

Covert Beamforming Design for Active RIS-Assisted NOMA-ISAC Systems

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Abstract—In this paper, an active reconfigurable intelligent surface (RIS) is deployed to assist covert communications in a non-orthogonal multiple access (NOMA) inspired integrated sensing and communication (ISAC) system. With the aid of the active RIS, a dual-function base-station (BS) serves a public user and a covert user under the NOMA principles while sensing multiple targets by using the superimposed sensing and communication signal. To maximize the covert rate of the NOMA-inspired ISAC (NOMA-ISAC) system, transmission beamforming at BS and reflection beamforming at active RIS are jointly optimized subject to QoS requirements of the NOMA public user, sensing beam pattern similarity constraint, and covertness level against the warden. To tackle the optimization variables coupled in the objective function and constraints, an alternating optimization algorithm is proposed, while the resulted non-convex fractional programming subproblems of transmission and reflection beamforming optimizations are solved by exploiting a penalized Dinkelbach transformation to obtain a high-quality rank-one solution. Numerical results demonstrate that the active RIS-assisted NOMA-ISAC scheme outperforms the passive RIS-assisted counterpart in terms of the covert rate and sensing beam pattern similarity.

Index Terms—Active reconfigurable intelligent surface (RIS), covert beamforming design, integrated sensing and communication (ISAC)

I. INTRODUCTION

With expanding of communication bandwidths and developments of multiple-input multiple-output (MIMO) techniques, waveforms are now suitable for radar target sensing at a reasonable range resolution [1]. Toward this trend, the concept of integrated sensing and communication (ISAC) has recently sparked heated discussions, particularly on the design of dual-functional waveforms [1]–[3]. Compared to orthogonal multiple access (OMA), non-orthogonal multiple access (NOMA) seems to be a viable solution for supporting multicast transmission and mitigating inter-user interference in NOMA-inspired ISAC (NOMA-ISAC) systems with limited wireless resources [4]. By adequately designing transmission beamforming, the superimposed NOMA signal can be exploited for guaranteeing an effective sensing quality while maximizing the communication throughput [5]. With the aid of successive interference cancellation (SIC), the transmit powers of the communication and sensing waveforms were well-designed in NOMA-ISAC systems, which ensures an accurate radar sensing and provides a new degree of freedom (DoF) for communications. However, due to the open and broadcasting natures of NOMA-ISAC

systems, the sensitive and privacy information embedded in waveforms is susceptible to interception and eavesdropping in the presence of malicious users.

A novel technology known as reconfigurable intelligent surface (RIS) has been developed to reflect wireless signal constructively or destructively, such that the arrived signal power on the desired directions can be strengthened or weakened by smartly adjusting the reflecting amplitudes and phase shifts [6]. Due to this property, RIS is often utilized to facilitate covert communications in NOMA systems [7], [8], which provides a higher level of security than the conventional physical layer security (PLS) by endeavoring to conceal the legitimate transmissions against wardens. Compared to covert communications in NOMA systems, where only the signal of the NOMA public user was utilized as a shield for covert transmissions [9], RIS-aided superposition transmissions can cost-effectively hide covert communication behaviors by leveraging the RIS's phase shift uncertainties and the non-orthogonal transmissions of the NOMA public user. Nevertheless, the reflecting link quality of the passive RIS-aided wireless communication system is limited due to the effect of “multiplicative fading”, leading to a negligible capacity gain. To overcome this limitation, the active RIS was proposed [10], [11]. The active RIS integrated amplifiers into the meta-elements to amplify the reflected signals, compensating for the substantial path loss of reflected links and obviating the need for complex and power-hungry radio frequency (RF) chain components at the base-station (BS) [10]. With the aid of the active RIS, the performance of PLS [12] and covert communication [13] can be improved by optimizing reflection beamforming at the active RIS and collaborating with the BS transmission optimization.

