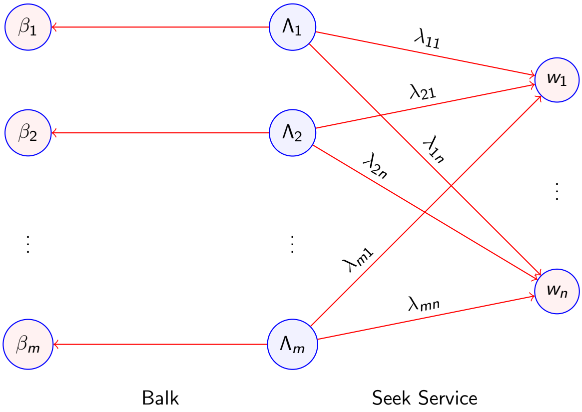
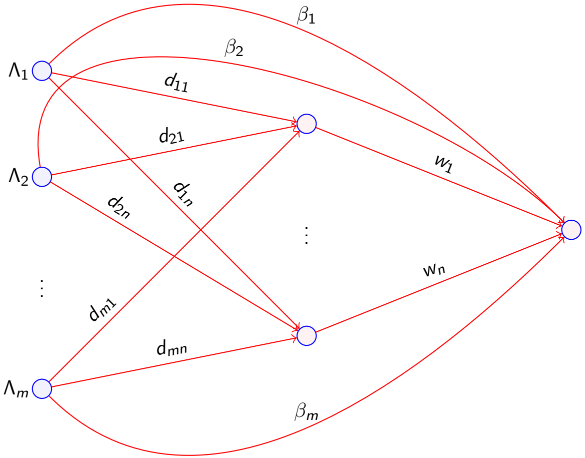
+Vincent.Knight
@drvinceknight
vincent-knight.com/Talks

