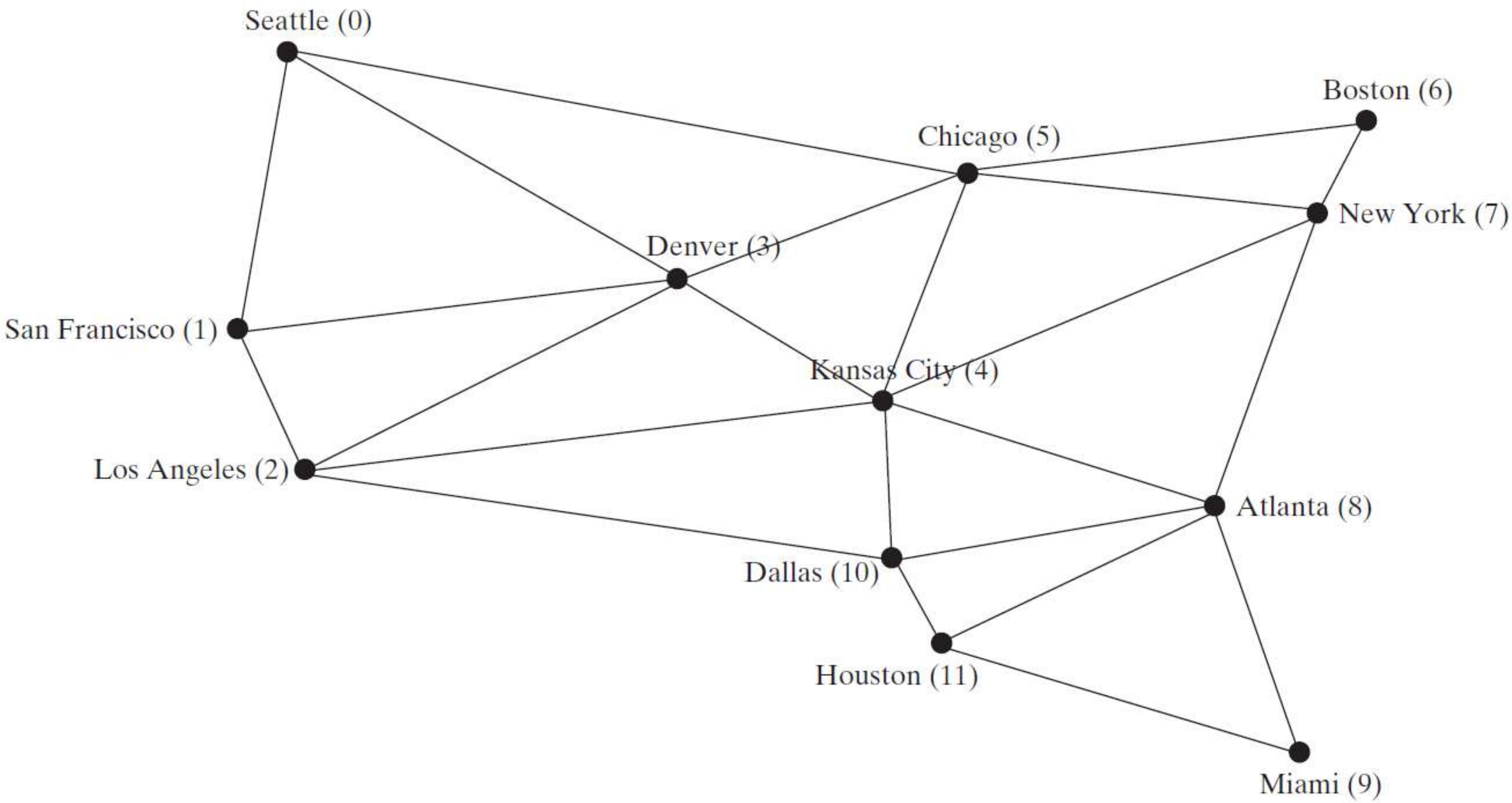
Chapter 28 Graphs and Applications

Objectives

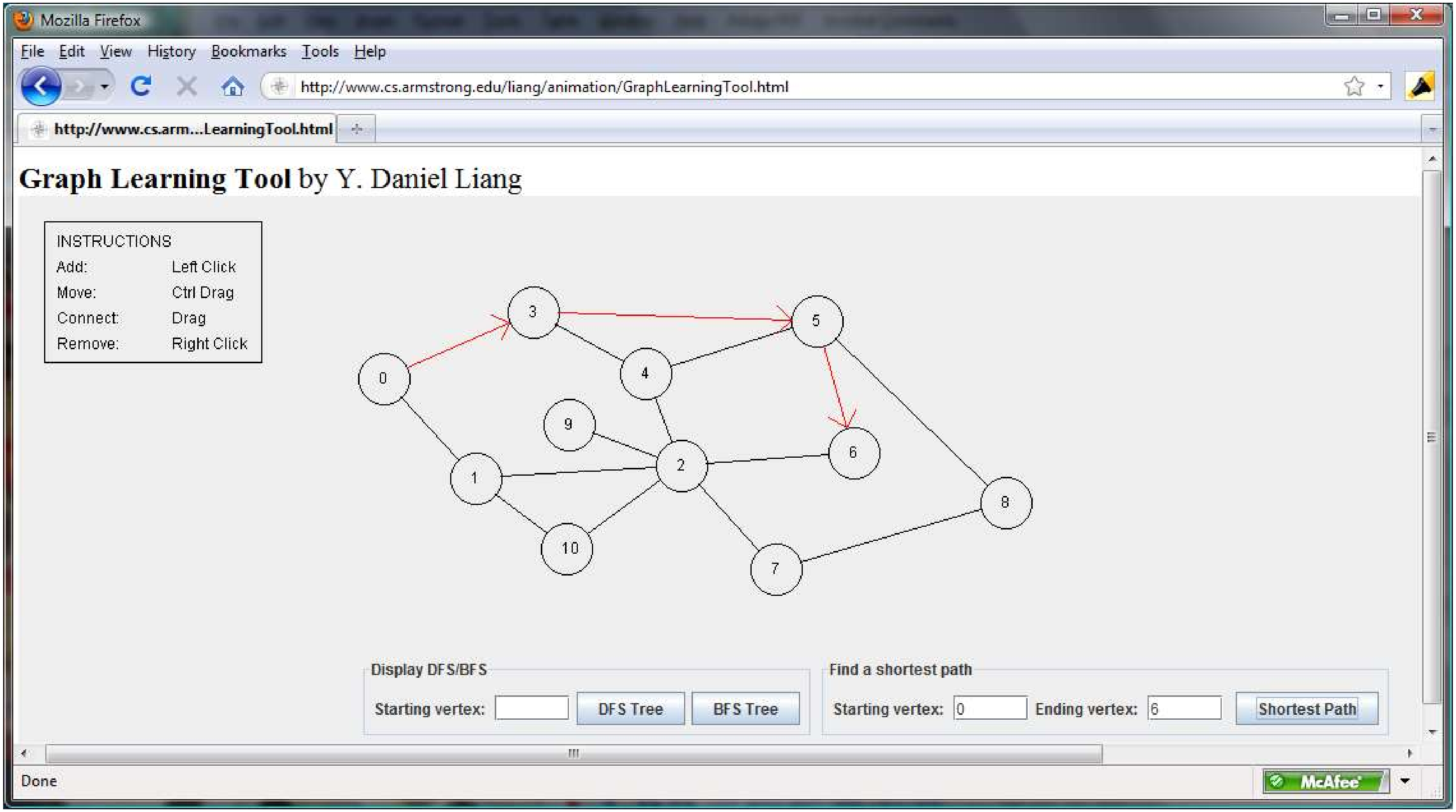
- To model real-world problems using graphs and explain the Seven Bridges of Königsberg problem (§28.1).
- To describe the graph terminologies: vertices, edges, simple graphs, weighted/unweighted graphs, and directed/undirected graphs (§28.2).
- To represent vertices and edges using lists, edge arrays, edge objects, adjacency matrices, and adjacency lists (§28.3).
- To model graphs using the **Graph** interface, the **AbstractGraph** class, and the **UnweightedGraph** class (§28.4).
- To display graphs visually (§28.5).
- To represent the traversal of a graph using the **AbstractGraph.Tree** class (§28.6).
- To design and implement depth-first search (§28.7).
- To solve the connected-circle problem using depth-first search (§28.8).
- To design and implement breadth-first search (§28.9).
- To solve the nine-tail problem using breadth-first search (§28.10).

