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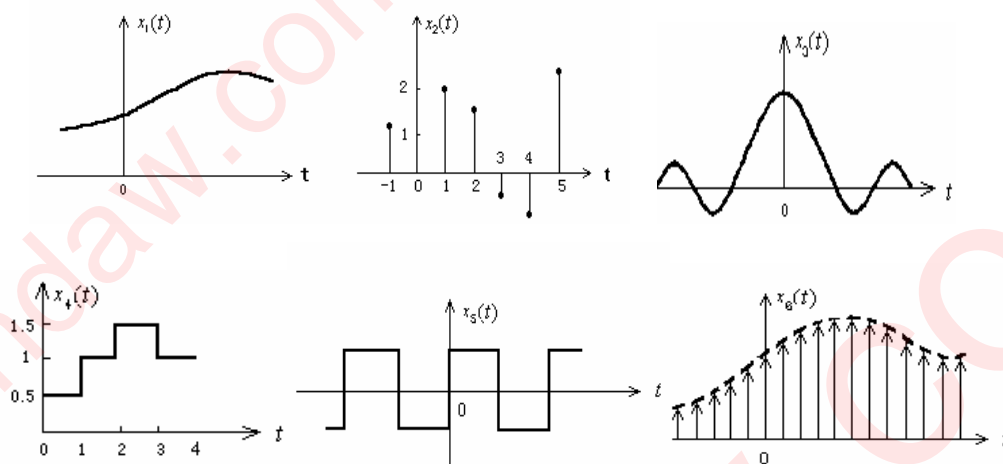
Khdaw团队一直秉承用心为大家服务的宗旨，以关注学生的学习生活为出发点，

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习题一 (P7)

1. 指出题图 1-1 所示各信号是连续时间信号？还是离散时间信号。



题图 1-1

解: $x_1(t), x_3(t), x_4(t), x_5(t)$ 是连续时间信号

$x_2(t), x_6(t)$ 是离散时间信号。

2. 判断下列各信号是否是周期信号，如果是周期信号，求出它的基波周期。

(1) $x(t) = 2 \cos(3t + \pi/4)$

(2) $x(n) = \cos(8\pi n/7 + 2)$

(3) $x(t) = e^{j(\pi-1)t}$

(4) $x(n) = e^{j(n/8-\pi)}$

(5) $x(n) = \sum_{m=0}^{\infty} [\delta(n-3m) - \delta(n-1-3m)]$

(6) $x(t) = \cos 2\pi t \times u(t)$

(7) $x(n) = \cos(n/4) \times \cos(n\pi/4)$

(8) $x(n) = 2 \cos(n\pi/4) + \sin(n\pi/8) - 2 \sin(n\pi/2 + \pi/6)$

分析:

(1) 离散时间复指数信号的周期性:

为了使 $e^{j\Omega n}$ 为周期性的，周期 $N > 0$ ，就必须有 $e^{j\Omega(n+N)} = e^{j\Omega n}$ ，因此有 $e^{j\Omega N} = 1$ 。

ΩN 必须为 2π 的整数倍，即必须有一个整数 m ，满足

$$\Omega N = 2\pi m$$

所以

$$\frac{\Omega}{2\pi} = \frac{m}{N}$$

因此，若 $\frac{\Omega}{2\pi}$ 为一有理数， $e^{j\Omega n}$ 为周期性的，否则，不为周期性的。

所以，周期信号 $e^{j\Omega n}$ 基波频率为: $\frac{2\pi}{N} = \frac{\Omega}{m}$ ，基波周期为: $N = m \frac{2\pi}{\Omega}$ 。

(2) 连续时间信号的周期性: (略)

答案:

- (1) 是周期信号, $T = \frac{2\pi}{3}$
 (2) 是周期信号, $T = \frac{7m}{4} = 7$
 (3) 是周期信号, $T = 2$
 (4) 不是周期信号
 (5) 不是周期信号
 (6) 不是周期信号
 (7) 不是周期信号
 (8) 是周期信号, $T = 16$

3. 试判断下列信号是能量信号还是功率信号。

- (1) $x_1(t) = Ae^{-t} \quad t \geq 0$ (2) $x_2(t) = A \cos(\omega_0 t + \theta)$
 (3) $x_3(t) = \sin 2t + \sin 2\pi t$ (4) $x_4(t) = e^{-t} \sin 2t$

解:

(1) $x_1(t) = Ae^{-t} \quad t \geq 0$

$$w = \lim_{T \rightarrow \infty} \int_0^T A^2 e^{-2t} dt = \lim_{T \rightarrow \infty} A^2 \left[\frac{1}{-2} e^{-2t} \right]_0^T$$

$$= \frac{A^2}{-2} \lim_{T \rightarrow \infty} (e^{-2T} - 1) = -\frac{A^2}{2} \lim_{T \rightarrow \infty} \left(\frac{1}{e^{2T}} - 1 \right) = \frac{A^2}{2}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_0^T A^2 e^{-2t} dt = -\frac{A^2}{2} \lim_{T \rightarrow \infty} \left(\frac{1}{2Te^{2T}} - \frac{1}{2T} \right) = 0$$

$\therefore x_1(t)$ 为能量信号

(2) $x_2(t) = A \cos(\omega_0 t + \theta)$

$$w = \infty \quad P = \frac{A^2}{2}$$

$$w = \lim_{T \rightarrow \infty} \int_{-T}^T A^2 \cos(\omega_0 t + \theta) dt$$

$$= A^2 \lim_{T \rightarrow \infty} \int_{-T}^T \frac{\cos(2\omega_0 t + 2\theta) + 1}{2} dt$$

$$= \frac{A^2}{2} \lim_{T \rightarrow \infty} \left[\frac{1}{2\omega_0} \sin(2\omega_0 t + 2\theta) + t \right]_{-T}^T$$

$$\begin{aligned}
 &= \frac{A^2}{2} \lim_{T \rightarrow \infty} \left[\frac{1}{2\omega_0} \sin(2\omega_0 T + 2\theta) - \frac{1}{2\omega_0} \sin(-2\omega_0 T + 2\theta) + 2T \right] \\
 &= \infty \\
 P &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x_2^2(t) dt \\
 &= \frac{A^2}{2} \lim_{T \rightarrow \infty} \left[\frac{\frac{1}{2\omega_0} \sin(2\omega_0 T + 2\theta) - \frac{1}{2\omega_0} \sin(-2\omega_0 T + 2\theta)}{2T} + 1 \right] \\
 &= \frac{A^2}{2} + \lim_{T \rightarrow \infty} \frac{\sin(2\omega_0 T + 2\theta) - \sin(-2\omega_0 T + 2\theta)}{4\omega_0 T} \\
 &= \frac{A^2}{2}
 \end{aligned}$$

$\therefore x_2(t)$ 为功率信号

$$(3) \quad x_3(t) = \sin 2t + \sin 2\pi t$$

$$\begin{aligned}
 w &= \lim_{T \rightarrow \infty} \int_{-T}^T (\sin 2t + \sin 2\pi t)^2 dt \\
 &= \lim_{T \rightarrow \infty} \int_{-T}^T (\sin^2 2t + 2 \sin 2t \sin 2\pi t + \sin^2 2\pi t) dt \\
 &= \lim_{T \rightarrow \infty} \int_{-T}^T \left[\frac{1 - \cos 4t}{2} + \frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{2} + \frac{1 - \cos 4\pi t}{2} \right] dt \quad \begin{matrix} \alpha = 2t \\ \beta = 2\pi t \end{matrix} \\
 &= \lim_{T \rightarrow \infty} \int_{-T}^T \left[1 - \frac{\cos 4t}{2} + \frac{\cos(\alpha + \beta) - \cos(\alpha - \beta)}{2} - \frac{\cos 4\pi t}{2} \right] dt \\
 &= \lim_{T \rightarrow \infty} \left[t - \frac{\sin 4t}{8} + \frac{\sin(2 + 2\pi)t}{(2 + 2\pi)2} - \frac{\sin(2 - 2\pi)t}{(2 - 2\pi)2} - \frac{\sin 4\pi t}{8\pi} \right]_{-T}^T \\
 &= \lim_{T \rightarrow \infty} \left[2T - \frac{\sin 4T}{8} + \frac{\sin(-4T)}{8} + \frac{\sin(2 + 2\pi)T}{4 + 4\pi} + \frac{\sin(2 + 2\pi)T}{4 + 4\pi} \right. \\
 &\quad \left. - \frac{\sin(2 - 2\pi)T}{4 - 4\pi} - \frac{\sin(2 - 2\pi)T}{4 - 4\pi} - \frac{\sin 4\pi T}{8} - \frac{\sin 4\pi T}{8} \right] \\
 &= \lim_{T \rightarrow \infty} \left[2T - \frac{\sin 4T}{4} + \frac{\sin(2 + 2\pi)T}{2 + 2\pi} - \frac{\sin(2 - 2\pi)T}{2 - 2\pi} - \frac{\sin 4\pi T}{4} \right] \\
 &= \infty
 \end{aligned}$$

$$\begin{aligned}
 P &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x_3^2(t) dt \\
 &= \lim_{T \rightarrow \infty} \left[1 - \frac{\sin 4T}{8T} + \frac{\sin(2+2\pi)T}{(2+2\pi)2T} - \frac{\sin(2-2\pi)T}{(2-2\pi)2T} - \frac{\sin 4\pi T}{8T} \right] \\
 &= 1
 \end{aligned}$$

$\therefore x_3(t)$ 为功率信号

$$(4) \quad x_4(t) = e^{-t} \sin 2t$$

$$\begin{aligned}
 w &= \lim_{t \rightarrow \infty} \int_{-T}^T e^{-t} \sin^2 2t dt \\
 &= \lim_{T \rightarrow \infty} \int_{-T}^T e^{-t} \frac{1 - \cos 4t}{2} dt \\
 &= \lim_{T \rightarrow \infty} \int_{-T}^T \frac{e^{-2t}}{2} dt - \lim_{t \rightarrow \infty} \int_{-T}^T e^{-2t} \cos 4t dt \\
 &= \lim_{T \rightarrow \infty} \left[\frac{e^{-2t}}{-4} \right]_{-T}^T - \lim_{T \rightarrow \infty} \int_{-T}^T e^{-2t} \cos 4t dt \\
 &= \lim_{T \rightarrow \infty} \left(\frac{e^{-2T}}{-4} + \frac{e^{2T}}{4} \right) - \lim_{T \rightarrow \infty} \int_{-T}^T e^{-2t} \cos 4t dt \\
 \because \int_{-T}^T e^{-2t} \cos 4t dt &= \frac{1}{5} \left[-\frac{1}{2} e^{-2t} \cos 4t + e^{-2t} \sin 4t \right]_{-T}^T \\
 \therefore w &= \lim_{T \rightarrow \infty} \left(\frac{e^{-2T}}{-4} + \frac{e^{2T}}{4} \right) - \lim_{T \rightarrow \infty} \frac{1}{5} \left[-\frac{1}{2} e^{-2t} \cos 4t + e^{-2t} \sin 4t \right]_{-T}^T \\
 &= \lim_{T \rightarrow \infty} \left(\frac{e^{-2T}}{-4} + \frac{e^{2T}}{4} \right) - \frac{1}{5} \lim_{T \rightarrow \infty} \left[-\frac{1}{2} e^{-2T} \cos 4T + e^{-2T} \sin 4T + \frac{1}{2} e^{2T} \cos 4T + e^{2T} \sin 4T \right] \\
 &= \lim_{T \rightarrow \infty} \left(\frac{e^{-2T}}{-4} + \frac{e^{2T}}{4} + \frac{1}{10} e^{-2T} \cos 4T - \frac{1}{5} e^{-2T} \sin 4T - \frac{1}{10} e^{2T} \cos 4T - \frac{1}{5} e^{2T} \sin 4T \right) \\
 &= \lim_{T \rightarrow \infty} e^{-2T} \left[-\frac{1}{4} + \frac{\cos 4T}{10} - \frac{\sin 4T}{5} \right] + \lim_{T \rightarrow \infty} e^{2T} \left[\frac{1}{4} - \frac{\cos 4T}{10} - \frac{\sin 4T}{5} \right] \\
 &= 0 + \infty \\
 P &= \lim_{T \rightarrow \infty} \frac{e^{-2T}}{2T} \left[-\frac{1}{4} + \frac{\cos 4T}{10} - \frac{\sin 4T}{5} \right] + \lim_{T \rightarrow \infty} \frac{e^{2T}}{2T} \left[\frac{1}{4} - \frac{\cos 4T}{10} - \frac{\sin 4T}{5} \right] \\
 &= 0 + \infty \\
 \therefore x_4(t) &\text{ 既非功率信号, 也非能量信号。}
 \end{aligned}$$

4. 对下列每一个信号求能量 E 和功率 P :

$$(1) x_1(t) = e^{-2t} u(t)$$

$$(2) x_2(t) = e^{j(2t+\pi/4)}$$

$$(3) x_3(t) = \cos t$$

$$(4) x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$(5) x_2[n] = e^{j(\pi/2n+\pi/8)}$$

$$(6) x_3[n] = \cos\left(\frac{\pi}{4}n\right)$$

解:

$$(1) P_\infty = 0, E_\infty = 1/4$$

$$(2) P_\infty = 1, E_\infty = \infty$$

$$(3) P_\infty = 1/2, E_\infty = \infty$$

$$(4) P_\infty = 0, E_\infty = 4/3$$

$$(5) P_\infty = 1, E_\infty = \infty$$

$$(6) P_\infty = 1/2, E_\infty = \infty$$

习 题

1. 应用冲激信号的抽样特性, 求下列各表达式的函数值。

$$(1) \int_{-\infty}^{\infty} f(t-t_0)\delta(t)dt = f(t-t_0)|_{t=0} = f(-t_0)$$

$$(2) \int_0^{\infty} (e^t + t)\delta(t+2)dt = 0 \quad (\text{注意积分的上, 下限})$$

$$(3) \int_{-\infty}^{\infty} f(t-t_0)\delta(t-t_0)dt = f(t-t_0)|_{t=t_0} = f(0)$$

$$(4) \int_{-\infty}^{\infty} (t + \sin t)\delta(t - \frac{\pi}{6})dt = t + \sin t|_{t=\frac{\pi}{6}} = \frac{\pi}{6} + \frac{1}{2}$$

$$(5) \int_0^{\infty} \delta(t-t_0)u(t - \frac{t_0}{2})dt = u(t - \frac{t_0}{2})|_{t=\frac{t_0}{2}} = u(\frac{t_0}{2}) = u(t_0)$$

$$(6) \int_{-\infty}^{\infty} e^{-j\omega t}(\delta(t) - \delta(t-t_0))dt = \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t)dt - \int_{-\infty}^{\infty} e^{-j\omega t}\delta(t-t_0)dt = 1 - e^{-j\omega t_0}$$

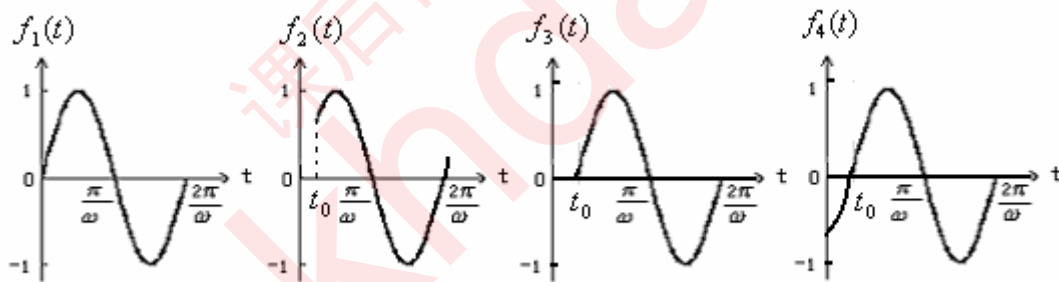
2. 绘出下列各时间函数的波形图, 注意它们的区别。

$$(1) f_1(t) = \sin \omega t u(t)$$

$$(2) f_2(t) = \sin \omega t u(t-t_0)$$

$$(3) f_3(t) = \sin \omega(t-t_0)u(t-t_0)$$

$$(4) f_4(t) = \sin \omega(t-t_0)u(t)$$



3. 连续时间信号 $x_1(t)$ 和 $x_2(t)$ 如图示, 试画出下列信号的波形。

$$(1) 2x_1(t)$$

$$(2) 0.5x_1(t)$$

$$(3) x_1(t-2)$$

$$(4) x_1(2t)$$

$$(5) x_1(2t+1) \text{ 和 } x_1(2t-1)$$

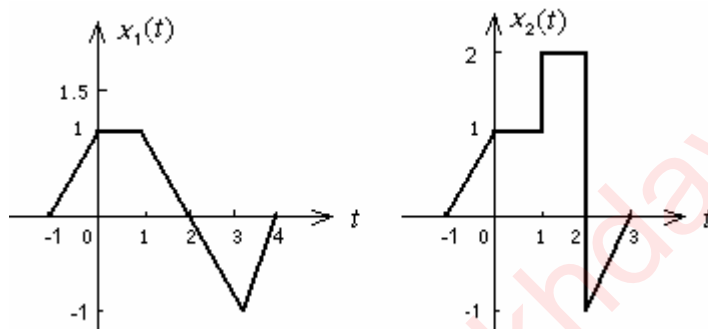
$$(6) x_1(-t-1)$$

$$(7) x_2(2-t/3)$$

$$(8) -x_2(-2t+1/2)$$

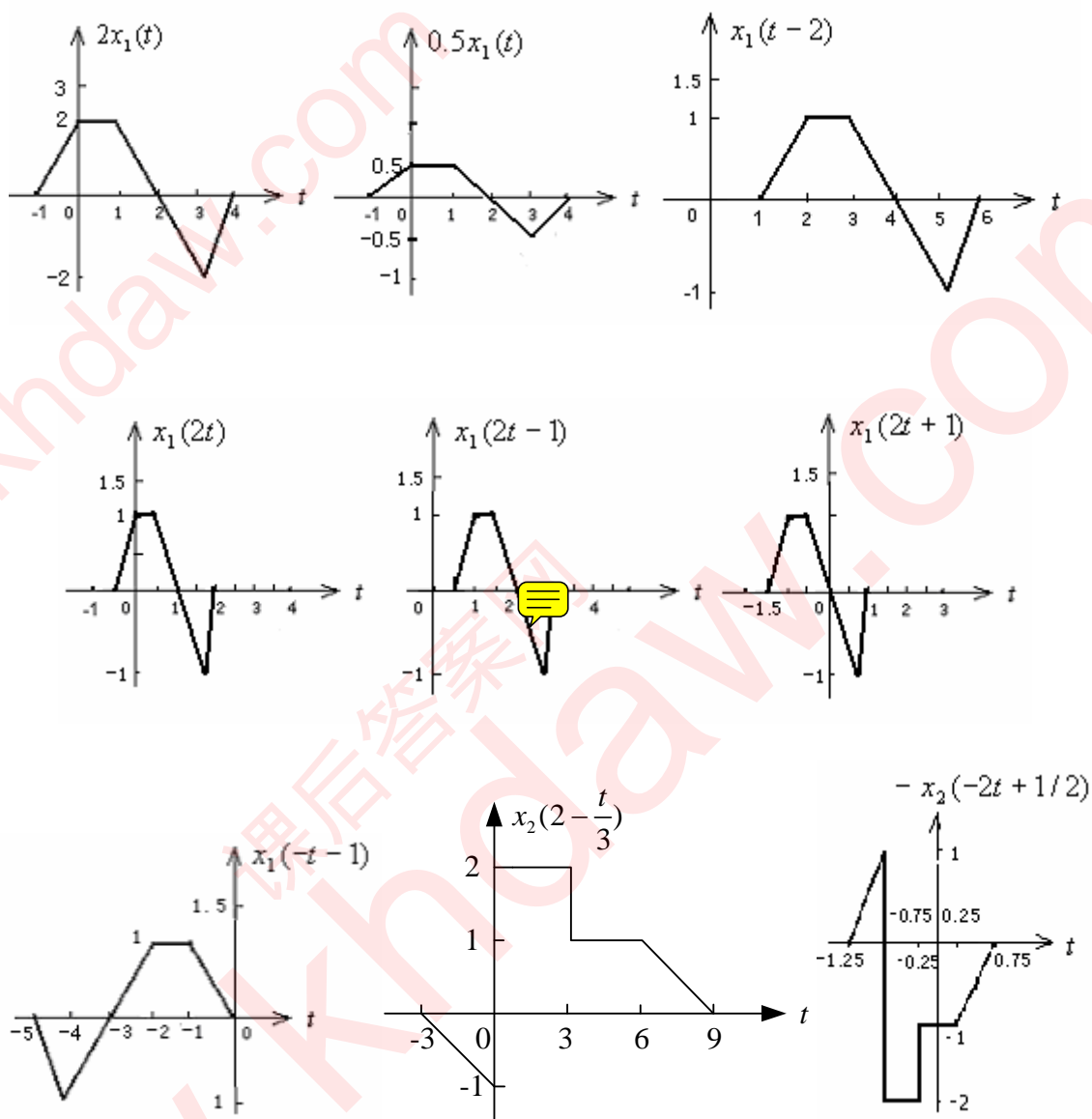
$$(9) x_1(t) \cdot x_2(t)$$

(10) 分别画出 $x_1'(t)$ 和 $x_2'(t)$ 的波形并写出相应的表达式。



题 3 图

解: (1)----(8)