$$\begin{pmatrix} (2,2) & (5,0) \\ (0,5) & (4,4) \end{pmatrix}$$

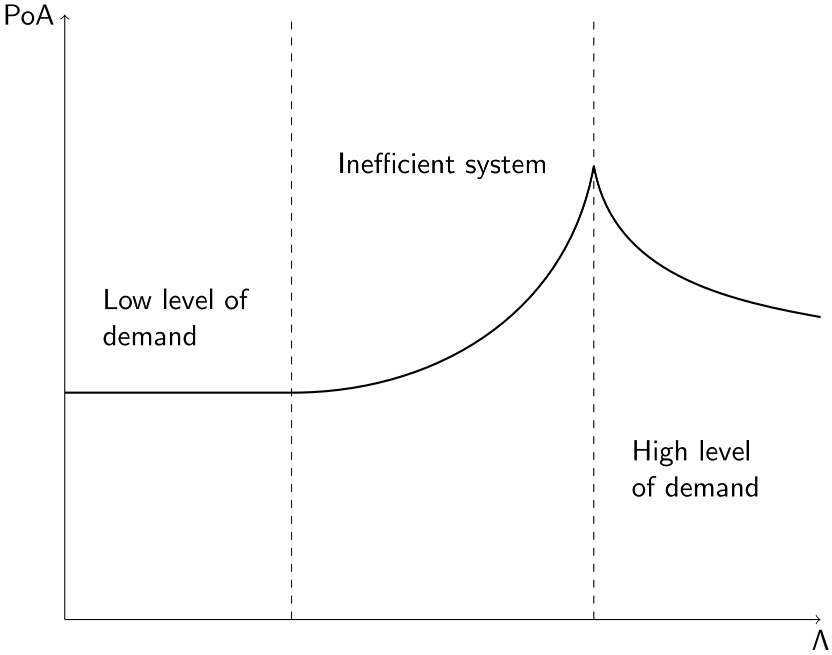


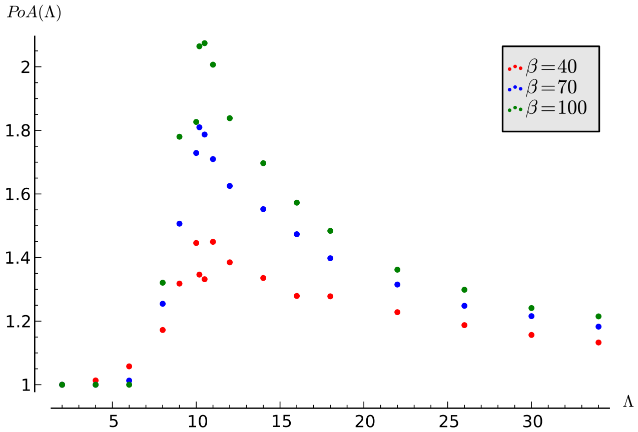
β_1
Λ_1
λ_{11}
w_1
λ_{21}
β_2
Λ_2
λ_{1n}
λ_{2n}
λ_{m1}
w_n
λ_{mn}
β_m
Λ_m
Balk
Seek Service

Parameter	Interpretation
$m \in \mathbb{Z}$	Number of sources
$n \in \mathbb{Z}$	Number of service centers
$\beta \in \mathbb{R}^m_{\geq 0}$	Worth of service
$\Lambda \in \mathbb{R}^m_{\geq 0}$	Demand rate
w_j for $j \in [n]$	A convex utility function
d_{ij} for $i \in [m]$, $j \in [n]$	Distance from source i to service center j
λ_{ij} for $i \in [m]$, $j \in [n]$	Traffic from source i to service center j



Λ_1
Λ_2
Λ_m
β_1
β_2
β_m
d_{11}
d_{21}
d_{1n}
d_{2n}
d_{m1}
d_{mn}
w_1
w_n

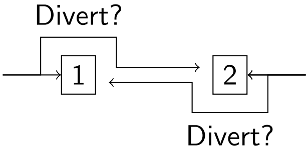

PoA
Inefficient system
Low level of demand
High level of demand
Λ

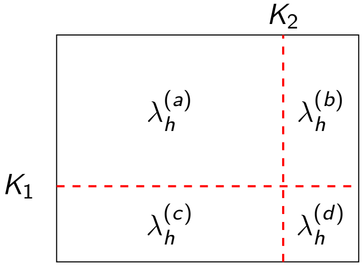


$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

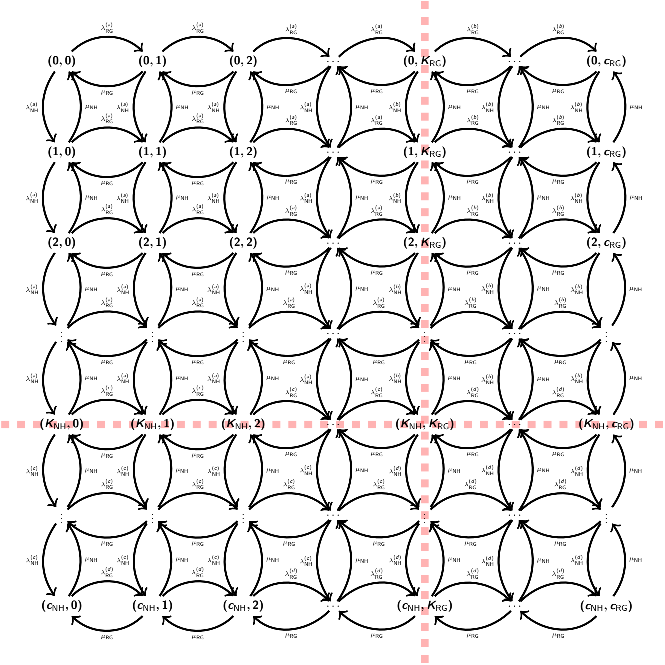
Price of Anarchy in Public Services *EJORS*, 2013.