Modeling Using Graphs

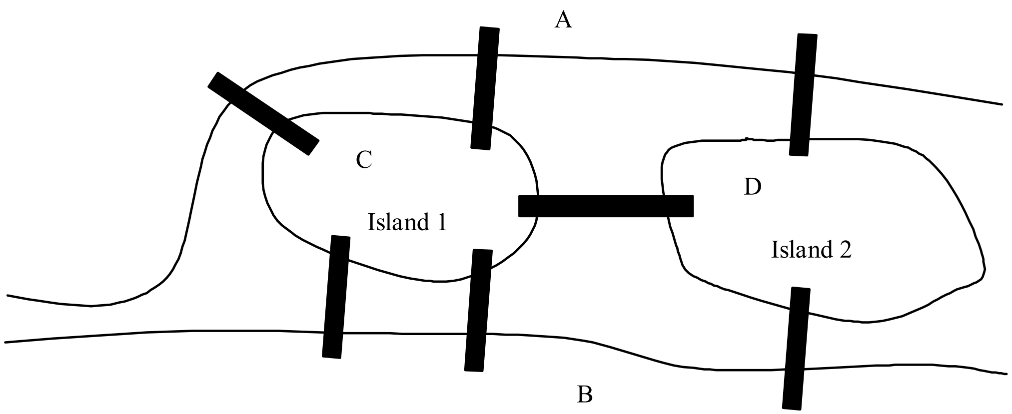


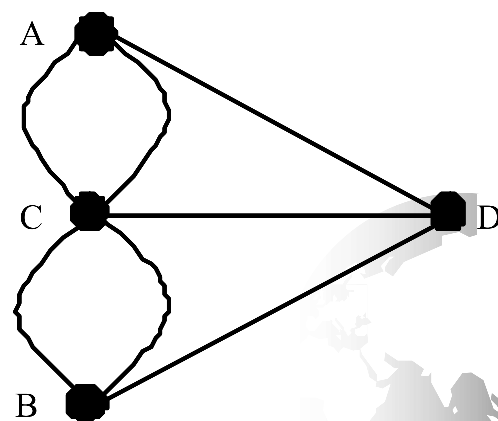
Graph Animation

www.cs.armstrong.edu/liang/animation/web/GraphLearningTool.html



Seven Bridges of Königsberg





Basic Graph Terminologies

What is a graph? G=(V, E)

Define a graph

Directed vs. undirected graphs

Weighted vs. unweighted graphs

Adjacent vertices

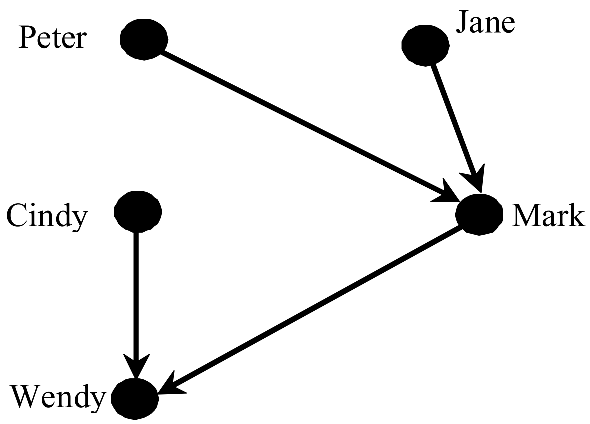
Incident

Degree

Neighbor

loop

Directed vs Undirected Graph



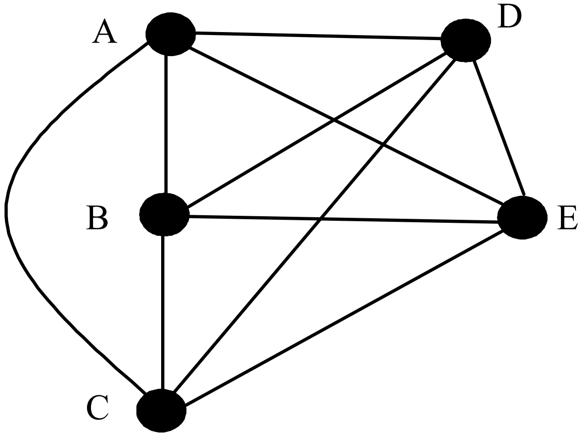
Basic Graph Terminologies

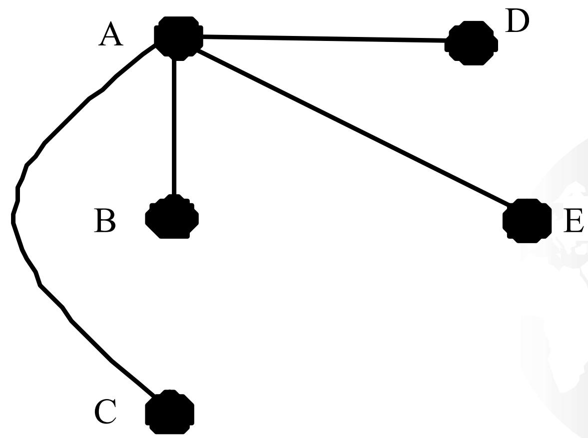
Parallel edge

Simple graph

Complete graph

Spanning tree





Representing Graphs

Representing Vertices

Representing Edges: Edge Array

Representing Edges: Edge Objects

Representing Edges: Adjacency Matrices

Representing Edges: Adjacency Lists

Representing Vertices

String[] vertices = {“Seattle“, “San Francisco“, “Los Angles”, “Denver”, “Kansas City”, “Chicago”, … };

