ECE/ME 238 : Advanced Control Design Lab

Lecture 2 Notes: Acrobot versus Pendubot

Acrobot: Stand-Up and Stabilization

- See paper by Mark Spong, 1995. (Posted on Canvas.)

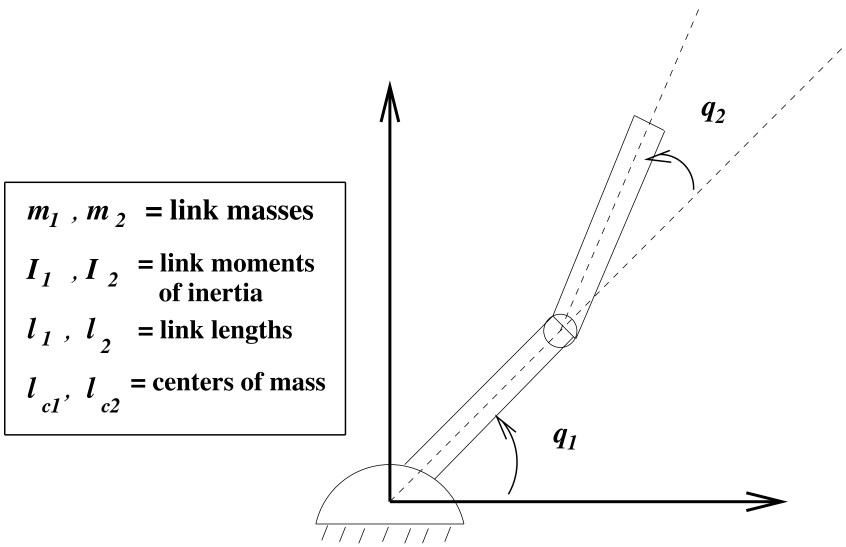

Figure 1: Two Link Robot

Consider a two-link, planar robot, as shown at left.

This can be used to model both:

- Acrobot
- Pendubot

Because there are d=2 degrees of freedom (DOFs), which are the absolute angle q1 and the relative angle q2, as depicted, there will also be d=2 equations of motion (EOMs), which can be derived via Newton's 2nd Law, e.g., via the Lagrangian approach for obtaining EOMs.

Two link arm: Equations of Motion

Example 1: Two Link Robot. Consider the two-link robot shown in Figure (1):

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + h_1 + \phi_1 = \tau_1 \quad (4)$$

Acrobot: tau1 = 0 (passive "shoulder")

$$d_{12}\ddot{q}_1 + d_{22}\ddot{q}_2 + h_2 + \phi_2 = \tau_2 \quad (5)$$

Pendubot: tau2 = 0 (passive "elbow")

where

$$\begin{aligned}
d_{11} &= m_1\ell_{c1}^2 + m_2(\ell_1^2 + \ell_{c2}^2 + 2\ell_1\ell_{c2}\cos(q_2)) + I_1 + I_2 \\
d_{22} &= m_2\ell_{c2}^2 + I_2 \\
d_{12} &= m_2(\ell_{c2}^2 + \ell_1\ell_{c2}\cos(q_2)) + I_2 \\
h_1 &= -m_2\ell_1\ell_{c2}\sin(q_2)\dot{q}_2^2 - 2m_2\ell_1\ell_{c2}\sin(q_2)\dot{q}_2\dot{q}_1 \\
h_2 &= m_2\ell_1\ell_{c2}\sin(q_2)\dot{q}_1^2 \\
\phi_1 &= (m_1\ell_{c1} + m_2\ell_1)g\cos(q_1) + m_2\ell_{c2}g\cos(q_1 + q_2) \\
\phi_2 &= m_2\ell_{c2}g\cos(q_1 + q_2)
\end{aligned}$$