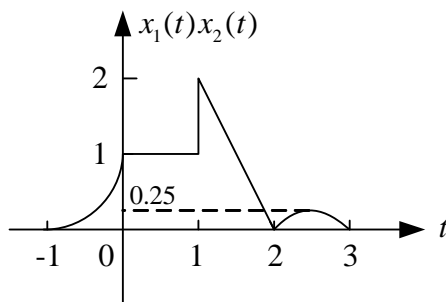


$$(7) \quad x_2(t) \rightarrow x_2(t+2) \rightarrow x_2(-t+2) \rightarrow x_2(2-\frac{t}{3})$$

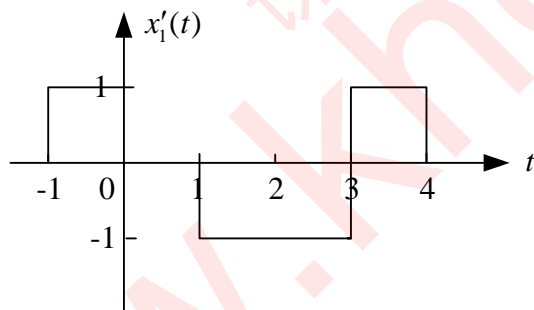
$$(9) \quad x_1(t) = \begin{cases} t+1, & -1 \leq t < 0 \\ 1, & 0 \leq t < 1 \\ -t+2, & 1 \leq t < 3 \\ t-4, & 3 \leq t \leq 4 \\ 0, & \text{其它} \end{cases}$$

$$x_2(t) = \begin{cases} t+1, & -1 \leq t < 0 \\ 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ t-3, & 2 \leq t \leq 3 \\ 0, & \text{其它} \end{cases}$$

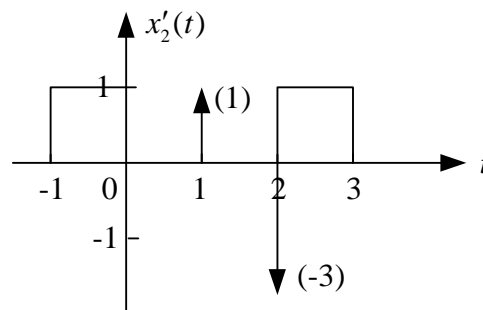
$$x_1(t)x_2(t) = \begin{cases} (t+1)^2, & -1 \leq t < 0 \\ 1, & 0 \leq t < 1 \\ -2t+4, & 1 \leq t < 2 \\ -(t-2.5)^2 + 0.25, & 2 \leq t \leq 3 \\ 0, & \text{其它} \end{cases}$$



$$(10) \quad x'_1(t) = \begin{cases} 1, & -1 \leq t < 0 \\ 0, & 0 \leq t < 1 \\ -1, & 1 \leq t < 3 \\ 1, & 3 \leq t \leq 4 \\ 0, & \text{其它} \end{cases}$$



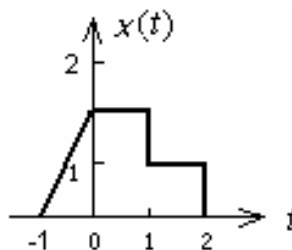
$$x'_2(t) = \begin{cases} 1, & -1 \leq t < 0 \\ 0, & 0 \leq t < 1 \\ \delta(t), & t = 1 \\ 0, & 1 < t < 2 \\ -3\delta(t), & t = 2 \\ 1, & 2 \leq t \leq 3 \\ 0, & \text{其它} \end{cases}$$



4. 已知 $x(t)$ 如题图 1-2 所示, 试画出 $y_1(t)$ 和 $y_2(t)$ 的波形。

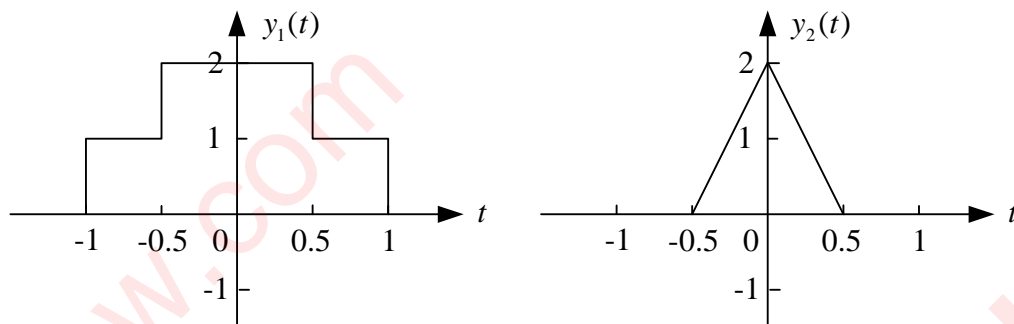
(1) $y_1(t) = x(2t)u(t) + x(-2t)u(-2t)$

(2) $y_2(t) = x(2t)u(-t) + x(-2t)u(t)$



题图 1-2

解:



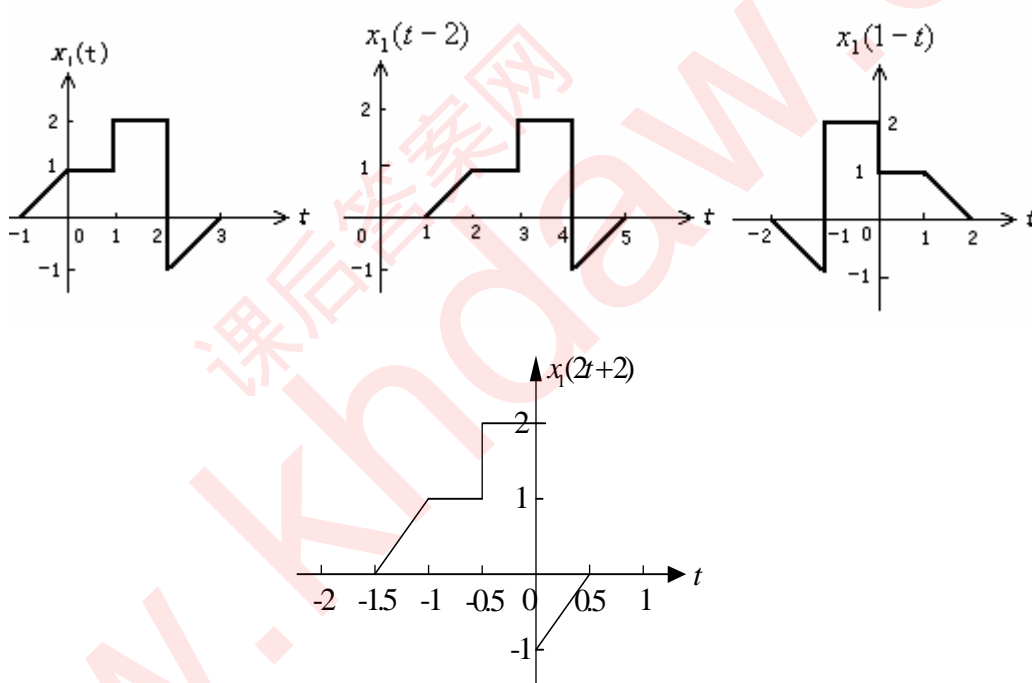
5. 已知连续时间信号 $x_1(t)$ 如题图 1-3 所示, 试画出下列各信号的波形图。

(1) $x_1(t-2)$

(2) $x_1(1-t)$

(3) $x_1(2t+2)$

解:



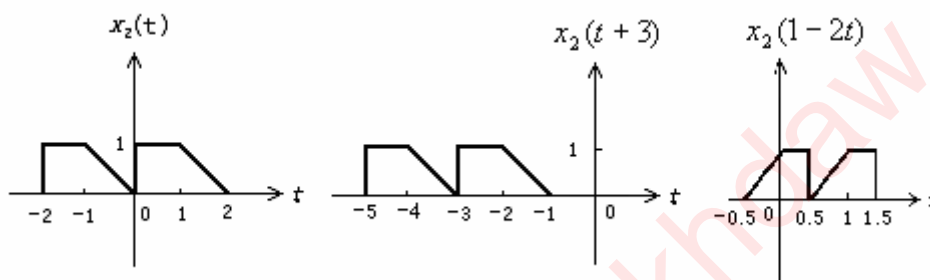
6. 根据题图 1-4 所示的信号 $x_2(t)$, 试画出下列各信号的波形图。

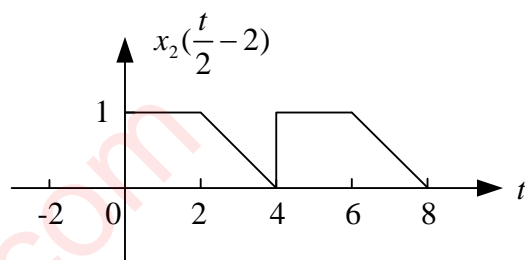
(1) $x_2(t+3)$

(2) $x_2(\frac{t}{2}-2)$

(3) $x_2(1-2t)$

解:





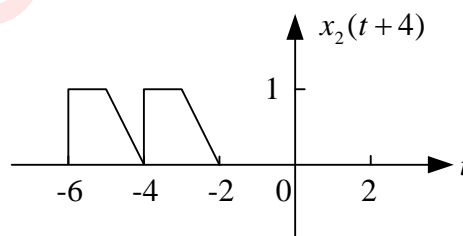
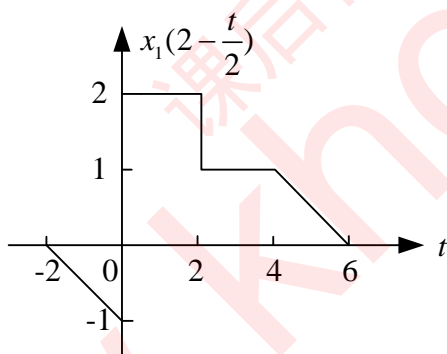
7. 根据题图 1-3 和题图 1-4 所示的 $x_1(t)$ 和 $x_2(t)$, 画出下列各信号的波形图。

(1) $x_1(t)x_2(-t)$ (2) $x_1(1-t)x_2(t-1)$ (3) $x_1(2-\frac{t}{2})x_2(t+4)$

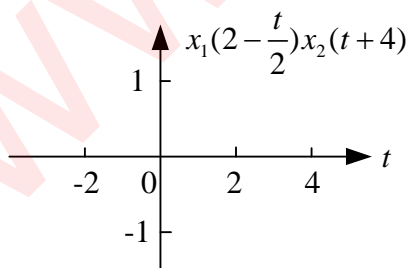
解: (1) $x_1(t)x_2(-t) = \begin{cases} t+2 & -2 \leq t < -1 \\ 1 & -1 < t \leq 0 \\ t & 0 < t \leq 1 \\ 1 & 1 < t \leq 2 \\ 0 & \text{others} \end{cases}$

(2) $x_1(1-t)x_2(t-1) = \begin{cases} 2 & -1 \leq t \leq 0 \\ 1-t & 0 < t \leq 1 \\ 2-t & 1 < t \leq 2 \\ 0 & \text{others} \end{cases}$

(3)

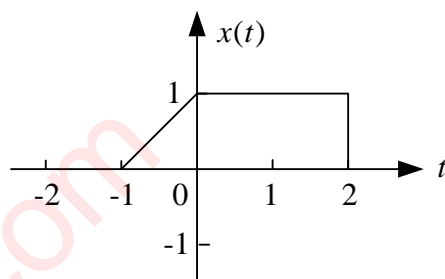


$x_1(2-\frac{t}{2})x_2(t+4) = 0$



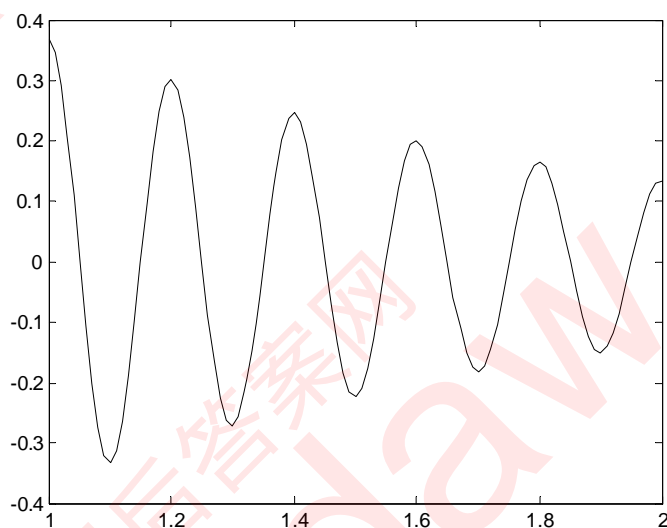
8. 已知信号 $x(5-2t)$ 的波形如题图 1-5 所示, 试画出 $x(t)$ 的波形图。

解: $x(5-2t) \rightarrow x(5-t) \rightarrow x(5+t) \rightarrow x(t)$



9. 画出下列各信号的波形图

(2) $x(t) = e^{-t} \cos 10\pi t [u(t-1) - u(t-2)]$



10. 已知信号 $x(t) = \sin t \times [u(t) - u(t - \pi)]$, 求

(1) $x_1(t) = \frac{d^2}{dt^2} x(t) + x(t)$

(2) $x_2(t) = \int_{-\infty}^t x(\tau) d\tau$

解: (1) $\frac{dx_1(t)}{dt} = \cos t [u(t) - u(t - \pi)] + \sin t [\delta(t) - \delta(t - \pi)]$

$$= \cos t [u(t) - u(t - \pi)]$$

$$x_1(t) = \frac{d^2}{dt^2} x(t) + x(t)$$

$$= -\sin t [u(t) - u(t - \pi)] + \cos t [\delta(t) - \delta(t - \pi)] + \sin t [u(t) - u(t - \pi)]$$

$$= \cos t [\delta(t) - \delta(t - \pi)]$$

$$= \delta(t) + \delta(t - \pi)$$

(2)(i) 当 $t < 0$ 时, $x_2(t) = \int_{-\infty}^t x(\tau) d\tau = \int_{-\infty}^t \sin \tau [u(\tau) - u(\tau - \pi)] d\tau = 0$

(ii) 当 $0 \leq t < \pi$ 时, $x_2(t) = \int_{-\infty}^t x(\tau) d\tau = \int_0^t \sin \tau d\tau = 1 - \cos t$

(iii) 当 $t \geq \pi$ 时, $x_2(t) = \int_{-\infty}^t x(\tau) d\tau = \int_0^\pi \sin \tau d\tau = 2$

$$\text{综上所述, } x_2(t) = \begin{cases} 0, t < 0 \\ 1 - \cos t, 0 \leq t < \pi \\ 2, t \geq \pi \end{cases}$$

11. 计算下列积分:

$$(1) \int_{-\infty}^{\infty} \sin t \cdot \delta(t - \frac{T_1}{2}) dt = \sin \frac{T_1}{2}$$

$$(2) \int_{-\infty}^{\infty} e^{-t} \times \delta(t + 2) dt = e^2$$

$$(3) \int_{-\infty}^{\infty} (t^3 + t + 2) \delta(t - 1) dt = 4$$

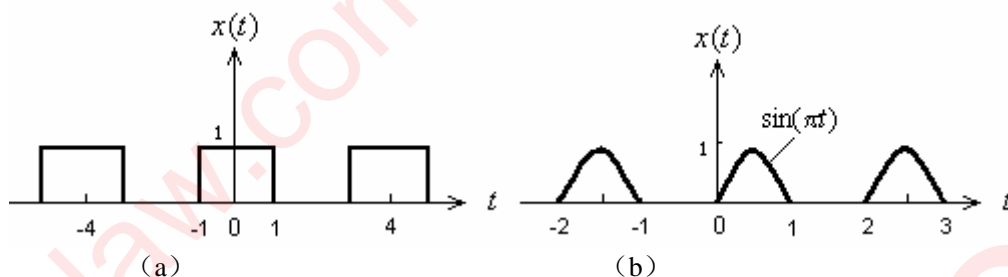
$$(4) \int_{-\infty}^{\infty} u(t - \frac{t_0}{2}) \delta(t - t_0) dt = u(\frac{t_0}{2})$$

$$(5) \int_{-\infty}^{\infty} e^{-\tau} \delta(\tau) d\tau = 1$$

$$(6) \int_{-1}^1 \delta(t^2 - 4) dt = 0$$

习 题(p61)

1. 用直接计算傅里叶系数的方法, 求题图 1-6 所示周期函数的傅里叶系数(三角形式或指数形式)。



题图 1-6 周期函数的傅里叶系数

解: (a) 周期为 $T_0 = 4$, $\omega_0 = \frac{\pi}{2}$, 信号在一个周期内的表达式为: $x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$

(1) 三角形式

$$a_0 = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) dt = \frac{1}{2} \int_{-1}^1 dt = 1$$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \cos(n\omega_0 t) dt \quad n = 1, 2, \dots$$

$$= \frac{1}{2} \int_{-1}^1 \cos(n\omega_0 t) dt$$

$$= \frac{1}{2} \frac{1}{n\omega_0} \sin(n\omega_0 t) \Big|_{-1}^1$$

$$= \frac{1}{n\omega_0} \sin(n\omega_0 t) \Big|_0^1$$

$$= \frac{\sin(n\omega_0)}{n\omega_0}$$

$$= Sa(n\omega_0)$$

$$= Sa\left(\frac{\pi n}{2}\right)$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) \sin(n\omega_0 t) dt \quad n = 1, 2, \dots$$

$$= \frac{1}{2} \int_{-1}^1 \sin(n\omega_0 t) dt$$

$$= 0$$

$$\text{所以, } x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} sa\left(\frac{\pi n}{2}\right) \cos\left(\frac{\pi n t}{2}\right) \quad n = 1, 2, \dots$$

(2) 指数形式

$$\begin{aligned}
 X(n\omega_0) &= \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-jn\omega_0 t} dt \\
 &= \frac{1}{4} \int_{-1}^1 e^{-jn\omega_0 t} dt \\
 &= \frac{1}{4} \left(-\frac{1}{jn\omega_0} e^{-jn\omega_0 t} \Big|_{-1}^1 \right) \\
 &= \frac{1}{4} \left(e^{jn\omega_0} - e^{-jn\omega_0} \right) \times \frac{1}{jn\omega_0} \\
 &= \frac{1}{2n\omega_0} \sin(n\omega_0) \\
 &= \frac{1}{2} \text{Sa}(n\omega_0) \\
 &= \frac{1}{2} \text{Sa}\left(\frac{\pi n}{2}\right)
 \end{aligned}$$

$$\text{所以, } x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{2} \text{sa}\left(\frac{\pi n}{2}\right) e^{j\frac{\pi n t}{2}}$$

(b) 周期为 $T_0 = 2, \omega_0 = \pi$, 信号在一个周期内的表达式为:

$$x(t) = \begin{cases} \sin(\pi t), & 0 < t < 1 \\ 0, & \text{其它} \end{cases}$$

(1) 三角形式

$$a_0 = \frac{2}{2} \int_0^1 x(t) dt = \int_0^1 \sin(\pi t) dt = -\frac{1}{\pi} \cos(\pi t) \Big|_0^1 = \frac{2}{\pi}$$

$$a_n = \frac{2}{2} \int_0^1 x(t) \cos(n\omega_0 t) dt \quad n = 1, 2, \dots$$

$$= \int_0^1 \sin(\pi t) \cos(n\omega_0 t) dt$$

$$= -\frac{1}{2(\pi + n\omega_0)} \cos(\pi + n\omega_0)t \Big|_0^1 - \frac{1}{2(\pi - n\omega_0)} \cos(\pi - n\omega_0)t \Big|_0^1$$

$$= -\frac{1}{2(\pi + n\omega_0)} \cos(\pi + n\omega_0) + \frac{1}{2(\pi + n\omega_0)} - \frac{1}{2(\pi - n\omega_0)} \cos(\pi - n\omega_0) + \frac{1}{2(\pi - n\omega_0)}$$

$$= \frac{\pi \cos(n\omega_0)}{\pi^2 - n^2 \omega_0^2} + \frac{\pi}{\pi^2 - n^2 \omega_0^2}$$

$$= \frac{\cos(\pi n)}{\pi(1 - n^2)} + \frac{1}{\pi(1 - n^2)}$$

$$= \frac{\cos(\pi n) + 1}{\pi(1 - n^2)}$$

$$\begin{aligned}
 b_n &= \int_0^1 \sin(\pi t) \sin(n\omega_0 t) dt \quad n=1, 2, \dots \\
 &= -\frac{1}{2(\pi + n\omega_0)} \sin(\pi + n\omega_0)t \Big|_0^1 + \frac{1}{2(\pi - n\omega_0)} \sin(\pi - n\omega_0)t \Big|_0^1 \\
 &= \frac{\pi}{\pi^2 - n^2\omega_0^2} \sin(n\omega_0) \\
 &= 0
 \end{aligned}$$

$$\text{所以, } x(t) = \frac{1}{\pi} + \sum_{n=1}^{\infty} \frac{\cos(\pi n) + 1}{\pi(1 - n^2)} \cos(n\pi t)$$

