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Optimal planning and control of a segway model taking into account spatial obstacles

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This contribution focuses on the optimal trajectory planning for a Segway model (inverted pendulum on two independently actuated wheels). Basis for this planning is the dynamical model for this under-actuated, non-holonomic multibody system. For the stabilization as well as the trajectory control, a partial input/output linearization is performed, where the orientation of the robot w.r.t. the horizontal plane and the inclination angle of the pendulum are used as output. The remaining non-linear part is linearized about the upright equilibrium position and stabilized with an LQR controller. The trajectory optimization is based on the partially linearized system, where the output (orientation and inclination) is parameterized by B-Splines. The system is required to move through predefined points in the horizontal plane. For the optimization the control points of the B-Splines serve as optimization variables and the overall energy of the robot serves as cost functional. Maximum motor velocities, motor torques and the ground reaction forces are the constraints. The latter are crucial when planning trajectories where the robot must pass (swing) under vertical obstacles while not losing ground contact. These maneuvers are characterized by high horizontal accelerations in order to lower the head of the robot and bring it back to upright equilibrium.

1 Mathematical modeling including ground reaction forces

In order to describe the dynamic behavior of the Segway model, consisting of 3 rigid bodies, shown in Fig. 1, the equations of motion are needed. Therefore the generalized coordinates $\mathbf{q} = \begin{pmatrix} x & y & \gamma & \theta & \xi & \eta \end{pmatrix}^\mathsf{T}$ are introduced, with the position and the orientation at the horizontal plane, x, y and γ, the inclination angle θ of the basis, and the relative rotation angles ξ and η of the wheels. Since ideal rolling of the wheels is assumed, non-holonomic constraints (constraints at velocity level) have to be considered. This leads to the introduction of a vector of minimal velocities $\dot{\mathbf{s}} = \begin{pmatrix} v_L & \dot\gamma & \dot\theta \end{pmatrix}^\mathsf{T}$, with the longitudinal velocity v_L and the angular velocities $\dot\gamma$ and $\dot\theta$. Likewise the driving torques can be combined to the vector of inputs $\mathbf{u} = \begin{pmatrix} M_1 & M_2 \end{pmatrix}^\mathsf{T}$. Using these definitions and by applying synthetic methods proposed in [1] the equations of motion in state space representation

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{\mathbf{q}} \\ \ddot{\mathbf{s}} \end{pmatrix} = \begin{pmatrix} \mathbf{H}(\mathbf{q})\dot{\mathbf{s}} \\ -\mathbf{M}(\mathbf{q})^{-1}\mathbf{g}(\mathbf{q},\dot{\mathbf{s}}) \end{pmatrix} + \begin{bmatrix} \mathbf{0}^{6\times 2} \\ \mathbf{M}(\mathbf{q})^{-1}\mathbf{B} \end{bmatrix} \mathbf{u} := \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} = \begin{pmatrix} \mathbf{f}_{12}(\gamma, v_L) \\ \begin{pmatrix} \dot\gamma & \dot\theta \end{pmatrix}^\mathsf{T} \\ \mathbf{f}_{56}(\dot{\mathbf{s}}) \\ f_7(\theta,\dot{\mathbf{s}}) \\ \mathbf{f}_{89}(\theta,\dot{\mathbf{s}}) \end{pmatrix} + \begin{bmatrix} \mathbf{0}^{2\times 2} \\ \mathbf{0}^{2\times 2} \\ \mathbf{0}^{2\times 2} \\ \mathbf{G}_7(\theta) \\ \mathbf{G}_{89}(\theta) \end{bmatrix} \mathbf{u} \quad (1)$$

can be obtained. The symmetric positive definite mass matrix is $\mathbf{M}(\mathbf{q})$, the state dependent generalized forces are collected in the vector $\mathbf{g}(\mathbf{q},\dot{\mathbf{s}})$, the constant input matrix is denoted with $\mathbf{B}$ and $\mathbf{H}(\mathbf{q})$ gives the kinematic relation $\dot{\mathbf{q}} = \mathbf{H}(\mathbf{q})\dot{\mathbf{s}}$.

