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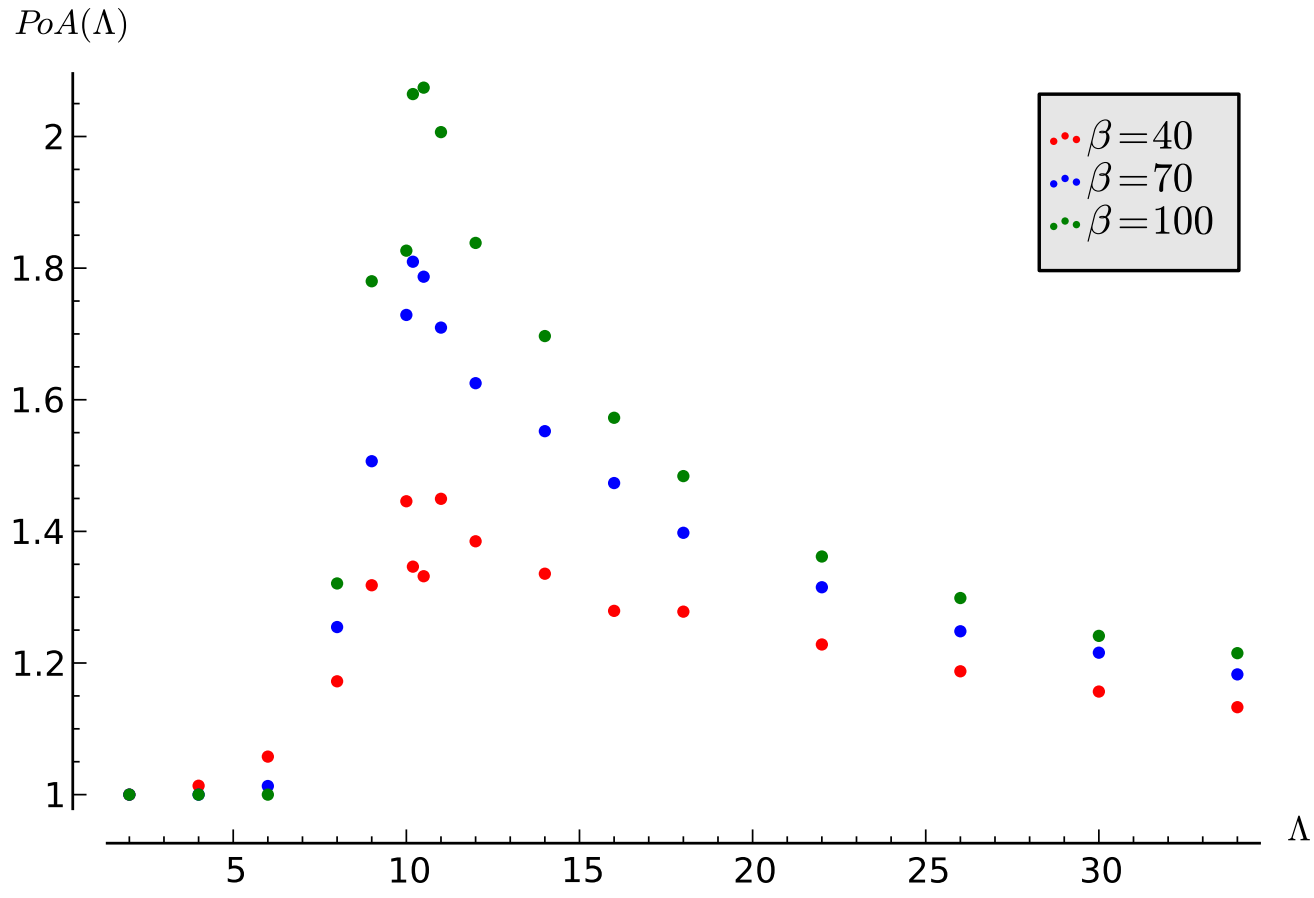
Professor Jeff Griffiths

$$\begin{pmatrix} (2,2) & (5,0) \\ (0,5) & (4,4) \end{pmatrix}$$


$PoA(\Lambda)$
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
Λ
$\beta = 40$
$\beta = 70$
$\beta = 100$

What about the controllers?

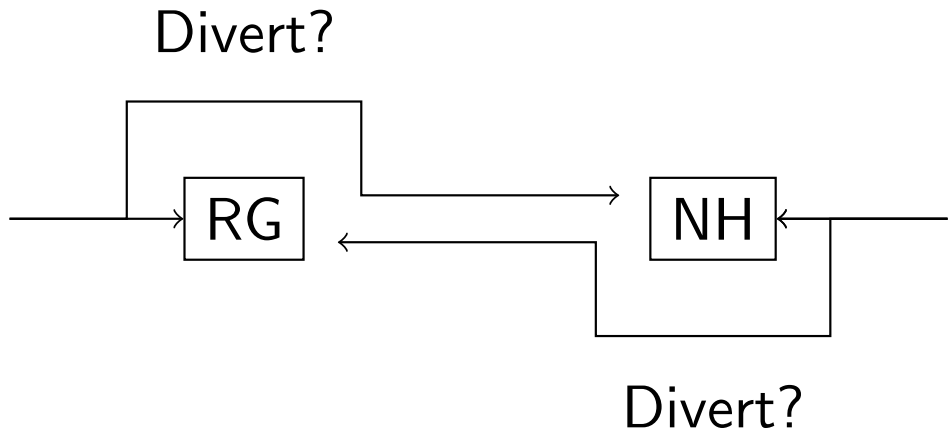
What about the controllers?