(b) 指数形式

$$\begin{aligned}
 X(n\omega_0) &= \frac{1}{2} \int_0^1 \sin(\pi t) e^{-jn\omega_0 t} dt \\
 &= \frac{1}{2} \frac{1}{\pi^2 - n^2\omega_0^2} e^{-jn\omega_0 t} [-jn\omega_0 \sin(\pi t) - \pi \cos(\pi t)] \Big|_0^1 \\
 &= \frac{\pi e^{-jn\omega_0}}{2(\pi^2 - n^2\omega_0^2)} + \frac{\pi}{2(\pi^2 - n^2\omega_0^2)} \\
 &= \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)} \\
 &= \frac{1 + \cos(n\pi)}{2\pi(1 - n^2)}
 \end{aligned}$$

$$\text{所以, } x(t) = \sum_{n=-\infty}^{\infty} \frac{1 + \cos(n\pi)}{2\pi(1 - n^2)} e^{jn\pi t}$$

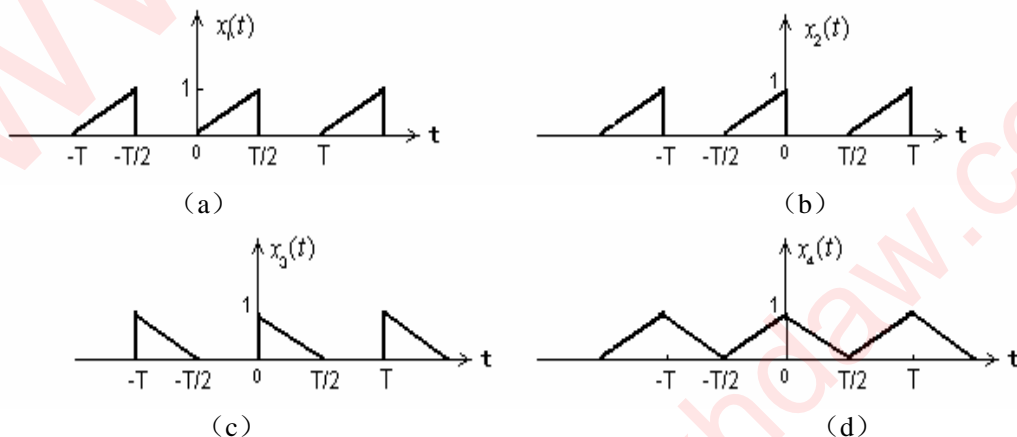
2. 如题图 1-7 所示是四个周期相同的信号

(1) 用直接求傅立叶系数的方法求题图 1-7a 所示信号的傅立叶级数 (三角形形式);

(2) 将题图 1-7a 的函数 $x_1(t)$ 左或右移 $T/2$, 就得到题图 1-7b 函数 $x_2(t)$, 利用(1)的结果求 $x_2(t)$ 的傅立叶级数;

(3) 利用以上结果求题图 1-7c 的函数 $x_3(t)$ 的傅立叶级数;

(4) 利用以上结果求题图 1-7d 的函数 $x_4(t)$ 的傅立叶级数。



题图 1-7

解: (1) $a_{01} = \frac{2}{T} \int_0^{\frac{T}{2}} \frac{2t}{T} dt = \frac{2}{T^2} t^2 \Big|_0^{\frac{T}{2}} = 0.5$

$$\begin{aligned} a_{n1} &= \frac{2}{T} \int_0^{\frac{T}{2}} \frac{2t}{T} \cos(n\omega_0 t) dt \quad n=1, 2, \dots \\ &= \frac{4}{T^2} \int_0^{\frac{T}{2}} t \cos(n\omega_0 t) dt \\ &= \frac{4}{T^2} \left[\left(\frac{1}{n\omega_0} \right)^2 \cos(n\omega_0 t) + \left(\frac{1}{n\omega_0} \right) t \sin(n\omega_0 t) \right] \Big|_0^{\frac{T}{2}} \\ &= \frac{4}{T^2} \left[\left(\frac{1}{n\omega_0} \right)^2 \cos(n\pi) + \left(\frac{T}{2n\omega_0} \right) \sin(n\pi) - \left(\frac{1}{n\omega_0} \right)^2 \right] \\ &= \frac{4}{T^2} \left[\left(\frac{T}{2\pi n} \right)^2 \cos(n\pi) + \left(\frac{T^2}{4\pi n} \right) \sin(n\pi) - \left(\frac{T}{2\pi n} \right)^2 \right] \\ &= \frac{1}{\pi^2 n^2} \cos(n\pi) + \frac{1}{\pi n} \sin(n\pi) - \frac{1}{\pi^2 n^2} \\ &= \frac{\cos(n\pi) - 1}{\pi^2 n^2} \end{aligned}$$

$$\begin{aligned} b_{n1} &= \frac{2}{T} \int_0^{\frac{T}{2}} \frac{2t}{T} \sin(n\omega_0 t) dt \quad n=1, 2, \dots \\ &= \frac{4}{T^2} \int_0^{\frac{T}{2}} t \sin(n\omega_0 t) dt \\ &= \frac{4}{T^2} \left[\left(\frac{1}{n\omega_0} \right)^2 \sin(n\omega_0 t) - \left(\frac{1}{n\omega_0} \right) t \cos(n\omega_0 t) \right] \Big|_0^{\frac{T}{2}} \\ &= \frac{4}{T^2} \left[\left(\frac{T}{2\pi n} \right)^2 \sin(n\pi) - \left(\frac{T^2}{4\pi n} \right) \cos(n\pi) \right] \\ &= \frac{1}{\pi^2 n^2} \sin(n\pi) - \frac{1}{\pi n} \cos(n\pi) \\ &= -\frac{\cos(n\pi)}{n\pi} \end{aligned}$$

所以,

$$\begin{aligned} x_1(t) &= \frac{a_{01}}{2} + \sum_{n=1}^{\infty} \left[a_{n1} \cos\left(\frac{2\pi n t}{T}\right) + b_{n1} \sin\left(\frac{2\pi n t}{T}\right) \right] \\ &= 0.25 + \sum_{n=1}^{\infty} \left[\frac{\cos(n\pi) - 1}{\pi^2 n^2} \cos\left(\frac{2\pi n t}{T}\right) - \frac{\cos(n\pi)}{n\pi} \sin\left(\frac{2\pi n t}{T}\right) \right] \end{aligned}$$

(2)

$$\begin{aligned} x_2(t) &= x_1\left(t + \frac{T}{2}\right) \\ &= \frac{a_{01}}{2} + \sum_{n=1}^{\infty} \left\{ a_{n1} \cos\left[\frac{2\pi n}{T}\left(t + \frac{T}{2}\right)\right] + b_{n1} \sin\left[\frac{2\pi n}{T}\left(t + \frac{T}{2}\right)\right] \right\} \\ &= \frac{a_{01}}{2} + \sum_{n=1}^{\infty} \left[a_{n1} \cos(\pi n) \cos\left(\frac{2\pi nt}{T}\right) + b_{n1} \cos(\pi n) \sin\left(\frac{2\pi nt}{T}\right) \right] \end{aligned}$$

所以, $a_{02} = a_{01} = 0.5$, $a_{n2} = a_{n1} \cos(\pi n) = \frac{\cos(n\pi) - 1}{\pi^2 n^2} \cos(n\pi) = \frac{1 - \cos(n\pi)}{\pi^2 n^2}$,

$$b_{n2} = b_{n1} \cos(n\pi) = -\frac{\cos(n\pi)}{n\pi} \cos(n\pi) = -\frac{1}{n\pi}, \quad n = 1, 2, \dots$$

$$x_2(t) = 0.25 + \sum_{n=1}^{\infty} \left[\frac{1 - \cos(n\pi)}{\pi^2 n^2} \cos\left(\frac{2\pi nt}{T}\right) - \frac{1}{n\pi} \sin\left(\frac{2\pi nt}{T}\right) \right].$$

(3)

$$\begin{aligned} x_3(t) &= x_2(-t) = 0.25 + \sum_{n=1}^{\infty} \left[\frac{1 - \cos(n\pi)}{\pi^2 n^2} \cos\left(-\frac{2\pi nt}{T}\right) - \frac{1}{n\pi} \sin\left(-\frac{2\pi nt}{T}\right) \right] \\ &= 0.25 + \sum_{n=1}^{\infty} \left[\frac{1 - \cos(n\pi)}{\pi^2 n^2} \cos\left(\frac{2\pi nt}{T}\right) + \frac{1}{n\pi} \sin\left(\frac{2\pi nt}{T}\right) \right] \end{aligned}$$

所以, $a_{03} = 0.5$, $a_{n3} = \frac{1 - \cos(n\pi)}{\pi^2 n^2}$, $b_{n3} = \frac{1}{n\pi}$, $n = 1, 2, \dots$

(4)

$$x_4(t) = x_2(t) + x_3(t) = 0.5 + \sum_{n=1}^{\infty} \frac{2(1 - \cos n\pi)}{\pi^2 n^2} \cos\left(\frac{2\pi nt}{T}\right)$$

所以, $a_{04} = 1$, $a_{n4} = \frac{2(1 - \cos n\pi)}{\pi^2 n^2}$, $b_{n4} = 0$, $n = 1, 2, \dots$

3. 实际中有一种利用非线性器件产生谐波的方法, 其脉冲波形如图 P3.3 所示;

- (1) 求脉冲波形中三次谐波的幅度;
- (2) 使三次谐波幅度为最大的最佳截止角 θ_0 的值。

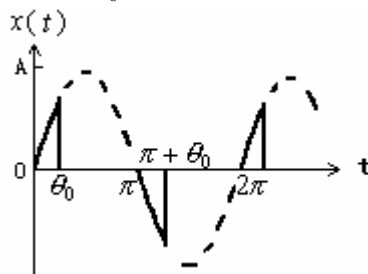


图 P3.3

解: 由信号图可知, 图示波形数学表达式为:

$$x(t) = A \sin \omega_0 t \cdot [u(t) - u(t - \theta_0) + u(t - \pi) - u(t - \pi - \theta_0)]$$

信号为周期信号, 周期为 $T = 2\pi$, $\omega_0 = 1$, 所以有

$$x(t) = A \sin t \cdot [u(t) - u(t - \theta_0) + u(t - \pi) - u(t - \pi - \theta_0)]$$

求解其傅立叶系数分别为：

$$\begin{aligned} a_0 &= \frac{2}{T_0} \int_0^{2\pi} x(t) dt = \frac{1}{\pi} \int_0^{2\pi} x(t) dt \\ &= \frac{A}{\pi} \int_0^{2\pi} \sin t [u(t) - u(t - \theta_0) + u(t - \pi) - u(t - \pi - \theta_0)] dt \\ &= \frac{A}{\pi} \left[\int_0^{2\pi} \sin t dt - \int_{\theta_0}^{2\pi} \sin t dt + \int_{\pi}^{2\pi} \sin t dt - \int_{\pi+\theta_0}^{2\pi} \sin t dt \right] \\ &= \frac{A}{\pi} \left[-\cos t \Big|_0^{2\pi} + \cos t \Big|_{\theta_0}^{2\pi} - \cos t \Big|_{\pi}^{2\pi} + \cos t \Big|_{\pi+\theta_0}^{2\pi} \right] \\ &= 0 \\ a_n &= \frac{2}{T_0} \int_0^{2\pi} x(t) \cos ntdt = \frac{A}{\pi} \int_0^{2\pi} x(t) \cos ntdt \\ &= \frac{A}{\pi} \int_0^{2\pi} \sin t \cos nt [u(t) - u(t - \theta_0) + u(t - \pi) - u(t - \pi - \theta_0)] dt \\ &= \frac{A}{\pi} \left[\int_0^{2\pi} \sin t \cos ntdt - \int_{\theta_0}^{2\pi} \sin t \cos ntdt + \int_{\pi}^{2\pi} \sin t \cos ntdt - \int_{\pi+\theta_0}^{2\pi} \sin t \cos ntdt \right] \\ &= \frac{A}{2\pi} \left[\int_0^{2\pi} (\sin(t+nt) + \sin(t-nt)) dt - \int_{\theta_0}^{2\pi} (\sin(t+nt) + \sin(t-nt)) dt \right. \\ &\quad \left. + \int_{\pi}^{2\pi} (\sin(t+nt) + \sin(t-nt)) dt - \int_{\pi+\theta_0}^{2\pi} (\sin(t+nt) + \sin(t-nt)) dt \right] \\ &= \frac{A}{2\pi} \left\{ \left[-\frac{1}{n+1} \cos(t+nt) - \frac{1}{1-n} \cos(t-nt) \right] \Big|_0^{2\pi} - \left[-\frac{1}{n+1} \cos(t+nt) - \frac{1}{1-n} \cos(t-nt) \right] \Big|_{\theta_0}^{2\pi} \right. \\ &\quad \left. + \left[-\frac{1}{n+1} \cos(t+nt) - \frac{1}{1-n} \cos(t-nt) \right] \Big|_{\pi}^{2\pi} - \left[-\frac{1}{n+1} \cos(t+nt) - \frac{1}{1-n} \cos(t-nt) \right] \Big|_{\pi+\theta_0}^{2\pi} \right\} \\ &= \frac{A}{2\pi} \left\{ \left[-\frac{\cos(2\pi+2n\pi)}{n+1} - \frac{\cos(2\pi-2n\pi)}{1-n} + \frac{\cos 0}{n+1} + \frac{\cos 0}{1-n} \right] \right. \\ &\quad \left. - \left[-\frac{\cos(2\pi+2n\pi)}{n+1} - \frac{\cos(2\pi-2n\pi)}{1-n} + \frac{\cos(\theta_0+n\theta_0)}{n+1} + \frac{\cos(\theta_0-n\theta_0)}{1-n} \right] \right. \\ &\quad \left. + \left[-\frac{\cos(2\pi+2n\pi)}{n+1} - \frac{\cos(2\pi-2n\pi)}{1-n} + \frac{\cos(\pi+n\pi)}{n+1} + \frac{\cos(\pi-n\pi)}{1-n} \right] \right. \\ &\quad \left. - \left[-\frac{\cos(2\pi+2n\pi)}{n+1} - \frac{\cos(2\pi-2n\pi)}{1-n} + \frac{\cos(\pi+\theta_0+n\pi+n\theta_0)}{n+1} + \frac{\cos(\pi+\theta_0-n\pi-n\theta_0)}{1-n} \right] \right\} \\ &= \frac{A}{2\pi} \left[\frac{1}{n+1} + \frac{1}{1-n} - \frac{\cos(n+1)\theta_0}{n+1} - \frac{\cos(1-n)\theta_0}{1-n} + \frac{\cos(n+1)\pi}{n+1} + \frac{\cos(1-n)\pi}{1-n} \right. \\ &\quad \left. - \frac{\cos(n+1)\pi \cdot \cos(\theta_0+n\theta_0) - \sin(n+1)\pi \cdot \sin(\theta_0+n\theta_0)}{n+1} \right. \\ &\quad \left. - \frac{\cos(1-n)\pi \cdot \cos(\theta_0-n\theta_0) - \sin(1-n)\pi \cdot \sin(\theta_0-n\theta_0)}{n+1} \right] \end{aligned}$$

即

$$a_n = \begin{cases} \frac{A}{2\pi} \left(\frac{2-2\cos(n+1)\theta_0}{n+1} + \frac{2-2\cos(1-n)\theta_0}{1-n} \right) & n=1,3,5,\dots \\ 0, & n=0,2,4,6,\dots \end{cases}$$

$$\begin{aligned} b_n &= \frac{A}{\pi} \int_0^{2\pi} \sin t \sin nt [u(t) - u(t-\theta_0) + u(t-\pi) - u(t-\pi-\theta_0)] dt \\ &= \frac{A}{\pi} \left[\int_0^{2\pi} \sin t \sin nt dt - \int_{\theta_0}^{2\pi} \sin t \sin nt dt + \int_{\pi}^{2\pi} \sin t \sin nt dt - \int_{\pi+\theta_0}^{2\pi} \sin t \sin nt dt \right] \\ &= -\frac{A}{2\pi} \left[\int_0^{2\pi} (\cos(t+nt) - \cos(t-nt)) dt - \int_{\theta_0}^{2\pi} (\cos(t+nt) - \cos(t-nt)) dt \right. \\ &\quad \left. + \int_{\pi}^{2\pi} (\cos(t+nt) - \cos(t-nt)) dt - \int_{\pi+\theta_0}^{2\pi} (\cos(t+nt) - \cos(t-nt)) dt \right] \\ &= -\frac{A}{2\pi} \left\{ \left[\frac{1}{n+1} \sin(t+nt) - \frac{1}{1-n} \sin(t-nt) \right] \Big|_0^{2\pi} - \left[\frac{1}{n+1} \sin(t+nt) - \frac{1}{1-n} \sin(t-nt) \right] \Big|_{\theta_0}^{2\pi} \right. \\ &\quad \left. + \left[\frac{1}{n+1} \sin(t+nt) - \frac{1}{1-n} \sin(t-nt) \right] \Big|_{\pi}^{2\pi} - \left[\frac{1}{n+1} \sin(t+nt) - \frac{1}{1-n} \sin(t-nt) \right] \Big|_{\pi+\theta_0}^{2\pi} \right\} \\ &= -\frac{A}{2\pi} \left[\frac{\sin(n+1)\theta_0}{n+1} - \frac{\sin(1-n)\theta_0}{1-n} + \frac{\sin(n+1)\theta_0 \cos(1+n)\pi}{n+1} - \frac{\sin(1-n)\theta_0 \cos(1-n)\pi}{1-n} \right] \end{aligned}$$

所以, 有

$$b_n = \begin{cases} -\frac{A}{\pi} \left(\frac{\sin(n+1)\theta_0}{n+1} - \frac{\sin(1-n)\theta_0}{1-n} \right) & n=1,3,5,\dots \\ -\frac{A}{2\pi} \left(\frac{\sin(n+1)\theta_0}{n+1} - \frac{\sin(1-n)\theta_0}{1-n} \right) & n=2,4,6,\dots \end{cases}$$

因此

$$\begin{aligned} a_3 &= \frac{A}{2\pi} \left[\frac{1-\cos 4\theta_0}{2} - 1 + \cos 2\theta_0 \right] \\ &= \frac{A}{4\pi} [-1 - \cos 4\theta_0 + 2\cos 2\theta_0] \end{aligned}$$

$$\begin{aligned} b_3 &= -\frac{A}{2\pi} \left[\frac{\sin 4\theta_0}{4} - \frac{\sin(-2\theta_0)}{-2} \right] \\ &= -\frac{A}{4\pi} [\sin 4\theta_0 - 2\sin 2\theta_0] \end{aligned}$$

$$A_3 = \sqrt{a_3^2 + b_3^2} = \frac{A}{2\pi} (1 - \cos 2\theta_0)$$

当截止角 $\theta_0 = \frac{\pi}{2}$ 时, 3 次谐波幅值取最大值, 即 $A_{3\max} = \frac{A}{\pi}$.

