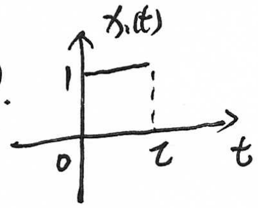
P101

22. 求傅里叶变换。

(a).

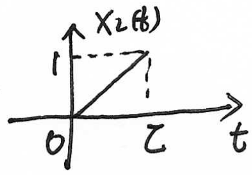


利用时移特性. $x_1(t)=g(t-\frac{\tau}{2})$

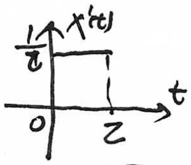
所以. $\mathcal{F}[g(t)] = E\tau\cdot Sa\frac{\omega\tau}{2} = \tau\, Sa\frac{\omega\tau}{2}$.

$\mathcal{F}[x(t)] = e^{-j\omega\cdot\frac{\tau}{2}}\mathcal{F}[g(t)] = e^{-j\omega\frac{\tau}{2}}\cdot\tau\cdot Sa\frac{\omega\tau}{2}$.

(b).



设 $x'(t)=\frac{dx_2(t)}{dt}$



因为 $x'(t)\longleftrightarrow E\cdot\tau\cdot Sa\frac{\omega\tau}{2}\cdot e^{-j\omega\frac{\tau}{2}} = Sa\frac{\omega\tau}{2}\cdot e^{-j\omega\frac{\tau}{2}}$.

所以. $X_2(t) = \frac{1}{j\omega}Sa\frac{\omega\tau}{2}\cdot e^{-j\omega\frac{\tau}{2}}$

23. (2). $x(t)=\frac{2a}{a^2+t^2}$ $-\infty<t<\infty$

$e^{-a|t|}\longleftrightarrow\frac{2a}{a^2+\omega^2}$ $a>0$

$f(\omega)=\frac{2a}{a^2+\omega^2}$

$F^{-1}[f(\omega)] = e^{-a|t|}$ $a>0$

$\therefore \frac{2a}{a^2+t^2}\longleftrightarrow 2\pi e^{-a|\omega|}$

24. (1). $f(t)=e^{-jt}\delta(t-2)$

$F(\omega)=\int_{-\infty}^{\infty}e^{-jt}\delta(t-2)\cdot e^{-j\omega t}dt = \int_{-\infty}^{\infty}\delta(t-2)e^{-j(\omega+1)t}dt = e^{-j\cdot 2(\omega+1)}$

(2). $f(t)=e^{-2t}u(t+1)$

$F(\omega)=\int_{-\infty}^{\infty}e^{-2t}u(t+1)e^{-j\omega t}dt=\int_{-1}^{\infty}e^{-(2+j\omega)t}dt = -\frac{e^{-(2+j\omega)t}}{2+j\omega}\Big|_{-1}^{\infty}$

$= \frac{e^{2+j\omega}}{2+j\omega}$

(5). $f(t) = u(\frac{t}{2}-1)$

$$F(w) = 2[\pi\delta(2w) + \frac{1}{2jw}]\cdot e^{-j\cdot 2w}$$

$$= [2\pi\delta(2w) + \frac{1}{jw}]\cdot e^{-j\cdot 2w}$$

26. (1). $tX(2t) \longleftrightarrow \frac{1}{2} j \frac{dX(\frac{w}{2})}{dw}$

(2). $(t-2)X(t) \longleftrightarrow tX(t) - 2X(t) \longleftrightarrow j\frac{dX(w)}{dw} - 2X(w)$

(3) $X(2t-5) \longleftrightarrow \frac{1}{2}X(t-\frac{5}{2}) \longleftrightarrow \frac{1}{2}X(\frac{w}{2})\cdot e^{-j\cdot\frac{5}{2}w}$.

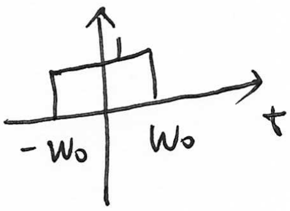
(8). $e^{jt}X(3-2t)$

$$X(3-2t) \longleftrightarrow \frac{1}{2}X(-\frac{w}{2}\cdot\frac{3}{2}) \longleftrightarrow \frac{1}{2}X(-\frac{w}{2})\cdot e^{-j\cdot\frac{3}{2}\cdot w}$$

$$e^{jt}\cdot X(3-2t) \longleftrightarrow \frac{1}{2}X(-\frac{w-1}{2})\cdot e^{-j\cdot\frac{3}{2}(w-1)} = \frac{1}{2}X(\frac{1-w}{2})\cdot e^{j\cdot\frac{3}{2}(1-w)}$$

27. (1). $X(w) = \begin{cases} 1 & |w|<w_0 \\ 0 & |w|>w_0 \end{cases}$

$X(t) = \begin{cases} 1 & |t|<w_0 \\ 0 & |t|>w_0 \end{cases}$



因为 $F[X(t)] = E\tau\cdot Sa\frac{w\tau}{2} = 2w_0\cdot Sa\frac{w\cdot 2w_0}{2} = 2w_0\cdot Sa(w\cdot w_0)$

所以. $F[2w_0 Sa(w_0 t)] = 2\pi X(w)$

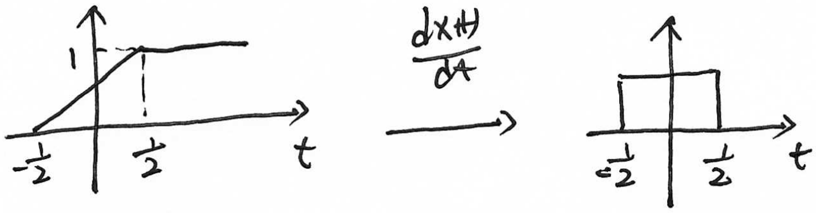
$\therefore F^{-1}[X(w)] = \frac{1}{2\pi}\cdot 2w_0\cdot Sa\frac{w_0 t}{2} = \frac{w_0}{\pi}\cdot Sa(w_0 t)$

(2). $X(w) = \delta(w+w_0) - \delta(w-w_0)$

因为. $e^{jw_0 t} \longleftrightarrow 2\pi\delta(w-w_0)$

所以 $x(t) = \frac{1}{2\pi}[e^{-jw_0 t} - e^{jw_0 t}]$

33、(1)、信号如图.



$$\frac{dx(t)}{dt} = g_{\tau=1}(t)$$

$$\frac{dx(t)}{dt} \leftrightarrow Sa(\frac{w}{2})$$

则 $x(t) = \int_{-\infty}^{t} \frac{dx(\tau)}{d\tau} d\tau \leftrightarrow \frac{1}{jw} Sa(\frac{w}{2}) + \pi\delta(w)$

$$X(w) = \frac{1}{jw} Sa\frac{w}{2} + \pi\delta(w)$$

(2). 因为 $1 \leftrightarrow 2\pi\delta(w)$

$$g(t) = x(t) - \frac{1}{2} \leftrightarrow \frac{1}{jw} Sa\frac{w}{2}.$$