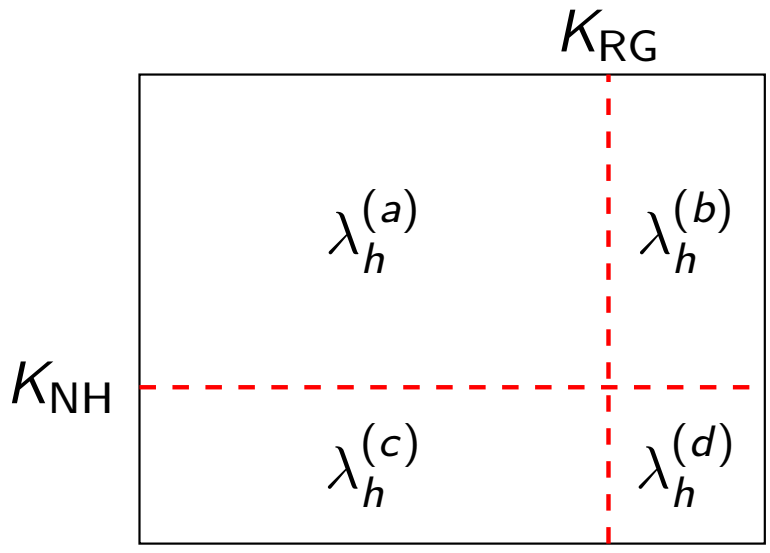
S. Deo and I. Gurvich. **Centralized vs. Decentralized Ambulance Diversion: A Network Perspective.** *Management Science*, 57(7):13001319, May 2011.

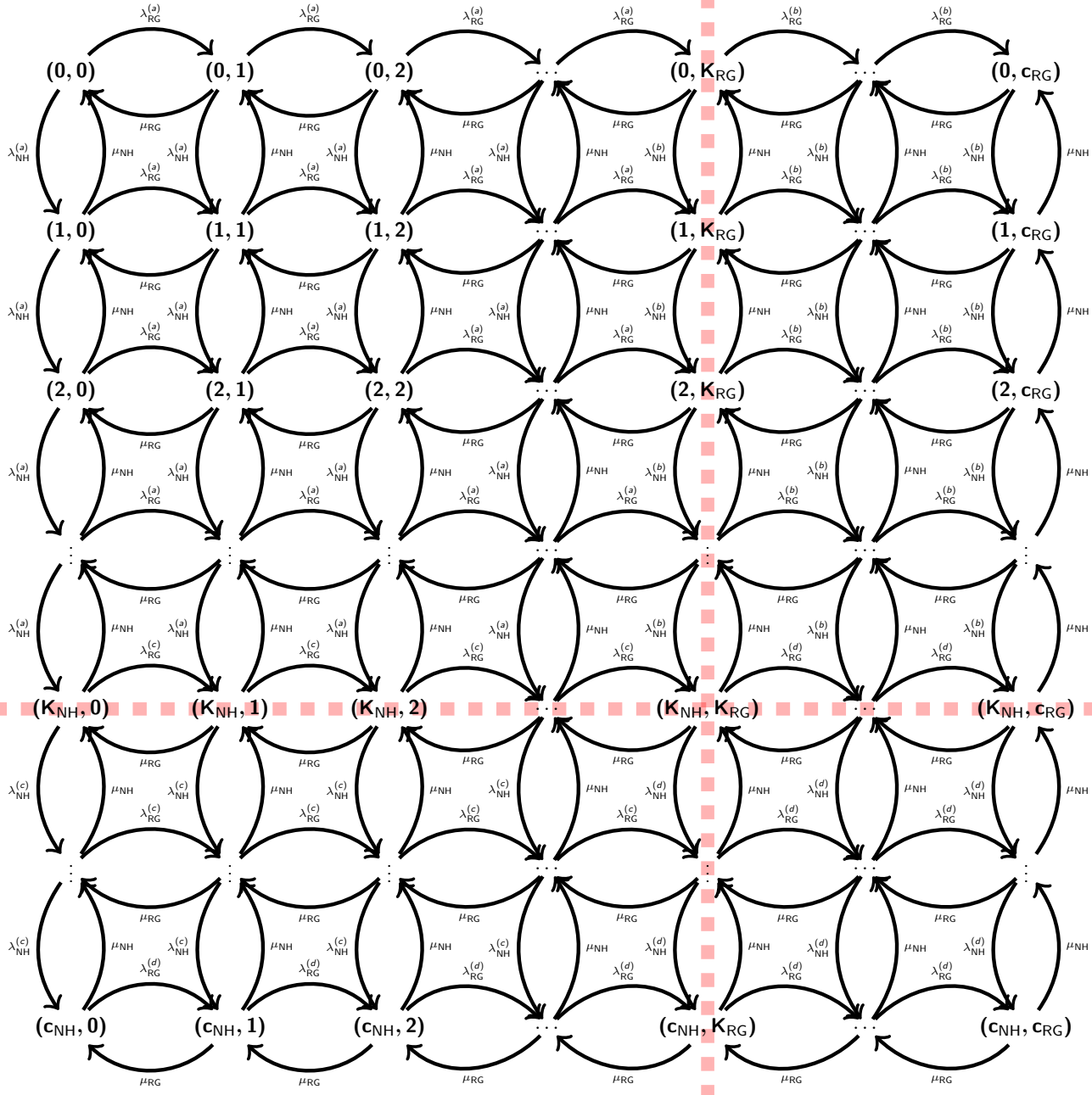
[INCLUDE ANY PLOT FROM PAPER WITH IZA]