4. 下列信号的傅立叶级数表达式。

(1) $x(t) = \cos 4t + \sin 6t$;

(2) $x(t)$ 是以 2 为周期的信号, 且 $x(t) = e^{-t}, -1 < t < 1$

解：(1) (i)三角形式：

两个周期信号相加后可否为周期信号？

假设 $x_1(t)$ 、 $x_2(t)$ 都是周期信号，对应的周期是 T_1, T_2 ，则它们的和是周期的，也即存在一个正数 T ，使得

$$x_1(t+T) + x_2(t+T) = x_1(t) + x_2(t)$$

当且仅当 T_1/T_2 是两个正整数 q, r 之比 q/r 时，上式才成立。如果 q, r 是互质的，则 $T = rT_1$ 是 $x_1(t) + x_2(t)$ 的基本周期。

$x(t) = \cos 4t + \sin 6t$ 的周期 $T_0 = \pi$ ， $\omega_0 = 2$

$$a_0 = 0, \quad a_n = \begin{cases} 1, & n = 2 \\ 0, & n \neq 2 \end{cases}, \quad b_n = \begin{cases} 1, & n = 3 \\ 0, & n \neq 3 \end{cases}$$

所以， $x(t)$ 的三角傅里叶级数仍为 $x(t) = \cos 4t + \sin 6t$

(ii)复指数形式：

$$X(n\omega_0) = \frac{1}{2}(a_n - jb_n) = \begin{cases} \frac{1}{2}, & n = \pm 2 \\ -\frac{1}{2}j, & n = 3 \\ \frac{1}{2}j, & n = -3 \\ 0, & \text{其它} \end{cases}$$

所以， $x(t)$ 的指数傅里叶级数 $x(t) = \frac{1}{2}(e^{j4t} + e^{-j4t}) - \frac{1}{2}j(e^{j6t} - e^{-j6t})$

(2) (i)三角形式：

$$a_0 = \frac{2}{2} \int_{-1}^1 e^{-t} dt = -e^{-t} \Big|_{-1}^1 = e - e^{-1}$$

$$a_n = \int_{-1}^1 e^{-t} \cos(n\pi t) dt \quad n = 1, 2, \dots$$

$$= \frac{1}{1+n^2\pi^2} e^{-t} [n\pi \sin(n\pi t) - \cos(n\pi t)] \Big|_{-1}^1$$

$$= \frac{(e - e^{-1}) \cos(n\pi)}{1+n^2\pi^2}$$

$$b_n = \int_{-1}^1 e^{-t} \sin(n\pi t) dt \quad n = 1, 2, \dots$$

$$= \frac{1}{1+n^2\pi^2} e^{-t} [-\sin(n\pi t) - n\pi \cos(n\pi t)] \Big|_{-1}^1$$

$$= \frac{(e - e^{-1})n\pi \cos(n\pi)}{1+n^2\pi^2}$$

$$\text{所以, } x(t) = \frac{e - e^{-1}}{2} + \sum_{n=1}^{\infty} \left[\frac{(e - e^{-1})\cos(n\pi)}{1+n^2\pi^2} \cos(n\pi t) + \frac{(e - e^{-1})n\pi \cos(n\pi)}{1+n^2\pi^2} \sin(n\pi t) \right]$$

(ii) 复指数形式:

$$X(n\omega_0) = \frac{1}{2}(a_n - jb_n) = \frac{(e - e^{-1})\cos(n\pi)}{2(1+n^2\pi^2)} - \frac{j(e - e^{-1})n\pi \cos(n\pi)}{2(1+n^2\pi^2)}$$

$$= \frac{(e - e^{-1})(1 - jn\pi)\cos(n\pi)}{2(1+n^2\pi^2)} = \frac{(e - e^{-1})\cos(n\pi)}{2(1 + jn\pi)}$$

$$\text{所以, } x(t) = \sum_{n=-\infty}^{\infty} \frac{(e - e^{-1})\cos(n\pi)}{2(1 + jn\pi)} e^{jn\pi t}$$

5. 设 $x(t)$ 是一个周期信号, 其基波周期为 T_0 , 傅立叶级数的系数为 A_k , 用 A_k 表示下列信号的傅里叶级数系数。

(1) $x(t - t_0)$

(2) $x(-t)$

(3) $x^*(t)$

(4) $\int_{-\infty}^t x(\tau) d\tau$, 假设 $A_0 = 0$

(5) $\frac{dx(t)}{dt}$

(6) $x(at), a > 0$, (要先确定该信号的周期)

解: 先确定是否为周期信号, 设 $x(t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k)$

$$(1) \quad x(t - t_0) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos[k\omega_0(t - t_0) + \varphi_k] = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos[k\omega_0 t + \varphi_k - k\omega_0 t_0]$$

$$A_{01} = A_0, \quad A_{k1} = A_k, \quad \varphi_{k1} = \varphi_k - k\omega_0 t_0, \quad k = 1, 2, \dots$$

$$(2) \quad x(-t) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos[k\omega_0(-t) + \varphi_k] = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t - \varphi_k)$$

$$A_{02} = A_0, \quad A_{k2} = A_k, \quad \varphi_{k2} = -\varphi_k, \quad k = 1, 2, \dots$$

$$(3) \quad x^*(t) = \frac{A_0^*}{2} + \sum_{k=1}^{\infty} A_k^* \cos(k\omega_0 t + \varphi_k)$$

$$A_{03} = A_0^*, \quad A_{k3} = A_k^*, \quad \varphi_{k3} = \varphi_k, \quad k = 1, 2, \dots$$

(4) $\int_{-\infty}^t x(\tau) d\tau$ 不一定为周期信号，所以不存在傅里叶级数。

$$\begin{aligned} (5) \quad \frac{dx(t)}{dt} &= \frac{d}{dt} \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \varphi_k) = \sum_{k=1}^{\infty} A_k \frac{d}{dt} \cos(k\omega_0 t + \varphi_k) \\ &= \sum_{k=1}^{\infty} -k\omega_0 A_k \sin(k\omega_0 t + \varphi_k) = \sum_{k=1}^{\infty} -k\omega_0 A_k \cos(k\omega_0 t + \varphi_k - \frac{\pi}{2}) \end{aligned}$$

$$A_{05} = 0, \quad A_{k5} = -k\omega_0 A_k, \quad \varphi_{k5} = \varphi_k - \frac{\pi}{2}, \quad k = 1, 2, \dots$$

(6) $x(t)$ 的周期为 $\frac{T_0}{a}$

$$x(at) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos(ka\omega_0 t + \varphi_k)$$

$$A_{06} = A_0, \quad A_{k6} = A_k, \quad \varphi_{k6} = \varphi_k, \quad \omega_{06} = a\omega_0, \quad k = 1, 2, \dots$$

$$x(at) = \frac{A_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_{05} t + \varphi_k)$$

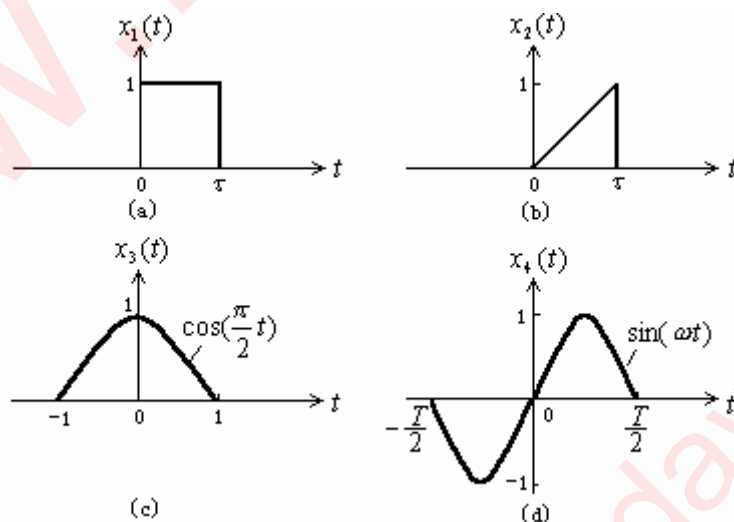
7. 计算下列连续时间周期信号（基波频率 $\omega_0 = \pi$ ）的傅立叶系数 a_k ：

$$x(t) = \begin{cases} 1.5 & 0 \leq t < 1 \\ -1.5 & 1 \leq t < 2 \end{cases}$$

解：

$$a_k = \int_0^2 x(t) \cos(n\pi t) dt = 1.5 \int_0^1 \cos(n\pi t) dt - 1.5 \int_1^2 \cos(n\pi t) dt = \frac{1.5}{n\pi} \sin(n\pi t) \Big|_0^1 - \frac{1.5}{n\pi} \sin(n\pi t) \Big|_1^2 = 0$$

9. 求题图 1-10 所示各信号的傅立叶变换。



题图 1-10

解: (a) $x_1(t) = \begin{cases} 1, & 0 < t < \tau \\ 0, & \text{其它} \end{cases}$

解法 1: 由定义 $X_1(\omega) = \int_{-\infty}^{+\infty} x_1(t) e^{-j\omega t} dt = \int_0^{\tau} e^{-j\omega t} dt = -\frac{1}{j\omega} e^{-j\omega t} \Big|_0^{\tau} = \frac{1 - e^{-j\omega\tau}}{j\omega}$

$$= \frac{e^{-\frac{j\omega\tau}{2}} (e^{\frac{j\omega\tau}{2}} - e^{-\frac{j\omega\tau}{2}})}{j\omega} = e^{-\frac{j\omega\tau}{2}} \tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$$

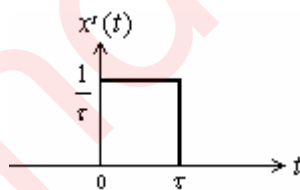
解法 2: 由傅里叶变换的时移特性以及 $g(t)$ 的傅里叶变换可得 $x_1(t) = g(t - \frac{\tau}{2})$, 所以

$$F[g(t)] = \tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$$

$$X_1(\omega) = e^{-\frac{j\omega\tau}{2}} \tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$$

(b) $x_2(t) = \begin{cases} \frac{t}{\tau}, & 0 < t < \tau \\ 0, & \text{其它} \end{cases}$

解法 1: 设 $x'(t) = \frac{dx_2}{dt}$, 如图所示



因为: $x' \xleftrightarrow{F} \text{Sa}\left(\frac{\omega\tau}{2}\right) e^{-j\frac{\tau}{2}\omega}$, 则

$$x_2(t) \xleftrightarrow{F} \frac{1}{j\omega} \text{Sa}\left(\frac{\omega\tau}{2}\right) e^{-j\frac{\tau}{2}\omega}$$

解法 2: 按照定义求解:

$$X_2(\omega) = \int_{-\infty}^{+\infty} x_2(t) e^{-j\omega t} dt = \frac{1}{\tau} \int_0^{\tau} t e^{-j\omega t} dt = \frac{1}{\tau} \left[\frac{1}{(-j\omega)^2} (-j\omega t - 1) e^{-j\omega t} \right]_0^{\tau} = \frac{(j\omega\tau + 1)e^{-j\omega\tau} - 1}{\omega^2 \tau}$$

(c) $x_3(t) = \begin{cases} \cos(\frac{\pi}{2}t), & 0 < t < 1 \\ 0, & \text{其它} \end{cases}$

$$X_3(\omega) = \int_{-\infty}^{\infty} x_3(t) e^{-j\omega t} dt = \int_{-1}^1 \cos\left(\frac{\pi}{2}t\right) e^{-j\omega t} dt$$

$$= \frac{\left[\frac{\pi}{2} e^{-j\omega t} \sin\left(\frac{\pi}{2}t\right) - j\omega \cos\left(\frac{\pi}{2}t\right) \right]_{-1}^1}{\left(\frac{\pi}{2}\right)^2 + (j\omega)^2} = \frac{2\pi}{\pi^2 - 4\omega^2} (e^{-j\omega} + e^{j\omega}) = \frac{4\pi \cos \omega}{\pi^2 - 4\omega^2}$$

$$(d) \quad x_4(t) = \begin{cases} \sin(\omega_0 t), & -\frac{T}{2} < t < \frac{T}{2}, \quad \omega_0 = \frac{2\pi}{T} \\ 0, & \text{其它} \end{cases}$$

$$X_4(\omega) = \int_{-\infty}^{\infty} x_4(t) e^{-j\omega t} dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} \sin(\omega_0 t) e^{-j\omega t} dt$$

$$= \frac{1}{(-j\omega)^2 + \omega_0^2} e^{-j\omega t} [-j\omega \sin(\omega_0 t) - \omega_0 \cos(\omega_0 t)] \Big|_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \frac{2\omega_0}{\omega^2 - \omega_0^2} (e^{j\omega \frac{T}{2}} - e^{-j\omega \frac{T}{2}}) = \frac{4j\omega_0 \sin(\frac{\omega T}{2})}{\omega^2 - \omega_0^2} = \frac{j8\pi \sin(\frac{\omega T}{2})}{T^2 \omega^2 - 4\pi^2}$$

10. 利用对偶性质求下列函数的傅立叶变换:

$$(1) \quad f(t) = \frac{\sin(2\pi(t-2))}{\pi(t-2)}, -\infty < t < \infty$$

$$(2) \quad f(t) = \frac{2a}{a^2 + t^2}, -\infty < t < \infty$$

$$(3) \quad f(t) = \left[\frac{\sin(2\pi t)}{2\pi t} \right]^2, -\infty < t < \infty$$

$$\text{解: (1)} \quad g(t) = \begin{cases} 1, & |t| < \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases} \longleftrightarrow \tau \text{Sa}\left(\frac{\omega\tau}{2}\right) = \tau \frac{\sin(\frac{\omega\tau}{2})}{\frac{\omega\tau}{2}}$$

由对偶特性

$$\tau \text{Sa}\left(\frac{t\tau}{2}\right) = \tau \frac{\sin(\frac{t\tau}{2})}{\frac{t\tau}{2}} \longleftrightarrow 2\pi g(-\omega) = 2\pi g(\omega) = \begin{cases} 2\pi, & |\omega| < \frac{\tau}{2} \\ 0, & |\omega| > \frac{\tau}{2} \end{cases}$$

令 $\tau = 4\pi$

$$\frac{4\pi \sin(2\pi t)}{2\pi t} \longleftrightarrow 2\pi g(\omega) = \begin{cases} 2\pi, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

$$\frac{\sin(2\pi t)}{\pi t} \longleftrightarrow g(\omega) = \begin{cases} 1, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

$$\frac{\sin[2\pi(t-2)]}{\pi(t-2)} \longleftrightarrow e^{-j2\omega} g(\omega) = \begin{cases} e^{-j2\omega}, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

$$(2) e^{-a|t|} \longleftrightarrow \frac{2a}{\omega^2 + a^2}, \quad a > 0$$

$$f(\omega) = \frac{2a}{a^2 + \omega^2}$$

$$F^{-1}[f(\omega)] = e^{-a|t|}, \quad a > 0$$

所以, 有

$$\frac{2a}{a^2 + t^2} \longleftrightarrow 2\pi e^{-a|\omega|}$$

$$(3) \text{ 由 (1) 可知: } \frac{\sin(2\pi t)}{2\pi t} \longleftrightarrow \frac{1}{2} g(\omega) = \begin{cases} \frac{1}{2}, & |\omega| < 2\pi \\ 0, & |\omega| > 2\pi \end{cases}$$

$$\left[\frac{\sin(2\pi t)}{2\pi t} \right]^2 \longleftrightarrow \frac{1}{2\pi} \frac{1}{2} g(\omega) * \frac{1}{2} g(\omega) = \frac{1}{8\pi} g(\omega) * g(\omega) = \frac{1}{8\pi} \int_{-\infty}^{+\infty} g(\tau) g(\omega - \tau) d\tau$$

$$= \begin{cases} \frac{1}{8\pi} (\omega + 4\pi), & -4\pi \leq \omega \leq 0 \\ \frac{1}{8\pi} (4\pi - \omega), & 0 \leq \omega \leq 4\pi \end{cases}$$

11. 求下列信号的傅立叶变换。

$$(1) f(t) = e^{-jt} \delta(t-2)$$

$$(2) f(t) = e^{-3(t-1)} \delta'(t-1)$$

$$(3) f(t) = \text{sgn}(t^2 - 9)$$

$$(4) f(t) = e^{-2t} u(t+1)$$

$$(5) f(t) = u\left(\frac{t}{2} - 1\right)$$

$$\text{解: (1) } F(\omega) = \int_{-\infty}^{\infty} e^{-jt} \delta(t-2) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \delta(t-2) e^{-j(\omega+1)t} dt = e^{-j2(1+\omega)}$$

$$(2) F(\omega) = \int_{-\infty}^{\infty} e^{-3(t-1)} \delta'(t-1) e^{-j\omega t} dt = - \frac{de^{-3(t-1)-j\omega t}}{dt} \Big|_{t=1} = (3+j\omega) e^{-j\omega}$$

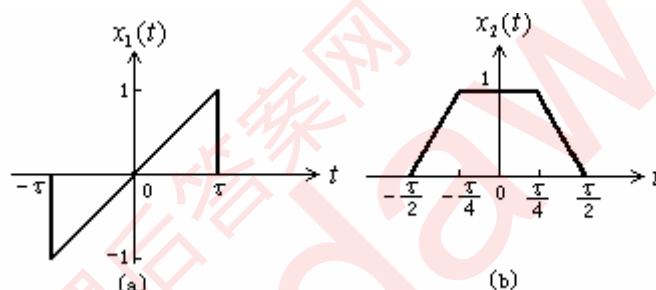
$$(3) f(t) = \text{sgn}(t^2 - 9) = u(t-3) + u(-t-3) - g_{\tau=6}(t)$$

$$\begin{aligned}
 F(\omega) &= e^{-j3\omega} \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] + e^{j3\omega} \left[\pi\delta(-\omega) - \frac{1}{j\omega} \right] - 6Sa(3\omega) \\
 &= 2\pi\delta(\omega) \cos 3\omega - \frac{2}{\omega} \sin 3\omega - 6Sa(3\omega) \\
 &= 2\pi\delta(\omega) \cos(3\omega) - 12sa(3\omega) \\
 &= 2\pi\delta(\omega) - 12Sa(3\omega)
 \end{aligned}$$

$$(4) F(\omega) = \int_{-\infty}^{\infty} e^{-2t} u(t+1) e^{-j\omega t} dt = \int_{-1}^{\infty} e^{-(2+j\omega)t} dt = -\frac{e^{-(2+j\omega)t}}{2+j\omega} \Big|_{-1}^{\infty} = \frac{e^{2+j\omega}}{2+j\omega}$$

$$(5) F(\omega) = 2 \left[\pi\delta(2\omega) + \frac{1}{2j\omega} \right] e^{-j2\omega} = \left[2\pi\delta(2\omega) + \frac{1}{j\omega} \right] e^{-j2\omega}$$

12. 试用时域积分性质，求题图 1-11 所示信号的频谱。



题图 1-11

$$\begin{aligned}
 \text{解: (1) } \frac{dx_1(t)}{dt} &= \frac{1}{\tau} g_{\tau=2\tau}(t) - [\delta(t-\tau) + \delta(t+\tau)] \\
 \frac{dx_1(t)}{dt} &\longleftrightarrow 2Sa(\omega\tau) - (e^{-j\omega\tau} + e^{j\omega\tau}) = 2Sa(\omega\tau) - 2\cos(\omega\tau)
 \end{aligned}$$

$$x_1(t) = \int_{-\infty}^t \frac{dx_1(\xi)}{d\xi} d\xi, \quad -\tau < t < \tau$$

$$X_1(\omega) = \frac{1}{j\omega} [2Sa(\omega\tau) - 2\cos(\omega\tau)] = \frac{2}{j\omega} [Sa(\omega\tau) - \cos(\omega\tau)]$$

13. 若已知 $f(t)$ 的傅立叶变换 $F(\omega)$ ，试求下列函数的频谱：

$$\begin{aligned}
 (1) & tf(2t) & (2) & (t-2)f(t) & (3) & t \frac{df(t)}{dt} \\
 (4) & f(1-t) & (5) & (1-t)f(1-t) & (6) & f(2t-5) \\
 (7) & \int_{-\infty}^{1-0.5t} f(\tau) d\tau & (8) & e^{jt} f(3-2t) & (9) & \frac{df(t)}{dt} * \frac{1}{\pi t}
 \end{aligned}$$

$$\text{解: (1) } tf(2t) \longleftrightarrow \frac{1}{2} j \frac{dF(\frac{\omega}{2})}{d\omega}$$

$$(2) (t-2)f(t) \longleftrightarrow j \frac{dF(\omega)}{d\omega} - 2F(\omega)$$

$$(3) \quad t \frac{df(t)}{dt} \leftrightarrow -F(\omega) - \omega \frac{dF(\omega)}{d\omega}$$

$$(4) \quad f(1-t) \leftrightarrow F(-\omega)e^{-j\omega}$$

$$(5) \quad (1-t)f(1-t) \leftrightarrow -j \frac{dF(-\omega)}{d\omega} e^{-j\omega}$$

$$(6) \quad f(2t-5) \leftrightarrow \frac{1}{2} F\left(\frac{\omega}{2}\right) e^{-j\frac{5}{2}\omega}$$

$$(7) \quad x(t) = \int_{-\infty}^t f(\tau) d\tau \longleftrightarrow \frac{1}{j\omega} F(\omega) + \pi F(0) \delta(\omega)$$

$$\begin{aligned} x(1-0.5t) &= \int_{-\infty}^{1-0.5t} f(\tau) d\tau \longleftrightarrow 2 \left[-\frac{F(-2\omega)}{2j\omega} + \pi F(0) \delta(-2\omega) \right] e^{-j2\omega} \\ &= e^{-j2\omega} \left[\frac{jF(-2\omega)}{\omega} + 2\pi F(0) \delta(2\omega) \right] \end{aligned}$$

$$(8) \quad f(3-2t) \longleftrightarrow \frac{1}{2} F\left(-\frac{\omega}{2}\right) e^{-j\frac{3}{2}\omega}$$

$$e^{jt} f(3-2t) \longleftrightarrow \frac{1}{2} F\left(-\frac{\omega-1}{2}\right) e^{-j\frac{3}{2}(\omega-1)} = \frac{1}{2} F\left(\frac{1-\omega}{2}\right) e^{j\frac{3}{2}(1-\omega)}$$

$$(9) \quad \frac{df(t)}{dt} \longleftrightarrow j\omega F(\omega), \quad \frac{1}{t} \longleftrightarrow -j\pi \operatorname{sgn}(\omega)$$

$$\frac{df(t)}{dt} * \frac{1}{\pi t} \longleftrightarrow \frac{1}{\pi} j\omega F(\omega) = \frac{1}{\pi} j\omega F(\omega) [-j\pi \operatorname{sgn}(\omega)] = \omega F(\omega) \operatorname{sgn}(\omega)$$

