2}{\min S}$$


from A, B
$\mathrm{PoA} = \dfrac{\tilde{c}_1 S \tilde{c}_2}{c_1^* S c_2^*} = \dfrac{\tilde{c}_1 S \tilde{c}_2}{\min S}$
from $f(\lambda)$

Effect of p1

Effect of lambda

Effect of C

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