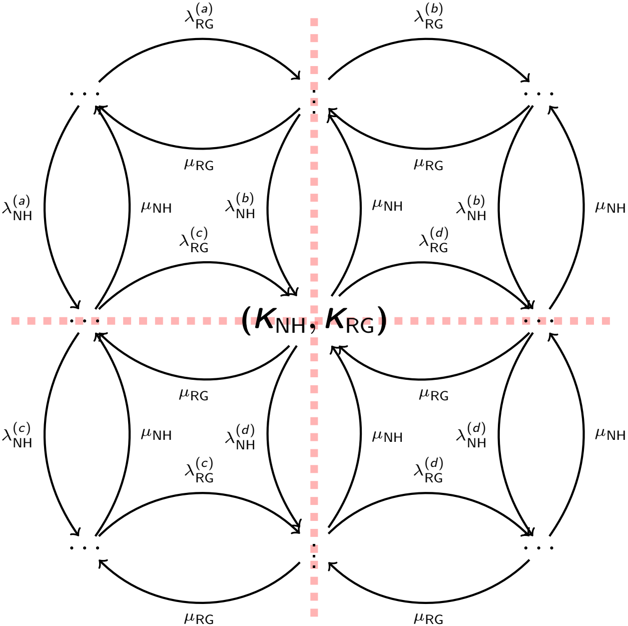
What about the controllers?


Divert?
1
2
Divert?


K_2
$\lambda_h^{(a)}$
$\lambda_h^{(b)}$
K_1
$\lambda_h^{(c)}$
$\lambda_h^{(d)}$

Parameter	Interpretation
$h \in \{1, 2\}$	CCU
c_h	Capacity of CCU
K_h	Cutoff strategy of CCU
λ_h^{area}	Demand rate
μ_h	Service rate of CCU


(0, 0)
(0, 1)
(0, 2)
(0, K_RG)
(0, c_RG)
(1, 0)
(1, 1)
(1, 2)
(1, K_RG)
(1, c_RG)
(2, 0)
(2, 1)
(2, 2)
(2, K_RG)
(2, c_RG)
(K_NH, 0)
(K_NH, 1)
(K_NH, 2)
(K_NH, K_RG)
(K_NH, c_RG)
(c_NH, 0)
(c_NH, 1)
(c_NH, 2)
(c_NH, K_RG)
(c_NH, c_RG)
μ_RG
μ_NH
λ_RG^(a)
λ_RG^(b)
λ_RG^(c)
λ_RG^(d)
λ_NH^(a)
λ_NH^(b)
λ_NH^(c)
λ_NH^(d)

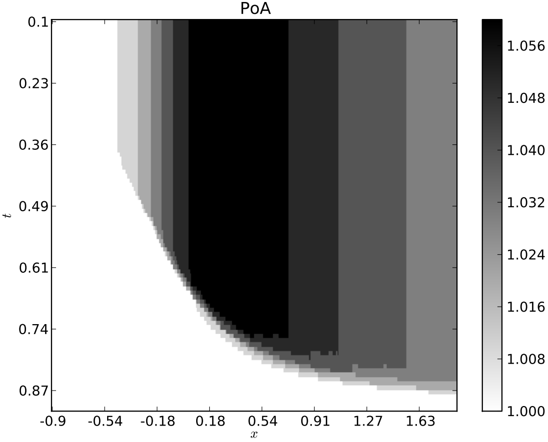


$\lambda_{\mathrm{RG}}^{(a)}$
$\lambda_{\mathrm{RG}}^{(b)}$
μ_{RG}
μ_{RG}
$\lambda_{\mathrm{NH}}^{(a)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(b)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(b)}$
μ_{NH}
$\lambda_{\mathrm{RG}}^{(c)}$
$\lambda_{\mathrm{RG}}^{(d)}$
$(K_{\mathrm{NH}}, K_{\mathrm{RG}})$
μ_{RG}
μ_{RG}
$\lambda_{\mathrm{NH}}^{(c)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(d)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(d)}$
μ_{NH}
$\lambda_{\mathrm{RG}}^{(c)}$
$\lambda_{\mathrm{RG}}^{(d)}$
μ_{RG}
μ_{RG}