14. 求下列函数的傅立叶逆变换:

$$(1) \quad X(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & |\omega| > \omega_0 \end{cases}$$

$$(2) \quad X(\omega) = \delta(\omega + \omega_0) - \delta(\omega - \omega_0)$$

$$(3) \quad X(\omega) = 2 \cos(3\omega)$$

$$(4) \quad X(\omega) = [u(\omega) - u(\omega - 2)]e^{-j\omega}$$

$$(5) \quad X(\omega) = \sum_{n=0}^2 \frac{2 \sin \omega}{\omega} e^{-j(2\pi+1)\omega}$$

解: (1) $X(t) = \begin{cases} 1 & |t| < \omega_0 \\ 0 & |t| > \omega_0 \end{cases}$

因为: $F[X(t)] = 2\omega_0 Sa(\omega\omega_0)$

所以, $F[2\omega_0 Sa(\omega_0 t)] = 2\pi X(\omega)$

$$\text{即: } F^{-1}[X(\omega)] = \frac{\omega_0}{\pi} \text{Sa}(\omega_0 t)$$

$$(2) X(\omega) = \delta(\omega + \omega_0) - \delta(\omega - \omega_0)$$

$$\text{因为: } e^{j\omega_0 t} \xleftrightarrow{F} 2\pi\delta(\omega - \omega_0)$$

$$\text{所以: } x(t) = \frac{1}{2\pi} e^{-j\omega_0 t} - \frac{1}{2\pi} e^{j\omega_0 t}$$

$$(3) X(\omega) = 2\cos(3\omega)$$

$$F[X(t)] = 2\pi\delta(\omega - 3) + 2\pi\delta(\omega + 3)$$

$$F[2\pi\delta(t - 3) + 2\pi\delta(t + 3)] = 4\pi\cos 3\omega$$

$$F^{-1}[2\cos(3\omega)] = \delta(t - 3) + \delta(t + 3)$$

$$(4) X(\omega) = [u(\omega) - u(\omega - 2)]e^{-j\omega}$$

$$F[u(t) - u(t - 2)] = \pi\delta(\omega) + \frac{1}{j\omega} - (\pi\delta(\omega) + \frac{1}{j\omega})e^{-2j\omega}$$

$$F[(u(t) - u(t - 2))e^{-jt}] = \pi\delta(\omega + 1) + \frac{1}{j(\omega + 1)} - (\pi\delta(\omega + 1) + \frac{1}{j(\omega + 1)})e^{-2j(\omega + 1)}$$

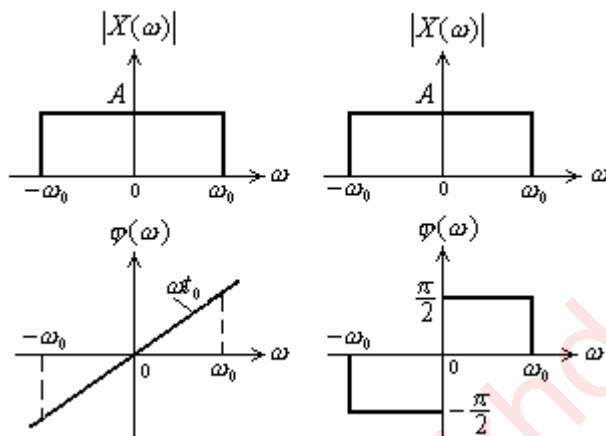
由傅立叶变换对称性, 得

$$F[X(t)] = 2\pi x(-\omega)$$

$$\begin{aligned} x(-\omega) &= \frac{1}{2\pi} \left[\pi\delta(\omega + 1) + \frac{1}{j(\omega + 1)} - [\pi\delta(\omega + 1) + \frac{1}{j(\omega + 1)}]e^{-2j(\omega + 1)} \right] \\ &= \frac{1}{2\pi} (1 - e^{-2j(\omega + 1)}) \left[\pi\delta(\omega + 1) + \frac{1}{j(\omega + 1)} \right] \end{aligned}$$

$$x(t) = \frac{1}{2\pi} (1 - e^{-2j(-t + 1)}) \left[\pi\delta(-t + 1) + \frac{1}{j(-t + 1)} \right]$$

15. 利用傅里叶变换的性质, 求题图 1-12 所示函数的傅里叶逆变换。



题图 1-12

解: (1) $X(\omega) = Ag_{2\omega_0}(\omega)e^{-j\omega t_0}$

$$g_{2\omega_0}(t) \longleftrightarrow 2\omega_0 Sa(\omega\omega_0)$$

由对偶性质

$$2\omega_0 Sa(\omega_0 t) \longleftrightarrow 2\pi g_{2\omega_0}(\omega)$$

$$\frac{A\omega_0}{\pi} Sa(\omega_0 t) \longleftrightarrow Ag_{2\omega_0}(\omega)$$

$$\frac{A\omega_0}{\pi} Sa[\omega_0(t+t_0)] \longleftrightarrow Ag_{2\omega_0}(\omega)e^{-j\omega t_0}$$

$$x(t) = \frac{A\omega_0}{\pi} Sa[\omega_0(t+t_0)]$$

$$(2) X(\omega) = -jA[u(\omega+\omega_0)-u(\omega)] + jA[u(\omega)-u(\omega-\omega_0)]$$

解法 1: 利用频域微积分特性

$$\begin{aligned} \frac{dX(\omega)}{d\omega} &= jA[-\delta(\omega+\omega_0)-\delta(\omega-\omega_0)+2\delta(\omega)] \\ &\longleftrightarrow \frac{jA}{2\pi}[-e^{-j\omega_0 t}-e^{j\omega_0 t}+2] = \frac{jA[1-\cos(\omega_0 t)]}{\pi} \end{aligned}$$

$$\text{因为 } \frac{dX(\omega)}{d\omega} \longleftrightarrow (-jt)x(t), \text{ 所以 } (-jt)x(t) = \frac{jA[1-\cos(\omega_0 t)]}{\pi}$$

$$x(t) = \frac{A[\cos(\omega_0 t)-1]}{\pi t}$$

解法 2: 利用对偶性和时域微积分特性

由对偶特性得

$$-jA[u(t+\omega_0)-u(t)] + jA[-u(t-\omega_0)+u(t)] \longleftrightarrow 2\pi x(-\omega)$$

对上式左边微分, 再由时域微分特性得

$$jA[-\delta(t+\omega_0)-\delta(t-\omega_0)+2\delta(t)] \longleftrightarrow j\omega 2\pi x(-\omega)$$

上式左边的傅里叶变换与右边相等

$$jA(-e^{j\omega\omega_0}-e^{-j\omega\omega_0}+2) = j\omega 2\pi x(-\omega)$$

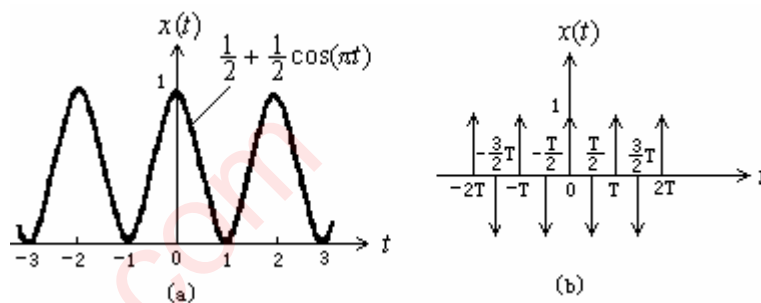
$$jA[-2\cos(\omega\omega_0)+2] = j\omega 2\pi x(-\omega)$$

$$x(-\omega) = \frac{A}{\pi\omega}(1-\cos\omega\omega_0)$$

将 t 代替 $-\omega$, 得

$$x(t) = \frac{A[\cos(\omega_0 t)-1]}{\pi t}$$

16. 试求题图 1-15 所示周期信号的频谱函数。图 1-15 (b) 中冲激函数的强度均为 1。



题图 1-15

解: (a) 解法 1: 由定义 $X(n\omega_0) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn\omega_0 t} dt$

$$\begin{aligned} X(n\pi) &= \frac{1}{2} \int_{-1}^1 \left[\frac{1}{2} + \frac{1}{2} \cos(\pi t) \right] e^{-jn\pi t} dt = \frac{1}{4} \int_{-1}^1 e^{-jn\pi t} dt + \frac{1}{4} \int_{-1}^1 \cos(\pi t) e^{-jn\pi t} dt \\ &= \frac{1}{4} \cdot \frac{1}{-jn\pi} e^{-jn\pi t} \Big|_{-1}^1 + \frac{1}{4} \cdot \frac{e^{-jn\pi t} [\pi \sin(\pi t) - jn\pi \cos(\pi t)]}{(-n^2\pi^2 + \pi^2)} \Big|_{-1}^1 \\ &= \frac{1}{2} \cdot \frac{1}{n\pi} \sin(n\pi) + \frac{1}{4} \cdot \frac{2n\pi \sin n\pi}{n^2\pi^2 - \pi^2} \\ &= \begin{cases} \frac{1}{2}, & n=0 \\ \frac{1}{4}, & n=\pm 1 \\ 0, & \text{其它} \end{cases} \end{aligned}$$

$$\begin{aligned} X(\omega) &= \frac{1}{2} \cdot 2\pi\delta(\omega) + \frac{1}{4} \cdot [2\pi\delta(\omega - \pi) + 2\pi\delta(\omega + \pi)] \\ &= \pi\delta(\omega) + \frac{\pi}{2}\delta(\omega - \pi) + \frac{\pi}{2}\delta(\omega + \pi) \end{aligned}$$

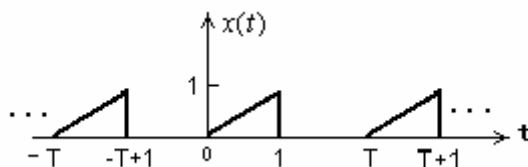
解法 2: 由傅里叶变换的性质和 $\cos(\pi t)$ 的傅里叶变换

$$\begin{aligned} \cos(\pi t) &\longleftrightarrow \pi [\delta(\omega + \pi) + \delta(\omega - \pi)] \\ \frac{1}{2} + \frac{1}{2} \cos(\pi t) &\longleftrightarrow \frac{1}{2} \cdot 2\pi\delta(\omega) + \frac{1}{2} \pi [\delta(\omega + \pi) + \delta(\omega - \pi)] \\ &= \pi\delta(\omega) + \frac{1}{2} \pi [\delta(\omega + \pi) + \delta(\omega - \pi)] \end{aligned}$$

$$(b) \quad x(t) = \delta_T(t) - \delta_T\left(t - \frac{T}{2}\right)$$

$$\begin{aligned}
 X(\omega) &= [\delta_T(t)] - e^{-\frac{j\omega T}{2}} [\delta_T(t)] \\
 &= \left(1 - e^{-\frac{j\omega T}{2}}\right) [\delta_T(t)] \\
 &= \frac{2\pi}{T} \left(1 - e^{-\frac{j\omega T}{2}}\right) \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{T}\right)
 \end{aligned}$$

20. 求题图 1-17 所示周期信号 $x(t)$ 的傅立叶变换。



题图 1-17

$$\begin{aligned}
 \text{解: } X(n\omega_0) &= \frac{1}{T} \int_0^1 t e^{-jn\omega_0 t} dt, \quad \omega_0 = \frac{2\pi}{T} \\
 &= \frac{1}{T} \left[\frac{1}{-n^2 \omega_0^2 T} (-jn\omega_0 t - 1) e^{-jn\omega_0 t} \right]_0^1 \\
 &= -\frac{1}{n^2 \omega_0^2 T} - \frac{1}{n^2 \omega_0^2 T} (-jn\omega_0 - 1) e^{-jn\omega_0} \\
 &= \frac{1}{n^2 \omega_0^2 T} [(jn\omega_0 + 1) e^{-jn\omega_0} - 1] \\
 X(\omega) &= \sum_{n=-\infty}^{\infty} 2\pi X(n\omega_0) \delta(\omega - n\omega_0) \\
 &= \sum_{n=-\infty}^{\infty} \left\{ 2\pi \frac{1}{n^2 \cdot \frac{4\pi^2}{T}} \left[\left(j \frac{2\pi n}{T} + 1 \right) e^{-\frac{j2\pi n}{T}} - 1 \right] \delta\left(\omega - \frac{2\pi n}{T}\right) \right\} \\
 &= \sum_{n=-\infty}^{\infty} \frac{T}{2\pi n^2} \left[\left(1 + j \frac{2\pi n}{T} \right) e^{-\frac{j2\pi n}{T}} - 1 \right] \delta\left(\omega - \frac{2\pi n}{T}\right)
 \end{aligned}$$

21. 考虑信号

$$x(t) = \begin{cases} 0, & t < -\frac{1}{2} \\ t + \frac{1}{2}, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 1, & t > \frac{1}{2} \end{cases}$$

(1) 利用傅里叶变换的微分和积分性质, 求 $X(\omega)$;

(2) $g(t) = x(t) - \frac{1}{2}$ 的傅里叶变换是什么?

解: (1) $\frac{dx(t)}{dt} = g_{\tau=1}(t)$

$$\frac{dx(t)}{dt} \longleftrightarrow Sa\left(\frac{\omega}{2}\right)$$

$$x(t) = \int_{-\infty}^t \frac{dx(\tau)}{d\tau} d\tau \longleftrightarrow \frac{1}{j\omega} sa\left(\frac{\omega}{2}\right) + \pi\delta(\omega)$$

$$X(\omega) = \frac{1}{j\omega} Sa\left(\frac{\omega}{2}\right) + \pi\delta(\omega)$$

(2) $g(t) = x(t) - \frac{1}{2}$

$$F[g(t)] = F[x(t)] - \pi\delta(\omega) = \frac{1}{j\omega} Sa\left(\frac{\omega}{2}\right)$$

习 题 (P78)

1. 定义计算下列信号的拉普拉斯变换及收敛域。

$$(1) e^{at}u(t), a > 0$$

$$(2) te^{at}u(t), a > 0$$

$$(3) e^{-at}u(-t), a > 0$$

$$(4) (\cos \omega_c t)u(-t)$$

$$(5) [\cos(\omega_c t + \theta)]u(t)$$

$$(6) [e^{-at} \sin(\omega_c t)]u(t), a > 0$$

$$(7) \delta(at - b), a \text{ 和 } b \text{ 为实数}$$

$$(8) x(t) = \begin{cases} e^{-2t}, & t > 0 \\ e^{3t}, & t < 0 \end{cases}$$

$$\text{解: (1) } X_{b1}(s) = \int_0^{\infty} e^{at} e^{-st} dt = \int_0^{\infty} e^{(a-s)t} dt = \frac{1}{a-s} e^{(a-s)t} \Big|_0^{\infty} = -\frac{1}{a-s} \quad (\sigma > a)$$

$$(2) X_{b2}(s) = \int_0^{\infty} te^{at} e^{-st} dt = \int_0^{\infty} te^{(a-s)t} dt = \frac{1}{a-s} (te^{(a-s)t} - \frac{1}{a-s} e^{(a-s)t}) \Big|_0^{\infty} = \frac{1}{(a-s)^2} \quad (\sigma > a)$$

$$(3) X_{b3}(s) = \int_{-\infty}^0 e^{-(a+s)t} dt = \frac{1}{a+s} e^{-(a+s)t} \Big|_{-\infty}^0 = \frac{1}{a+s} \quad (\sigma < -a)$$

$$(4) X_{b4}(s) = \int_{-\infty}^0 \cos \omega_c t e^{-st} dt = \frac{\omega_c e^{-st} \sin \omega_c t - se^{-st} \cos \omega_c t}{s^2 + \omega_c^2} \Big|_{-\infty}^0 = -\frac{s}{s^2 + \omega_c^2} \quad (\sigma < 0)$$

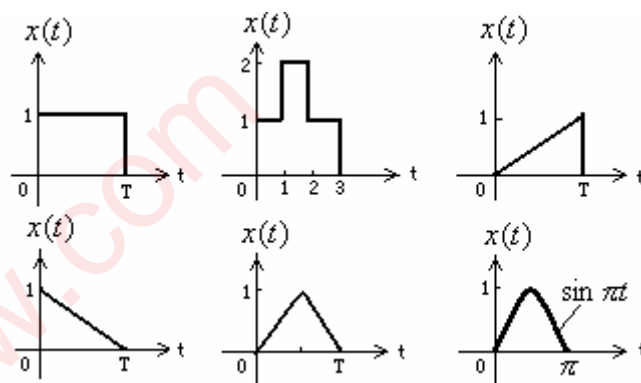
$$(5) X_{b5}(s) = \int_0^{\infty} \cos(\omega_c t + \theta) e^{-st} dt = \frac{\omega_c e^{-st} \sin(\omega_c t + \theta) - se^{-st} \cos(\omega_c t + \theta)}{s^2 + \omega_c^2} \Big|_0^{\infty} \\ = \frac{s \cos \theta - \omega_c \sin \theta}{s^2 + \omega_c^2} \quad (\sigma > 0)$$

$$(6) X_{b6}(s) = \int_0^{\infty} \sin \omega_c t \cdot e^{-(s+a)t} dt = \frac{\omega_c}{(s+a)^2 + \omega_c^2} \quad (\sigma > 0)$$

$$(7) X_{b7}(s) = \int_{-\infty}^{\infty} \delta(at - b) e^{-st} dt = \int_{-\infty}^{\infty} \delta(\tau) e^{-s\frac{\tau+b}{a}} \frac{1}{a} d\tau = \frac{1}{a} e^{-\frac{b}{a}s} \quad (\sigma \in R)$$

$$(8) X_b(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = \int_{-\infty}^0 e^{3t} e^{-st} dt + \int_0^{\infty} e^{-2t} e^{-st} dt \\ = \frac{1}{3-s} e^{(3-s)t} \Big|_{-\infty}^0 - \frac{1}{2+s} e^{-(2+s)t} \Big|_0^{\infty} \\ = \frac{1}{3-s} + \frac{1}{2+s} \\ = \frac{5}{(2+s)(3-s)} \quad (-2 < \sigma < 3)$$

2. 用定义计算题图 1-18 所示各信号的拉普拉斯变换。



题图 1-18

解: (1) $X_{b1}(s) = \int_0^T e^{-st} dt = \frac{1}{-s} e^{(-s)t} \Big|_0^T = \frac{1 - e^{-sT}}{s} \quad (\sigma \in R)$

$$\begin{aligned} (2) \quad X_{b2}(s) &= \int_{-\infty}^{\infty} x_2(t) e^{-st} dt = \int_0^1 e^{-st} dt + \int_1^2 2e^{-st} dt + \int_2^3 e^{-st} dt \\ &= \int_0^3 e^{-st} dt + \int_1^2 e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_0^3 + -\frac{1}{s} e^{-st} \Big|_1^2 \\ &= \frac{1 - e^{-3s}}{s} + \frac{e^{-s} - e^{-2s}}{s} = \frac{1 + e^{-s} - e^{-2s} - e^{-3s}}{s} \end{aligned}$$

(收敛域为除坐标原点外的整个 S 平面)

$$\begin{aligned} (3) \quad X_{b3}(s) &= \int_0^T \frac{1}{T} t e^{-st} dt = \frac{1}{T} \left[-\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st} \right] \Big|_0^T \\ &= \frac{1}{T} \left(-\frac{1}{s} T e^{-sT} - \frac{1}{s^2} e^{-sT} + \frac{1}{s^2} \right) \quad (\sigma \in R) \end{aligned}$$

$$\begin{aligned} (4) \quad X_{b4}(s) &= \int_0^T \left[-\frac{1}{T} t + b \right] e^{-st} dt = \left\{ -\frac{1}{T} \left[-\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st} \right] + b \left[-\frac{1}{s} e^{-st} \right] \right\} \Big|_0^T \\ &= e^{-sT} \left[\frac{1}{s} + \frac{1}{Ts^2} - \frac{1}{Ts^2} - \frac{b}{s} \right] + \frac{b}{s} \quad (\sigma \in R) \end{aligned}$$

$$\begin{aligned} (5) \quad X_{b5}(s) &= \int_0^{T/2} \frac{2}{T} t e^{-st} dt + \int_{T/2}^T \left(-\frac{2}{T} t + 2 \right) e^{-st} dt \\ &= \frac{2}{T} \left[-\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st} \right] \Big|_0^{T/2} - \frac{2}{T} \left[-\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st} \right] \Big|_{T/2}^T + 2 \left[-\frac{1}{s} e^{-st} \right] \Big|_{T/2}^T \\ &= \frac{2}{Ts^2} (-2e^{-sT/2} + e^{-sT} + 1) \quad (\sigma \in R) \end{aligned}$$

$$\begin{aligned}
 X_{b6}(s) &= \int_0^{\pi} \sin \pi t e^{-st} dt = \left. -\frac{1}{s} \sin \pi t \cdot e^{-st} + \frac{1}{\pi s} \cos \pi t \cdot e^{-st} \right|_0^{\pi} \\
 (6) \quad &= \frac{1 + \frac{1}{\pi^2 s}}{1 + \frac{1}{\pi^2 s}} \\
 &= \frac{-\pi^2 \sin \pi^2 \cdot e^{-\pi s} + \pi \cos \pi^2 \cdot e^{-\pi s} - \pi}{\pi^2 s + 1} \quad (\sigma \in R)
 \end{aligned}$$

