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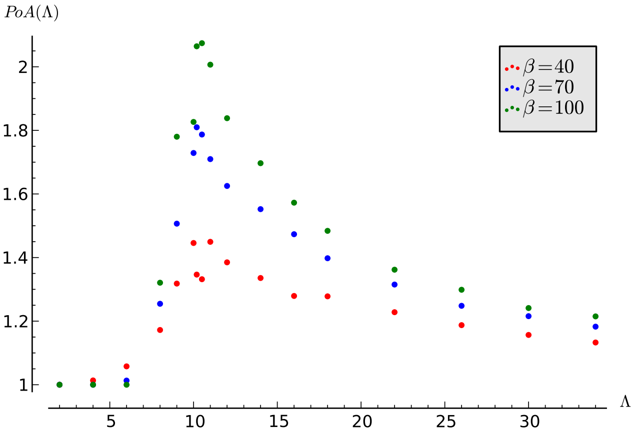
$$\begin{pmatrix} (2,2) & (5,0) \\ (0,5) & (4,4) \end{pmatrix}$$


$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

What about the controllers?

What about the controllers?

S. Deo and I. Gurvich. **Centralized vs. Decentralized Ambulance Diversion: A Network Perspective.** *Management Science*, 57(7):13001319, May 2011.

[INCLUDE ANY PLOT FROM PAPER WITH IZA]

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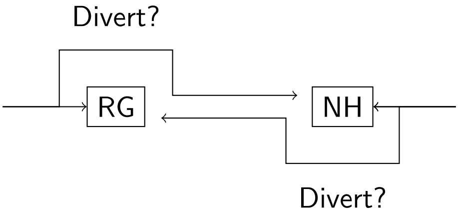
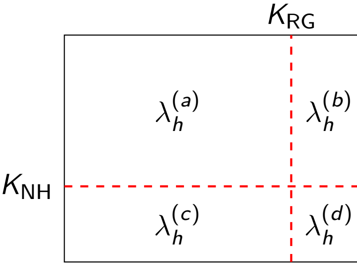
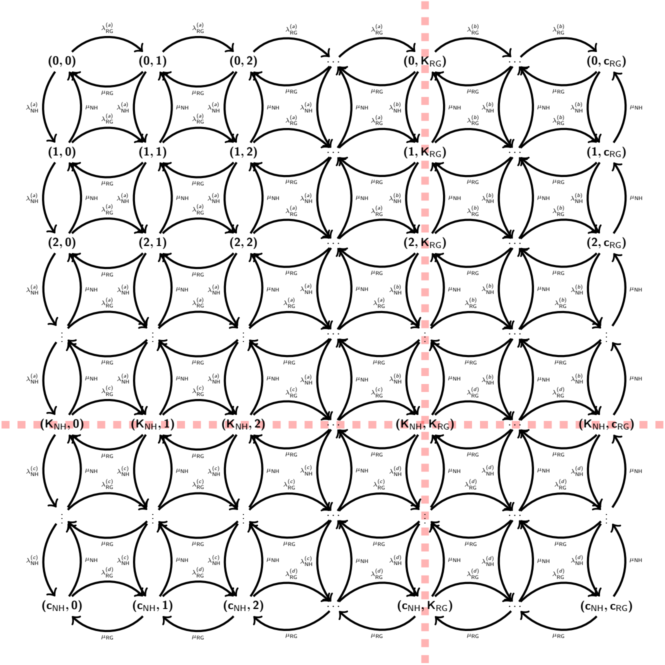
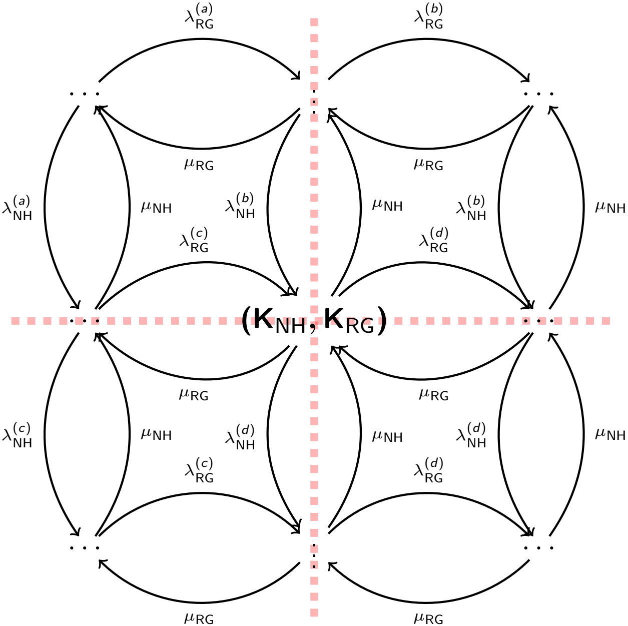
Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one., *Submitted to Anaesthesia*