City[] vertices = {city0, city1, … };

public class City {

}

List<String> vertices;

Representing Edges: Edge Array

int[][] edges = {{0, 1}, {0, 3} {0, 5}, {1, 0}, {1, 2}, … };

Representing Edges: Edge Object

```
public class Edge {
  int u, v;
  public Edge(int u, int v) {
    this.u = u;
    this.v = v;
}

List<Edge> list = new ArrayList<>();
list.add(new Edge(0, 1)); list.add(new Edge(0, 3)); …
```

Representing Edges: Adjacency Matrix

```
int[][] adjacencyMatrix = {
  {0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0}, // Seattle
  {1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0}, // San Francisco
  {0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0}, // Los Angeles
  {1, 1, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0}, // Denver
  {0, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0}, // Kansas City
  {1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 0, 0}, // Chicago
  {0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0}, // Boston
  {0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0}, // New York
  {0, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1}, // Atlanta
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1}, // Miami
  {0, 0, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1}, // Dallas
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0}  // Houston
};
```

Representing Edges: Adjacency Vertex List

```
List<Integer>[] neighbors = new List[12];
```

Seattle	neighbors[0]	1	3	5			
San Francisco	neighbors[1]	0	2	3			
Los Angeles	neighbors[2]	1	3	4	10		
Denver	neighbors[3]	0	1	2	4	5	
Kansas City	neighbors[4]	2	3	5	7	8	10
Chicago	neighbors[5]	0	3	4	6	7	
Boston	neighbors[6]	5	7				
New York	neighbors[7]	4	5	6	8		
Atlanta	neighbors[8]	4	7	9	10	11	
Miami	neighbors[9]	8	11				
Dallas	neighbors[10]	2	4	8	11		
Houston	neighbors[11]	8	9	10			

```
List<List<Integer>> neighbors = new ArrayList<>();
```

Representing Edges: Adjacency Edge List

```
List<Edge>[] neighbors = new List[12];
```

Seattle	neighbors[0]	Edge(0, 1)	Edge(0, 3)	Edge(0, 5)			
San Francisco	neighbors[1]	Edge(1, 0)	Edge(1, 2)	Edge(1, 3)			
Los Angeles	neighbors[2]	Edge(2, 1)	Edge(2, 3)	Edge(2, 4)	Edge(2, 10)		
Denver	neighbors[3]	Edge(3, 0)	Edge(3, 1)	Edge(3, 2)	Edge(3, 4)	Edge(3, 5)	
Kansas City	neighbors[4]	Edge(4, 2)	Edge(4, 3)	Edge(4, 5)	Edge(4, 7)	Edge(4, 8)	Edge(4, 10)
Chicago	neighbors[5]	Edge(5, 0)	Edge(5, 3)	Edge(5, 4)	Edge(5, 6)	Edge(5, 7)	
Boston	neighbors[6]	Edge(6, 5)	Edge(6, 7)				
New York	neighbors[7]	Edge(7, 4)	Edge(7, 5)	Edge(7, 6)	Edge(7, 8)		
Atlanta	neighbors[8]	Edge(8, 4)	Edge(8, 7)	Edge(8, 9)	Edge(8, 10)	Edge(8, 11)	
Miami	neighbors[9]	Edge(9, 8)	Edge(9, 11)				
Dallas	neighbors[10]	Edge(10, 2)	Edge(10, 4)	Edge(10, 8)	Edge(10, 11)		
Houston	neighbors[11]	Edge(11, 8)	Edge(11, 9)	Edge(11, 10)			

Representing Adjacency Edge List Using ArrayList

```java
List<ArrayList<Edge>> neighbors = new ArrayList<>();
neighbors.add(new ArrayList<Edge>());
neighbors.get(0).add(new Edge(0, 1));
neighbors.get(0).add(new Edge(0, 3));
neighbors.get(0).add(new Edge(0, 5));
neighbors.add(new ArrayList<Edge>());
neighbors.get(1).add(new Edge(1, 0));
neighbors.get(1).add(new Edge(1, 2));
neighbors.get(1).add(new Edge(1, 3));
...
...
neighbors.get(11).add(new Edge(11, 8));
neighbors.get(11).add(new Edge(11, 9));
neighbors.get(11).add(new Edge(11, 10));
```

Modeling Graphs

«interface» *Graph<V>*	The generic type V is the type for vertices.
+getSize(): int	Returns the number of vertices in the graph.
+getVertices(): List<V>	Returns the vertices in the graph.
+getVertex(index: int): V	Returns the vertex object for the specified vertex index.
+getIndex(v: V): int	Returns the index for the specified vertex.
+getNeighbors(index: int): List<Integer>	Returns the neighbors of vertex with the specified index.
+getDegree(index: int): int	Returns the degree for a specified vertex index.
+printEdges(): void	Prints the edges.
+clear(): void	Clears the graph.
+addVertex(v: V): boolean	Returns true if v is added to the graph. Returns false if v is already in the graph.
+addEdge(u: int, v: int): boolean	Adds an edge from u to v to the graph throws IllegalArgumentException if u or v is invalid. Returns true if the edge is added and false if (u, v) is already in the graph.
+addEdge(e: Edge): boolean	Adds an edge into the adjacency edge list.
+remove(v: V): boolean	Removes a vertex from the graph.
+remove(u: int, v: int): boolean	Removes an edge from the graph.
+dfs(v: int): UnWeightedGraph<V>.SearchTree	Obtains a depth-first search tree starting from v.
+bfs(v: int): UnWeightedGraph<V>.SearchTree	Obtains a breadth-first search tree starting from v.

UnweightedGraph<V>	
#vertices: List<V>	Vertices in the graph.
#neighbors: List<List<Edge>>	Neighbors for each vertex in the graph.
+UnweightedGraph()	Constructs an empty graph.
+UnweightedGraph(vertices: V[], edges: int[][])	Constructs a graph with the specified edges and vertices stored in arrays.
+UnweightedGraph(vertices: List<V>, edges: List<Edge>)	Constructs a graph with the specified edges and vertices stored in lists.
+UnweightedGraph(edges: int[][], numberOfVertices: int)	Constructs a graph with the specified edges in an array and the integer vertices 1, 2, ...
+UnweightedGraph(edges: List<Edge>, numberOfVertices: int)	Constructs a graph with the specified edges in a list and the integer vertices 1, 2, ….