Although PLS has been studied in NOMA-ISAC systems to improve the secure performance [14], emerging researches are needed on covert communications for NOMA-ISAC systems with the aim of achieving a higher security level. On the other hand, active RIS has not yet been applied to assist covert communications in NOMA-ISAC systems and how to exploit reflection beamforming to improve the covert communication performance for NOMA-ISAC systems is still unknown. Motivated by the above observations, we propose to deploy an active RIS to assist covert communications in a NOMA-ISAC system. In the considered NOMA-ISAC system, both

and removes the corresponding interference by exploiting SIC followed by the detection of its own signal s_b , while treating the sensing signal as noise. Grace detects s_g by treating the signals related to s_b and s_s as noise. In this paper, we assume that the sensing signal cannot be cancelled by Bob and Grace. Then, the achievable rates of Bob corresponding to the transmissions of s_g and s_b are given by $R_{b,s_g} = \log_2(1 + \gamma_{b,s_g})$ and $R_{b,s_b} = \log_2(1 + \gamma_{b,s_b})$, respectively, with

$$\gamma_{b,s_g} = \frac{|\mathbf{g}_b^H \mathbf{w}_g|^2}{|\mathbf{g}_b^H \mathbf{w}_b|^2 + \|\mathbf{g}_b^H \mathbf{W}_s\|^2 + \|\mathbf{h}_{rb}^H \Phi\|^2 \sigma_r^2 + \sigma_b^2} \quad (5)$$

and

$$\gamma_{b,s_b} = \frac{|\mathbf{g}_b^H \mathbf{w}_b|^2}{\|\mathbf{g}_b^H \mathbf{W}_s\|^2 + \|\mathbf{h}_{rb}^H \Phi\|^2 \sigma_r^2 + \sigma_b^2}. \quad (6)$$

Furthermore, the achievable rate of Grace is given by

$$R_{g,s_g} = \log_2 \left(1 + \frac{|\mathbf{g}_g^H \mathbf{w}_g|^2}{|\mathbf{g}_g^H \mathbf{w}_b|^2 + \|\mathbf{g}_g^H \mathbf{W}_s\|^2 + \|\mathbf{h}_{rg}^H \Phi\|^2 \sigma_r^2 + \sigma_g^2} \right). \quad (7)$$

The monitoring of Willie involves two hypotheses: \mathcal{H}_0 indicates the absence of the covert transmission from Alice to Bob, and \mathcal{H}_1 indicates the occurrence of the covert transmission from Alice to Bob. Under \mathcal{H}_0 and \mathcal{H}_1 , the received signals at Willie can be respectively expressed as:

$$\mathcal{H}_0 : y_w = \mathbf{g}_w^H (\mathbf{w}_g s_g + \mathbf{W}_s \mathbf{s}_s) + \mathbf{h}_{rw}^H \Phi \mathbf{z}_r + z_w \quad (8)$$

and

$$\mathcal{H}_1 : y_w = \mathbf{g}_w^H \mathbf{x} + \mathbf{h}_{rw}^H \Phi \mathbf{z}_r + z_w. \quad (9)$$

B. Coverttness Requirement

Assuming Willie uses the Neyman-Pearson criterion to detect the covert transmission, the optimal decision rule for minimizing the detection error probability (DEP) is the likelihood ratio test, i.e.,

$$\frac{p_1(y_w)}{p_0(y_w)} \underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\geq}} 1, \quad (10)$$

where \mathcal{D}_0 and \mathcal{D}_1 denote Willie's binary decisions endorsing \mathcal{H}_0 and \mathcal{H}_1 , respectively, $p_0(y_w) = \frac{1}{\pi \sigma_0^2} e^{-|y_w|^2 / \sigma_0^2}$ and $p_1(y_w) = \frac{1}{\pi \sigma_1^2} e^{-|y_w|^2 / \sigma_1^2}$ denote the likelihood functions for Willie's received signals under \mathcal{H}_0 and \mathcal{H}_1 , respectively, with $\sigma_0^2 = |\mathbf{g}_w^H \mathbf{w}_g|^2 + \|\mathbf{g}_w^H \mathbf{W}_s\|^2 + \|\mathbf{h}_{rw}^H \Phi\|^2 \sigma_r^2 + \sigma_w^2$ and $\sigma_1^2 = \|\mathbf{g}_w^H \mathbf{W}\|^2 + \|\mathbf{h}_{rw}^H \Phi\|^2 \sigma_r^2 + \sigma_w^2$, respectively. With respect to Willie's received signal power, the optimal decision rule for Willie can be rewritten as:

$$P_w \underset{\mathcal{D}_0}{\overset{\mathcal{D}_1}{\geq}} \phi^*, \quad (11)$$

where $P_w \stackrel{L \rightarrow \infty}{=} \frac{1}{L} \sum_{l=1}^L |y_w(l)|^2$ with l denoting the index of the transmission block and $\phi^* = \frac{\sigma_0^2 \sigma_1^2}{\sigma_0^2 - \sigma_1^2} \ln \frac{\sigma_1^2}{\sigma_0^2} > 0$ denotes the chosen optimal detection threshold. With the optimal detection rule, we can derive the minimum DEP achieved by Willie under the two hypotheses as [15]:

$$\xi^* = \Pr(\mathcal{D}_1 | \mathcal{H}_0) + \Pr(\mathcal{D}_1 | \mathcal{H}_1)$$

$$= 1 + \left(\frac{\sigma_1^2}{\sigma_0^2} \right)^{-\frac{\sigma_1^2}{\sigma_1^2 - \sigma_0^2}} - \left(\frac{\sigma_1^2}{\sigma_0^2} \right)^{-\frac{\sigma_0^2}{\sigma_1^2 - \sigma_0^2}}, \quad (12)$$

where $\Pr(\mathcal{D}_1 | \mathcal{H}_0)$ and $\Pr(\mathcal{D}_0 | \mathcal{H}_1)$ denote the probabilities of false alarm and missed detection, respectively. To facilitate the design of covert communications, we introduce a lower bound on ξ^* [15]:

$$\xi^* \geq 1 - \sqrt{\frac{1}{2} \mathcal{D}(p_0(y_w) || p_1(y_w))}, \quad (13)$$

where $\mathcal{D}(p_0(y_w) || p_1(y_w))$ refers to the Kullback-Leibler (KL) divergence, which can be calculated as follows:

$$\mathcal{D}(p_0(y_w) || p_1(y_w)) = \ln \left(\frac{\sigma_1^2}{\sigma_0^2} \right) + \frac{\sigma_0^2}{\sigma_1^2} - 1. \quad (14)$$

To ensure coverttness, the minimum DEP of Willie should satisfy $\xi^* \geq 1 - \varepsilon$, where $\varepsilon > 0$ is the desired level of coverttness. In this work, we set a tighter constraint $\mathcal{D}(p_0(y_w) || p_1(y_w)) \leq 2\varepsilon^2$ to ensure coverttness, taking into account the lower bound on ξ^* in (13). According to (14), by applying the monotonicity of the function $f(\lambda) = \ln \lambda + \frac{1}{\lambda} - 1$ in the interval $[1, \infty)$, the coverttness constraint can be reformulated as follows:

$$\begin{aligned} |\mathbf{g}_w^H \mathbf{w}_b|^2 + (1 - \kappa) (|\mathbf{g}_w^H \mathbf{w}_g|^2 + \|\mathbf{g}_w^H \mathbf{W}_s\|^2 + \|\mathbf{h}_{rw}^H \Phi\|^2 \sigma_r^2) \\ \leq (\kappa - 1) \sigma_w^2, \end{aligned} \quad (15)$$

where κ is the unique root of $f(\lambda) = 2\varepsilon^2$ in the interval $[1, \infty)$.

C. Sensing Requirement

In the NOMA-ISAC system, the transmitted NOMA signal can be exploited for target sensing [4]. In this work, we consider the beampattern similarity as the metric for evaluating the sensing performance by comparing the designed beampattern with the desired one [3]. Let $\mathbf{a}(\theta) = [1, e^{j2\pi\delta \sin(\theta)}, \dots, e^{j2\pi(M-1)\delta \sin(\theta)}]^T$ denote the transmit/receive steering vector at the direction θ with δ being the normalized antenna spacing. We define the transmit beampattern $P(\theta)$ as the transmit signal power distribution at the direction angle, which is given by [3]

$$P(\theta) = \mathbb{E}\{|\mathbf{a}^H(\theta) \mathbf{W} \mathbf{x}|^2\} = \mathbf{a}^H(\theta) \mathbf{W} \mathbf{W}^H \mathbf{a}(\theta). \quad (16)$$

The mean squared error (MSE) of the beampattern matching is used to evaluate the beampattern similarity, which measures the difference between the actual transmission beampattern and the desired beampattern, i.e.,

$$\mathcal{E}(\alpha, \mathbf{W}) \triangleq \frac{1}{V} \sum_{v=1}^V |\alpha \hat{P}(\theta_v) - \mathbf{a}^H(\theta_v) \mathbf{W} \mathbf{W}^H \mathbf{a}(\theta_v)|^2. \quad (17)$$

In (17), θ_v denotes the v th sampled angle over $[-\frac{\pi}{2}, \frac{\pi}{2}]$, α is a scaling factor and $\hat{P}(\theta_v)$ is the desired beampattern defined as a square waveform at the target direction $\hat{\theta}_q$, i.e.,

$$\hat{P}(\theta_v) = \begin{cases} 1, & \exists q \in \mathcal{Q}, |\theta_v - \hat{\theta}_q| < \frac{\Delta\theta}{2}, \\ 0, & \text{otherwise,} \end{cases} \quad (18)$$

where $\hat{\theta}_q$ denotes the direction of the q th target and $\Delta\theta$ denotes the width of desired beampattern at each estimation angle.