Divert?
RG
NH
Divert?


K_{RG}
K_{NH}
$\lambda_h^{(a)}$
$\lambda_h^{(b)}$
$\lambda_h^{(c)}$
$\lambda_h^{(d)}$

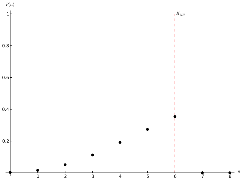


$(0,0)$ $(0,1)$ $(0,2)$ $(0,K_{RG})$ $(0,c_{RG})$
$(1,0)$ $(1,1)$ $(1,2)$ $(1,K_{RG})$ $(1,c_{RG})$
$(2,0)$ $(2,1)$ $(2,2)$ $(2,K_{RG})$ $(2,c_{RG})$
$(K_{NH},0)$ $(K_{NH},1)$ $(K_{NH},2)$ (K_{NH},K_{RG}) (K_{NH},c_{RG})
$(c_{NH},0)$ $(c_{NH},1)$ $(c_{NH},2)$ (c_{NH},K_{RG}) (c_{NH},c_{RG})
$\lambda_{RG}^{(a)}$ $\lambda_{RG}^{(b)}$ $\lambda_{RG}^{(c)}$ $\lambda_{RG}^{(d)}$
$\lambda_{NH}^{(a)}$ $\lambda_{NH}^{(b)}$ $\lambda_{NH}^{(c)}$ $\lambda_{NH}^{(d)}$
μ_{RG} μ_{NH}

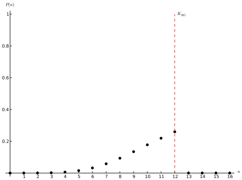


$\lambda_{\mathrm{RG}}^{(a)}$
$\lambda_{\mathrm{RG}}^{(b)}$
μ_{RG}
μ_{RG}
$\lambda_{\mathrm{NH}}^{(a)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(b)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(b)}$
μ_{NH}
$\lambda_{\mathrm{RG}}^{(c)}$
$\lambda_{\mathrm{RG}}^{(d)}$
$(\mathrm{K}_{\mathrm{NH}}, \mathrm{K}_{\mathrm{RG}})$
μ_{RG}
μ_{RG}
$\lambda_{\mathrm{NH}}^{(c)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(d)}$
μ_{NH}
$\lambda_{\mathrm{NH}}^{(d)}$
μ_{NH}
$\lambda_{\mathrm{RG}}^{(c)}$
$\lambda_{\mathrm{RG}}^{(d)}$
μ_{RG}
μ_{RG}

$(K_{\mathrm{NH}}, K_{\mathrm{RG}}) = (6, 12)$:

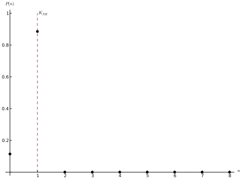


$h = NH$

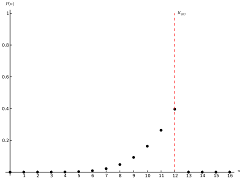


$h = RG$

$(K_{\mathrm{NH}}, K_{\mathrm{RG}}) = (1, 12)$:



$h = \mathrm{NH}$



$h = \mathrm{RG}$

For all $h \in \{\mathrm{NH}, \mathrm{RG}\}$ minimise:

$$(U_h - t)^2$$

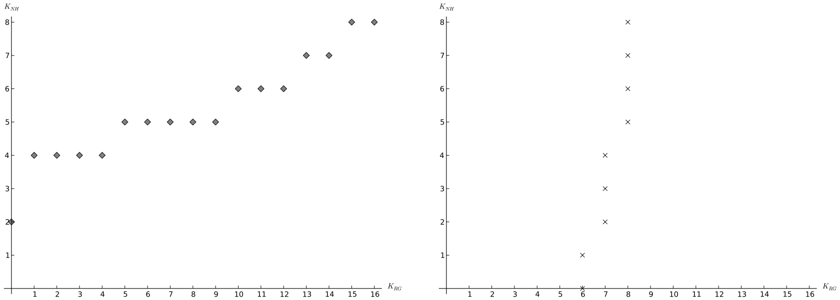
Subject to:

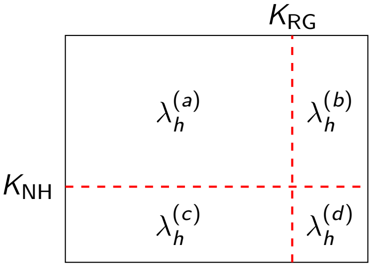
$$\begin{aligned} 0 \leq & K_h \leq c_h \\ & K_h \in \mathbb{Z} \end{aligned}$$

$$A = \begin{pmatrix} U_{\mathsf{NH}}(1,1) - t & \dots & U_{\mathsf{NH}}(1,c_{\mathsf{RG}}) - t \\ U_{\mathsf{NH}}(2,1) - t & \dots & U_{\mathsf{NH}}(2,c_{\mathsf{RG}}) - t \\ \vdots & \ddots & \vdots \\ U_{\mathsf{NH}}(c_{\mathsf{NH}},1) - t & \dots & U_{\mathsf{NH}}(c_{\mathsf{NH}},c_{\mathsf{RG}}) - t \end{pmatrix}$$

$$B = \begin{pmatrix} U_{\mathsf{RG}}(1,1) - t & \dots & U_{\mathsf{RG}}(1,c_{\mathsf{RG}}) - t \\ U_{\mathsf{RG}}(2,1) - t & \dots & U_{\mathsf{RG}}(2,c_{\mathsf{RG}}) - t \\ \vdots & \ddots & \vdots \\ U_{\mathsf{RG}}(c_{\mathsf{RG}},1) - t & \dots & U_{\mathsf{RG}}(c_{\mathsf{RG}},c_{\mathsf{RG}}) - t \end{pmatrix}$$

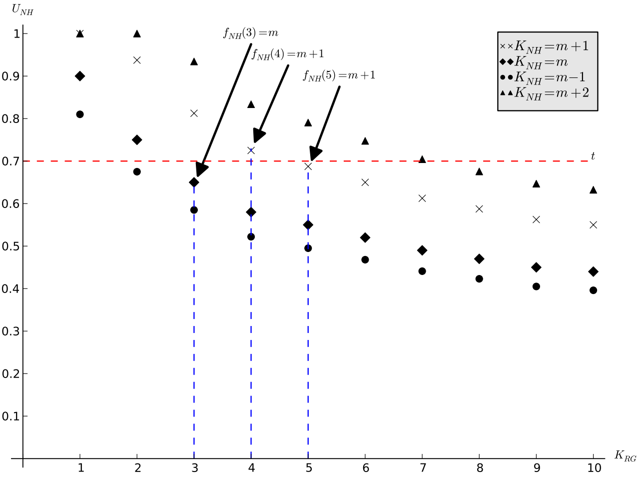
Theorem.
Let $f_h(k) : [1, c_{\bar{h}}] \to [1, c_h]$ be the best response of player $h \in \{\mathrm{NH}, \mathrm{RG}\}$ to the diversion threshold of $\bar{h} \neq h$ ($\bar{h} \in \{\mathrm{NH}, \mathrm{RG}\}$). If $f_h(k)$ is a non-decreasing function in k then the game has at least one Nash Equilibrium in Pure Strategies.


K_{NH}
K_{RG}
K_{NH}
K_{RG}

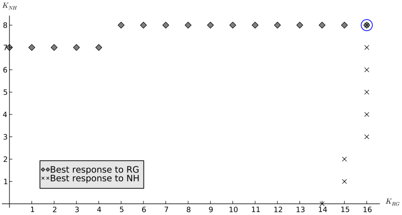


Lemma.

