Foldscheme Collection

Design and Development of Folding Mechanism

E²F | **RWTH**AACHEN UNIVERSITY

3(4R), MiuraOri

Main directional motion:	two translations
Geometric space formula:	3(4R)
Degree of freedom (DOF):	F = 1 + 0 (for each additional)

Structure:
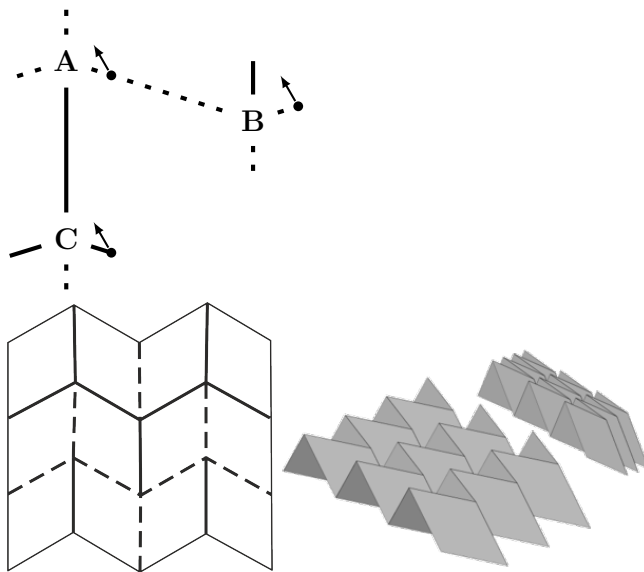
Common name: Miura-Ori [Miura, 1985]

Vertices information:

$A(4R) = \vee(\pi - \alpha) \vee (\pi - \alpha) \vee (\alpha) \wedge (\alpha) = 2\pi$
$B(4R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - \alpha) \vee (\pi - \alpha) = 2\pi$
$C(4R) = \wedge(\pi - \alpha) \wedge (\pi - \alpha) \wedge (\alpha) \vee (\alpha) = 2\pi$

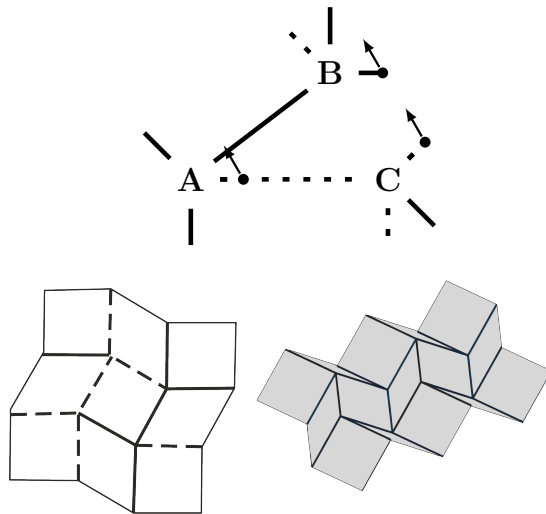**Foldline information:**

$\overline{AB} = l_1$

Combinations:

- Shift A to C
- Invert
- Mirror $\overline{AC}$

Key Figures:

$K_{\mathrm{TpF}}(\alpha) = 0.0086\alpha^2 - 0.11\alpha + 13$ (examplarily, is application specific)

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3(4R), Barreto's Mars

Main directional motion:	two translations
Geometric space formula:	3(4R)
Degree of freedom (DOF):	F = 1 + 0 (for each additional)

Structure:

Common name: Barreto's Mars [Barreto, 1997]

Vertices information:

$$A(4R) = \vee(\alpha) \wedge (\tfrac{\pi}{2}) \vee (\pi - \alpha) \wedge (\tfrac{\pi}{2}) = 2\pi$$
$$B(4R) = \wedge(\tfrac{\pi}{2}) \vee (\alpha) \wedge (\tfrac{\pi}{2}) \wedge (\pi - \alpha) = 2\pi$$
$$C(4R) = \wedge(\pi - \alpha) \wedge (\tfrac{\pi}{2}) \wedge (\alpha) \vee (\tfrac{\pi}{2}) = 2\pi$$

