

18. (1). $X(n) = a^{|n|}$

$$X(z) = \sum_{n=-\infty}^{\infty} a^{|n|} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} a^{-n} z^n + \sum_{n=0}^{\infty} a^n z^{-n}$$

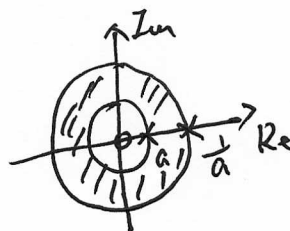
$$= \frac{az}{1-az} + \frac{1}{1-az^{-1}}$$

$$= \frac{(1-a^2)z^{-1}}{(z^{-1}-a)(1-az^{-1})}$$

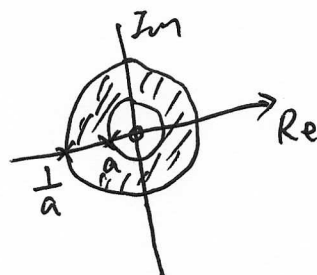
收敛域为 $|az| < 1$ 且 $|az^{-1}| < 1$ 且 $|a| < 1$.

收敛域为 $|a| < 1$ 且 $|a| < |z| < \frac{1}{|a|}$

零点为 $z=0$, 极点为 $z_1 = \frac{1}{a}$, $z_2 = a$.



$0 < a < 1$



$-1 < a < 0$

(2). $X(n) = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & n < 0, n > N-1 \end{cases}$

$$X(z) = \sum_{n=0}^{N-1} z^{-n} = \frac{1-z^{-N}}{1-z^{-1}}$$

零点为 $z=1$, 极点为 $z=1$. 收敛域为 $|z| > 0$.

B1. $X(n) = \begin{cases} n & 0 \leq n \leq N \\ 2N-n & N+1 \leq n \leq 2N \\ 0 & \text{其他} \end{cases}$

$$X(n) = n[u(n) - u(n-N)] + (2N-n)[u(n-N) - u(n-2N)]$$

$$= nu(n) - nu(n-N) + (2N-n)u(n-N) - (2N-n)u(n-2N)$$

$$= nu(n) - 2(n-N)u(n-N) + (n-2N)u(n-2N)$$

$$X(z) = \frac{z^{-1}}{(1-z^{-1})^2} - \frac{2z^{-N}z^{-1}}{(1-z^{-1})^2} + \frac{z^{-2N}z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{z^{-1}(1-z^{-N})^2}{(1-z^{-1})^2}$$

收敛域为 $1 < |z| < \infty$

零点 $z=1$, 极点 $z=0$, $z=1$

(p). $X(z) = n. \quad (n \geq 0)$

$$X(z) = \sum_{n=0}^{\infty} X(n) z^{-n} = \sum_{n=1}^{\infty} n z^{-n} = \frac{z^{-1}}{(1-z^{-1})^2}$$

零点为 $z=0$, 极点为 $z_1=z_2=1$

收敛域为 $|z| \geq 1$.

20. (1). $|z| > 2$.

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{-1}{1-2z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

因为 $|z| > 1$, 所以 $|2z^{-1}| < 1, |0.5z^{-1}| < 1$

$$\therefore X(z) = -\sum_{n=0}^{\infty} 2^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n} = \sum_{n=0}^{\infty} (2^{-n} - 2^n) z^{-n}$$

$X(n) = (2^{-n} - 2^n) u(n)$ 为右边序列.

(2). $|z| < 0.5$

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{0.5z}{1-0.5z} - \frac{2z}{1-2z}$$

因为 $|z| < 0.5$, 所以 $|0.5z| < 1, |2z| < 1$

$$X(z) = -\sum_{n=1}^{\infty} 2^n z^n + \sum_{n=1}^{\infty} 0.5^n z^n = \sum_{n=-\infty}^{\infty} (-2^{-n} + 2^n) z^n$$

$X(n) = (-2^{-n} + 2^n) u(-n-1)$ 为左边序列.

(3). $0.5 < |z| < 2$.

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{0.5z}{1-0.5z} - \frac{1}{1-0.5z^{-1}}$$

因为 $0.5 < |z| < 2$

$$X(z) = \sum_{n=1}^{\infty} 0.5^n z^n + \sum_{n=0}^{\infty} 0.5^n z^{-n} = \sum_{n=-\infty}^{\infty} 2^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n}$$

$X(n) = 2^n u(-n-1) + 0.5^n u(n)$ 为双边序列

$$28. (1) X(z) = \frac{8z-19}{z^2-5z+6}$$

$$\frac{X(z)}{z} = \frac{8z-19}{z(z-2)(z-3)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z-3}$$

$$A = \frac{X(z)}{z} \cdot z \Big|_{z=0} = \frac{8z-19}{(z-2)(z-3)} \Big|_{z=0} = -\frac{19}{6}$$

$$B = \frac{X(z)}{z} \cdot (z-2) \Big|_{z=2} = \frac{8z-19}{z(z-3)} \Big|_{z=2} = \frac{3}{2}$$

$$C = \frac{X(z)}{z} (z-3) \Big|_{z=3} = \frac{8z-19}{z(z-2)} \Big|_{z=3} = \frac{5}{3}$$

$$\therefore X(z) = -\frac{16}{9} \frac{z}{z} + \frac{3}{2} \frac{z}{z-2} + \frac{5}{3} \frac{z}{z-3}$$

设收敛域为 $|z| > 3$, 则

$$X(n) = -\frac{16}{9} \delta(n) + \frac{3}{2} 2^n \cdot u(n) + \frac{5}{3} 3^n u(n)$$

若收敛域为 $2 < |z| < 3$, 则

$$X(n) = -\frac{16}{9} \delta(n) + \frac{3}{2} 2^n u(n) + \frac{5}{3} (-3)^n \cdot u(-n-1)$$

若收敛域为 $|z| < 2$, 则

$$X(n) = -\frac{16}{9} \delta(n) + \frac{3}{2} \cdot (-2)^n u(-n-1) + \frac{5}{3} (-3)^n u(-n-1)$$

例：求下列序列的Z变换。

1). $x(n) = (-1)^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} (-1)^n u[n] z^{-n} = \sum_{n=0}^{\infty} (-z^{-1})^n$$

$$|-z^{-1}| < 1 \quad \text{即} \quad |z| > 1$$

$$X(z) = \frac{1}{1+z^{-1}} = \frac{z}{z+1}$$

2). $x(n) = \left(\frac{1}{2}\right)^{n+1} u[n+3]$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n+1} u[n+3] z^{-n} = \frac{1}{2} \sum_{n=-3}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$0 < \left|\frac{1}{2} z^{-1}\right| < 1 \quad \text{即} \quad \frac{1}{2} < |z| < \infty$$

$$X(z) = \frac{1}{2} \cdot \frac{\left(\frac{1}{2} z^{-1}\right)^{-3}}{1 - \frac{1}{2} z^{-1}} = \frac{\left(\frac{1}{2}\right)^{-2} z^3}{1 - \frac{1}{2} z^{-1}} = \frac{4z^4}{z - \frac{1}{2}} \quad \frac{1}{2} < |z| < \infty$$

例 3. $f_{\max} = 4 \text{ kHz}$, $\Delta f < 8 \text{ Hz}$

1). 采样频率. $f_s \geq 2f_{\max} = 8 \text{ kHz}$

2). $T_s \leq \frac{1}{f_s} = \frac{1}{8 \text{ K}} = 0.125 \times 10^{-3} \text{ s}$

3). $N = \frac{f_s}{\Delta f_s} = \frac{8 \text{ kHz}}{8 \text{ Hz}} = 1000$