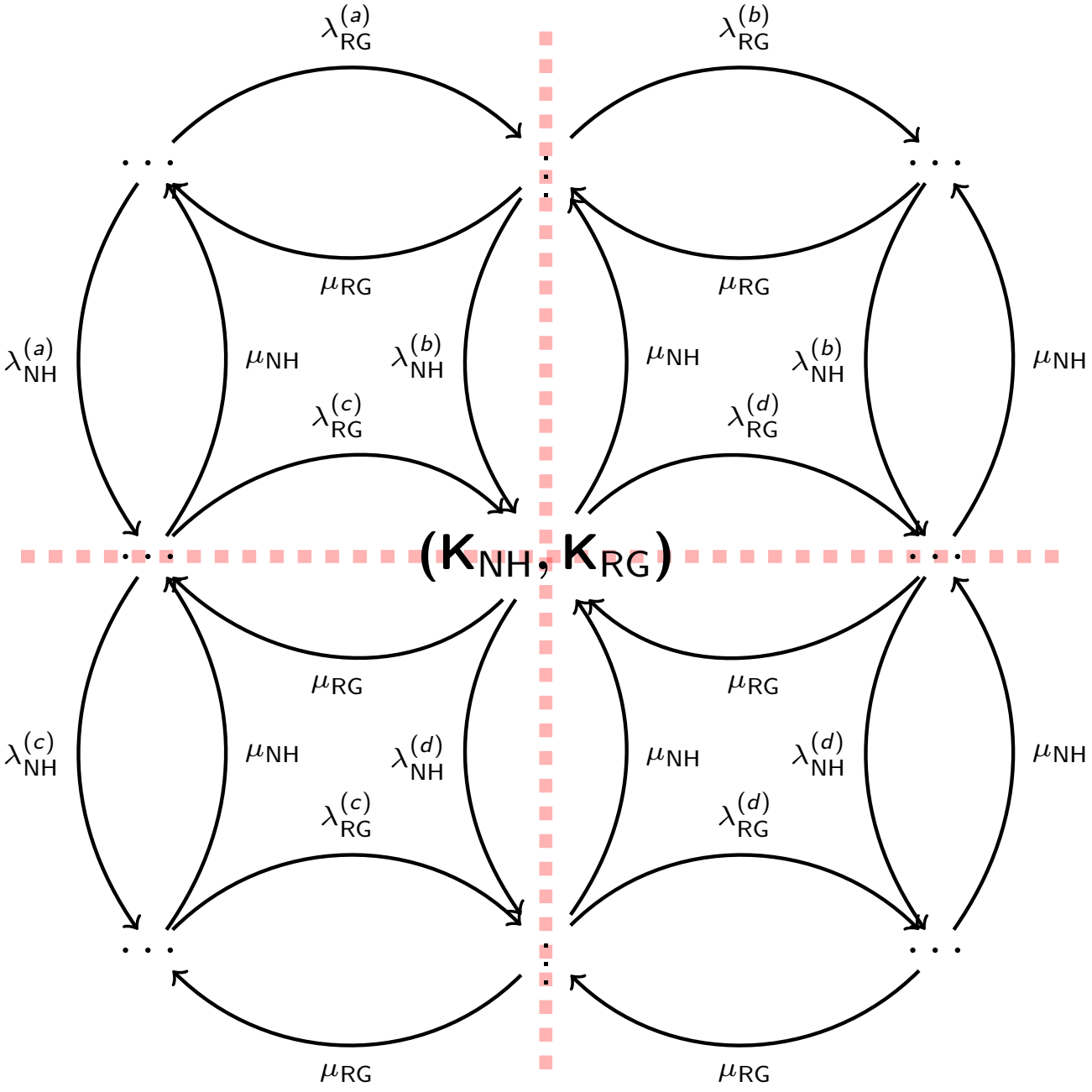
[INCLUDE ANY PLOT FROM PAPER WITH IZA]

Mathematical modelling of patient flows to predict critical care capacity required following the merger of two District General Hospitals into one., *Submitted to Anaesthesia*

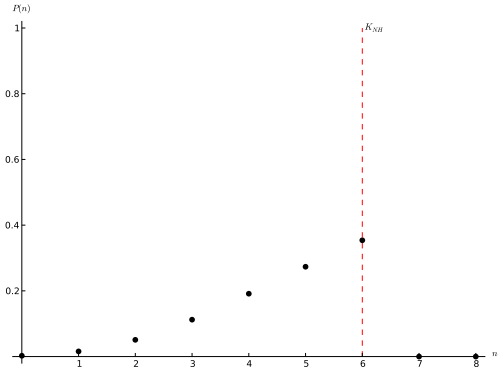

Divert?
RG
NH
Divert?