3. 确定时间函数 $x(t)$ 的拉普拉斯变换、零极点及其收敛域。

- (1) $x(t) = e^{-2t} u(t) + e^{-3t} u(t)$ (2) $x(t) = e^{-4t} u(t) + e^{-5t} (\sin 5t) u(t)$
 (3) $x(t) = e^{2t} u(-t) + e^{3t} u(-t)$ (4) $x(t) = t e^{-2|t|}$
 (5) $x(t) = |t| e^{-2|t|}$ (6) $x(t) = |t| e^{2t} u(-t)$
 (7) $x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{其它} \end{cases}$ (8) $x(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \end{cases}$
 (9) $x(t) = \delta(t) + u(t)$ (10) $x(t) = \delta(3t) + u(3t)$

解:

(1) $x(t) = e^{-2t} u(t) + e^{-3t} u(t)$

$$\begin{aligned}
 X_{b1}(s) &= \int_0^{\infty} e^{-2t} e^{-st} dt + \int_0^{\infty} e^{-3t} e^{-st} dt \\
 &= -\frac{1}{s+2} e^{-(s+2)t} \Big|_0^{\infty} - \frac{1}{s+3} e^{-(s+3)t} \Big|_0^{\infty} \\
 &= \frac{1}{s+2} + \frac{1}{s+3} \quad (\sigma > -3)
 \end{aligned}$$

(2) $x(t) = e^{-4t} u(t) + e^{-5t} (\sin 5t) u(t)$

$$\begin{aligned}
 X_{b2}(s) &= \int_0^{\infty} e^{-4t} e^{-st} dt + \int_0^{\infty} e^{-5t} \sin 5t \cdot e^{-st} dt \\
 &= \frac{1}{s+4} + \frac{1}{5(s+5)^2 + 1} \quad (\sigma > -4)
 \end{aligned}$$

(3) $x(t) = e^{2t} u(-t) + e^{3t} u(-t)$

$$\begin{aligned}
 X_{b3}(s) &= \int_{-\infty}^0 e^{2t} e^{-st} dt + \int_{-\infty}^0 e^{3t} e^{-st} dt \\
 &= \frac{1}{2-s} e^{-(s-2)t} \Big|_{-\infty}^0 - \frac{1}{s-3} e^{-(s-3)t} \Big|_{-\infty}^0 \\
 &= \frac{1}{2-s} + \frac{1}{3-s} \quad (\sigma < 3)
 \end{aligned}$$

(4) $x(t) = t e^{-2|t|}$

$$\begin{aligned}
 X_{b4}(s) &= \int_{-\infty}^0 t e^{2t} e^{-st} dt + \int_0^{\infty} t e^{-2t} e^{-st} dt \\
 &= -\frac{1}{(2-s)^2} + \frac{1}{(s+2)^2} \quad (-2 < \sigma < 2)
 \end{aligned}$$

(5) $x(t) = |t| e^{-2|t|}$

$$\begin{aligned} X_{b5}(s) &= \int_{-\infty}^0 (-te^{2t} e^{-st}) dt + \int_0^{\infty} te^{-2t} e^{-st} dt \\ &= -\frac{1}{(2-s)^2} + \frac{1}{(s+2)^2} \quad (-2 < \sigma < 2) \end{aligned}$$

(6) $x(t) = |t|e^{2t}u(-t)$

$$\begin{aligned} X_{b6}(s) &= -\int_{-\infty}^0 te^{2t} e^{-st} dt \\ &= \frac{1}{(2-s)^2} \quad (\sigma < 2) \end{aligned}$$

(7) $x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

$$X_{b7}(s) = \int_0^1 e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_0^1 = \frac{1-e^{-s}}{s}$$

零极点均为 0，收敛域为整个 S 平面。

$$\begin{aligned} (8) X_{b8}(s) &= \int_{-\infty}^{\infty} x_8(t) e^{-st} dt = \int_0^1 te^{-st} dt + \int_1^2 (2-t)e^{-st} dt \\ &= -\frac{1}{s} \left[te^{-st} + \frac{1}{s} e^{-st} \right] \Big|_0^1 - \frac{2}{s} e^{-st} \Big|_1^2 + \frac{1}{s} \left[te^{-st} + \frac{1}{s} e^{-st} \right] \Big|_1^2 \\ &= \left(\frac{e^{-s}-1}{s} \right)^2 \end{aligned}$$

二重零点 $s=0$ ，二重极点 $s=0$ ，收敛域为除坐标原点外的整个 S 平面。

(9) $x(t) = \delta(t) + u(t)$

$$X_{b9}(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt = 1 + \frac{1}{s}$$

零点 $s=-1$ ，极点 $s=0$ ，收敛域为 $\sigma > 0$ 。

$$(10) [x_8(t)] = [\delta(3t) + u(3t)] = [\delta(3t)] + [u(3t)] = \frac{1}{3} + \frac{1}{3} \frac{1}{s/3} = \frac{s+3}{3s}$$

零点 $s=-3$ ，极点 $s=0$ ，收敛域为 $\sigma > 0$ 。

5. 若已知 $u(t)$ 的拉普拉斯变换为 $\frac{1}{s}$ ，收敛域为 $\Re\{s\} > 0$ ，试利用拉氏变换的性质，求下列信号的拉氏变换式及其收敛域。

(1) $[\cos(\omega_c t)]u(t)$ (2) $[\sin(\omega_c t) + \cos(\omega_c t)]u(t)$

(3) $[e^{-at} \cos(\beta t)]u(t)$ (4) $[t \cos(\omega_c t)]u(t)$

(5) $[te^{-at} \cos(\omega_c t)]u(t)$ (6) $e^{-t}u(t-T)$

(7) $te^{-t}u(t-T)$ (8) $t\delta'(t)$

$$(9) t^2 \delta''(t)$$

$$(10) \sum_{k=0}^{\infty} a^k \delta(t - kT)$$

解: (5) $e^{-at} \cos(\omega_c t) u(t) \longleftrightarrow \frac{s+a}{(s+a)^2 + \omega_c^2}$

利用双边拉普拉斯变换的复频域微分特性

$$[te^{-at} \cos(\omega_c t)]u(t) \longleftrightarrow -\frac{d}{ds} \left[\frac{s+a}{(s+a)^2 + \omega_c^2} \right] = \frac{(s+a)^2 - \omega_c^2}{[(s+a)^2 + \omega_c^2]^2}$$

收敛域为 $\sigma > -a$ 。

$$(16) (1 - e^{-at})u(t) = u(t) - e^{-at}u(t) \longleftrightarrow \frac{1}{s} - \frac{1}{s+a}$$

利用双边拉普拉斯变换的复频域积分特性

$$t^{-1}(1 - e^{-at})u(t) \longleftrightarrow \int_s^\tau \left(\frac{1}{\tau} - \frac{1}{\tau+a} \right) d\tau = \ln \left| \frac{\tau}{\tau+a} \right| \Big|_s^\infty = \ln \left| \frac{s}{s+a} \right|$$

若 $a > 0$, 收敛域为 $\sigma > 0$; 若 $a < 0$, 收敛域为 $\sigma > -a$ 。

6. 求下列函数的拉普拉斯反变换:

$$(1) \frac{1}{s^2 + 9} \quad \text{Re}\{s\} > 0$$

$$(2) \frac{s}{s^2 + 9} \quad \text{Re}\{s\} < 0$$

$$(3) \frac{s+1}{(s+1)^2 + 9} \quad \text{Re}\{s\} < -1$$

$$(4) \frac{3s}{(s^2 + 1)(s^2 + 4)} \quad \text{Re}\{s\} > 0$$

$$(5) \frac{s+1}{s^2 + 5s + 6} \quad -3 < \text{Re}\{s\} < -2$$

$$(6) \frac{s+2}{s^2 + 7s + 12} \quad -4 < \text{Re}\{s\} < -3$$

$$(7) \frac{(s+1)^2}{s^2 - s + 1} \quad \text{Re}\{s\} > \frac{1}{2}$$

$$(8) \frac{s^2 - s + 1}{(s+1)^2} \quad \text{Re}\{s\} > -1$$

$$(9) \frac{s^2 + 4s + 5}{s^2 + 3s + 2} \quad \text{Re}\{s\} > -1$$

$$(10) \frac{s^2 - s + 1}{s^3 - s^2} \quad \text{Re}\{s\} > 1$$

解: (1)

$$(7) \frac{(s+1)^2}{s^2 - s + 1} = 1 + \frac{\frac{3}{2} - \frac{\sqrt{3}}{2}j}{s - \frac{1+\sqrt{3}}{2}} + \frac{\frac{3}{2} + \frac{\sqrt{3}}{2}j}{s - \frac{1-\sqrt{3}}{2}} \quad \text{Re}\{s\} > \frac{1}{2}$$

$$\left[\frac{(s+1)^2}{s^2 - s + 1} \right] = \delta(t) + \left(\frac{3}{2} - \frac{\sqrt{3}}{2}j \right) e^{\frac{1+\sqrt{3}}{2}t} u(t) + \left(\frac{3}{2} + \frac{\sqrt{3}}{2}j \right) e^{\frac{1-\sqrt{3}}{2}t} u(t)$$

$$(8) \frac{s^2 - s + 1}{(s+1)^2} = 1 + \frac{-3s}{(s+1)^2} = 1 + \frac{-3}{s+1} + \frac{3}{(s+1)^2} \quad \text{Re}\{s\} > -1$$

$$\left[\frac{s^2 - s + 1}{(s+1)^2} \right] = [\delta(t) - 3e^{-t} + 3te^{-t}]u(t)$$

$$(9) \frac{s^2 + 4s + 5}{s^2 + 3s + 2} = 1 + \frac{2}{s+1} - \frac{1}{s+2} \quad (\text{部分分式展开})$$

$$[1] = \delta(t)$$

$$\left[\frac{2}{s+1} \right] = 2e^{-t}u(t)$$

$$\left[\frac{1}{s+2} \right] = e^{-2t}u(t)$$

$$\left[\frac{s^2 + 4s + 5}{s^2 + 3s + 2} \right] = \delta(t) + 2e^{-t}u(t) - e^{-2t}u(t) = \delta(t) + (2e^{-t} - e^{-2t})u(t)$$

$$(10) \frac{s^2 - s + 1}{s^3 - s^2} \quad \operatorname{Re}\{s\} > 1$$

$$\frac{s^2 - s + 1}{s^3 - s^2} = \frac{1}{s-1} - \frac{1}{s^2} \quad \operatorname{Re}\{s\} > 1$$

$$\left[\frac{s^2 - s + 1}{s^3 - s^2} \right] = e^t u(t) - tu(t)$$

11. 已知信号 $x(t)$ 的拉普拉斯变换为 $X(s) = \frac{s+2}{s^2+4s+5}$, 试求下列信号的拉普拉斯变换。

$$(1) x(2t-1)u(2t-1) \quad (2) tx(t) \quad (3) e^{-3t}x(t)$$

$$(4) \frac{dx(t)}{dt} \quad (5) 2x(t/4) + 3x(5t) \quad (6) x(t)\cos 7t$$

$$\text{解: (6)} \quad [X(s)] = \left[\frac{s+2}{s^2+4s+5} \right] = \left[\frac{s+2}{(s+2)^2+1} \right] = e^{-2t} \cos tu(t) (\sigma > -2)$$

$$\text{所以, } x(t)\cos 7t = e^{-2t} \cos tu(t)\cos 7t = \frac{1}{2}e^{-2t}u(t)[\cos 8t + \cos 6t]$$

$$= \frac{1}{2}e^{-2t} \cos 8tu(t) + \frac{1}{2}e^{-2t} \cos 6tu(t)$$

$$[x(t)\cos 7t] = \frac{1}{2} [e^{-2t} \cos 8tu(t)] + \frac{1}{2} [e^{-2t} \cos 6tu(t)]$$

$$= \frac{1}{2} \frac{s+2}{(s+2)^2+64} + \frac{1}{2} \frac{s+2}{(s+2)^2+36}$$

$$= \frac{1}{2}(s+2) \left[\frac{1}{(s+2)^2 + 64} + \frac{1}{(s+2)^2 + 36} \right] \quad (\sigma > -2)$$

$$\text{或 } [x(t)\cos 7t] = \frac{1}{2} \left[\frac{s+2+j}{(s+2+j)^2 + 49} + \frac{s+2-j}{(s+2-j)^2 + 49} \right] \quad (\sigma > -2)$$

13. 由下列各象函数求原函数的傅立叶变换 $X(\omega)$ 。

$$(1) \frac{1}{s} \quad (2) \frac{2}{s^2 + 1} \quad (3) \frac{s+2}{s^2 + 4s + 8} \quad (4) \frac{s}{(s+4)^2}$$

$$\text{解: (3)} \quad \frac{s+2}{s^2 + 4s + 8} = \frac{s+2}{(s+2)^2 + 2^2}$$

其收敛域为 $\sigma > -2$ ，因此 $j\omega$ 轴在 $X_b(s)$ 的收敛域内，所以

$$X(\omega) = X_b(s) \Big|_{s=j\omega} = \frac{j\omega + 2}{(j\omega)^2 + 4j\omega + 8} = \frac{2 + j\omega}{8 - \omega^2 + 4j\omega}$$

14. 设 $f(t)u(t) \leftrightarrow F(s)$ ，且有实常数 $a>0$, $b>0$ ，试证：

$$(1) \quad f(at-b)u(at-b) \leftrightarrow \frac{1}{a} e^{-\frac{b}{a}s} F\left(\frac{s}{a}\right)$$

$$(2) \quad \frac{1}{a} e^{-\frac{b}{a}s} f\left(\frac{t}{a}\right)u(t) \leftrightarrow F(as+b)$$

证明：(1) 由拉普拉斯变换的定义

$$[f(at-b)u(at-b)] = \int_{-\infty}^{+\infty} f(at-b)u(at-b)e^{-st} dt$$

$$\begin{aligned} \text{令 } at-b = \tau \\ t = \frac{\tau+b}{a} \end{aligned} \int_{-\infty}^{+\infty} f(at-b)u(at-b)e^{-st} dt = \int_{-\infty}^{+\infty} f(\tau)u(\tau) e^{-s\left(\frac{\tau+b}{a}\right)} d\left(\frac{\tau+b}{a}\right)$$

$$= \frac{1}{a} e^{-\frac{b}{a}s} \int_{-\infty}^{+\infty} f(\tau)u(\tau) e^{-\frac{s}{a}\tau} d\tau = \frac{1}{a} e^{-\frac{b}{a}s} F\left(\frac{s}{a}\right)$$

$$\text{(利用 } F(s) = \int_{-\infty}^{+\infty} f(\tau)u(\tau) e^{-s\tau} d\tau \text{)。}$$

(2) 由拉普拉斯变换的定义

$$\left[\frac{1}{a} e^{-\frac{b}{a}s} f\left(\frac{t}{a}\right)u(t) \right] = \int_{-\infty}^{+\infty} \frac{1}{a} e^{-\frac{b}{a}s} f\left(\frac{t}{a}\right)u(t) e^{-st} dt$$

$$\begin{aligned} \text{令 } \frac{t}{a} = \tau \\ t = a\tau \end{aligned} \int_{-\infty}^{+\infty} \frac{1}{a} e^{-b\tau} f(\tau)u(a\tau) e^{-sa\tau} d(a\tau) = \int_{-\infty}^{+\infty} f(\tau)u(a\tau) e^{-(as+b)\tau} d\tau$$

$$= \int_{-\infty}^{+\infty} f(\tau)u(\tau) e^{-(as+b)\tau} d\tau = F(as+b)$$

(利用 $u(a\tau) = (\tau)(a > 0)$, $F(s) = \int_{-\infty}^{+\infty} f(\tau)u(\tau)e^{-s\tau}d\tau$)

15. 求下列象函数 $X(s)$ 的原函数的初值 $x(0_+)$ 和终值 $x(\infty)$ 。

$$(1) X(s) = \frac{2s+3}{(s+1)^2} \quad (2) X(s) = \frac{3s+1}{s(s+1)}$$

解: (1) $x(0_+) = \lim_{s \rightarrow \infty} sX(s) = \lim_{s \rightarrow \infty} \frac{s(2s+3)}{(s+1)^2} = 2,$

$$x(\infty) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{s(2s+3)}{(s+1)^2} = 0.$$

$$(2) x(0_+) = \lim_{s \rightarrow \infty} sX(s) = \lim_{s \rightarrow \infty} \frac{s(3s+1)}{s(s+1)} = 3,$$

$$x(\infty) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{s(3s+1)}{s(s+1)} = 1.$$

16. 设信号 $x(t)$ 的有理拉普拉斯变换具有两个极点 $s=-1$ 和 $s=-3$ 。若 $g(t) = e^{2t}x(t)$, 其傅立叶变换 $G(\omega)$ 收敛, 请问 $x(t)$ 是否是左边的、右边的、或是双边的?

解: $[x(t)] = X_b(s) = \frac{N(s)}{(s+1)(s+3)}$

$$[g(t)] = G(s) = X_b(s-2) = \frac{N(s-2)}{(s-1)(s+1)}$$

所以 $G(s)$ 的极点为 $s_1 = -1, s_2 = 1$ 。由于 $G(\omega)$ 收敛, 所以 $G(s)$ 存在, 并且其收敛域包含 $j\omega$ 轴或以 $j\omega$ 轴为边界, 再根据有理拉普拉斯变换的收敛域特点 (P.79), 可知 $G(s)$ 的收敛域为 $-1 < \sigma < 1$, 由拉普拉斯变换的收敛域的基本特点 (P.72), 可知 $g(t) = e^{2t}x(t)$ 为双边信号, $x(t) = e^{-2t}g(t)$ 也为双边信号。

17. 已知信号 $e^{-at}u(t)$ 的拉普拉斯变换为 $\frac{1}{s+a}$, 其中 $\Re\{s\} > \Re\{-a\}$ 。求

$$X(s) = \frac{2(s+2)}{s^2+7s+12}, \Re\{s\} > -3 \text{ 的反变换。}$$

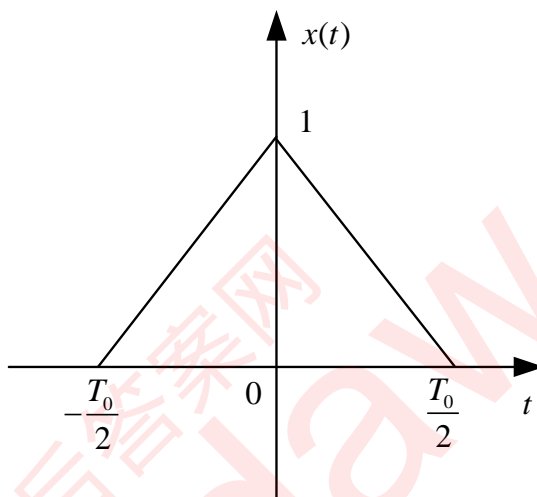
解: $X(s) = \frac{2(s+2)}{s^2+7s+12} = \frac{-2}{s+3} + \frac{4}{s+4}$ (部分分式法展开)

$$[X(s)] = -2e^{-3t}u(t) + 4e^{-4t}u(t)$$

习题 (P155)

1. 已知三角脉冲如题图 3-1 所示, 试求

- (1) 三角脉冲的频谱;
- (2) 画出对 $x(t)$ 以等间隔 $T_0/8$ 进行理想采样所构成的采样信号 $x_s(t)$ 的频谱 $X_s(\omega)$;
- (3) 将 $x(t)$ 以周期 T_0 重复, 构成周期信号 $x_p(t)$, 画出对 $x_p(t)$ 以 $T_0/8$ 进行理想采样所构成的采样信号 $x_{ps}(t)$ 的频谱 $X_{ps}(\omega)$;
- (4) 若已知 $x(t)$ 的频谱函数 $X(\omega)$, 对 $X(\omega)$ 进行频率采样, 若想不失真地恢复信号 $x(t)$, 需满足哪些条件?