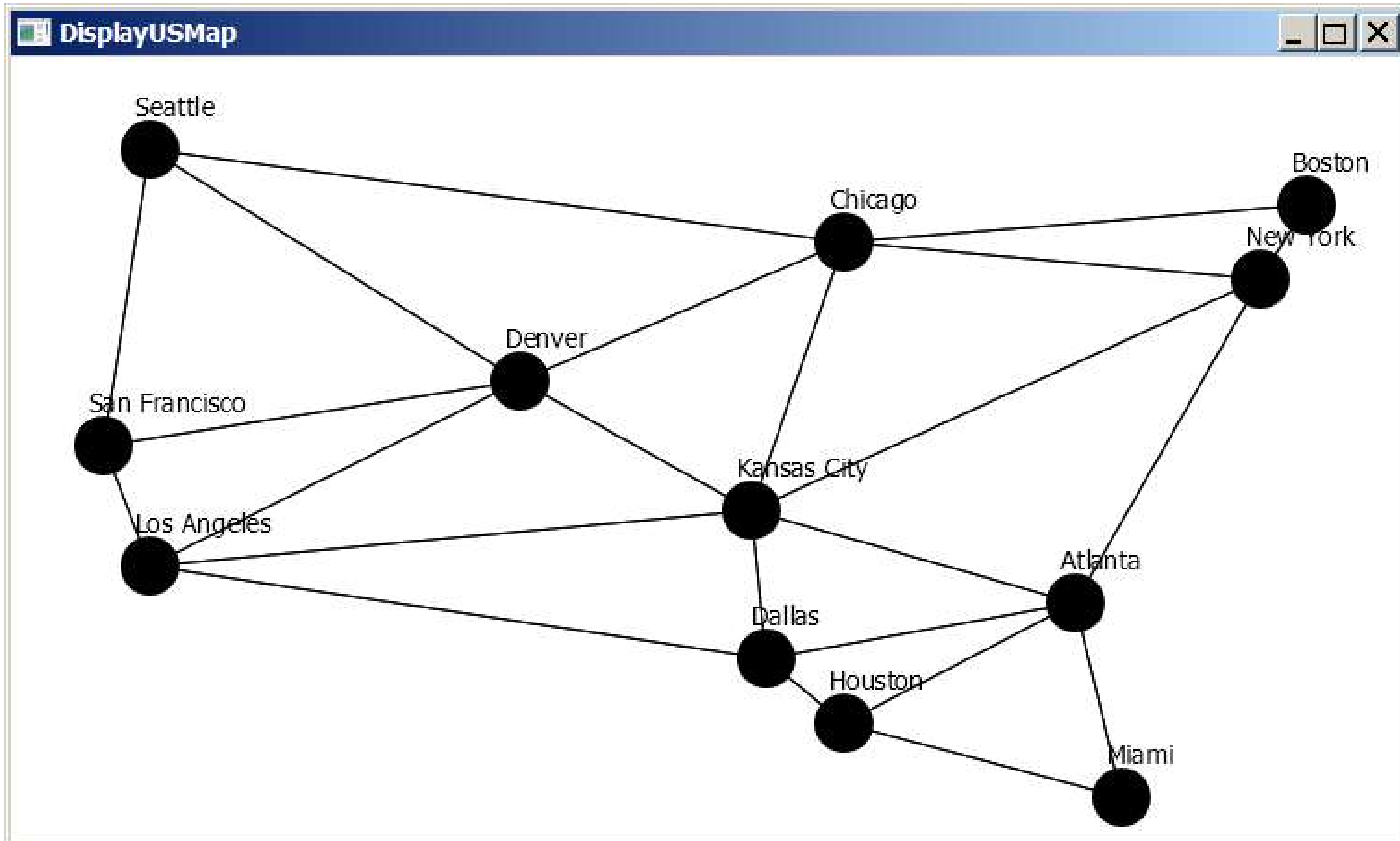
Graph

UnweightedGraph

TestGraph

Run

Graph Visualization



GraphView | Displayable | DisplayUSMap | Run

Graph Traversals

Depth-first search and breadth-first search

Both traversals result in a spanning tree, which can be modeled using a class.

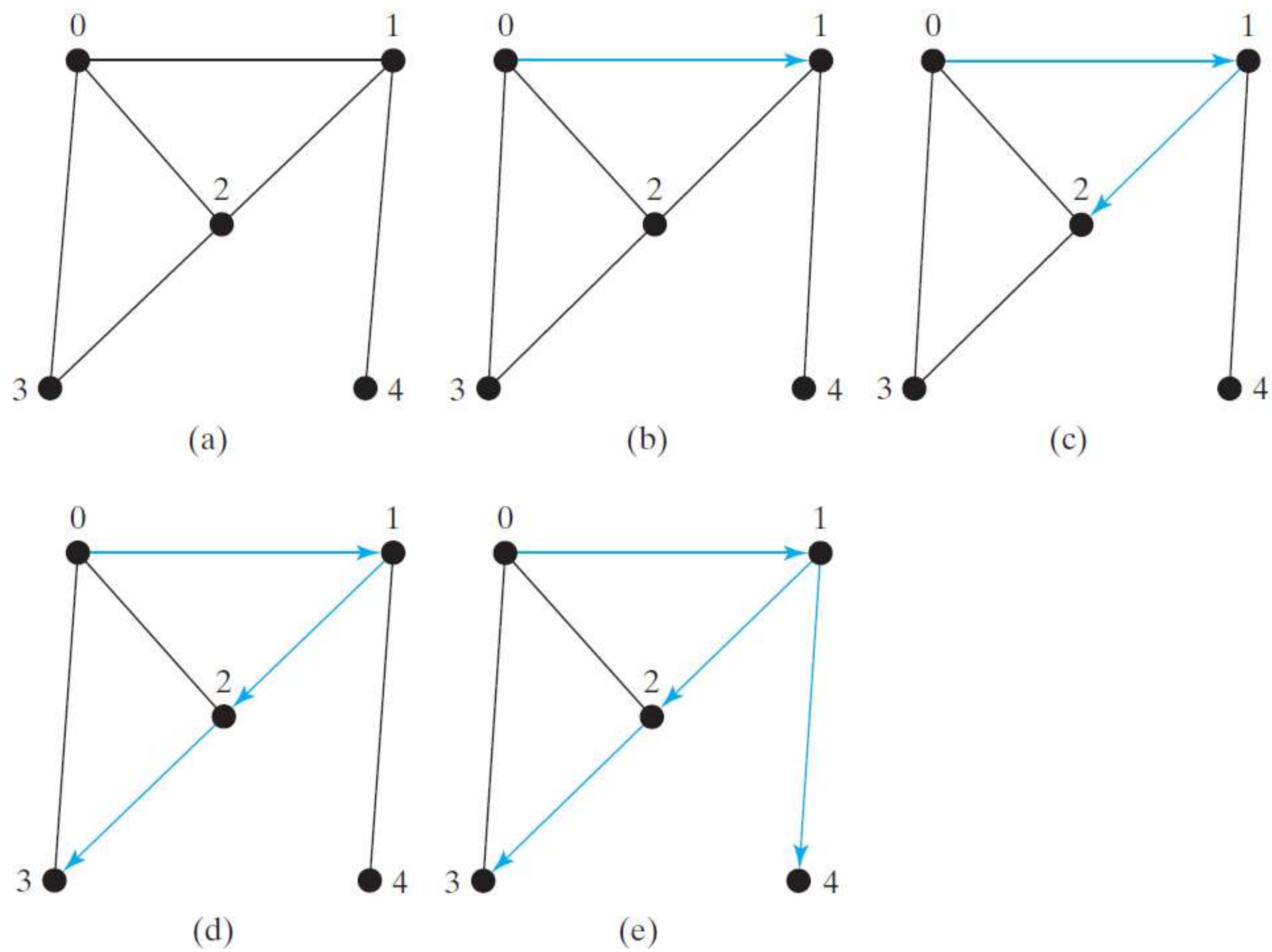
UnweightedGraph<V>.SearchTree	
-root: int	The root of the tree.
-parent: int[]	The parents of the vertices.
-searchOrder: List<Integer>	The orders for traversing the vertices.
+SearchTree(root: int, parent: int[], searchOrder: List<Integer>)	Constructs a tree with the specified root, parent, and searchOrder.
+getRoot(): int	Returns the root of the tree.
+getSearchOrder(): List<Integer>	Returns the order of vertices searched.
+getParent(index: int): int	Returns the parent for the specified vertex index.
+getNumberOfVerticesFound(): int	Returns the number of vertices searched.
+getPath(index: int): List<V>	Returns a list of vertices from the specified vertex index to the root.
+printPath(index: int): void	Displays a path from the root to the specified vertex.
+printTree(): void	Displays tree with the root and all edges.