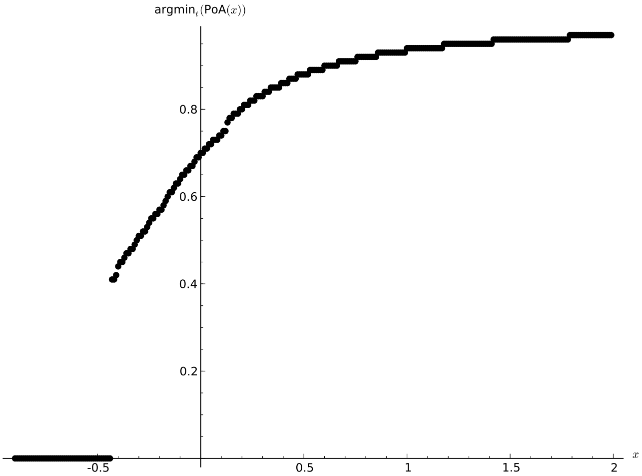
Theorem.
Let $f_h(k) : [1, c_{\bar{h}}] \to [1, c_h]$ be the best response of player $h \in \{\mathrm{NH}, \mathrm{RG}\}$ to the diversion threshold of $\bar{h} \neq h$ ($\bar{h} \in \{\mathrm{NH}, \mathrm{RG}\}$). If $f_h(k)$ is a non-decreasing function in k then the game has at least one Nash Equilibrium in Pure Strategies.

$$A = \begin{pmatrix} (U_{\mathrm{NH}}(1,1) - t)^2 & \dots & (U_{\mathrm{NH}}(1, c_{\mathrm{RG}}) - t)^2 \\ (U_{\mathrm{NH}}(2,1) - t)^2 & \dots & (U_{\mathrm{NH}}(2, c_{\mathrm{RG}}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathrm{NH}}(c_{\mathrm{NH}},1) - t)^2 & \dots & (U_{\mathrm{NH}}(c_{\mathrm{NH}}, c_{\mathrm{RG}}) - t)^2 \end{pmatrix}$$

$$B = \begin{pmatrix} (U_{\mathrm{RG}}(1,1) - t)^2 & \dots & (U_{\mathrm{RG}}(1, c_{\mathrm{RG}}) - t)^2 \\ (U_{\mathrm{RG}}(2,1) - t)^2 & \dots & (U_{\mathrm{RG}}(2, c_{\mathrm{RG}}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathrm{RG}}(c_{\mathrm{RG}},1) - t)^2 & \dots & (U_{\mathrm{RG}}(c_{\mathrm{RG}}, c_{\mathrm{RG}}) - t)^2 \end{pmatrix}$$



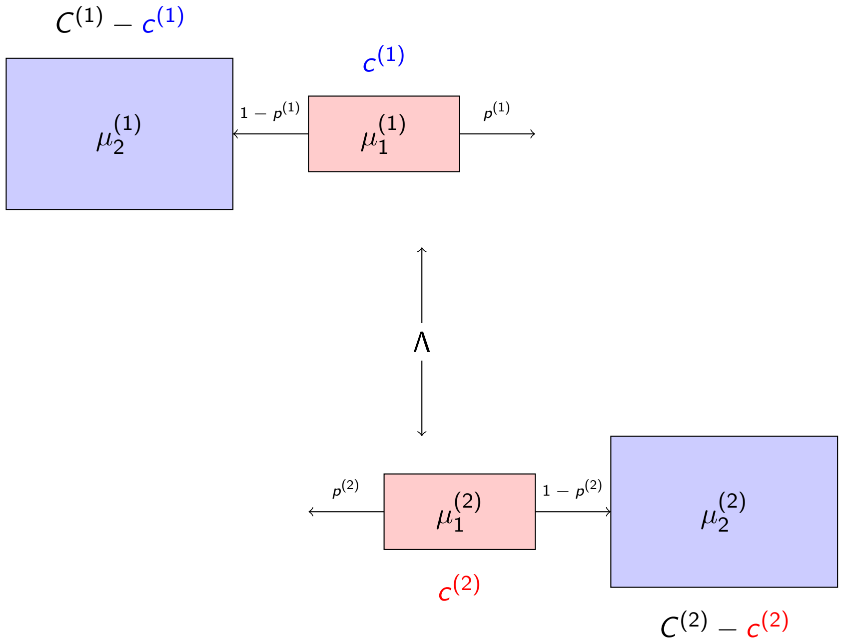
PoA
t
0.1
0.23
0.36
0.49
0.61
0.74
0.87
x
-0.9
-0.54
-0.18
0.18
0.54
0.91
1.27
1.63
1.056
1.048
1.040
1.032
1.024
1.016
1.008
1.000