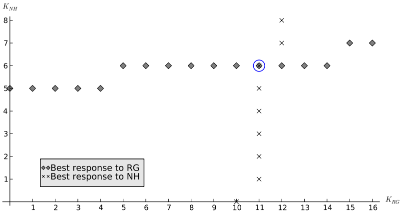
- If $\lambda_{\mathrm{NH}}^{(a)} \leq \lambda_{\mathrm{NH}}^{(b)}$ and $\lambda_{\mathrm{NH}}^{(c)} \leq \lambda_{\mathrm{NH}}^{(d)}$ then $f_{\mathrm{NH}}(k)$ is a non-decreasing function in k.
- If $\lambda_{\mathrm{RG}}^{(a)} \leq \lambda_{\mathrm{RG}}^{(c)}$ and $\lambda_{\mathrm{RG}}^{(b)} \leq \lambda_{\mathrm{RG}}^{(d)}$ then $f_{\mathrm{RG}}(k)$ is a non-decreasing function in k.



U_{NH}
K_{RG}
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1 2 3 4 5 6 7 8 9 10
$f_{NH}(3) = m$
$f_{NH}(4) = m+1$
$f_{NH}(5) = m+1$
t
$K_{NH} = m+1$
$K_{NH} = m$
$K_{NH} = m-1$
$K_{NH} = m+2$

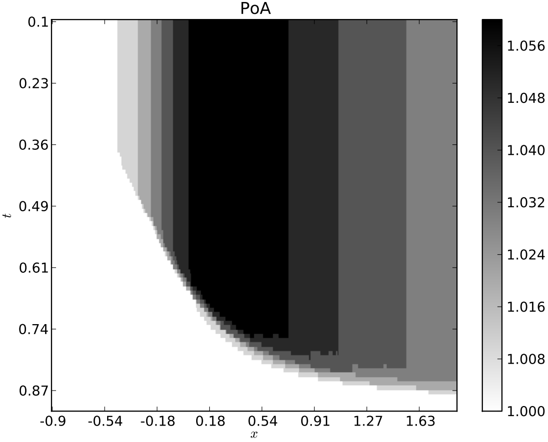

K_{NH}
K_{RG}
Best response to RG
Best response to NH

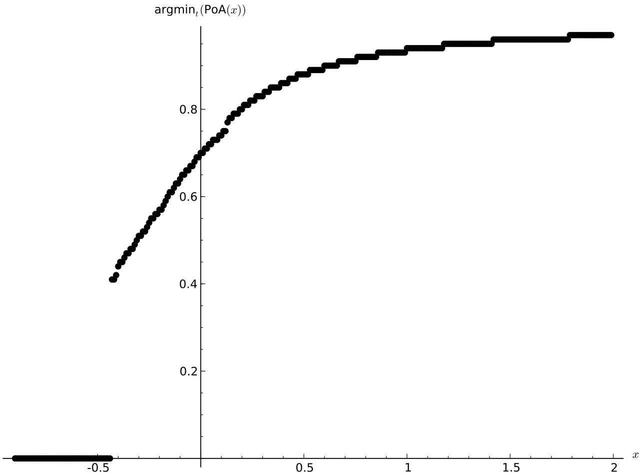
$(t = 0.8)$


K_{NH}
K_{RG}
Best response to RG
Best response to NH

$(t = 0.6)$

$$\mathrm{PoA} = \frac{T^*}{\widetilde{T}}$$


PoA
0.1
0.23
0.36
0.49
0.61
0.74
0.87
t
-0.9
-0.54
-0.18
0.18
0.54
0.91
1.27
1.63
x
1.056
1.048
1.040
1.032
1.024
1.016
1.008
1.000



$\mathrm{argmin}_t(\mathrm{PoA}(x))$
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

Conclusions

- Developed a strategic form game representation of CCU interaction;
- Proved structural properties of equilibrium behaviour;
- Identified a potential justified approach to obtaining policies.

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Measuring the Price of Anarchy in Critical Care Unit Interactions, *Submitted to OMEGA*

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