Depth-First Search

The depth-first search of a graph is like the depth-first search of a tree discussed in §25.2.3, “Tree Traversal.” In the case of a tree, the search starts from the root. In a graph, the search can start from any vertex.

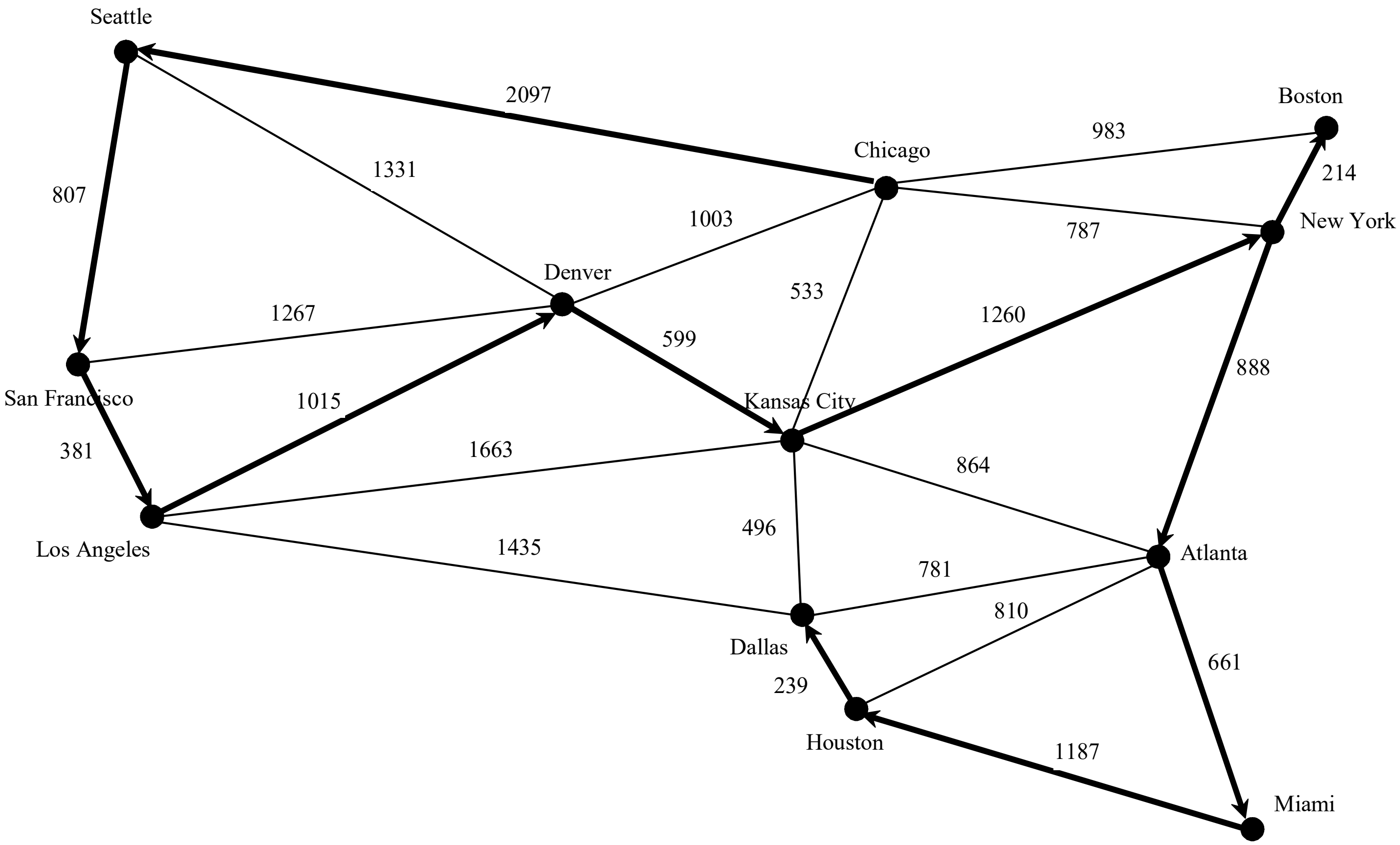
```
Input: G = (V, E) and a starting vertex v
Output: a DFS tree rooted at v
Tree dfs(vertex v) {
  visit v;
  for each neighbor w of v
    if (w has not been visited) {
      set v as the parent for w;
      dfs(w);
    }
}
```

Depth-First Search Example



(a) (b) (c) (d) (e)

