Course project: Reconstruction of flow field for a lid driven cavity flow

Contributed by Dr. Khemraj Shukla
Division of Applied Mathematics, Brown University

Project statement

The lid-driven cavity is a standard test case for verifying the accuracy of new computational methods for incompressible Navier–Stokes equations. The governing equation for steady state incompressible flow for a cavity reads

$$(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}, \qquad \text{in } \Omega \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \text{in } \Omega, \tag{2}$$

$$\tag{3}$$

with prescribed boundary conditions as shown in Figure 3.

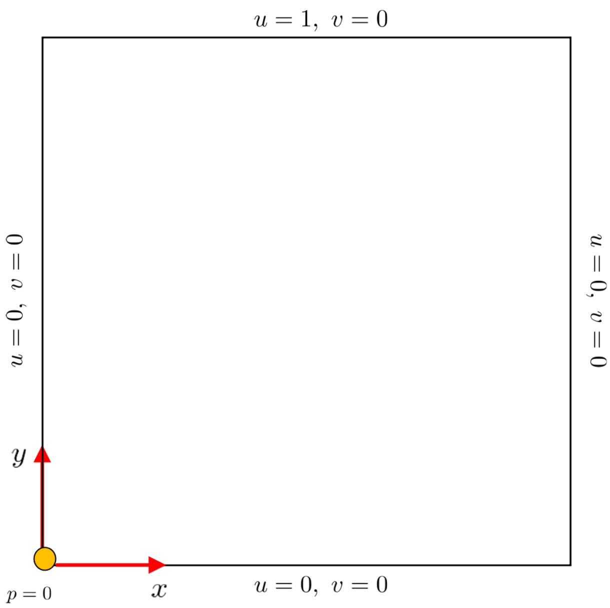

Figure 1: Domain and boundary condition for cavity flow

Figure 3 represents the domain with extent $[0, 1] \otimes [0, 1]$ and no-slip boundary conditions are applied at left, bottom and right boundaries. The top boundary is moving with constant velocity in the positive x direction. The solutions of $\boldsymbol{u} = [u, v]$ and p for $Re = 100$ is given in Figure 2.

Dataset

The data for $\boldsymbol{u}$ obtained by solving the Equation 1 using spectral element method is provided. The data is provided in ".mat" format. The data can be loaded using *scipy* package of python, e.g. *scipy.io.loadmat*.

Tasks

1. **Forward problem:** Compute the solution of Equation 1 for $\boldsymbol{u}$ using Physics-informed neural network [1] $Re = 100$ with boundary conditions as described in Figure 1.

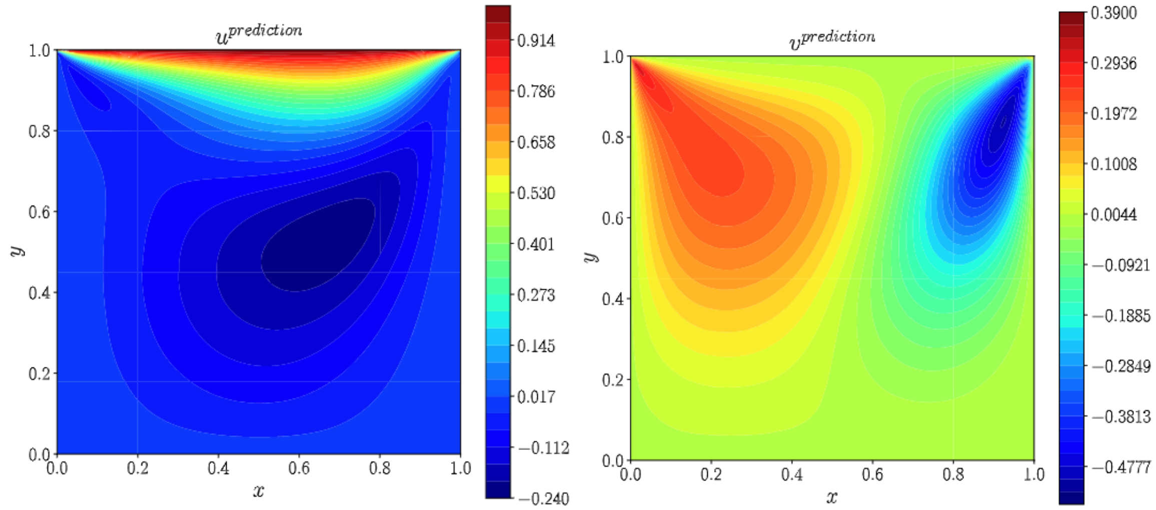

Figure 2: u, v, and p for a lid driven cavity flow

2. In this task randomly sample 10,20,30, and 40 data points in a patch inside domain (as shown in Figure 3) for $\boldsymbol{u}$ and p and reconstruct the flow field $u = [u, v]$ for entire domain. Report the L2-norm of relative errors between reconstructed and actual fields. Write a brief summary on your observations.

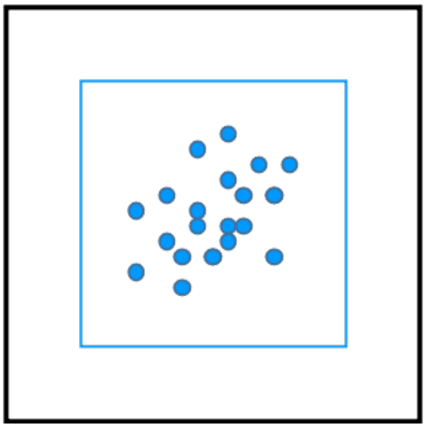

Figure 3: An example of patch and sample strategy.

Programming Options

You may use TensorFlow, PyTorch, or Modulus for completing the tasks.

References

[1] M. Raissi, P. Perdikaris, and G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, 2019.