III. COVERT BEAMFORMING DESIGN

The aim of the covert beamforming design is to maximize the covert rate R_{b,s_b} by jointly optimizing the transmission beamforming and reflection beamforming, subject to the power budget constraints of Alice and active RIS, Grace's QoS requirement, sensing beampattern similarity requirement, and pre-defined covertness level. Considering the additional constraints imposed by the NOMA-ISAC system, the covert rate maximization problem is formulated as follows:

$$(P1) : \max_{\alpha, \mathbf{W}, \Phi} R_{b,s_b} \quad (19a)$$

$$\text{s.t. } \|\mathbf{W}\|_F^2 \leq P_a^{\max}, \quad (19b)$$

$$R_{g,s_g} \leq R_{b,s_b}, \quad \|\mathbf{w}_g\|^2 \geq \|\mathbf{w}_b\|^2, \quad (19c)$$

$$R_{g,s_g} \geq R_g^{\min}, \quad (19d)$$

$$\frac{1}{V} \sum_{v=1}^V |\alpha \hat{P}(\theta_v) - \mathbf{a}^H(\theta_v) \mathbf{W} \mathbf{W}^H \mathbf{a}(\theta_v)|^2 \leq \epsilon, \quad (19e)$$

$$|\Phi_{n,n}| \leq \eta_n, \quad (3), \quad (15). \quad (19f)$$

In problem (P1), (19b) is Alice's maximum transmit power constraint, (19c) guarantees successful SIC at Bob, (19d) ensures that Grace achieves the minimum target rate R_g^{\min} to meet its QoS requirements, (19e) guarantees the desired beampattern similarity, and constraints in (19f) denote the limitations of the maximum reflecting amplitude of n th element of Φ , reflection power budget at the active RIS, and covertness level, respectively. However, due to the coupling between the optimization variables \mathbf{W} and Φ in problem (P1), the joint optimization of \mathbf{W} and Φ is infeasible. To address this challenge, we propose to decouple problem (P1) into two sub-problems of optimizing the transmission beamforming and reflection beamforming, respectively. Considering that maximizing R_{b,s_b} is equivalent to maximizing γ_{b,s_b} , we drop the $\log(\cdot)$ function and propose a penalized Dinkelbach approach to recast the resulting non-convex fractional programming (FP) sub-problems with rank-one constraints to a convex form. Furthermore, we design an AO algorithm to maximize the covert rate. The details of our proposed scheme are presented in the following subsections.

A. Transmission Beamforming Optimization

For any given feasible Φ , problem (P1) can be reduced to optimize \mathbf{W} only. To facilitate solving the covert rate maximization problem, we introduce the auxiliary variables $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$, $i \in \{g, b\}$, $\tilde{\mathbf{W}}_s = \mathbf{W}_s \mathbf{W}_s^H$, $\Upsilon_k = \mathbf{g}_k \mathbf{g}_k^H$, $\mathbf{H}_{rk} = \mathbf{h}_{rk} \mathbf{h}_{rk}^H$, $\Psi = \Phi \Phi^H$ and $\Gamma = \bar{\Gamma}^H \bar{\Gamma}$ with $\bar{\Gamma} = \Phi \mathbf{G}$. One can obtain that $|\mathbf{g}_k^H \mathbf{w}_i|^2 = \text{tr}(\Upsilon_k \mathbf{W}_i)$, $\|\mathbf{g}_k^H \mathbf{W}_s\|^2 = \text{tr}(\Upsilon_k \tilde{\mathbf{W}}_s)$, $\|\mathbf{h}_{rk}^H \Phi\|^2 = \text{tr}(\mathbf{H}_{rk} \Psi)$, and $\|\Phi \mathbf{G} \mathbf{W}\|_F^2 + \|\Phi\|_F^2 \sigma_r^2 = \text{tr}(\Gamma \tilde{\mathbf{W}}) + \text{tr}(\Psi) \sigma_r^2$ with $\tilde{\mathbf{W}} = \mathbf{W}_g + \mathbf{W}_b + \tilde{\mathbf{W}}_s$. After dropping the $\log(\cdot)$ function, the optimization problem with respect to \mathbf{W} can be formulated as:

$$(P2) : \max_{\alpha, \mathbf{W}_g, \mathbf{W}_b, \tilde{\mathbf{W}}_s} \frac{\text{tr}(\Upsilon_b \mathbf{W}_b)}{\text{tr}(\Upsilon_b \tilde{\mathbf{W}}_s) + \text{tr}(\mathbf{H}_{rb} \Psi) \sigma_r^2 + \sigma_b^2} \quad (20a)$$

$$\text{s.t. } \text{tr}(\tilde{\mathbf{W}}) \leq P_a^{\max}, \quad \text{tr}(\mathbf{W}_g) \geq \text{tr}(\mathbf{W}_b), \quad (20b)$$

$$\gamma_{\text{th}} \left(\text{tr}(\Upsilon_g (\mathbf{W}_b + \tilde{\mathbf{W}}_s)) + \text{tr}(\mathbf{H}_{rg} \Psi) \sigma_r^2 + \sigma_g^2 \right) \leq \text{tr}(\Upsilon_g \mathbf{W}_g), \quad (20c)$$

$$\frac{1}{V} \sum_{v=1}^V |\alpha \hat{P}(\theta_v) - \mathbf{a}^H(\theta_v) \tilde{\mathbf{W}} \mathbf{a}(\theta_v)|^2 \leq \epsilon, \quad (20d)$$

$$\text{tr}(\Gamma \tilde{\mathbf{W}}) + \text{tr}(\Psi) \sigma_r^2 \leq P_r^{\max}, \quad (20e)$$

$$\text{tr}(\Upsilon_w \mathbf{W}_b) + (1 - \kappa) \left(\text{tr}(\Upsilon_w (\mathbf{W}_g + \tilde{\mathbf{W}}_s)) + \text{tr}(\mathbf{H}_{rw} \Psi) \sigma_r^2 \right) \leq (\kappa - 1) \sigma_w^2, \quad (20f)$$

$$\mathbf{W}_g \succeq 0, \quad \mathbf{W}_b \succeq 0, \quad \tilde{\mathbf{W}}_s \succeq 0, \quad (20g)$$

$$\text{rank}(\mathbf{W}_g) = 1, \quad \text{rank}(\mathbf{W}_b) = 1, \quad (20h)$$

In problem (P2), constraint (20c) represents the QoS requirements of Grace with $\gamma_{\text{th}} = 2^{R_g^{\min}} - 1$. To address the rank-one constraint (20h), we introduce a penalty term $\frac{1}{\iota_1} \sum_{i \in \{g, b\}} (\|\mathbf{W}_i\|_* + \widehat{\mathbf{W}}_i^{(t)})$ in the denominator of (20a), where ι_1 is a penalty factor, $\|\cdot\|_*$ denotes the nuclear norm, and $\widehat{\mathbf{W}}_i^{(t)}$ is the convex upper bound of the non-convex term $-\|\mathbf{W}_i\|_2$ with $\|\cdot\|_2$ standing for the spectral norm, i.e., $-\|\mathbf{W}_i\|_2 \leq \widehat{\mathbf{W}}_i^{(t)}$. Here, if $\iota_1 \rightarrow 0$, the exactly rank-one metrics can be guaranteed by maximizing the fractional objective function due to the fact $\text{rank}(\mathbf{W}_i) = 1$ is equivalent to $\|\mathbf{W}_i\|_* - \|\mathbf{W}_i\|_2 = 0$. One can obtain the convex upper bound $\widehat{\mathbf{W}}_i^{(t)}$ by leveraging the first-order Taylor expansion at the point $\mathbf{W}_i^{(t)}$, which is expressed as $\widehat{\mathbf{W}}_i^{(t)} \triangleq -\|\mathbf{W}_i^{(t)}\|_2 - \text{tr}[\mathbf{q}_{\max, i}^{(t)} (\mathbf{q}_{\max, i}^{(t)})^H (\mathbf{W}_i - \mathbf{W}_i^{(t)})]$, where $\mathbf{W}_i^{(t)}$ is the solution obtained in the t th iteration and $\mathbf{q}_{\max, i}^{(t)}$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{W}_i^{(t)}$. With the added penalty term, the objective function of problem (P2) is expressed as:

$$\frac{\text{tr}(\Upsilon_b \mathbf{W}_b)}{\text{tr}(\Upsilon_b \tilde{\mathbf{W}}_s) + \text{tr}(\mathbf{H}_{rb} \Psi) \sigma_r^2 + \sigma_b^2 + \frac{1}{\iota_1} \sum_{i \in \{g, b\}} (\|\mathbf{W}_i\|_* + \widehat{\mathbf{W}}_i^{(k)})}. \quad (21)$$