K_{RG}
K_{NH}
$\lambda_h^{(a)}$
$\lambda_h^{(b)}$
$\lambda_h^{(c)}$
$\lambda_h^{(d)}$

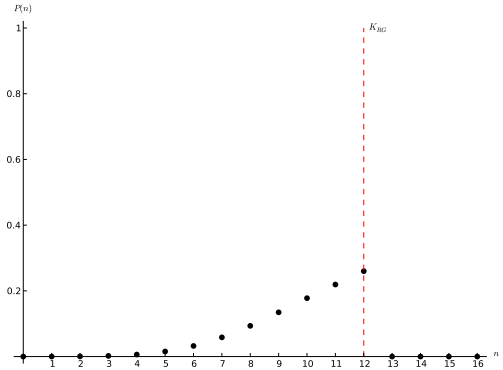

(0, 0)
(0, 1)
(0, 2)
(0, K_RG)
(0, c_RG)
(1, 0)
(1, 1)
(1, 2)
(1, K_RG)
(1, c_RG)
(2, 0)
(2, 1)
(2, 2)
(2, K_RG)
(2, c_RG)
(K_NH, 0)
(K_NH, 1)
(K_NH, 2)
(K_NH, K_RG)
(K_NH, c_RG)
(c_NH, 0)
(c_NH, 1)
(c_NH, 2)
(c_NH, K_RG)
(c_NH, c_RG)


$\lambda^{(a)}_{\mathrm{RG}}$
$\lambda^{(b)}_{\mathrm{RG}}$
μ_{RG}
μ_{NH}
$\lambda^{(a)}_{\mathrm{NH}}$
$\lambda^{(b)}_{\mathrm{NH}}$
$\lambda^{(c)}_{\mathrm{RG}}$
$\lambda^{(d)}_{\mathrm{RG}}$
$(\mathbf{K}_{\mathrm{NH}}, \mathbf{K}_{\mathrm{RG}})$
$\lambda^{(c)}_{\mathrm{NH}}$
$\lambda^{(d)}_{\mathrm{NH}}$

$(K_{\mathrm{NH}}, K_{\mathrm{RG}}) = (6, 12)$:

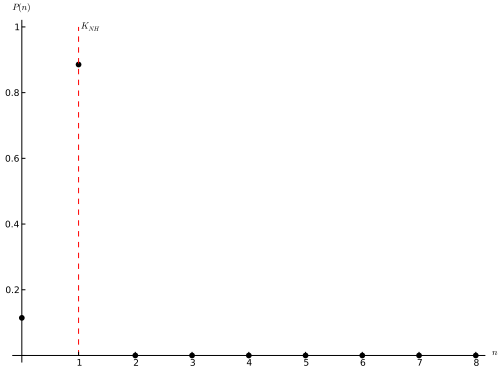


$h = NH$

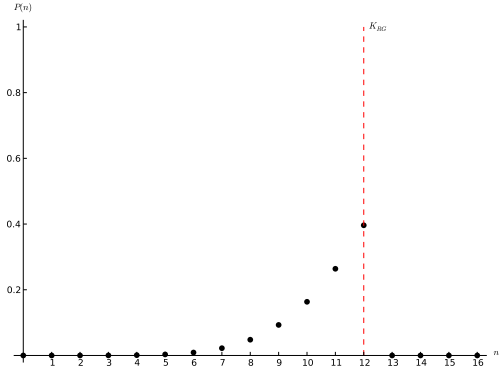


$h = RG$

$(K_{\mathrm{NH}}, K_{\mathrm{RG}}) = (1, 12)$:



$h = \mathrm{NH}$



$h = \mathrm{RG}$

For all $h \in \{\text{NH}, \text{RG}\}$ minimise:

$$(U_h - t)^2$$

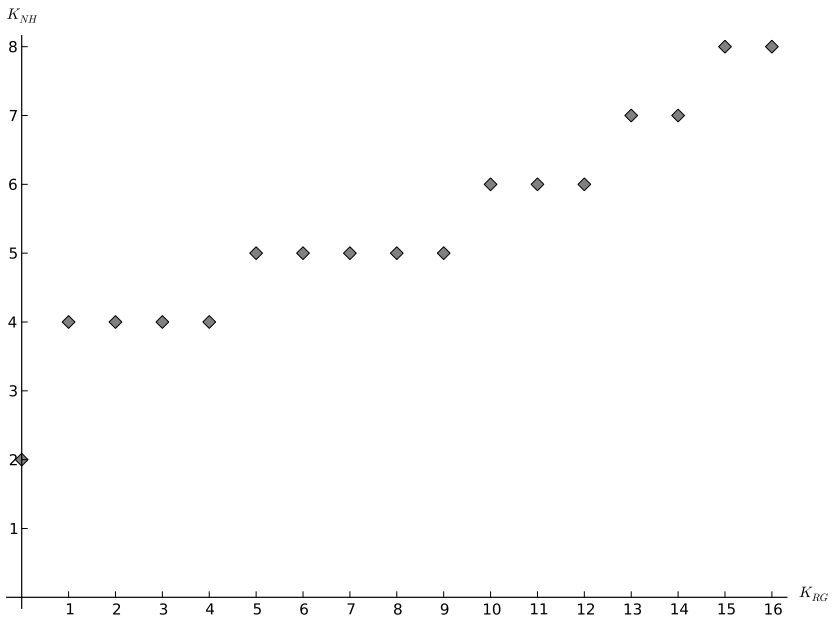
Subject to:

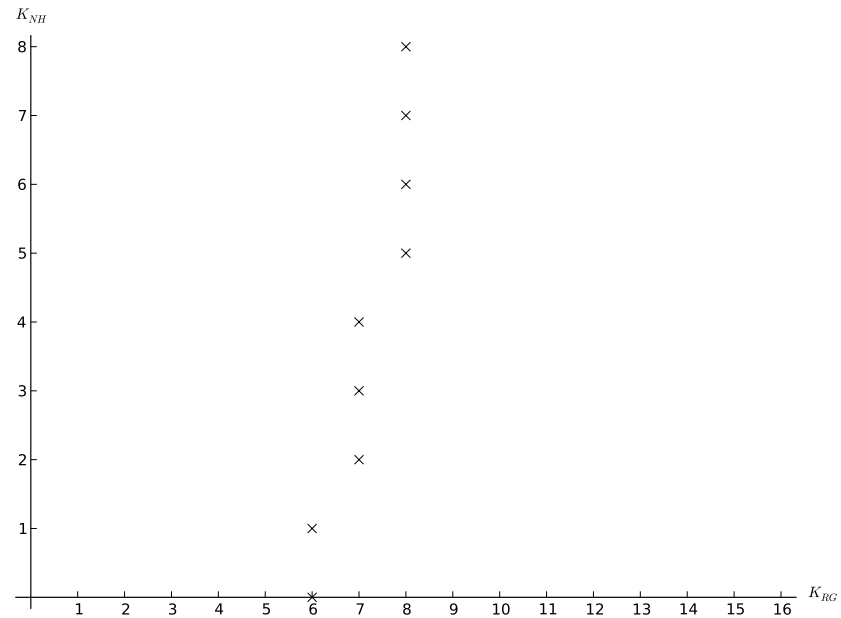
$$\begin{aligned} 0 \leq & K_h \leq c_h \\ & K_h \in \mathbb{Z} \end{aligned}$$

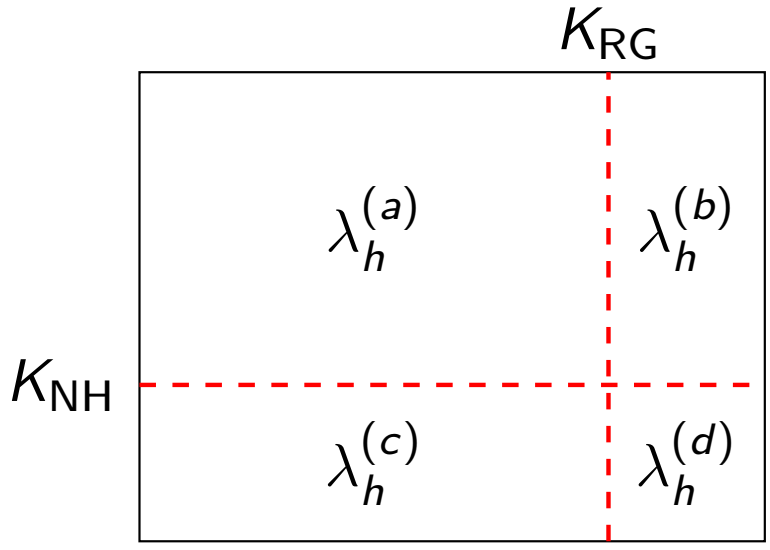
$$A = \begin{pmatrix} (U_{\mathrm{NH}}(1,1) - t)^2 & \dots & (U_{\mathrm{NH}}(1, c_{\mathrm{RG}}) - t)^2 \\ (U_{\mathrm{NH}}(2,1) - t)^2 & \dots & (U_{\mathrm{NH}}(2, c_{\mathrm{RG}}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathrm{NH}}(c_{\mathrm{NH}},1) - t)^2 & \dots & (U_{\mathrm{NH}}(c_{\mathrm{NH}}, c_{\mathrm{RG}}) - t)^2 \end{pmatrix}$$

$$B = \begin{pmatrix} (U_{\mathrm{RG}}(1,1) - t)^2 & \dots & (U_{\mathrm{RG}}(1, c_{\mathrm{RG}}) - t)^2 \\ (U_{\mathrm{RG}}(2,1) - t)^2 & \dots & (U_{\mathrm{RG}}(2, c_{\mathrm{RG}}) - t)^2 \\ \vdots & \ddots & \vdots \\ (U_{\mathrm{RG}}(c_{\mathrm{RG}},1) - t)^2 & \dots & (U_{\mathrm{RG}}(c_{\mathrm{RG}}, c_{\mathrm{RG}}) - t)^2 \end{pmatrix}$$

Theorem.
Let $f_h(k) : [1, c_{\bar{h}}] \to [1, c_h]$ be the best response of player $h \in \{\text{NH}, \text{RG}\}$ to the diversion threshold of $\bar{h} \neq h$ ($\bar{h} \in \{\text{NH}, \text{RG}\}$). If $f_h(k)$ is a non-decreasing function in k then the game has at least one Nash Equilibrium in Pure Strategies.