Depth-First Search Example



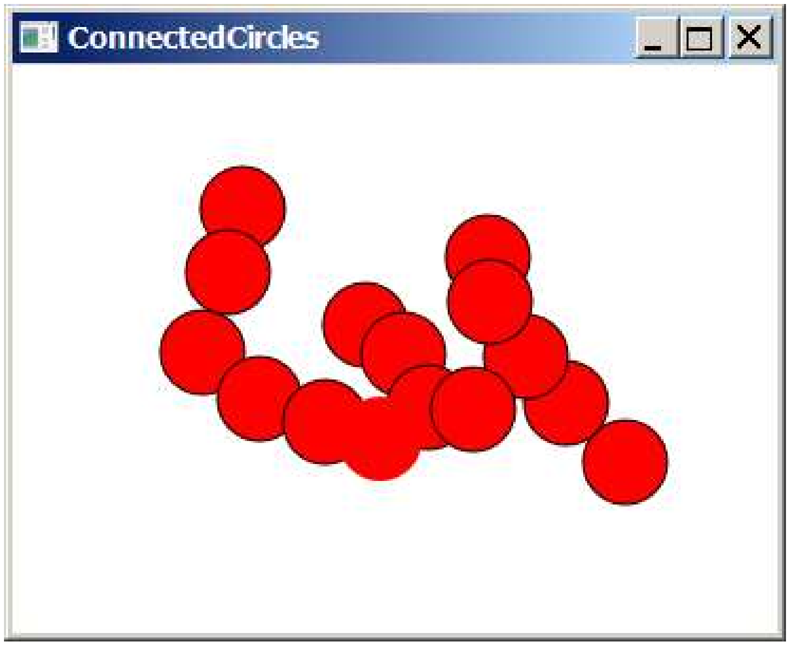
TestDFS

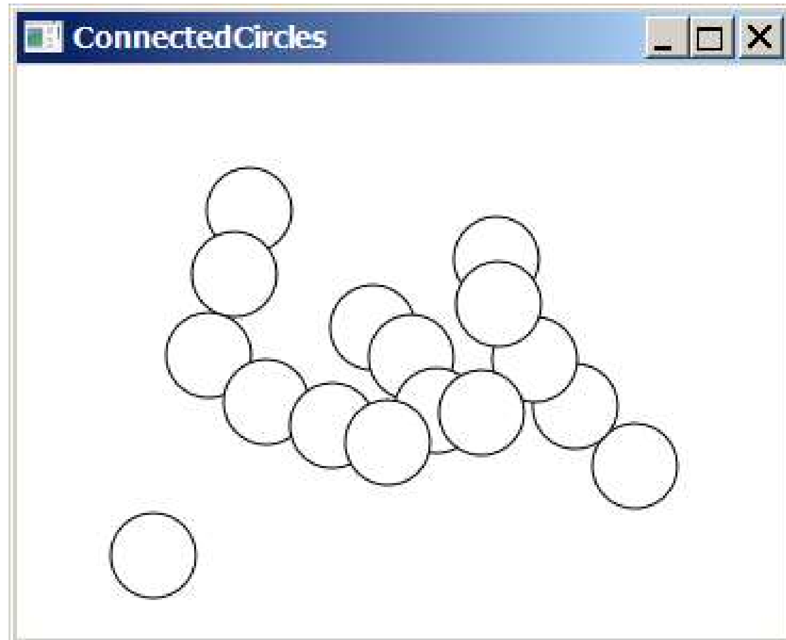
Run

Applications of the DFS

- Detecting whether a graph is connected. Search the graph starting from any vertex. If the number of vertices searched is the same as the number of vertices in the graph, the graph is connected. Otherwise, the graph is not connected.
- Detecting whether there is a path between two vertices.
- Finding a path between two vertices.
- Finding all connected components. A connected component is a maximal connected subgraph in which every pair of vertices are connected by a path.
- Detecting whether there is a cycle in the graph.
- Finding a cycle in the graph.

The Connected Circles Problem





ConnectedCircles Run

Breadth-First Search

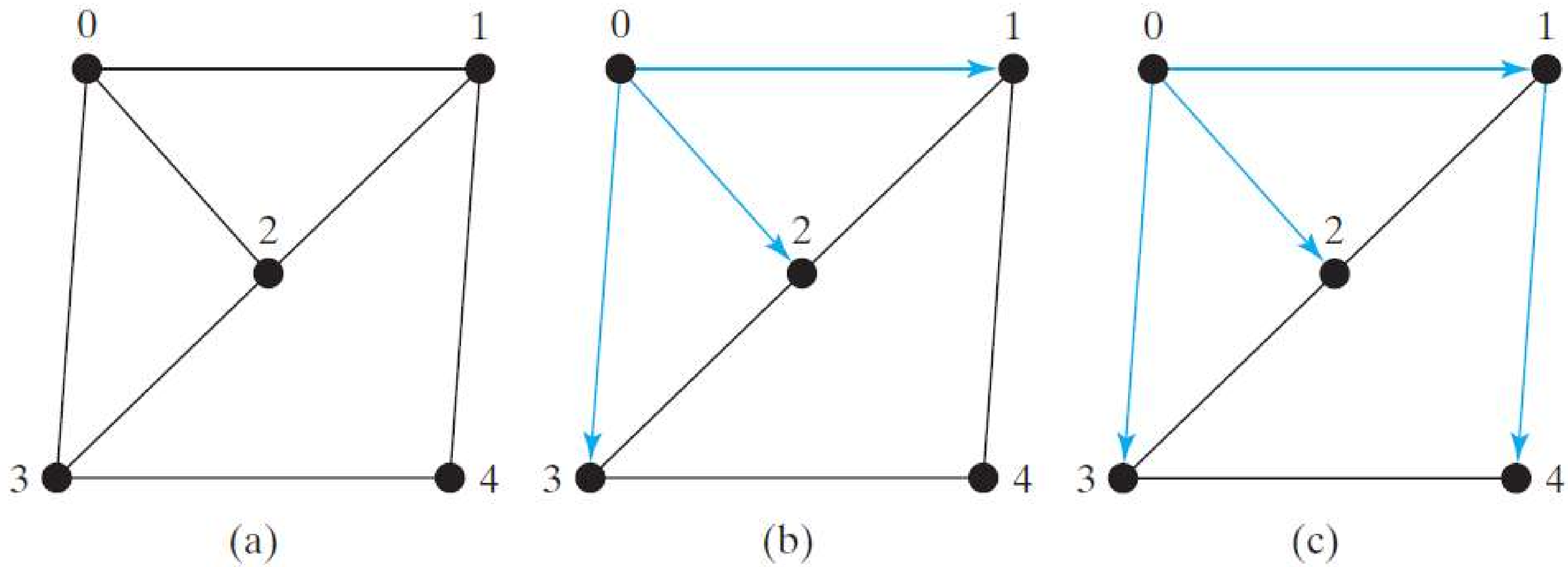
The breadth-first traversal of a graph is like the breadth-first traversal of a tree discussed in §25.2.3, "Tree Traversal." With breadth-first traversal of a tree, the nodes are visited level by level. First the root is visited, then all the children of the root, then the grandchildren of the root from left to right, and so on.

Breadth-First Search Algorithm

Input: G = (V, E) and a starting vertex v
Output: a BFS tree rooted at v

```
bfs(vertex v) {
  create an empty queue for storing vertices to be visited;
  add v into the queue;
  mark v visited;
  while the queue is not empty {
    dequeue a vertex, say u, from the queue
    process u;
    for each neighbor w of u
      if w has not been visited {
        add w into the queue;
        set u as the parent for w;
        mark w visited;
      }
  }
}
```