Assuming ideal rolling conditions for the wheels, $\dot{\mathbf{s}}$ serves as generalized velocity. Therefore it has to be examined, whether the actual reaction forces between the wheels and the ground comply with that assumption. These ground reaction forces $\mathbf{f}_R = \begin{pmatrix} F_{R_1,x} & F_{R_2,x} & F_{R,y} & F_{R_1,z} & F_{R_2,z} \end{pmatrix}$ can be expressed as functions of $\mathbf{q}$, $\dot{\mathbf{s}}$ and $\ddot{\mathbf{s}}$. Here $F_{R_i,x}$ and $F_{R_i,z}$ denote the ${}_Rx$- and ${}_Rz$-component of the force acting on the i-th wheel and $F_{R,y}$ denotes the accumulated ${}_Ry$-component for both wheels, since this component can not be easily separated.

2 Input/output linearization, optimal trajectory planning and control

For the purpose of trajectory planning and optimization it is beneficial to describe the system state and the needed driving torques over time only by a limited number of parameters. Furthermore the calculation effort for solving the differential equations can be reduced significantly by applying an input/output linearization with respect to the chosen virtual output $\mathbf{y} = \begin{pmatrix} \gamma & \theta \end{pmatrix}^\mathsf{T}$. This approach derived from [2] results in the input transformation $\mathbf{v} = \mathbf{f}_{89}(\theta,\dot{\mathbf{s}}) + \mathbf{G}_{89}(\theta)\mathbf{u}$ with a well defined inverse transformation $\mathbf{u} = \mathbf{G}_{89}^{-1}(\theta)\left(\mathbf{v} - \mathbf{f}_{89}(\theta,\dot{\mathbf{s}})\right)$. The transformed system $\dot{\mathbf{x}} = \bar{\mathbf{f}}(\mathbf{x}) + \bar{\mathbf{G}}(\mathbf{x})\mathbf{v}$ with $\bar{\mathbf{f}}(\mathbf{x}) = \begin{pmatrix} \mathbf{f}_{12}^\mathsf{T}(\gamma, v_L) & \dot\gamma & \dot\theta & \mathbf{f}_{56}^\mathsf{T}(\dot{\mathbf{s}}) & \bar f_7(\theta,\dot\gamma,\dot\theta) & \mathbf{0}^{1\times 2} \end{pmatrix}^\mathsf{T}$ and $\bar{\mathbf{G}}(\mathbf{x}) = \begin{bmatrix} \mathbf{0}^{2\times 6} & \bar{\mathbf{G}}_7^\mathsf{T}(\theta) & \mathbf{I}^{2\times 2} \end{bmatrix}^\mathsf{T}$ is partially linear and can be solved step by step. The differential equations for the states γ and θ are reduced to double integrators and are trivially solved by choosing a sufficiently differentiable function for the virtual output $\mathbf{y}$. By doing so the solution of the differential equation for

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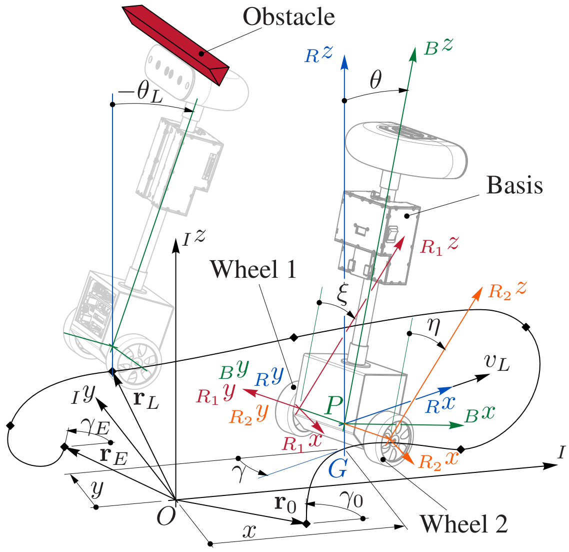

Fig. 1: Segway with reference frames, minimal coordinates and an exemplary trajectory including an obstacle