Since the above single-ratio concave-convex fractional objective function still hinders a direct solution, we further utilize the Dinkelbach transformation [16] to convert it into a concave form. In particular, by defining $f_1(\mathbf{W}_b) = \text{tr}(\Upsilon_b \mathbf{W}_b)$, and $f_2(\tilde{\mathbf{W}}_s, \mathbf{W}_i) = \text{tr}(\Upsilon_b \tilde{\mathbf{W}}_s) + \text{tr}(\mathbf{H}_{rb} \Psi) \sigma_r^2 + \sigma_b^2 + \frac{1}{\iota_1} \sum_{i \in \{g, b\}} (\|\mathbf{W}_i\|_* + \widehat{\mathbf{W}}_i^{(k)})$, we obtain the following transformed optimization problem:

$$(P2.1) : \max_{\alpha, \mathbf{W}_g, \mathbf{W}_b, \tilde{\mathbf{W}}_s} f_1(\mathbf{W}_b) - u_1 f_2(\tilde{\mathbf{W}}_s, \mathbf{W}_i) \quad (22a)$$

$$\text{s.t. } (20b) - (20g), \quad (22b)$$

where the auxiliary variable u_1 achieves the optimal value when $u_1^{(\ell+1)} = f_1^{(\ell)}(\mathbf{W}_b) / f_2^{(\ell)}(\tilde{\mathbf{W}}_s, \mathbf{W}_i)$ in the ℓ th iteration. It is evident that problem (P2.1) is a convex optimization problem and can be readily solved using existing convex optimization solvers such as CVX.

B. Reflection Beamforming Optimization

For any given \mathbf{W} , problem (P1) can be reduced to optimize Φ only. To handle the non-convexity of the objective function and constraints (19c), (19d), and (19f) with respect to Φ and formulate a tractable covert rate maximization problem, we first introduce $\bar{\mathbf{u}} = [\phi_1, \dots, \phi_N]$ with ϕ_n , $\forall n \in \{1, \dots, N\}$, being the n -th diagonal element in Φ and $\bar{\mathbf{\Lambda}}_{k,j} = [\text{diag}(\mathbf{h}_{rk}^H \mathbf{G} \mathbf{w}_j; \mathbf{h}_{ak}^H \mathbf{w}_j)]$ with \mathbf{w}_j being the j th, $j \in \{1, \dots, 2 + M\}$, column of the beamforming matrix \mathbf{W} and construct $\mathbf{\Lambda}_{k,j} = \bar{\mathbf{\Lambda}}_{k,j} \bar{\mathbf{\Lambda}}_{k,j}^H$ and $\mathbf{U} = \mathbf{u} \mathbf{u}^H$ with $\mathbf{u} = [\bar{\mathbf{u}}, 1]^H$, which results in the expression $|\mathbf{g}_k^H \mathbf{w}_j|^2 = \text{tr}(\mathbf{\Lambda}_{k,j} \mathbf{U})$. Then, by applying the change of variables $\mathbf{\Omega}_k = \text{diag}(|\mathbf{h}_{rk}|^2, \dots, |\mathbf{h}_{rk}|^2, 0)$, $\mathbf{\Pi} = \text{diag}([1_{1 \times N}, 0]^T)$, and $\mathbf{S}_j = \text{diag}(|\mathbf{s}_j|_1^2, \dots, |\mathbf{s}_j|_N^2, 0)$ with $\mathbf{s}_j = \mathbf{G} \mathbf{w}_j$, we obtain that $\|\Phi \mathbf{G} \mathbf{W}\|_F^2 = \sum_{j=1}^{2+M} \text{tr}(\mathbf{S}_j \mathbf{U})$, $\|\mathbf{h}_{rk}^H \Phi\|^2 = \text{tr}(\mathbf{\Omega}_k \mathbf{U})$, and $\|\Phi\|_F^2 = \text{tr}(\mathbf{\Pi} \mathbf{U})$. After taking the ratio term γ_{b,s_b} out of the $\log(\cdot)$ function, we can formulate the reflection beamforming optimization problem as:

$$(P3) : \max_{\mathbf{U}} \frac{\text{tr}(\mathbf{\Lambda}_{b,2} \mathbf{U})}{\sum_{j=3}^{2+M} \text{tr}(\mathbf{\Lambda}_{b,j} \mathbf{U}) + \text{tr}(\mathbf{\Omega}_b \mathbf{U}) \sigma_r^2 + \sigma_b^2} \quad (23a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{\Lambda}_{g,1} \mathbf{U}) \leq \text{tr}(\mathbf{\Lambda}_{b,1} \mathbf{U}), \quad (23b)$$

$$\text{tr}(\mathbf{\Lambda}_{g,1} \mathbf{U}) \geq \gamma_{th} \left(\sum_{j=2}^{2+M} \text{tr}(\mathbf{\Lambda}_{g,j} \mathbf{U}) + \text{tr}(\mathbf{\Omega}_g \mathbf{U}) \sigma_r^2 + \sigma_g^2 \right), \quad (23c)$$

$$\sum_{j=1}^{2+M} \text{tr}(\mathbf{S}_j \mathbf{U}) + \text{tr}(\mathbf{\Pi} \mathbf{U}) \sigma_r^2 \leq P_r^{\max}, \quad (23d)$$

$$\begin{aligned} \text{tr}(\mathbf{\Lambda}_{w,2} \mathbf{U}) + (1 - \kappa) \left(\sum_{j=1, j \neq 2}^{2+M} \text{tr}(\mathbf{\Lambda}_{w,j} \mathbf{U}) + \text{tr}(\mathbf{\Omega}_w \mathbf{U}) \sigma_r^2 \right) \\ \leq (\kappa - 1) \sigma_w^2, \end{aligned} \quad (23e)$$

$$|\mathbf{U}_{n,n}| \leq \eta_n^2, \quad \forall n, \quad |\mathbf{U}_{N+1, N+1}| = 1, \quad (23f)$$

$$\text{rank}(\mathbf{U}) = 1, \quad \mathbf{U} \succeq \mathbf{0}. \quad (23g)$$

Similar to the approach adopted in (21), we add a penalty-term $\frac{1}{\iota_2} (\|\mathbf{U}\|_* + \hat{\mathbf{U}}^{(t)})$ to the objection function's denominator to address the non-convexity of rank-1 constraint generated by variable changes in (23g). Here, ι_2 is a control factor, and $\hat{\mathbf{U}}^{(t)}$ is the upper bound on $-\|\mathbf{U}\|_2$ with closed form $\hat{\mathbf{U}}^{(t)} = -\|\mathbf{U}\|_2^{(t)} - \text{tr}(\mathbf{u}_{\max}^{(t)} (\mathbf{u}_{\max}^{(t)})^H (\mathbf{U} - \mathbf{U}^{(t)}))$, where $\mathbf{u}_{\max}^{(t)}$ represents the eigenvector corresponding to the largest eigenvalue of $\mathbf{U}^{(t)}$ in the t th solution. As a result, the new objective function can be expressed as:

$$\frac{\text{tr}(\mathbf{\Lambda}_{b,2} \mathbf{U})}{\sum_{j=3}^{2+M} \text{tr}(\mathbf{\Lambda}_{b,j} \mathbf{U}) + \text{tr}(\mathbf{\Omega}_b \mathbf{U}) \sigma_r^2 + \sigma_b^2 + \frac{1}{\iota_2} (\|\mathbf{U}\|_* + \hat{\mathbf{U}}^{(t)})}. \quad (24)$$

By maximizing this fractional objective function, we ensure that $\text{rank}(\mathbf{U}) = 1$ as $\iota_2 \rightarrow 0$, as $\|\mathbf{U}\|_* - \|\mathbf{U}\|_2 = 0$ is equivalent to $\text{rank}(\mathbf{U}) = 1$. Since the concave-convex single-ratio objective in (24) still hinders a direct solution, we apply the Dinkelbach transformation to convert it into a concave form. Specifically, let $f_3(\mathbf{U}) = \text{tr}(\mathbf{\Lambda}_{b,2} \mathbf{U})$ and $f_4(\mathbf{U}) =$