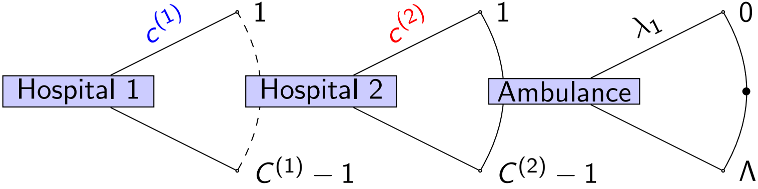


argmin$_t$(PoA(x))
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

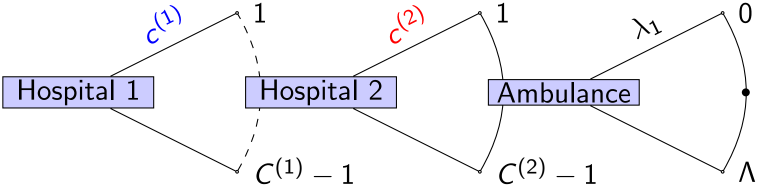
Measuring the Price of Anarchy in Critical Care Unit Interactions, *Submitted to OMEGA*


UNED ACHOSION BRYS
EMERGENCY UNIT
AMBULANCE
EMERGENCY AMBULANCE
AMBULANCE
AMBULANCE


$C^{(1)} - c^{(1)}$
$c^{(1)}$
$\mu_2^{(1)}$
$1 - p^{(1)}$
$\mu_1^{(1)}$
$p^{(1)}$
Λ
$p^{(2)}$
$\mu_1^{(2)}$
$1 - p^{(2)}$
$\mu_2^{(2)}$
$c^{(2)}$
$C^{(2)} - c^{(2)}$

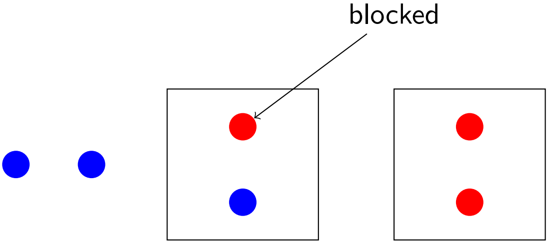


$c^{(1)}$
1
Hospital 1
$C^{(1)} - 1$
$c^{(2)}$
1
Hospital 2
$C^{(2)} - 1$
λ_1
0
Ambulance
Λ