$$
\begin{array}{ll}
\min\limits_{\mathbf{p}_{opt}} \left(\displaystyle\int_{t_0}^{t_E} \|\mathbf{u}(\tau, \mathbf{p}_{opt})\|_2^2 \, \mathrm{d}\tau \right) & \\
\text{s.t.} & \\
\|\mathbf{r}(t_i, \mathbf{p}_{opt}) - \mathbf{r}_i\|_2^2 = 0 & i \in \mathcal{WP} \\
\gamma(t_i, \mathbf{p}_{opt}) - \gamma_i = 0 & i = \{0, E\} \\
v_L(t_i, \mathbf{p}_{opt}) = 0 & i = \{0, E\} \\
|\theta_L| - |\theta(t_L, \mathbf{p}_{opt})| < 0 & \\
\mathbf{u}_{min} < \mathbf{u}(t, \mathbf{p}_{opt}) < \mathbf{u}_{max} & t \in [t_0, t_E] \\
\boldsymbol{\omega}_{M,min} < \boldsymbol{\omega}_M(t, \mathbf{p}_{opt}) < \boldsymbol{\omega}_{M,max} & t \in [t_0, t_E] \\
\theta_{min} < \theta(t, \mathbf{p}_{opt}) < \theta_{max} & t \in [t_0, t_E] \\
\mathbf{f}_{R,min} < \mathbf{f}_R(t, \mathbf{p}_{opt}) < \mathbf{f}_{R,max} & t \in [t_0, t_E]
\end{array}
$$

Fig. 2: Optimization problem

v_L reduces to an integration over time. Now with calculated v_L the same holds true for the remaining states. In summary, all the system states including their derivatives, the driving torques and also the ground reaction forces can be defined by an at least two times differentiable ansatz for the virtual output $\mathbf{y}$.

As of now the virtual output $\mathbf{y}$ is defined by B-splines of degree three in order to avoid discontinuities for the driving torques. So the derivatives of the splines can be expressed explicitly by the control points of the splines $\bar{\boldsymbol{\gamma}}$ and $\bar{\boldsymbol{\theta}}$ as shown in [3]. These control points as well as the needed constants of integration $\mathbf{q}_0$ and $v_{L,0}$ can be used as optimization variables $\mathbf{p}_{opt} = \begin{pmatrix} \mathbf{q}_0^\mathsf{T} & v_{L,0} & \bar{\boldsymbol{\gamma}}^\mathsf{T} & \bar{\boldsymbol{\theta}}^\mathsf{T} \end{pmatrix}^\mathsf{T}$ in the optimization problem shown in Fig. 2. The goal is to minimize the overall energy between the starting time t_0 and the end time t_E of the trajectory and to avoid the obstacle shown in Fig. 1. This problem definition is similar to the one handled in [4]. The position at the surface plane is specified by a vector $\mathbf{r} = \begin{pmatrix} x & y \end{pmatrix}^\mathsf{T}$. The desired movement is defined by discrete waypoints $\mathbf{r}_i$ at the points in time t_i with $i \in \mathcal{WP} := \{0, 1, \ldots, L, \ldots, E\}$. The initial and terminal orientation angle is denoted by γ_0 and γ_E and the initial and terminal velocity v_L is set to zero. In order to avoid the obstacle the magnitude of the inclination angle θ has to be greater than the minimum feasible angle θ_L at the according waypoint $\mathbf{r}_L$. Furthermore the system limits for the motor torques $\mathbf{u}$ as well as the motor speeds $\boldsymbol{\omega}_M = \begin{pmatrix} \dot{\xi} & \dot{\eta} \end{pmatrix}^\mathsf{T}$ must not be exceeded. Additionally the magnitude of θ is limited by lower and upper boundaries in order to exclude mathematical solutions from the outset, which contain a pendulum swing below the ground level. Finally the ground reaction forces $\mathbf{f}_R$ have to be limited in such a way, that the $F_{R_i,z}$ components with $i \in \{1, 2\}$ are limited to compressive forces and the remaining components are limited with respect to the friction cone. This optimization problem is solved with an active set SQP algorithm.

The states and motor torques given by the optimized trajectory serve as input values for the trajectory tracking control composed of a feed forward control and a stabilizing feed back controller. Therefore the state of the input/output linearized system is reduced to $\mathbf{x}_R = \begin{pmatrix} v_{L,I}, \gamma, \theta, v_L, \dot{\gamma}, \dot{\theta} \end{pmatrix}^\mathsf{T}$ with the actual curve length of the trajectory $v_{L,I}$. The remaining nonlinear part of the reduced system is linearized at the upper equilibrium and a discrete LQR controller is designed. Afterwards the output of this linear controller in addition with a feed forward control for the transformed input $\mathbf{v}$ has to be transformed back to the original input $\mathbf{u}$. Because there only exist measurements for the relative angles of the wheels ξ and η, the inclination angle θ and the respective angular velocity $\dot{\theta}$, a discrete Kalman observer is used to get the full system state. For the design of this observer the original system is again linearized at the upper equilibrium.

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