Foldline information:

$$\overline{AB} = l_1$$
$$\overline{AC} = l_2$$
$$\overline{AB} = \overline{CD}$$
$$\overline{AF} = \overline{BC}$$

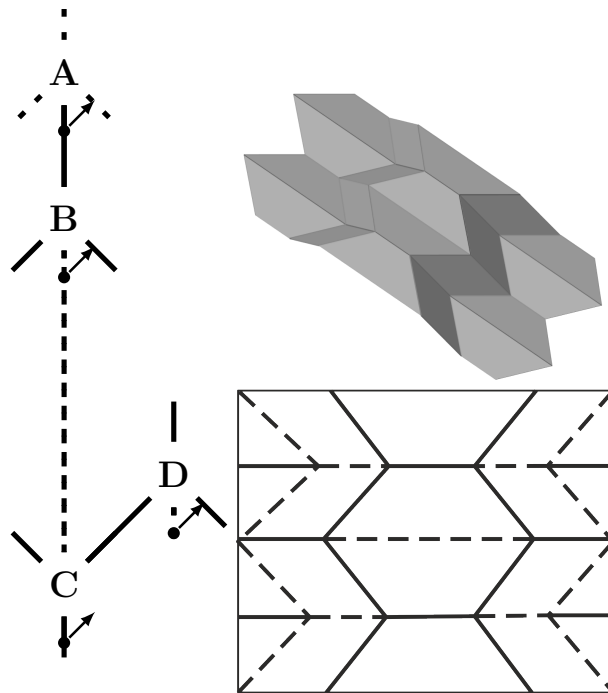**Combinations:**

•Rotate A by π, shift to B, invert

Key Figures:

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6(4R), Tachi-Miura

Main directional motion:	two translations
Geometric space formula:	6(4R)
Degree of freedom (DOF):	F = 1 + 0 (for each additional)

Structure:

Common name: Tachi-Miura [Tachi 2010]

Vertices information:

$A(4\underline{R}6\underline{R}) = \wedge(\alpha) \vee (\pi - \alpha) \vee (\pi - \alpha) \vee (\alpha) = 2\pi$

$B(4\underline{R}6\underline{R}) = \vee\underline{\wedge}(\alpha)\wedge\underline{\vee}(\pi - \alpha)\wedge\underline{\vee}(\pi - \alpha)\wedge\underline{\vee}(\alpha) = 2\pi$

$C(4\underline{R}6\underline{R}) = \wedge\underline{\vee}(\pi - \alpha) \wedge (\alpha)\vee\underline{\wedge}(\alpha)\wedge\underline{\vee}(\pi - \alpha) = 2\pi$

$D(4\underline{R}6\underline{R}) = \vee\underline{\wedge}(\alpha)\wedge\underline{\vee}(\pi - \alpha)\wedge\underline{\vee}(\pi - \alpha)\wedge\underline{\vee}(\alpha) = 2\pi$

$\cancel{E(4\underline{R}6\underline{R}) = \vee(\pi - \alpha) \wedge (\alpha) \wedge (\alpha) \wedge (\pi - \alpha) = 2\pi}$

Foldline information:

$\overline{AB} = l_1$
$\overline{BC} = l_2$
$\overline{CD} = l_3$

Combinations:

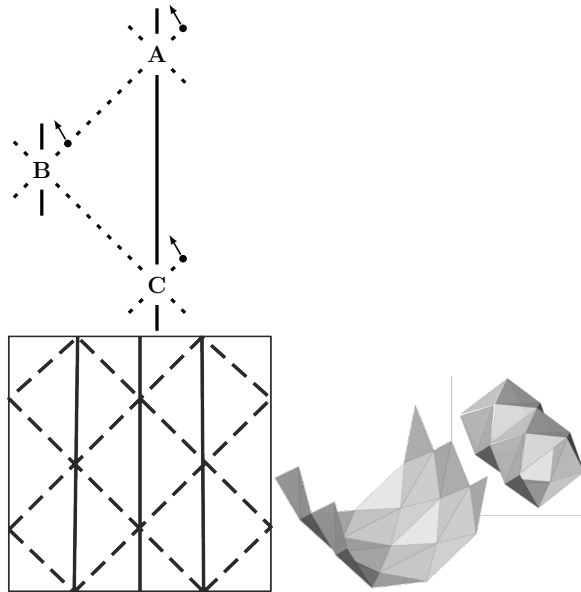• Rotate B by π, shift B to C
• Shift A to D