Breadth-First Search Example

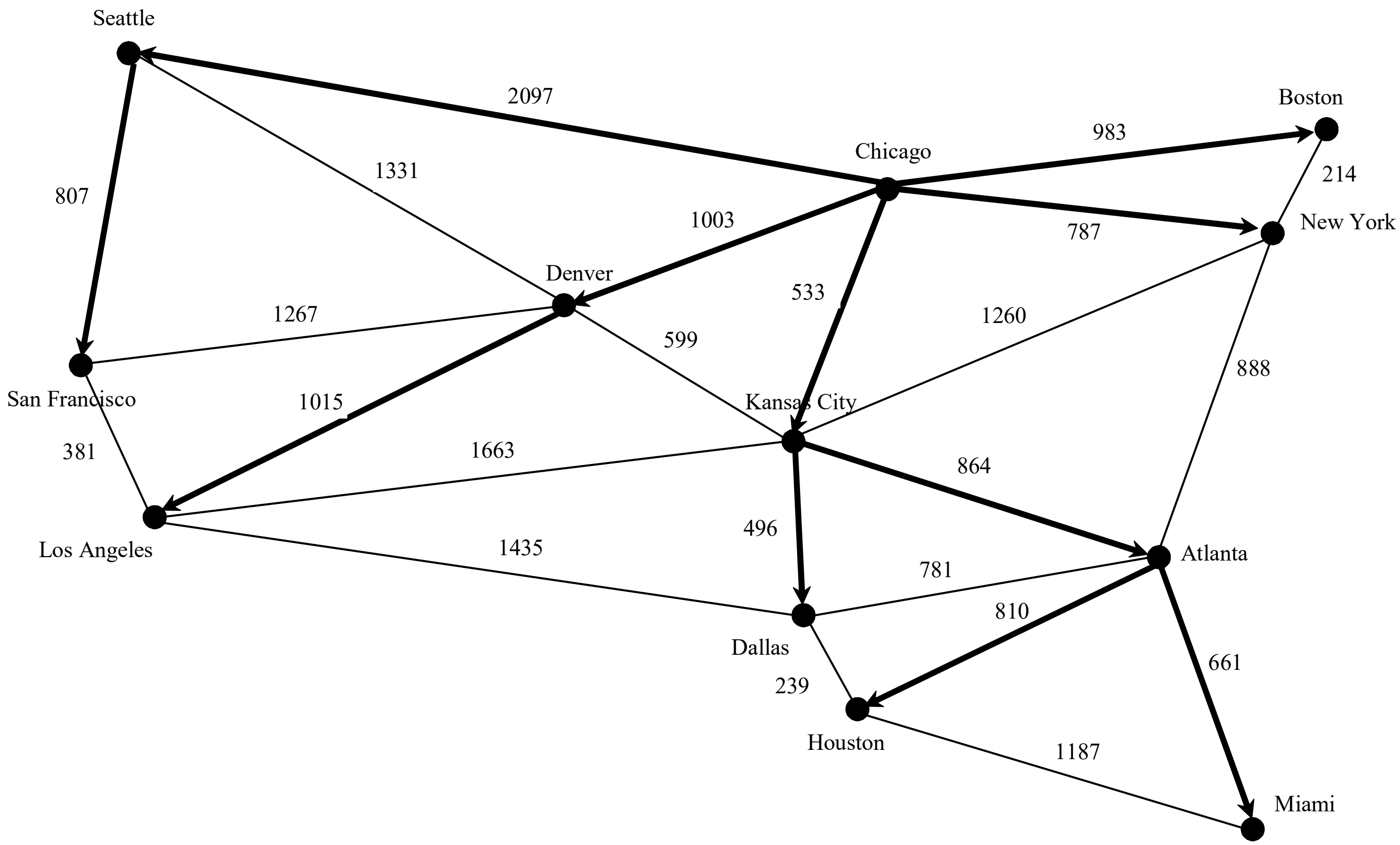


Queue: 0 isVisited[0] = true

Queue: 1 2 3 isVisited[1] = true, isVisited[2] = true, isVisited[3] = true

Queue: 2 3 4 isVisited[4] = true

Breadth-First Search Example



TestBFS Run

Applications of the BFS

- Detecting whether a graph is connected. A graph is connected if there is a path between any two vertices in the graph.
- Detecting whether there is a path between two vertices.
- Finding a shortest path between two vertices. You can prove that the path between the root and any node in the BFS tree is the shortest path between the root and the node.
- Finding all connected components. A connected component is a maximal connected subgraph in which every pair of vertices are connected by a path.
- Detecting whether there is a cycle in the graph.
- Finding a cycle in the graph.
- Testing whether a graph is bipartite. A graph is bipartite if the vertices of the graph can be divided into two disjoint sets such that no edges exist between vertices in the same set.

The Nine Tail Problem

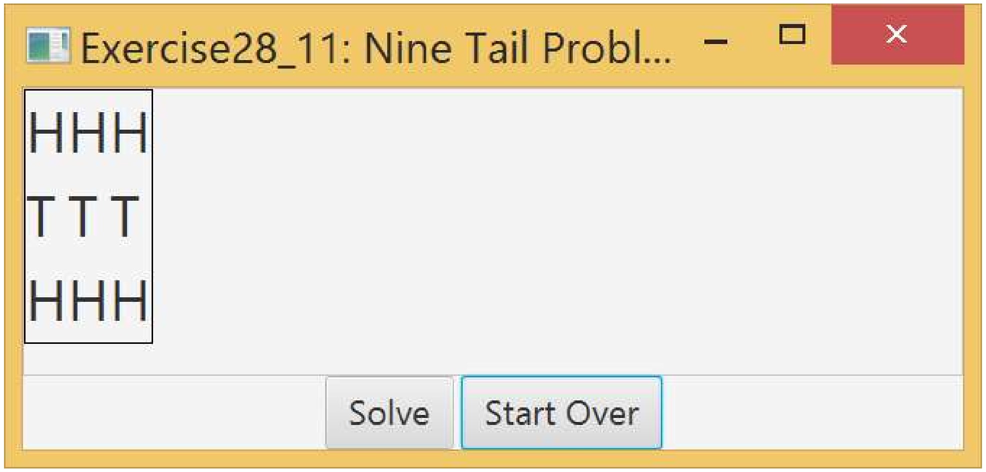
The problem is stated as follows. Nine coins are placed in a three by three matrix with some face up and some face down. A legal move is to take any coin that is face up and reverse it, together with the coins adjacent to it (this does not include coins that are diagonally adjacent). Your task is to find the minimum number of the moves that lead to all coins face down.