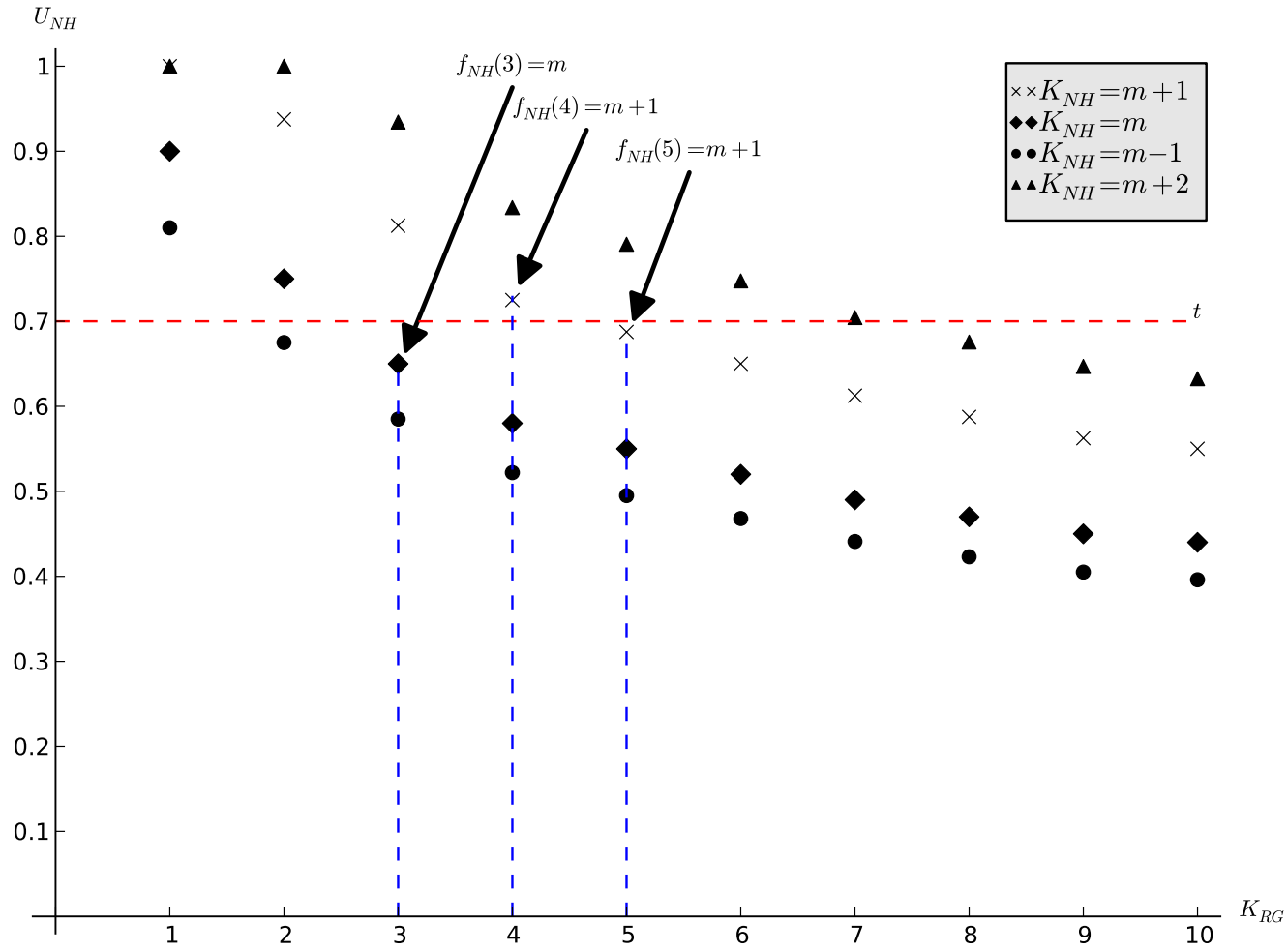

K_{NH}
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
K_{RG}

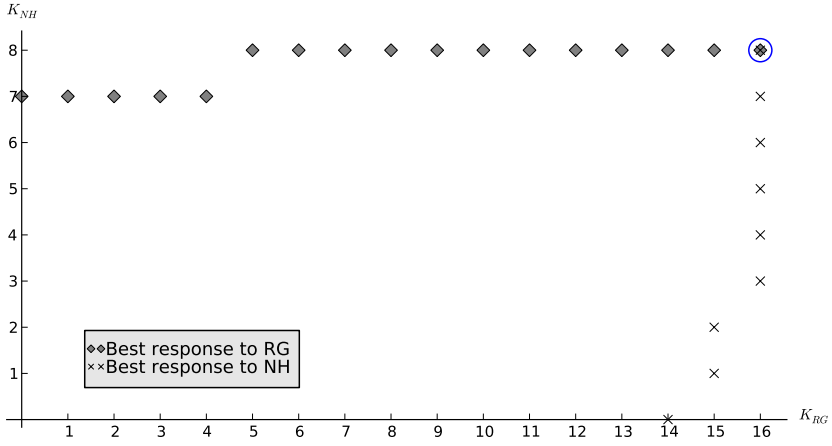

K_{NH}
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
K_{RG}



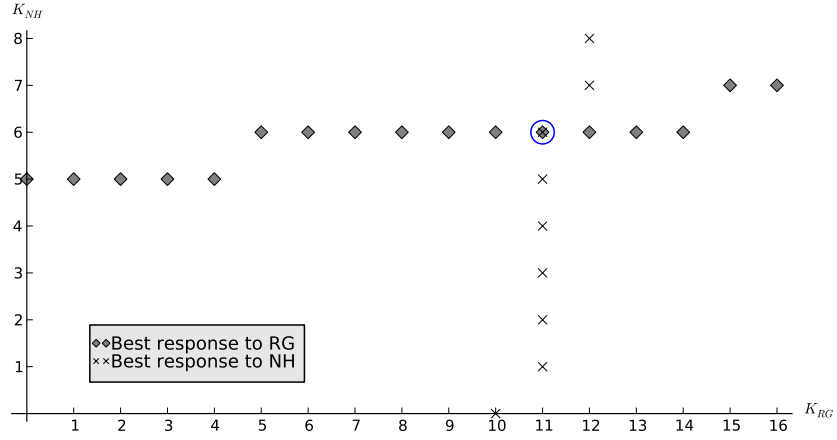
Lemma.

- If $\lambda_{\mathrm{NH}}^{(a)} \leq \lambda_{\mathrm{NH}}^{(b)}$ and $\lambda_{\mathrm{NH}}^{(c)} \leq \lambda_{\mathrm{NH}}^{(d)}$ then $f_{\mathrm{NH}}(k)$ is a non-decreasing function in k.
- If $\lambda_{\mathrm{RG}}^{(a)} \leq \lambda_{\mathrm{RG}}^{(c)}$ and $\lambda_{\mathrm{RG}}^{(b)} \leq \lambda_{\mathrm{RG}}^{(d)}$ then $f_{\mathrm{RG}}(k)$ is a non-decreasing function in k.


U_{NH}
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$f_{NH}(3)=m$
$f_{NH}(4)=m+1$
$f_{NH}(5)=m+1$
t
$\times\times K_{NH}=m+1$
$\blacklozenge\blacklozenge K_{NH}=m$
$\bullet\bullet K_{NH}=m-1$
$\blacktriangle\blacktriangle K_{NH}=m+2$
1
2
3
4
5
6
7
8
9
10
K_{RG}