Key Figures:

3(6R), Yoshimura

Main directional motion:	cylindrical, globally self-intersecting
Geometric space formula:	3(6R)
Degree of freedom (DOF):	F = 6 + 1 (for each additional)

Structure:

Common name: Yoshimura [Yoshimura, 1955]

Vertices information:

$A(6R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) \vee (\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) = 2\pi$
$B(6R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) \vee (\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) = 2\pi$
$C(6R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) \vee (\alpha) \wedge (\alpha) \vee (\pi - 2\alpha) = 2\pi$

Foldline information:

$\overline{AC} = l_1$
$\overline{BC} = \overline{AB} = \cos(\alpha) \cdot l_1$

Combinations:

• Shift from A to C
• Rotate around A with π
• Rotate around B with π
• Rotate around C with π
• Mirror along $\overline{AC}$

Key Figures:

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4(4R), Chicken Wire Tessellation

Main directional motion:	cylindrical, globally self-intersecting
Geometric space formula:	4(4R)
Degree of freedom (DOF):	F = 1 + 0 (for each additional)

Structure:
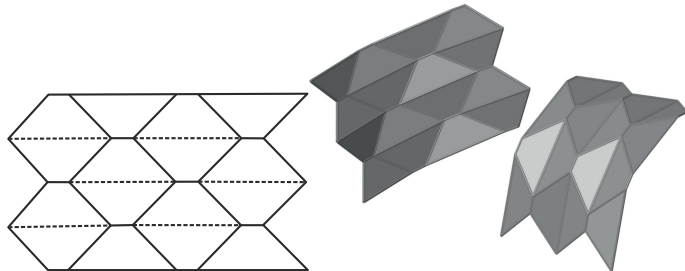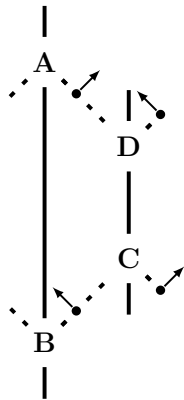
Common name: Chicken Wire [Miura 1969]

Vertices information:

$A(4R) = \vee(\pi - \alpha) \wedge (\pi - \alpha) \vee (\alpha) \wedge (\alpha) = 2\pi$
$B(4R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - \alpha) \wedge (\pi - \alpha) = 2\pi$
$C(4R) = \vee(\pi - \alpha) \wedge (\pi - \alpha) \vee (\alpha) \wedge (\alpha) = 2\pi$
$D(4R) = \vee(\alpha) \wedge (\alpha) \vee (\pi - \alpha) \wedge (\pi - \alpha) = 2\pi$

Foldline information:

$\overline{AB} = l_1$
$\overline{BC} = \overline{AD} = l_2$
$\overline{CD} = l_1 - 2 \cdot \cos(\alpha) \cdot l_2$

Combinations:

• Mirror along $\overline{AB}$
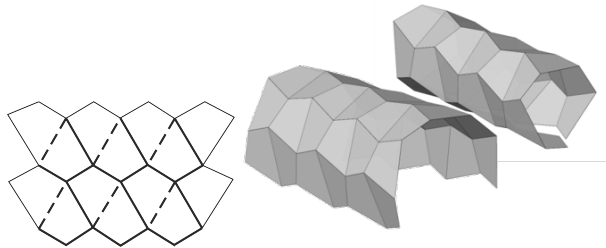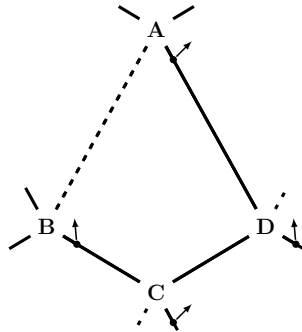• Shift from A to C
• Shift from D to B