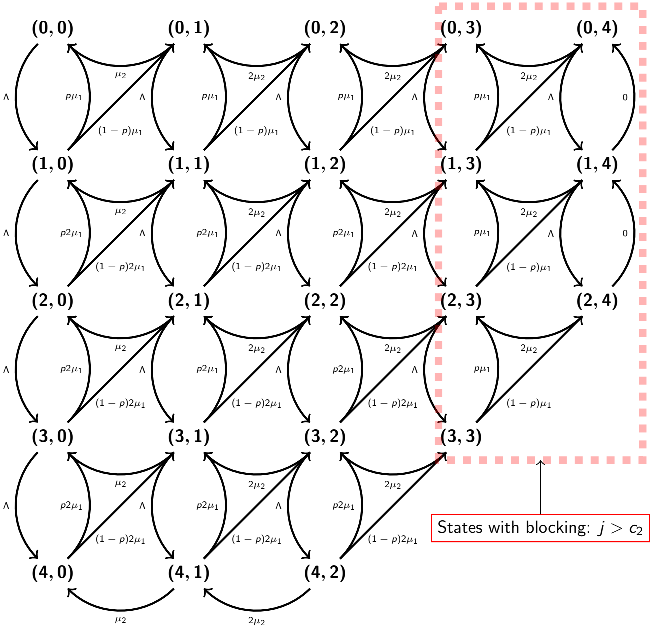


$$\left(|u_1^{(1)} - u_2^{(1)}|, |u_1^{(2)} - u_2^{(2)}|, |w^{(1)} - w^{(2)}|\right)$$

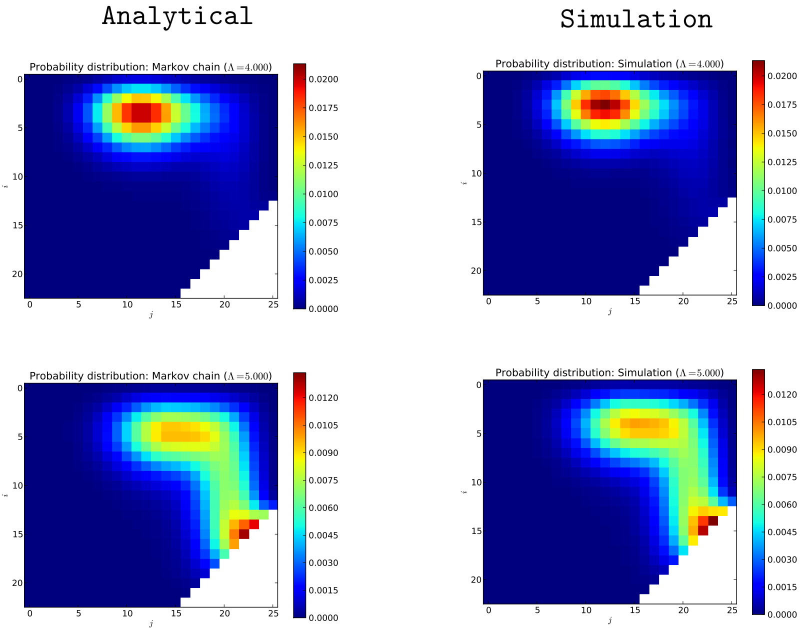
$$S = \left\{(i,j) \in \mathbb{Z}^2_{\geq 0} \mid 0 \leq j \leq c_1 + c_2,\ 0 \leq i \leq c_1 + N - \max(j - c_2, 0)\right\}$$



$$q_{(i_1,j_1),(i_2,j_2)} = \begin{cases} \Lambda, & \text{if } \delta = (1,0) \\ \min(c_1 - \max(j_1 - c_2, 0), i_1)(1-p)\mu_1, & \text{if } \delta = (-1,1) \\ \min(c_1 - \max(j_1 - c_2, 0), i_1)p\mu_1, & \text{if } \delta = (-1,0) \\ \min(c_2, j_1)\mu_2, & \text{if } \delta = (0,-1) \end{cases}$$



(0, 0)
(0, 1)
(0, 2)
(0, 3)
(0, 4)
(1, 0)
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(2, 0)
(2, 1)
(2, 2)
(2, 3)
(2, 4)
(3, 0)
(3, 1)
(3, 2)
(3, 3)
(4, 0)
(4, 1)
(4, 2)
Λ
$p\mu_1$
μ_2
$2\mu_2$
$(1-p)\mu_1$
$p2\mu_1$
$(1-p)2\mu_1$
0
States with blocking: $j > c_2$



Analytical
Simulation
Probability distribution: Markov chain ($\Lambda = 4.000$)
Probability distribution: Simulation ($\Lambda = 4.000$)
Probability distribution: Markov chain ($\Lambda = 5.000$)
Probability distribution: Simulation ($\Lambda = 5.000$)
i
j