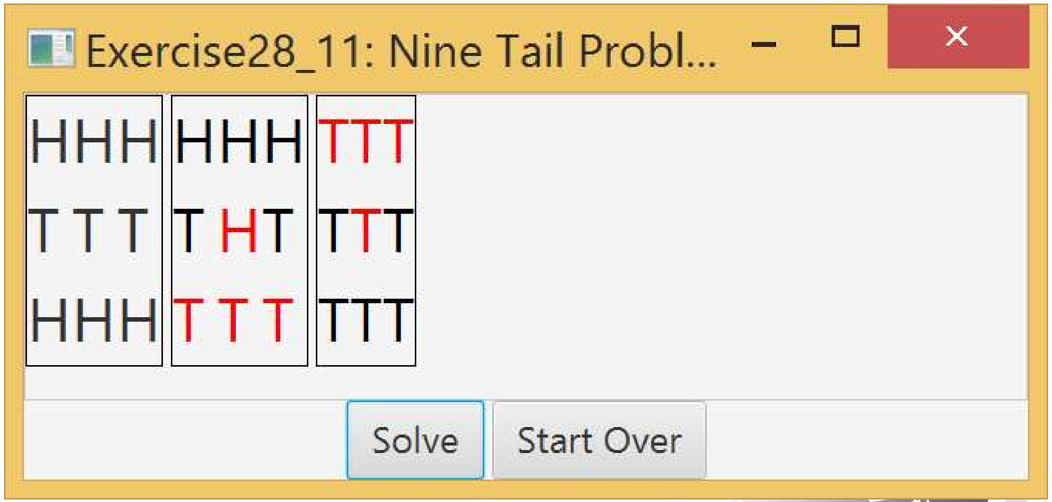
H	H	H
T	T	T
H	H	H

H	H	H
T	H	T
T	T	T

T	T	T
T	T	T
T	T	T

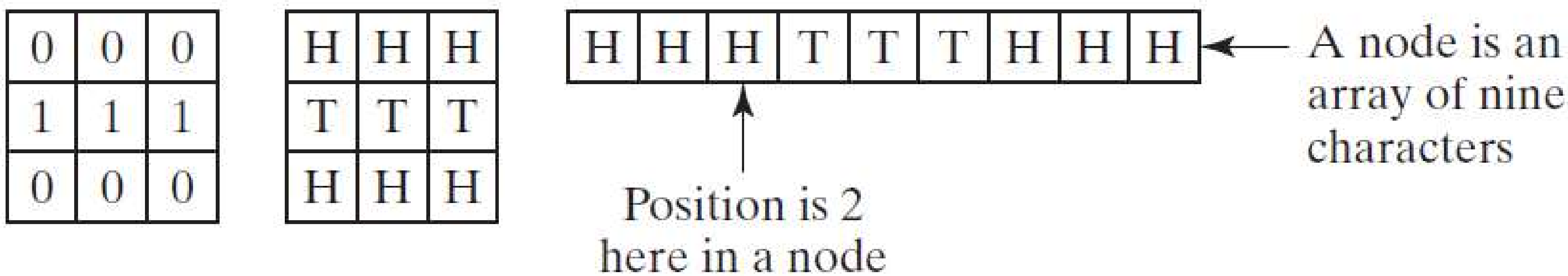
Nine Tail GUI Demo



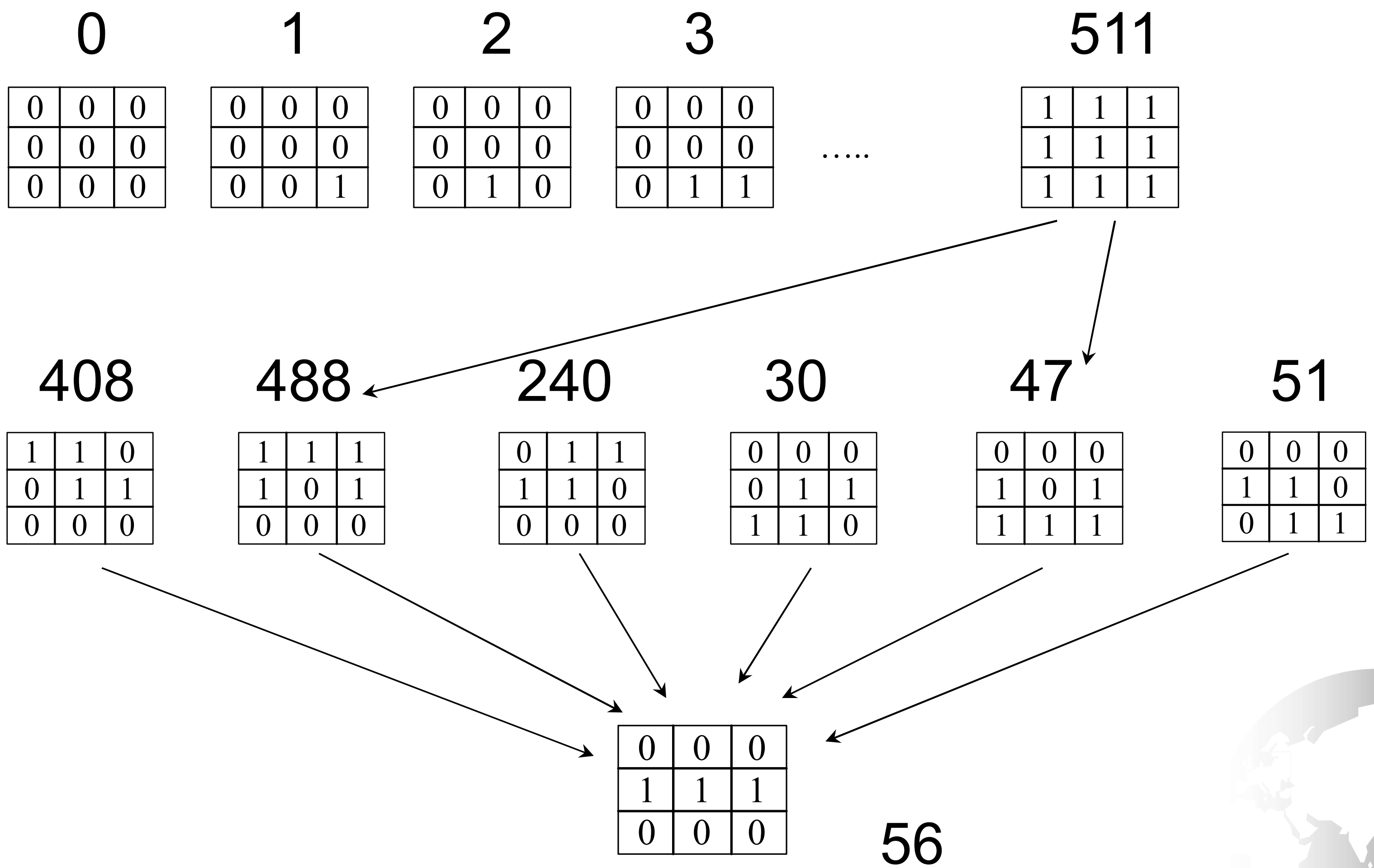


Nine Tail GUI

Representing Nine Coins



Model the Nine Tail Problem



NineTailModel

NineTailModel	
`#tree: AbstractGraph<Integer>.Tree`	A tree rooted at node 511.
`+NineTailModel()`	Constructs a model for the nine tails problem and obtains the tree.
`+getShortestPath(nodeIndex: int): List<Integer>`	Returns a path from the specified node to the root. The path returned consists of the node labels in a list.
`-getEdges(): List<AbstractGraph.Edge>`	Returns a list of `Edge` objects for the graph.
`+getNode(index: int): char[]`	Returns a node consisting of nine characters of Hs and Ts.
`+getIndex(node: char[]): int`	Returns the index of the specified node.
`+getFlippedNode(node: char[], position: int): int`	Flips the node at the specified position and returns the index of the flipped node.
`+flipACell(node: char[], row: int, column: int): void`	Flips the node at the specified row and column.
`+printNode(node: char[]): void`	Displays the node on the console.

NineTailModel NineTail Run