Key Figures:

$K_{\text{TpF}}(l) = -1.61 \cdot l^{-0.79} + 6.12$ (examplarily, is application specific)

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4(4R), Huffman Grid

Main directional motion:	screwing
Geometric space formula:	4(4R)
Degree of freedom (DOF):	F = 1 + 0 (for each additional)

Structure:

Common name: Huffman Grid [Huffmann, 1976]

Vertices information:

$$A(4R) = \wedge(\frac{\pi}{2}) \wedge (\pi - \alpha) \wedge (\frac{\pi}{2}) \vee (\alpha) = 2\pi$$
$$B(4R) = \wedge(\frac{\pi}{2}) \vee (\alpha) \wedge (\frac{\pi}{2}) \wedge (\pi - \alpha) = 2\pi$$
$$C(4R) = \wedge(\frac{\pi}{2}) \wedge (\pi - \alpha) \wedge (\frac{\pi}{2}) \vee (\alpha) = 2\pi$$
$$D(4R) = \wedge(\frac{\pi}{2}) \vee (\alpha) \wedge (\frac{\pi}{2}) \wedge (\pi - \alpha) = 2\pi$$

Foldline information:

$$\overline{AB} = \overline{AD} = l_1$$
$$\overline{BC} = \overline{CD} = l_2$$

Combinations:

• Shift from B to A and shift around π
• Shift from A to C
• Shift from B to D

Key Figures:

Screwing Foldschemes

3(6R), Origami Spring
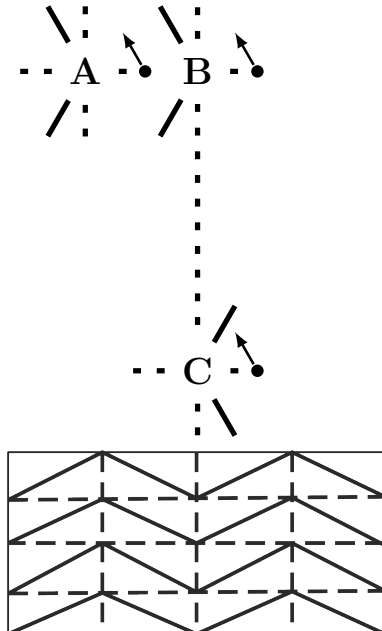
Main directional motion:	screwing
Geometric space formula:	3(6R)
Degree of freedom (DOF):	$F = 7 + x$ (for each additional)

Structure:

Common name: Origami spring

Vertices information:

$$A(6R) = \vee(\frac{\pi}{2}) \vee (\alpha) \wedge (\frac{\pi}{2} - \alpha) \vee (\frac{\pi}{2} - \alpha) \wedge (\alpha) \vee (\frac{\pi}{2}) = 2\pi$$

$$B(6R) = \vee(\frac{\pi}{2}) \vee (\alpha) \wedge (\frac{\pi}{2} - \alpha) \vee (\frac{\pi}{2} - \alpha) \wedge (\alpha) \vee (\frac{\pi}{2}) = 2\pi$$

$$C(6R) = \vee(\frac{\pi}{2} - \alpha) \wedge (\alpha) \vee (\frac{\pi}{2}) \vee (\frac{\pi}{2}) \vee (\alpha) \wedge (\frac{\pi}{2} - \alpha) = 2\pi$$

Foldline information:

$\overline{AB} = l_1$
$\overline{BC} = l_2$

Combinations:
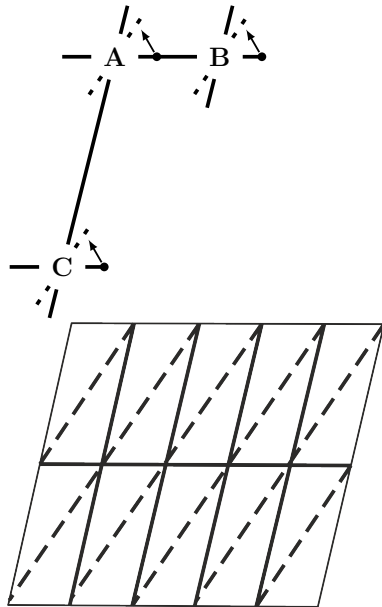
• Shift from A to B
• Mirror along $\overline{AB}$

Key Figures:

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Screwing Foldschemes

3(6R), Octagonal Tube Spring

Main directional motion:	screwing
Geometric space formula:	3(6R)
Degree of freedom (DOF):	$F = 7 + x$ (for each additional)

Structure:

Common name: Octagonal Tube Spring

Vertices information:

$A(4R) = \wedge(\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) \wedge (\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) = 2\pi$
$B(4R) = \wedge(\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) \wedge (\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) = 2\pi$
$C(4R) = \wedge(\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) \wedge (\alpha) \vee (\beta) \wedge (\pi - \alpha - \beta) = 2\pi$

Foldline information:

$\overline{AB} = l_1$
$\overline{AC} = l_2$

Combinations:

- Shift from A to B
- Shift from A to C
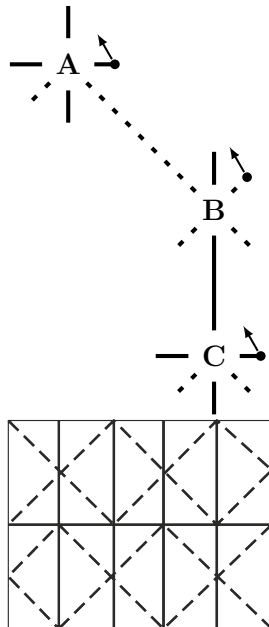- Rotate around A with π
- Rotate around B with π
- Rotate around C with π

Key Figures:

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Spherical Foldschemes

3(6R), MagicBall

Main directional motion:	spherical
Geometric space formula:	3(6R)
Degree of freedom (DOF):	F = 7 + x (for each additional)

Structure:

Common name: Magic Ball

Vertices information:

$$A(6R) = \wedge(\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) = 2\pi$$

$$B(6R) = \vee(\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{2}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{2}) = 2\pi$$

$$C(6R) = \wedge(\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) = 2\pi$$

Foldline information:

$$\overline{AB} = \frac{1}{\sqrt{2 \cdot l_1^2}}$$

$$\overline{BC} = l_1$$

Combinations:

- Shift from A to C
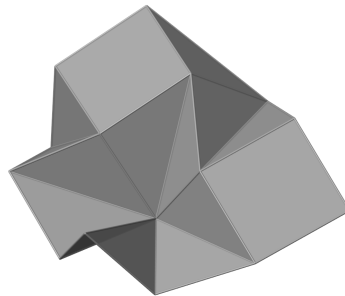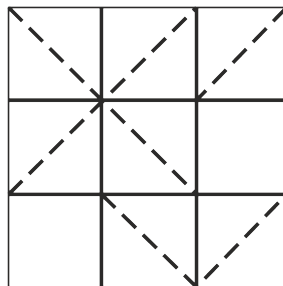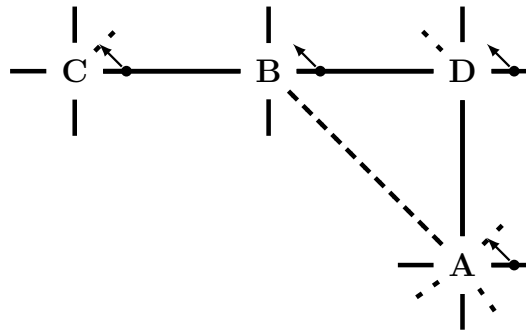- Rotate around B with π
- Mirror along $\overline{BC}$

Key Figures:

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Spherical Foldschemes

3(5R)1(8R), Waterbomb

Main directional motion:	spherical
Geometric space formula:	3(5R) 1(8R)
Degree of freedom (DOF):	missing

Structure:

Common name: Waterbomb

Vertices information:

$$A(8R) = \vee(\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{4}) = 2\pi$$

$$B(5R) = \wedge \underline{\vee}(\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{4}) \vee \underline{\wedge}(\tfrac{\pi}{4}) = 2\pi$$

$$C(5R) = \wedge(\tfrac{\pi}{4}) \vee (\tfrac{\pi}{4}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) = 2\pi$$

$$D(5R) = \wedge(\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{4}) \vee \underline{\wedge}(\tfrac{\pi}{4}) \wedge \underline{\vee}(\tfrac{\pi}{2}) \wedge (\tfrac{\pi}{2}) = 2\pi$$

Foldline information:

$$\overline{AD} = \overline{BD} = \overline{CB} = l_1$$
$$\overline{AB}\, \tfrac{1}{\sqrt{2 \cdot l_1^2}}$$
$$\overline{AD} = \sqrt{\dfrac{1}{2 + 2\cos(2\alpha)}}$$
$$\overline{AD} = \overline{BD}$$

Combinations:

- Rotate around A with $\tfrac{\pi}{2}$
- Rotate around A with π
- Mirror along $\overline{AD}$ ~~Shift from K to E~~
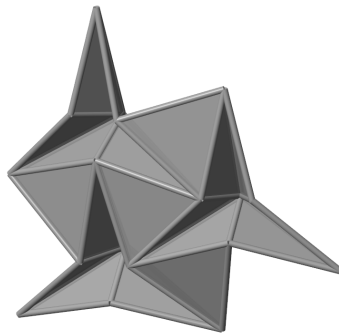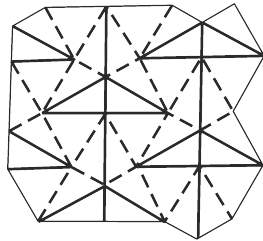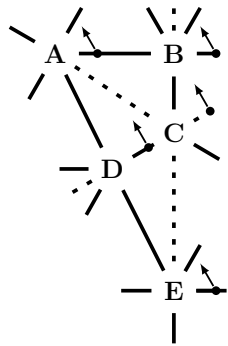- ~~Shift from B to H~~ Shift from A to D

Key Figures:

~~none~~

5(6R), Triangular Waterbomb

Main directional motion:	spherical
Geometric space formula:	5(6R)
Degree of freedom (DOF):	F fehlt!

Structure:

Common name: Triangular Waterbomb [Resch, 1960]

Vertices information:

$A(6R) = \wedge(\frac{\pi}{3}) \wedge (\frac{\pi}{2}) \wedge (\frac{\pi}{2}) \wedge (\frac{\pi}{3}) \wedge (\frac{\pi}{6}) \vee (\frac{\pi}{6}) = 2\pi$

$B(6R) = \wedge(\frac{\pi}{3}) \wedge (\frac{\pi}{6}) \vee (\frac{\pi}{6}) \wedge (\frac{\pi}{3}) \wedge (\frac{\pi}{2}) \wedge (\frac{\pi}{2}) = 2\pi$

$C(6R) = \vee(\frac{\pi}{3}) \wedge (\frac{\pi}{3}) \vee (\frac{\pi}{3}) \wedge (\frac{\pi}{3}) \vee (\frac{\pi}{3}) \wedge (\frac{\pi}{3}) = 2\pi$

$D(6R) = \wedge(\frac{\pi}{2}) \wedge (\frac{\pi}{3}) \wedge (\frac{\pi}{6}) \vee (\frac{\pi}{6}) \wedge (\frac{\pi}{3}) \wedge (\frac{\pi}{2}) = 2\pi$

$E(6R) = \wedge(\frac{\pi}{3}) \wedge (\frac{\pi}{6}) \vee (\frac{\pi}{6}) \wedge (\frac{\pi}{3}) \wedge (\frac{\pi}{2}) \wedge (\frac{\pi}{2}) = 2\pi$