题图 3-1

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$(1) \quad F[x(t)] = \frac{T_0}{2} Sa^2\left(\frac{\omega T_0}{4}\right)$$

$$(2) \quad \begin{aligned} X_s(\omega) &= \sum_{n=-\infty}^{\infty} 4Sa^2\left[\frac{T_0}{4}\omega - 4n\pi\right] \\ &= \sum_{n=-\infty}^{\infty} 4Sa^2\left[\frac{T_0}{4}(\omega - n\omega_s)\right] \end{aligned}$$

$$\omega_s = \frac{2\pi}{T} = \frac{16\pi}{T_0}$$

$$\begin{aligned} X_s(\omega) &= \frac{1}{2\pi} X(\omega) * P(\omega) \\ &= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} 4Sa^2\left[\frac{T_0}{4}\omega - 4n\pi\right] * \\ &= \frac{1}{2\pi} \frac{T_0}{2} Sa^2\left(\frac{\omega T_0}{4}\right) * \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) \end{aligned}$$

$$P(\omega) = \omega_s \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) = \frac{16\pi}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{16n\pi}{T_s}\right)$$

(3)

$$F[x(t)e^{j\omega_0 t}] = \frac{1}{2\pi} X(\omega) \cdot 2\pi\delta(\omega - \omega_0) = X(\omega) \cdot \delta(\omega - \omega_0) = X(\omega - \omega_0)$$

$$X(\omega) = \frac{T_0}{2} Sa^2\left(\frac{n\omega_0 T_0}{4}\right)$$

$$6. (1) \quad x(\Omega) = \frac{16e^{3j\Omega}}{2 - e^{-j\Omega}}$$

$$7. \quad \frac{1}{n\pi} \left(\sin \frac{3}{4} n\pi - \sin \frac{1}{4} n\pi \right)$$

8. 设 $x(n) \xrightarrow{F} X(e^{j\Omega})$, 试求下列序列的傅里叶变换:

$$(1) x(\alpha n) \quad (2) x^*(\alpha n)$$

其中, * 表示共轭, α 为任意常数。

解: (1) $x(\alpha n)$

$$F[x(\alpha n)] = \sum_{n=-\infty}^{\infty} x(\alpha n) e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{\Omega}{\alpha} n} = X(e^{j\frac{\Omega}{\alpha}})$$

$$(2) x^*(\alpha n)$$

$$F[x^*(\alpha n)] = X^*(e^{-j\frac{\Omega}{\alpha}})$$

10. 求下列周期序列的傅立叶级数

$$(1) \quad (\alpha^n u(n)) * \tilde{\delta}_8(n) \quad (0 < \alpha < 1)$$

$$(2) \quad \cos\left(\frac{\pi}{4} n\right)$$

$$\text{解: (1) } (\alpha^n u(n)) * \tilde{\delta}_8(n) \quad (0 < \alpha < 1)$$

$$\text{令 } \tilde{x}_8(n) = [a^n u(n)] * \tilde{\delta}_8(n)$$

$$\text{则 } \tilde{X}_8(k) = \sum_{n=0}^7 \tilde{x}_8(n) e^{-j\frac{\pi}{4} kn} = \sum_{n=0}^7 a^n e^{-j\frac{\pi}{4} kn}$$

$$\tilde{x}_8(n) = \frac{1}{8} \sum_{k=0}^7 [\sum_{n=0}^7 a^n e^{-j\frac{\pi}{4}kn}] e^{j\frac{\pi}{4}kn}$$

$$(2) \cos(\frac{\pi}{4}n)$$

$$\text{令 } \tilde{x}_8(n) = \cos(\frac{\pi}{4}n)$$

$$\text{则 } \tilde{X}_8(k) = \sum_{n=0}^7 \tilde{x}_8(n) e^{-j\frac{\pi}{4}kn} = \sum_{n=0}^7 \cos(\frac{\pi}{4}n) e^{-j\frac{\pi}{4}kn}$$

$$\tilde{x}_8(n) = \frac{1}{8} \sum_{k=0}^7 [\sum_{n=0}^7 \cos(\frac{\pi}{4}n) e^{-j\frac{\pi}{4}kn}] e^{j\frac{\pi}{4}kn}$$

12. 设 $x_a(t)$ 是周期连续时间信号,

$$x_a(t) = A \cos(200\pi t) + B \cos(500\pi t)$$

以采样频率 $f_s = 1\text{KHz}$ 对其进行采样, 计算采样信号 $x(n) = x_a(t)|_{t=nT_s}$ 的 DFS。

$$\text{解: } X(k\Omega_0) = \frac{1}{20} (A \cos(\frac{n\pi}{5}) + B \cos(\frac{n\pi}{2})) e^{-jk\frac{\pi}{10}n}$$

19. 求下列序列的 Z 变换, 并画出极零图和收敛区域。

$$(1) x(n) = a^{|n|}$$

解:

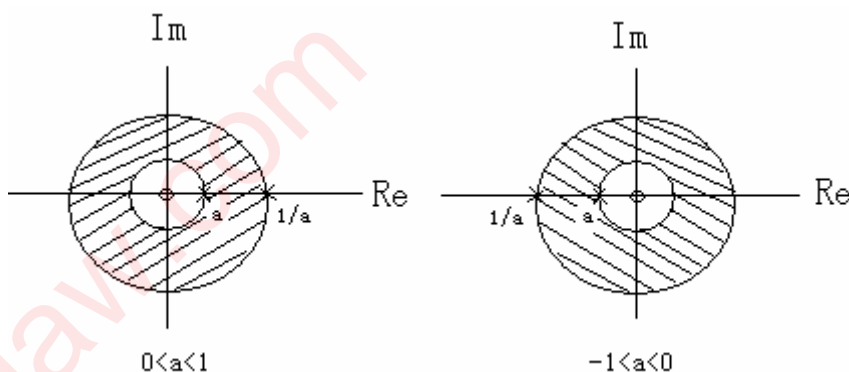
$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} a^{|n|} z^{-n} \\ &= \sum_{n=-\infty}^{-1} a^{-n} z^{-n} + \sum_{n=0}^{\infty} a^n z^{-n} \\ &= \frac{az}{1-az} + \frac{1}{1-az^{-1}} \\ &= \frac{(1-a^2)z^{-1}}{(z^{-1}-a)(1-az^{-1})} \end{aligned}$$

收敛域为 $|az| < 1$, 且 $|az^{-1}| < 1$, 且 $|a| < 1$

所以收敛域为 $|a| < 1$, 且 $|a| < |z| < \frac{1}{|a|}$ 。

零点为 $z=0$, 极点为 $z_1 = \frac{1}{a}$, $z_2 = a$

收敛区域为下图阴影部分。



$$(2) \quad x(n) = \begin{cases} 1, & 0 \leq n \leq N-1 \\ 0, & n < 0, n > N-1 \end{cases}$$

解:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{N-1} z^{-n} = \frac{1 - z^{-N}}{1 - z^{-1}}$$

零点为 $z=1$, 极点为 $z=1$, 收敛域为 $|z| > 0$ 。

$$(3) \quad x(n) = \begin{cases} n, & 0 \leq n \leq N \\ 2N - n, & N+1 \leq n \leq 2N \\ 0, & \text{其它 } n \end{cases}$$

解:

$$\begin{aligned} x(n) &= n[u(n) - u(n-1)] + (2N-n)[u(n-N) - u(n-2N)] \\ &= nu(n) - nu(n-N) + (2N-n)u(n-N) - (2N-n)u(n-2N) \\ &= nu(n) - 2(n-N)u(n-N) + (n-2N)u(n-2N) \end{aligned}$$

$$\begin{aligned} X(z) &= \frac{z^{-1}}{(1-z^{-1})^2} - \frac{2z^{-N}z^{-1}}{(1-z^{-1})^2} + \frac{z^{-2N}z^{-1}}{(1-z^{-1})^2} \\ &= \frac{z^{-1}(1-z^{-N})^2}{(1-z^{-1})^2} \end{aligned}$$

收敛域为 $1 < |z| \leq \infty$

零点 $z=1$, 极点 $z=0, z=1$ 。

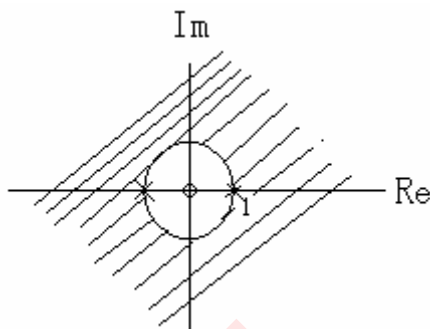
$$(4) \quad x(n) = n \quad (n \geq 0)$$

解:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=1}^{\infty} nz^{-n} = \frac{z^{-1}}{(1-z^{-1})^2}$$

零点为 $z=0$ ，极点为 $z_1 = z_2 = 1$

收敛域为 $|z| > 1$ 。



$$(5) \quad x(n) = \frac{1}{n!} \quad (n \geq 0)$$

$$(6) \quad x(n) = \cos an, \quad (n \geq 0) \quad (a \text{ 为常数})$$

21. 设 $x(n) \xleftrightarrow{z} \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}}$ ，试问 $x(n)$ 在以下三种收敛域下，哪一种是

左边序列、哪一种是右边序列、哪一种是双边序列？并求出各对应的 $x(n)$ 。

$$(1) \quad |z| > 2$$

$$(2) \quad |z| < 0.5$$

$$(3) \quad 0.5 < |z| < 2$$

解：(1) $|z| > 2$

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{-1}{1-2z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

因为 $|z| > 2$ ，所以 $|2z^{-1}| < 1, |0.5z^{-1}| < 1$

$$X(z) = -\sum_{n=0}^{\infty} 2^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n} = \sum_{n=0}^{\infty} (2^{-n} - 2^n) z^{-n}$$

$x(n) = (2^{-n} - 2^n)u(n)$ ，为右边序列

$$(2) \quad |z| < 0.5$$

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{0.5z}{1-0.5z} - \frac{2z}{1-2z}$$

因为 $|z| < 0.5$ ，所以 $|0.5z| < 1, |2z| < 1$

$$X(z) = -\sum_{n=1}^{\infty} 2^n z^n + \sum_{n=1}^{\infty} 0.5^n z^n = \sum_{n=-\infty}^{-1} (-2^{-n} + 2^n) z^{-n}$$

$x(n) = (-2^{-n} + 2^n)u(-n-1)$ ，为左边序列

(3) $0.5 < |z| < 2$

$$X(z) = \frac{-3z^{-1}}{2-5z^{-1}+2z^{-2}} = \frac{0.5z}{1-0.5z} + \frac{1}{1-0.5z^{-1}}$$

因为 $0.5 < |z| < 2$

$$X(z) = \sum_{n=1}^{\infty} 0.5^n z^n + \sum_{n=0}^{\infty} 0.5^n z^{-n} = \sum_{n=-\infty}^{-1} 2^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n}$$

$x(n) = 2^n u(-n-1) + 0.5^n u(n)$ ，为双边序列

22. 求下列 $F(z)$ 的 Z 反变换：

$$(1) \frac{1 - az^{-1}}{z^{-1} - a}, \quad |z| > \frac{1}{a}$$

$$(2) \frac{1 + z^{-1}}{1 - z^{-1} 2 \cos \Omega_0 + z^{-2}}, \quad |z| > 1$$

$$(3) \frac{z^{-n_0}}{1 + z^{-n_0}}, \quad |z| > 1, n_0 \text{ 为某整数}$$

解：

$$(1) \frac{1 - az^{-1}}{z^{-1} - a}, \quad |z| > \frac{1}{a}$$

$$\text{因为 } X(z) = \frac{z^{-1} - a^{-1}}{1 - (az)^{-1}} = \frac{z^{-1}}{1 - (az)^{-1}} - \frac{a^{-1}}{1 - (az)^{-1}} = \sum_{n=1}^{\infty} a^{-n+1} z^{-n} - \sum_{n=0}^{\infty} a^{-(n+1)} z^{-n}$$

$$\text{所以 } x(n) = a^{-n+1} u(n-1) - a^{-(n+1)} u(n)$$

$$(2) \frac{1 + z^{-1}}{1 - z^{-1} 2 \cos \Omega_0 + z^{-2}}, \quad |z| > 1$$

$$\text{因为 } X(z) = \frac{1 - z^{-1} \cos \Omega_0}{1 - 2z^{-1} \cos \Omega_0 + z^{-2}} + \frac{z^{-1} \sin \Omega_0}{1 - 2z^{-1} \cos \Omega_0 + z^{-2}} \cdot \frac{1 + \cos \Omega_0}{\sin \Omega_0}$$

$$\text{所以 } x(n) = \frac{\sin(n+1)\Omega_0}{\sin \Omega_0} u(n+1) + \frac{\sin n\Omega_0}{\sin \Omega_0} u(n)$$

$$(3) \frac{z^{-n_0}}{1+z^{-n_0}}, |z| > 1, n_0 \text{ 为某整数}$$

A. 当 $n_0 = 0$ 时, $X(z) = 0.5$, 则 $x(n) = 0.5\delta(n)$

B. 当 $n_0 \neq 0$ 时,

$$X(z) = -\sum_{n=1}^{\infty} (-z^{-n_0})^n = -\sum_{n=1}^{\infty} (-1)^n z^{-n_0 n} = -\sum_{n'=n_0}^{\infty} (-1)^{\frac{n'}{n_0}} z^{-n'} = -\sum_{n=n_0}^{\infty} (-1)^{\frac{n}{n_0}} z^{-n}$$

$$x(n) = (-1)^{\frac{n+n_0}{n_0}} u(n-n_0)$$

参考习题:

1. 求下列相应序列的频谱

$$(1) e^{-an} \cos \Omega_0 n \cdot u(n)$$

$$(2) r_N(n)$$

$$(3) 1/(1-az^{-1}), 0 < a < 1$$

$$(4) 1/(1-z^{-1}2a \cos \Omega_0 + z^{-2}a^2), 0 < a < 1$$

解:

$$(4) 1/(1-z^{-1}2a \cos \Omega_0 + z^{-2}a^2), 0 < a < 1$$

$$X(z) = \frac{1}{1-z^{-1}2a \cos \Omega_0 + z^{-2}a^2}$$

$$a < |z| \leq \infty$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$\text{令 } z = e^{j\Omega}, \text{ 则}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\Omega n}$$

$$X(e^{j\Omega}) = \frac{1}{1 - e^{-j\Omega} 2a \cos \Omega_0 + e^{-sj\Omega} a^2}$$

2. 设 $x(n) \xrightarrow{F} X(e^{j\Omega})$, 求 $|X(e^{j\Omega})|^2$ 的傅立叶反变换。

解:

$$|X(e^{j\Omega})|^2 = X(e^{j\Omega}) \cdot X^*(e^{j\Omega})$$

$$\begin{aligned} F^{-1}[|X(e^{j\Omega})|^2] &= F^{-1}[X(e^{j\Omega}) \cdot X^*(e^{j\Omega})] \\ &= F^{-1}[X(e^{j\Omega})] * F^{-1}[X^*(e^{j\Omega})] \\ &= x(n) * x^*(-n) \end{aligned}$$

3、试求 $\frac{2t}{t^2-1} \cos \frac{\pi t}{2}$ 的不失真采样的最大采样周期

解： 设 $x_1(t) = \frac{2t}{t^2 - 1} \leftrightarrow X_1(\omega)$

$x_2(t) = \cos \frac{\pi t}{2} \leftrightarrow X_2(\omega)$

$X(\omega) = F[x_1(t)x_2(t)] = X_1(\omega) * X_2(\omega)$

$\therefore F\left[\frac{1}{t}\right] = -j\pi \operatorname{sgn}(\omega)$

$\therefore F\left[\frac{1}{t+1}\right] = e^{j\omega} [-j\pi \operatorname{sgn}(\omega)]$

$F\left[\frac{1}{t-1}\right] = e^{-j\omega} [-j\pi \operatorname{sgn}(\omega)]$

$\therefore F\left[\frac{2t}{t^2 - 1}\right] = F\left[\frac{1}{t+1}\right] + F\left[\frac{1}{t-1}\right] = e^{j\omega} [-j\pi \operatorname{sgn}(\omega)] + e^{-j\omega} [-j\pi \operatorname{sgn}(\omega)]$

$= 2 \cos \omega [-j\pi \operatorname{sgn}(\omega)]$

而 $F\left[\cos \frac{\pi t}{2}\right] = \pi \left[\delta\left(\omega + \frac{\pi}{2}\right) + \delta\left(\omega - \frac{\pi}{2}\right)\right]$

$\therefore X(\omega) = X_1(\omega) * X_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \pi \left[\delta\left(\tau + \frac{\pi}{2}\right) + \delta\left(\tau - \frac{\pi}{2}\right)\right] \cdot 2 \cos(\omega - \tau) [-j\pi \operatorname{sgn}(\omega - \tau)] d\tau$

$= \frac{1}{2\pi} \left[\pi \times 2 \cos(\omega - \tau) (-j\pi \operatorname{sgn}(\omega - \tau)) \Big|_{\tau = -\frac{\pi}{2}} + \pi \times 2 \cos(\omega - \tau) (-j\pi \operatorname{sgn}(\omega - \tau)) \Big|_{\tau = \frac{\pi}{2}} \right]$

$= j\pi \sin(\omega) \left[\operatorname{sgn}\left(\omega + \frac{\pi}{2}\right) - \operatorname{sgn}\left(\omega - \frac{\pi}{2}\right) \right]$

$= \begin{cases} 2j\pi \sin(\omega) & |\omega| < \frac{\pi}{2} \\ 0 & |\omega| \geq \frac{\pi}{2} \end{cases}$

$\therefore \omega_m = \frac{\pi}{2}$

\therefore 最小采样频率为 $\Omega_s = 2\omega_m = \pi$

\therefore 最大采样周期为 $T = \frac{2\pi}{\Omega_s} = \frac{2\pi}{\pi} = 2$

4、一个理想采样系统频率为 $\Omega_s = 8\pi$ ，采样后经过低通 $L(\omega)$ 还原：

$L(\omega) = \begin{cases} \cos \frac{\omega}{8\pi} & |\omega| < 4\pi \\ 0 & |\omega| \geq 4\pi \end{cases}$ 今有输入 $x(t) = \frac{\sin \pi t}{\pi t}$ ，问输出信号 $y(t)$ 有没有失真？是

什么失真？

$$\text{解:} \because x(t) = \frac{\sin \pi t}{\pi t} = \text{Sa}(\pi t)$$

$$\therefore X(\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & |\omega| \geq \pi \end{cases}$$

$$\therefore X''(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\Omega_s) = 4 \sum_{n=-\infty}^{\infty} X(\omega - n \times 8\pi)$$

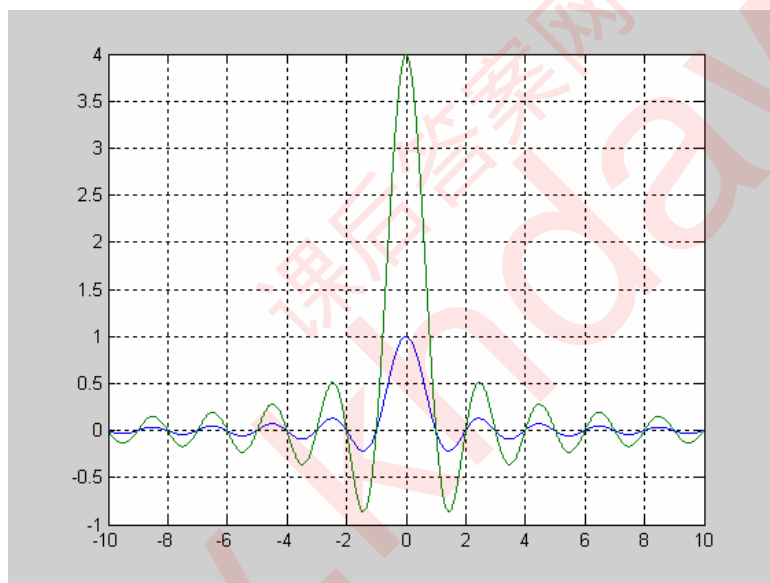
$$\therefore Y(\omega) = L(\omega)X''(\omega) = L(\omega)4 \sum_{n=-\infty}^{\infty} X(\omega - n \times 8\pi) = 4 \cos \frac{\omega}{8\pi} X(\omega)$$

$$\therefore F^{-1}\left[\cos \frac{\omega}{8\pi}\right] = \frac{1}{2} \left[\delta\left(-t + \frac{1}{8\pi}\right) + \delta\left(-t - \frac{1}{8\pi}\right) \right]$$

$$\therefore y(t) = 4F^{-1}\left[\cos \frac{\omega}{8\pi}\right] * x(t) = 2 \int_{-\infty}^{\infty} \left[\delta\left(-\tau + \frac{1}{8\pi}\right) + \delta\left(-\tau - \frac{1}{8\pi}\right) \right] \cdot \text{Sa}((t-\tau)\pi) d\tau$$

$$2 \left[\text{Sa}\left((t-\tau)\pi\right) \Big|_{\tau=\frac{1}{8\pi}} + \text{Sa}\left((t-\tau)\pi\right) \Big|_{\tau=-\frac{1}{8\pi}} \right] = 2 \left[\text{Sa}\left(\pi t - \frac{1}{8}\right) + \text{Sa}\left(\pi t + \frac{1}{8}\right) \right]$$