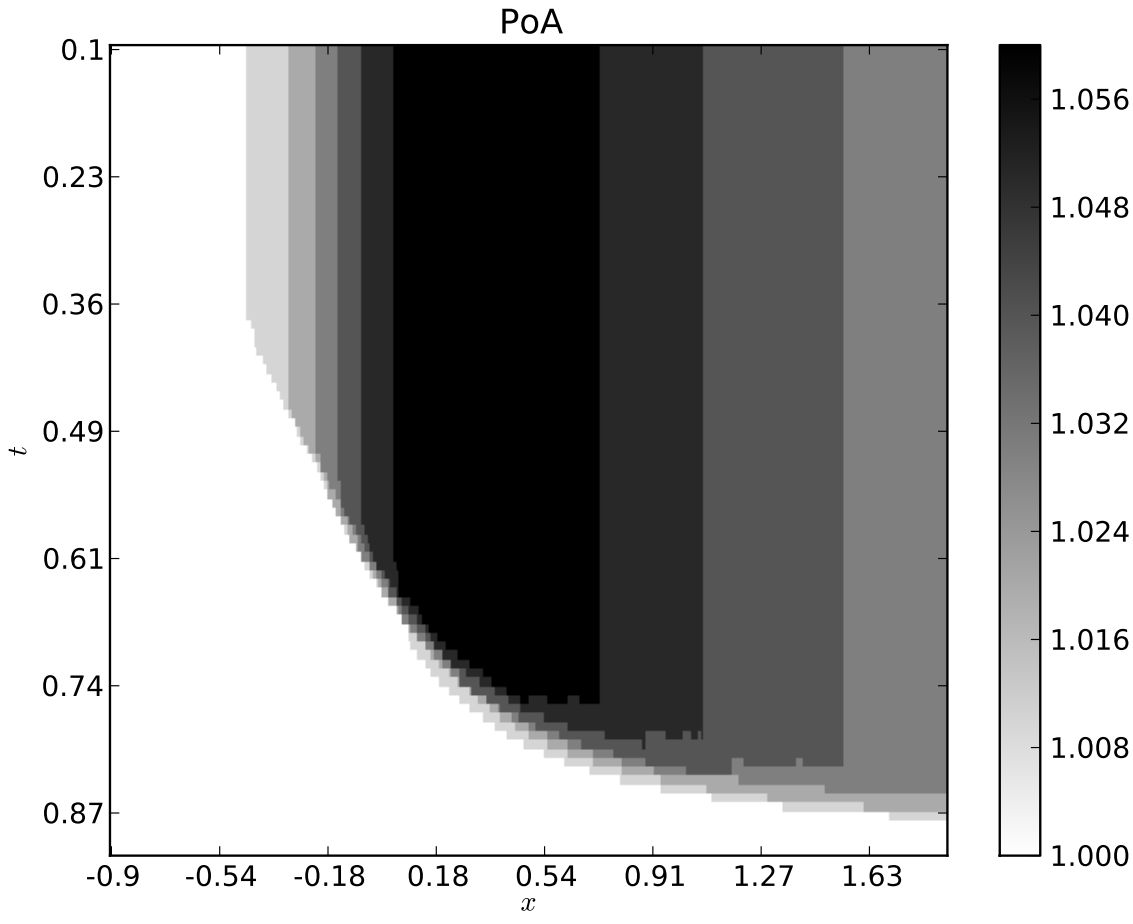

K_{NH}
K_{RG}
Best response to RG
Best response to NH

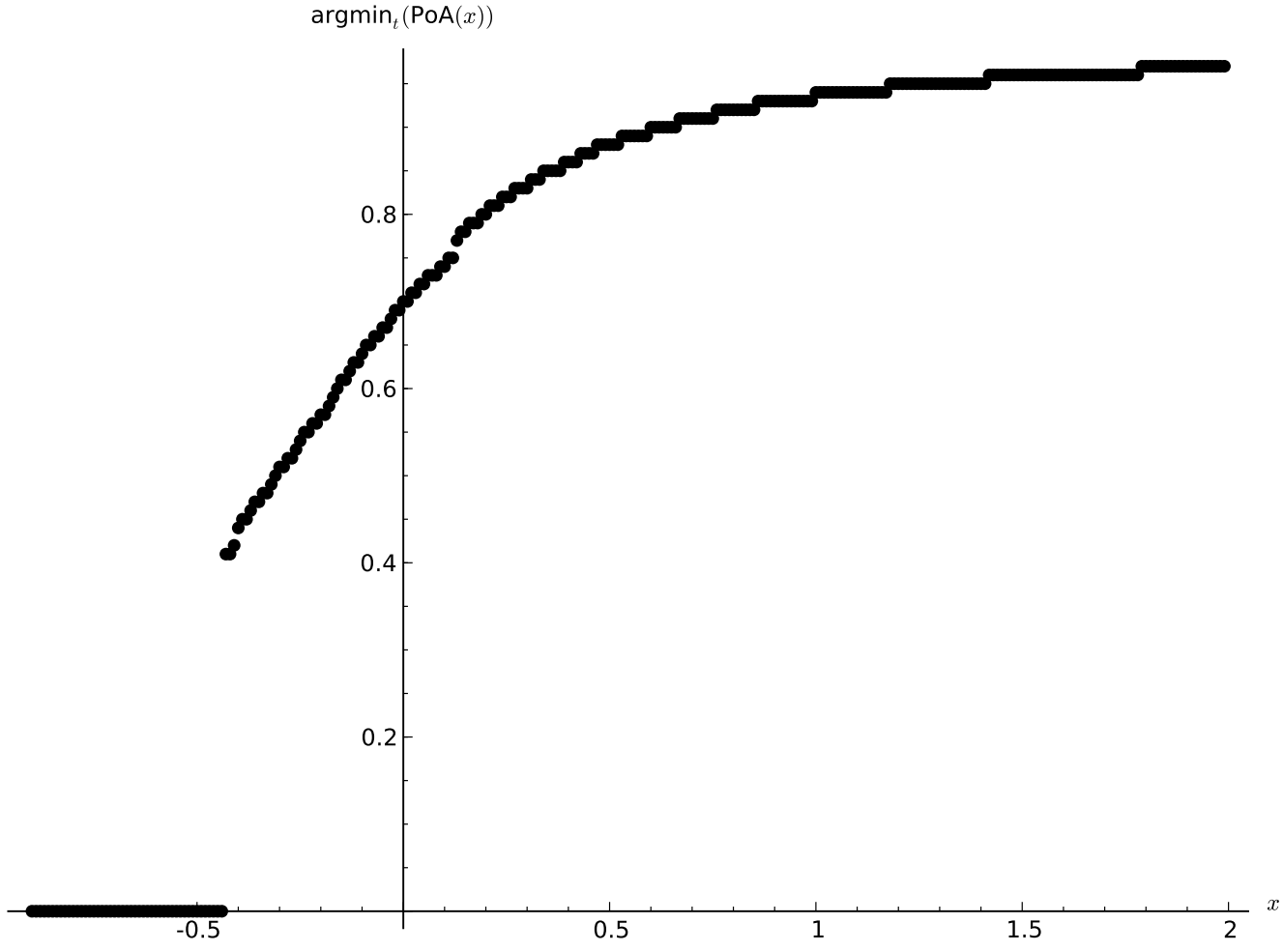
$(t = 0.8)$


K_{NH}
K_{RG}
Best response to RG
Best response to NH

$(t = 0.6)$

$$\mathrm{PoA} = \frac{T^*}{\widetilde{T}}$$


PoA
0.1
0.23
0.36
0.49
0.61
0.74
0.87
t
-0.9
-0.54
-0.18
0.18
0.54
0.91
1.27
1.63
x
1.056
1.048
1.040
1.032
1.024
1.016
1.008
1.000


argmin$_t$(PoA(x))
0.8
0.6
0.4
0.2
-0.5
0.5
1
1.5
2
x

Conclusions

- Developed a strategic form game representation of CCU interaction;
- Proved structural properties of equilibrium behaviour;
- Identified a potential justified approach to obtaining policies.

Conclusions

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Measuring the Price of Anarchy in Critical Care Unit Interactions, *Submitted to OMEGA*

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