Foldline information:

$\overline{AB} = l_1$

$\overline{AC} = \overline{CE} = \frac{2}{3}\sqrt{3AB}$

$\overline{AD} = \overline{DE} = \overline{AB}$

$\overline{BC} = \overline{CD} = \frac{1}{3}\sqrt{3AB}$

Combinations:

- Shift from A to E
- Rotate around $\frac{2\pi}{3}$
- Mirror along $\overline{BC}$
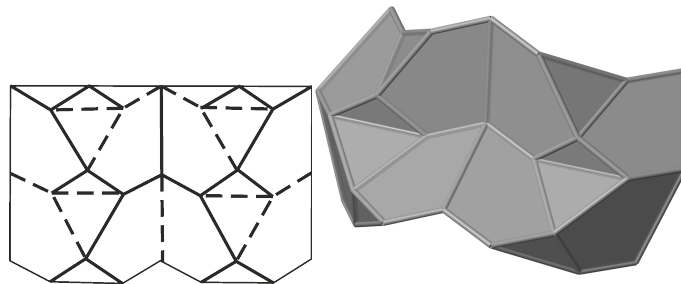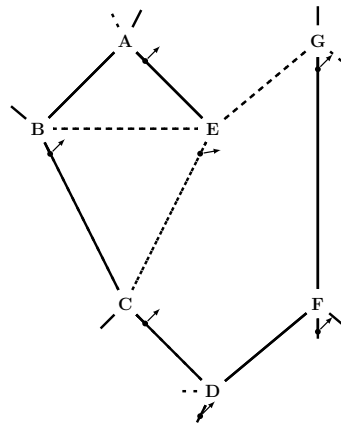- Mirror along $\overline{CE}$

Key Figures:

7(4R), Level Shifter

Main directional motion:	spherical
Geometric space formula:	7(4R)
Degree of freedom (DOF):	F = 1 + 0 (each additional)

Structure:

Common name: Level Shifter [Evans 2015]

Vertices information:

$$A(4R) = \wedge(\frac{\pi}{2}) \wedge (2\alpha) \vee (\frac{\pi}{2}) \wedge (\pi - 2\alpha) = 2\pi$$

$$B(4R) = \wedge(\frac{\pi}{2} - \alpha) \vee (2\alpha) \vee (\frac{\pi}{2} + \alpha) \wedge (\pi - \alpha) = 2\pi$$

$$C(4R) = \wedge(\frac{\pi}{2}) \vee (2\alpha) \wedge (\frac{\pi}{2}) \wedge (\pi - 2\alpha) = 2\pi$$

$$D(4R) = \wedge(\frac{\pi}{2} - \alpha) \wedge (\pi - \alpha) \wedge (\alpha) \wedge (\frac{\pi}{2} - \alpha) = 2\pi$$

$$E(4R) = \vee(\frac{\pi}{2} + \alpha) \vee (\pi - \alpha) \wedge (\alpha) \vee (\frac{\pi}{2} - \alpha) = 2\pi$$

$$F(4R) = \wedge(\frac{\pi}{2} - \alpha) \wedge (\pi - (\frac{\pi}{2} - \alpha)) \wedge (\pi - (\frac{\pi}{2} - \alpha)) \vee (\pi - (\frac{\pi}{2} - \alpha)) = 2\pi$$

$$G(4R) = \wedge(\frac{\pi}{2} - \alpha) \vee (\pi - (\frac{\pi}{2} - \alpha)) \wedge (\pi - (\frac{\pi}{2} - \alpha)) \vee (\pi - (\frac{\pi}{2} - \alpha)) = 2\pi$$

Foldline information:

$$\overline{AB} = \overline{AE} = \overline{CD} = l_1 = \overline{DF}$$

$$\overline{BE} = \sqrt{2 \cdot l_1^2 - 2 \cdot l_1^2 \cdot \cos(2 \cdot \alpha)}$$

$$\overline{BC} = \overline{CE} = \sqrt{\frac{\overline{BE}^2}{2} / (1 - \cos(\pi - 2 \cdot \alpha))}$$

$$\overline{EG} = \overline{DF} = l_4$$

$$\overline{FG} = \sqrt{\overline{CD}^2 + \overline{CE}^2}$$

Combinations:

• Rotate around A with π
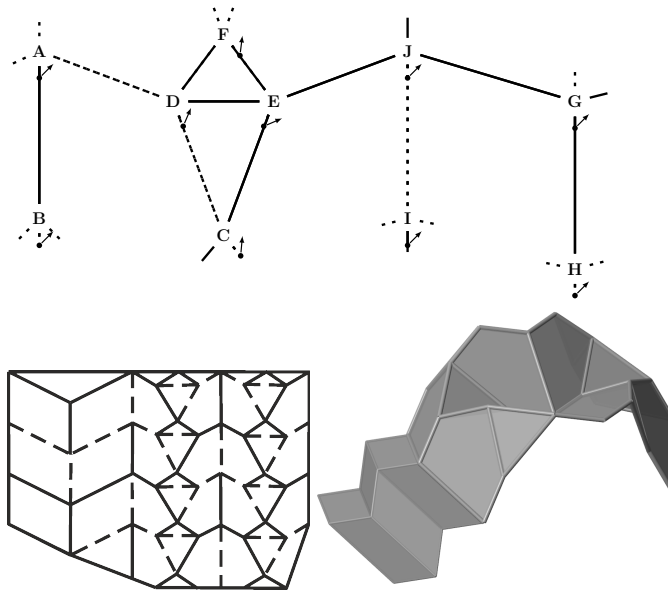• Shift from A to C
• Mirror along $\overline{FG}$

Key Figures:

Spherical Foldschemes

10(4R), Combination of Miura-Ori and Level Shifter

Main directional motion:	spherical
Geometric space formula:	10(4R)
Degree of freedom (DOF):	F = 1 + 0 (each additional)

Structure:

Common name: Combination of Miura-Ori and Level-Shifter [Evans 2010]

Vertices information:

$$A(4R) = \wedge(\alpha) \vee (\pi - \alpha) \vee (\pi - \alpha) \vee (\alpha) = 2\pi$$
$$B(4R) = \vee(\alpha) \wedge (\pi - \alpha) \wedge (\pi - \alpha) \wedge (\alpha) = 2\pi$$
$$C(4R) = \vee(\kappa) \wedge (\zeta) \vee (\kappa) \vee (\zeta) = 2\pi$$
$$D(4R) = \vee(\delta) \wedge (\gamma) \vee (\beta) \vee (\epsilon) = 2\pi$$
$$E(4R) = \wedge(\epsilon) \wedge (\beta) \vee (\gamma) \wedge (\delta) = 2\pi$$
$$F(4R) = \vee(\kappa) \vee (\zeta) \wedge (\kappa) \vee (\zeta) = 2\pi$$
$$G(4R) = \wedge(\pi - \alpha) \wedge (\alpha) \vee (\alpha) \wedge (\pi - \alpha) = 2\pi$$
$$H(4R) = \vee(\pi - \alpha) \vee (\alpha) \wedge (\alpha) \vee (\pi - \alpha) = 2\pi$$
$$I(4R) = \wedge(\alpha) \vee (\pi - \alpha) \vee (\pi - \alpha) \vee (\alpha) = 2\pi$$
$$J(4R) = \vee(\alpha) \wedge (\pi - \alpha) \wedge (\pi - \alpha) \wedge (\alpha) = 2\pi$$

Foldline information:

$$\overline{AB} = \overline{JI} = \overline{GH} = l_1$$
$$\overline{AD} = \overline{EJ} = l_2$$
$$\overline{DF} = \overline{FE} = l_3$$
$$\overline{DE} = l_4$$
$$\overline{DC} = \overline{CE} = l_5$$
$$\overline{GJ} = = l_6$$

Combinations:

- Mirror along $\overline{GH}$
- Shift from H' to H
- Shift from F to C

Key Figures:

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