$\therefore y(t)$ 应该有失真。



产生幅度失真。

5. 试指出如下序列的因果性及稳定性。

$$(1) 2^n u(-n)$$

解:

$$x(n) = 2^n u(-n)$$

$$x(n)u(n) = 2^n u(-n)u(n) = 2^n \Big|_{n=0} = 1$$

所以

$$x(n) \neq x(n)u(n)$$

所以 $x(n)$ 不是因果信号。

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^0 2^n = \sum_{n=0}^{\infty} 2^{-n} = 2 < \infty$$

所以 $x(n)$ 是稳定信号。

$$(2) \frac{1}{n} u(n)$$

$$\text{解: } x(n)u(n) = \frac{1}{n} u(n)u(n) = \frac{1}{n} u(n) = x(n)$$

所以 $x(n)$ 是因果信号。

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=0}^{\infty} \frac{1}{n} \text{ 发散}$$

所以 $x(n)$ 是不稳定信号。

6、设 $h(n) = \{-2, 2, 0, 1, 5\}$, $x(n) = \{2, 1, 6, 1, -1, 4\}$, 求 $y(n) = h(n) * x(n)$

$$\begin{array}{rcccccccc} \text{解:} & 2 & 1 & 6 & 1 & -1 & 4 & & \\ \times & -2 & 2 & 0 & 1 & 5 & & & \\ \hline & -4 & -2 & -12 & -2 & 2 & -8 & & \\ & & 4 & 2 & 12 & 2 & -2 & 8 & \\ & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 2 & 1 & 6 & 1 & -1 & 4 \\ + & & & & & 10 & 5 & 30 & 5 & -5 & 20 \\ \hline & -4 & 2 & -10 & 12 & 15 & 1 & 39 & 4 & -1 & 20 \end{array}$$

$$\text{即 } y(n) = \{-4, 2, -10, 12, 15, 1, 39, 4, -1, 20\}$$

7. 分别以 4、8 为周期，将 $x(n) = \{x(-1), x(0), x(1), x(2), x(3), x(4)\} = \{-1, 0, 1, 2, 3, 4\}$ 周期化，

求其周期信号 $\tilde{x}_N(n)$ 。

解： (1) $N = 4, \tilde{x}_4(n) = \{4, 1, 2, 2\}$

(2) $N = 8, \tilde{x}_8(n) = \{0, 1, 2, 3, 4, 0, 0, -1\}$

习题 (P202)

1. 某线性时不变系统, 当激励为图 P3.1(a)所示三个形状相同的波形时, 其零状态响应 $y_1(t)$ 如图 P3.1(b)所示。试求当激励为图 P3.1(c)所示的 $x_2(t)$ [每个波形与图(a)中的任一形状相同] 时的零状态响应 $y_2(t)$ 。

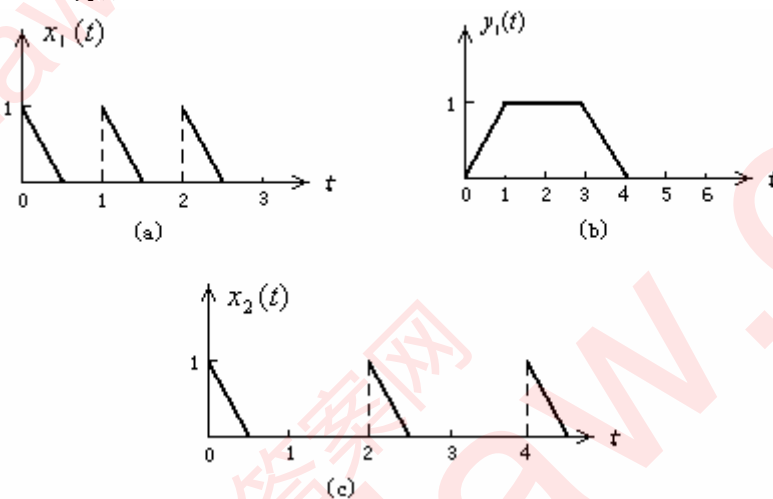


图 P3.1

解:

$$\begin{aligned} x_2(t) &= x_1(t) - x_1(t-1) + x_1(t-2) \\ \therefore y_2(t) &= y_1(t) - y_1(t-1) + y_1(t-2) \end{aligned}$$

令

$$\begin{aligned} \alpha(t) &= t \cdot [u(t) - u(t-1)] + (-t+2)[u(t-1) - u(t-2)] \\ &= tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2) \end{aligned}$$

$$\begin{aligned} y_2(t) &= \alpha(t) + \alpha(t-2) + \alpha(t-4) \\ &= tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2) + (t-2)u(t-2) \\ &\quad - 2(t-3)u(t-3) + (t-4)u(t-4) + (t-4)u(t-4) - 2(t-5)u(t-5) + (t-6)u(t-6) \\ &= tu(t) - 2(t-1)u(t-1) + 2(t-2)u(t-2) - 2(t-3)u(t-3) + 2(t-4)u(t-4) \\ &\quad - 2(t-5)u(t-5) + (t-6)u(t-6) \end{aligned}$$

6. 某一线性时不变系统, 在相同的初始条件下,
 若当激励为 $x(t)$ 时, 其全响应为 $y_1(t) = (2e^{-3t} + \sin 2t)U(t)$;
 若当激励为 $2x(t)$, 其全响应为 $y_2(t) = (e^{-3t} + 2\sin 2t)U(t)$ 。
 求: (1) 初始条件不变, 当激励为 $x(t-t_0)$ 时的全响应 $y_3(t)$, t_0 为大于零的实常数;
 (2) 初始条件增大 1 倍, 当激励为 $0.5x(t)$ 时的全响应 $y_4(t)$ 。

解: 因为系统是线性时不变的,

$$\therefore y_1(t) = y_1'(t) + y_1''(t) = (2e^{-3t} + \sin 2t)u(t)$$

其中 $y_1'(t)$ 是零输入响应, $y_1''(t)$ 是零状态响应

$$\text{则 } y_2(t) = y_1'(t) + y_2''(t) = y_1'(t) + 2y_1''(t) = (e^{-3t} + 2\sin 2t)u(t)$$

$$\therefore y_1''(t) = y_2(t) - y_1(t) = (-e^{-3t} + \sin 2t)u(t)$$

$$y_1'(t) = 3e^{-3t}u(t)$$

当初始条件不变, 激励为 $x_1(t-t_0)$ 时,

$$y_3(t) = y_1'(t) + y_1''(t-t_0) = 3e^{-3t}u(t) + [-e^{-3(t-t_0)} + \sin 2(t-t_0)]u(t-t_0)$$

(2)、当初始条件增大一倍, 激励为 $0.5x_1(t)$ 时, 有:

$$\begin{aligned} y_4(t) &= 2y_1'(t) + 0.5y_1''(t) = 6e^{-3t}u(t) + (-0.5e^{-3t} + 0.5\sin 2t)u(t) \\ &= (5.5e^{-3t} + 0.5\sin 2t)u(t) \end{aligned}$$

9. 如图 P3.4 所示系统是由几个子系统组合而成, 各子系统的冲激响应分别为

$$h_1(t) = U(t) \quad (\text{积分器})$$

$$h_2(t) = \delta(t-1) \quad (\text{单位延时器})$$

$$h_3(t) = -\delta(t) \quad (\text{倒相器})$$

求总系统的冲激响应 $h(t)$ 。

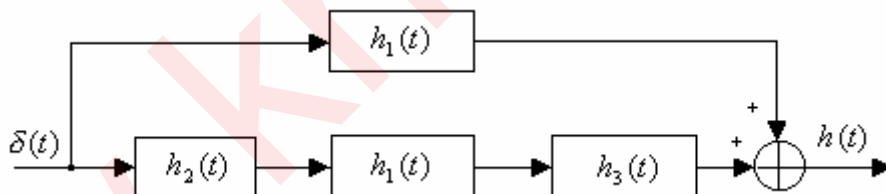


图 P3.4

解:

$$\begin{aligned} &h_2(t) * h_1(t) * h_3(t) + h_1(t) \\ &= u(t) - \delta(t-1) * \delta(t) * u(t) \\ &= u(t) - u(t-1) \end{aligned}$$

11. 考虑一个线性时不变系统 S 和一信号 $x(t) = 2e^{-3t}u(t-1)$, 若

$$x(t) \rightarrow y(t)$$

和

$$\frac{dx(t)}{dt} \rightarrow -3y(t) + e^{-2t}u(t)$$

求系统 S 的单位冲激响应 $h(t)$ 。

$$\text{解: } \because y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} 2e^{-3\tau}u(\tau-1)h(t-\tau)d\tau$$

$$\text{又} \because -3y(t) + e^{-2t}u(t) = \frac{dx(t)}{dt} * h(t)$$

$$\text{把} y(t) \text{代入有} -3 \int_{-\infty}^{\infty} 2e^{-3\tau}u(\tau-1)h(t-\tau)d\tau + e^{-2t}u(t) = \int_{-\infty}^{\infty} \frac{dx(\tau)}{d\tau} \cdot h(t-\tau)d\tau =$$

$$\int_{-\infty}^{\infty} \frac{d[2e^{-3\tau}u(\tau-1)]}{d\tau} \cdot h(t-\tau)d\tau = \int_{-\infty}^{\infty} [-6e^{-3\tau}u(\tau-1) + 2e^{-3\tau}\delta(\tau-1)] \cdot h(t-\tau)d\tau$$

$$\text{即: } \int_{-\infty}^{\infty} 2e^{-3\tau}\delta(\tau-1)h(t-\tau)d\tau = e^{-2t}u(t)$$

$$\therefore 2e^{-3}h(t-1) = e^{-2t}u(t)$$

$$h(t-1) = \frac{1}{2}e^3e^{-2t}u(t)$$

$$h(t) = \frac{1}{2}e^{-2t+1}u(t+1)$$

19. 在图 P3.14 (a) 所示系统中, 已知 $H_1(\omega)$ 如图 P3.14 (c) 所示; $h_2(t)$ 的波形如图 P3.14

(b) 所示; $f(t) = \sum_{n=-\infty}^{\infty} \delta(t-n), n=0, \pm 1, \pm 2, \dots$ 。求零状态响应 $y(t)$ 。

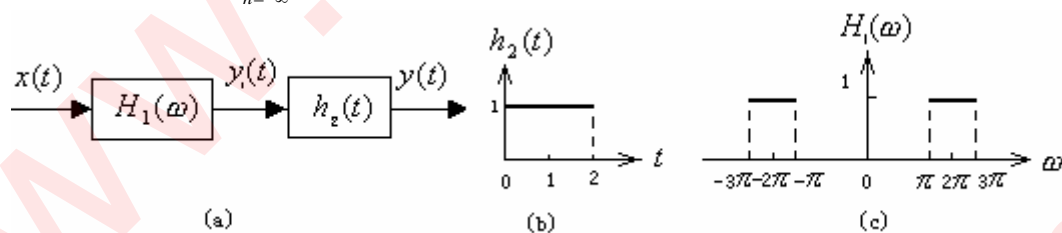


图 P3.14

$$\text{解: } H_2(\omega) = \frac{2e^{-j\omega} \sin \omega}{\omega}$$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n)$$

$$Y(\omega) = H_1(\omega)H_2(\omega)X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} H_1(2\pi n)H_2(2\pi n)\delta(\omega - 2\pi n) = 0$$

$$\therefore y(t) = 0$$

21. 有一因果线性时不变滤波器，其频率响应 $H(\omega)$ 如图 P3.16 所示。对以下给定的输入，求经过滤波后的输出 $y(t)$ 。

(1) $x(t) = e^{jt}$ (2) $x(t) = (\sin \omega_0 t)u(t)$

(3) $X(\omega) = \frac{1}{(j\omega)(6+j\omega)}$ (4) $X(\omega) = \frac{1}{2+j\omega}$

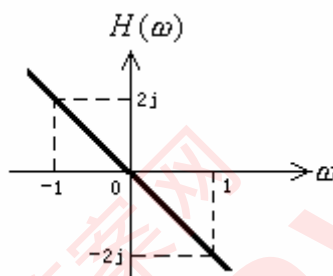


图 P3.16

解：由图 3.16 可知

$$H(\omega) = -2j\omega$$

$$H(s) = -2s$$

(1) $x(t) = e^{jt}$

$$x(t) = e^{jt} = \cos t + j \sin t$$

$$X(s) = \frac{s}{s^2 + 1} + j \frac{1}{s^2 + 1}$$

$$Y(s) = X(s)H(s) = -2 + \frac{2}{s^2 + 1} + j \frac{2s}{s^2 + 1}$$

$$y(t) = -2\delta(t) + 2\sin tu(t) + j2\cos tu(t)$$

(2) $x(t) = (\sin \omega_0 t)u(t)$

$$X(s) = \frac{\omega_0}{s^2 + \omega_0^2}$$

$$Y(s) = X(s)H(s) = -2s \frac{\omega_0}{s^2 + \omega_0^2} = -2\omega_0 \frac{s}{s^2 + \omega_0^2}$$

$$y(t) = -2\omega_0 \cos \omega_0 t u(t)$$

(3) $X(\omega) = \frac{1}{(j\omega)(6+j\omega)}$

$$Y(\omega) = X(\omega)H(\omega) = \frac{-2j\omega}{j\omega(6+j\omega)} = \frac{-2}{6+j\omega}$$

$$y(t) = -2e^{-6t}u(t)$$

$$(4) \quad X(\omega) = \frac{1}{2 + j\omega}$$

$$Y(\omega) = X(\omega)H(\omega) = \frac{-2j\omega}{2 + j\omega} = -2 + \frac{4}{2 + j\omega}$$

$$y(t) = -2\delta(t) + 4e^{-2t}u(t)$$

24. 已知如图 P3.18 所示系统。

- (1) 求 $H(s) = \frac{Y(s)}{F(s)}$;
- (2) 求冲激响应 $h(t)$ 与阶跃响应 $g(t)$;
- (3) 若 $f(t) = U(t-1) - U(t-2)$, 求零状态响应 $y(t)$ 。

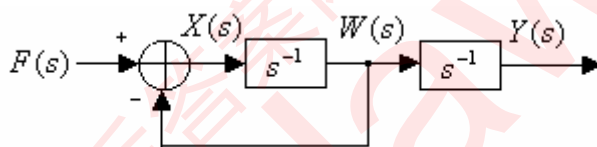


图 P3.18

解:

$$(1) \quad H(s) = \frac{1}{s} \cdot \frac{1}{1 + \frac{1}{s}} = \frac{1}{s(s+1)}$$

$$(2) \quad h(t) = u(t) - e^{-t}u(t) = (1 - e^{-t})u(t) \quad (\text{冲激响应 收敛域 } \sigma > 0)$$

$$G(s) = F(s)H(s) = \frac{1}{s} \cdot \frac{1}{s(s+1)} = \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2}$$

$$g(t) = (e^{-t} - 1 + t)u(t) \quad (\text{阶跃响应 收敛域 } \sigma > 0)$$

(3) 根据线性时不变系统的性质:

$$y(t) = g(t-1) - g(t-2) = (e^{-t+1} + t - 2)u(t-1) - (e^{-t+2} + t - 3)u(t-2)$$

参考习题:

1. 如图 P3.9(a)所示为理想低通滤波器系统, 已知激励

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), n = 0, \pm 1, \pm 2, \dots, T = 10^{-3} \text{ s};$$

系统的 $H(\omega) = 2G_{2\omega_m}(\omega)e^{-j\omega t_0}$ ，如图 P3.9 (b)所示， $\omega_m = 10^4 \text{ rad/s}$ 。求响应 $y(t)$ 。

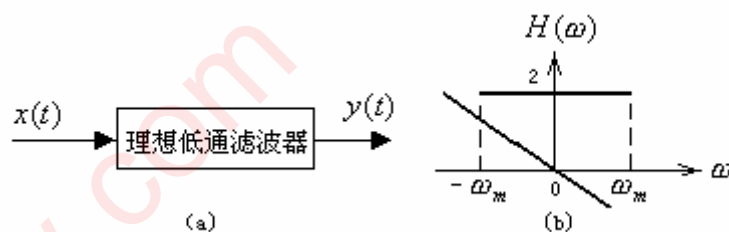


图 P3.9

解:
$$X(n\omega_0) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t) e^{-jn\omega_0 t} dt = \frac{1}{T}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} 2\pi \frac{1}{T} \delta(\omega - n\omega_0) = \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$$

$$Y(\omega) = 2\omega_0 [\delta(\omega) + \delta(\omega - \omega_0) e^{-j\omega_0 t_0} + \delta(\omega + \omega_0) e^{j\omega_0 t_0}]$$

$$y(t) = 2\omega_0 \left[\frac{1}{2\pi} + \frac{1}{\pi} \cos \omega_0 t_0 \cos \omega_0 t + \frac{1}{\pi} \sin \omega_0 t_0 \sin \omega_0 t \right]$$

$$= \frac{\omega_0}{\pi} [1 + 2 \cos \omega_0 (t - t_0)] = \frac{2}{T} [1 + 2 \cos \frac{2\pi}{T} (t - t_0)]$$

$$= 2000 [1 + 2 \cos 2000\pi (t - t_0)]$$

2. 已知系统频率特性 $H(\omega) = \frac{j\omega}{-\omega^2 + j5\omega + 6}$ ，系统的初始状态 $y(0) = 2, y'(0) = 1$ ，激励

$x(t) = e^{-t}U(t)$ 。求全响应 $y(t)$ 。

解:

$$H(\omega) = \frac{j\omega}{-\omega^2 + 5j\omega + 6}$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{j\omega}{-\omega^2 + j5\omega + 6} = \frac{j\omega}{(j\omega)^2 + j5\omega + 6}$$

$$y''(t) + 5y'(t) + 6y(t) = x'(t)$$

$$(s^2 + 5s + 6)Y(s) - 2s - 11 = sX(s) = \frac{s}{s+1}$$

$$\begin{aligned} Y(s) &= \frac{2s^2 + 14s + 11}{(s+1)(s+2)(s+3)} \\ &= \frac{-0.5}{s+1} + \frac{9}{s+2} + \frac{-12.5}{s+3} \end{aligned}$$

$$y(t) = -0.5e^{-t} + 9e^{-2t} - 6.5e^{-3t}$$

3. 考虑一个离散时间系统其输入 $x(n]$ 和输出 $y(n]$ 关系为

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

式中 n_0 为某一有限正整数。

(1) 系统是线性的吗?

(2) 系统是时不变的吗?

(3) 若 $x(n]$ 为有界且界定为一有限整数 B (即对全部 n , $|x(n)] < B$), 可以证明 $y(n]$ 是被界定到某一有限数 C 。因此可以得出该系统是稳定的。请用 B 和 n_0 来表示 C 。

解: (1)、 $x_1(k) \rightarrow y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k)$

$$x_2(k) \rightarrow y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_2(k)$$

$$x_3(k) = ax_1(k) + bx_2(k)$$

其中 a, b 为任意常数

$$\begin{aligned} \therefore y_3(n) &= \sum_{k=n-n_0}^{n+n_0} x_3(k) = \sum_{k=n-n_0}^{n+n_0} [ax_1(k) + bx_2(k)] = a \sum_{k=n-n_0}^{n+n_0} x_1(k) + b \sum_{k=n-n_0}^{n+n_0} x_2(k) \\ &= ay_1(n) + by_2(n) \end{aligned}$$

因此系统是线性的。

$$(2)、 y_1(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k) \quad y_1(n - n_1) = \sum_{k=n-n_0-n_1}^{n+n_0-n_1} x_1(k)$$

$$x_2(k) = x_1(k - n_1)$$

$$\therefore y_2(n) = \sum_{k=n-n_0}^{n+n_0} x_1(k - n_1) \xrightarrow{\text{令 } k' = k - n_1} \sum_{k'=n-n_0-n_1}^{n+n_0-n_1} x_1(k')$$

$$= \sum_{k=n-n_0-n_1}^{n+n_0-n_1} x_1(k) = y_1(n - n_1)$$

所以系统是时不变的。

$$(3)、 |y(n)] = \left| \sum_{k=n-n_0}^{n+n_0} x(k) \right| \leq \sum_{k=n-n_0}^{n+n_0} |x(k)] < \sum_{k=n-n_0}^{n+n_0} B = (2n_0 + 1)B = C$$

$\therefore |y(n)] < C$ 且 $C = (2n_0 + 1)B$ 系统是稳定的。