Algorithm 1 AO Algorithm for Optimizing \mathbf{W} and \mathbf{U}

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1: Initialize  $s \leftarrow 0$ ,  $\mathbf{U}^{(0)}$  and  $\mathbf{W}^{(0)}$ 
2: repeat  $s \leftarrow s + 1$ 
3:    $t \leftarrow 0$ 
4:   repeat  $t \leftarrow t + 1$ 
5:     Initialize  $u_1^{(0)} > 0$ ,  $\ell \leftarrow 0$ 
6:     repeat  $\ell \leftarrow \ell + 1$ 
7:       Update  $\mathbf{W}^{(\ell)}$  by solving (P2.1)
8:       Update  $u_1^{(\ell+1)} = f_1^{(\ell)}(\mathbf{W}_b)/f_2^{(\ell)}(\bar{\mathbf{W}}_s, \mathbf{W}_i)$ 
9:       until  $f_1^{(\ell)}(\mathbf{W}_b) - u_1^{(\ell)} f_2^{(\ell)}(\bar{\mathbf{W}}_s, \mathbf{W}_i) \leq 0$ 
10:       $\mathbf{W}^{(t)} \leftarrow \mathbf{W}^{(\ell)}$ ,  $t_1 \leftarrow c_1 t_1$ 
11:      until  $\frac{1}{c_1} \sum_{i \in \{g, b\}} (\|\mathbf{W}_i\|_* + \bar{\mathbf{W}}_i^{(t)}) \leq \xi_3$ 
12:       $\mathbf{W}^{(s)} \leftarrow \mathbf{W}^{(t)}$ 
13:       $t \leftarrow 0$ 
14:      repeat  $t \leftarrow t + 1$ 
15:        Initialize  $u_2^{(0)} > 0$ ,  $\ell \leftarrow 0$ 
16:        repeat  $\ell \leftarrow \ell + 1$ 
17:          Update  $\mathbf{U}^{(\ell)}$  by solving (P3.1)
18:          Update  $u_2^{(\ell+1)} = f_3^{(\ell)}(\mathbf{U})/f_4^{(\ell)}(\mathbf{U})$ 
19:          until  $f_3^{(\ell)}(\mathbf{U}) - u_2^{(\ell)} f_4^{(\ell)}(\mathbf{U}) \leq 0$ 
20:           $\mathbf{U}^{(t)} \leftarrow \mathbf{U}^{(\ell)}$ ,  $\iota_2 \leftarrow c_2 \iota_2$ 
21:          until  $\frac{1}{c_2} (\|\mathbf{U}\|_* + \hat{\mathbf{U}}^{(t)}) \leq \xi_2$ 
22:           $\mathbf{U}^{(s)} \leftarrow \mathbf{U}^{(t)}$ 
23:        until  $R_{b,s_b}^{(s)} - R_{b,s_b}^{(s-1)} \leq \xi_1$ 

```

$\sum_{j=3}^{2+M} \text{tr}(\mathbf{\Lambda}_{b,j} \mathbf{U}) + \text{tr}(\mathbf{\Omega}_b \mathbf{U}) \sigma_r^2 + \sigma_b^2 + \frac{1}{\iota_2} (\|\mathbf{U}\|_* + \hat{\mathbf{U}}^{(t)})$, the transformed optimization problem can be expressed as:

$$(P3.1) : \max_{\mathbf{U}} f_3(\mathbf{U}) - u_2 f_4(\mathbf{U}) \quad (25a)$$

$$\text{s.t.} \quad (23b), (23c), (23d), (23e), (23f), \quad \mathbf{U} \succeq \mathbf{0}, \quad (25b)$$

where the auxiliary variable u_2 is updated by $u_2^{(\ell+1)} = f_3^{(\ell)}(\mathbf{U})/f_4^{(\ell)}(\mathbf{U})$ in the ℓ th iteration. Now, problem (P3.1) is convex, \mathbf{U} can be optimized via existed CVX solver.

The proposed AO algorithm is summarized in Algorithm 1. To prevent the optimization falling into local optima caused by the zero penalty-term during initialization, we set $u^{(0)} > 0$ in the proposed penalized Dinkelbach approach. The overall computational complexity of Algorithm 1 is characterized by $\mathcal{O}[I_A(I_{P_1} I_D(3M^{3.5}) + I_{P_2} I_D(N+1)^{3.5})]$, where I_A and I_D denote the iteration numbers of the AO loop and Dinkelbach loop, respectively, and I_{P_1} and I_{P_2} denote the iteration numbers of the penalty-terms in problems (P2.1) and (P3.1), respectively.

IV. NUMERICAL RESULTS

Numerical results are provided in this section to validate the system performance of the active RIS-assisted NOMA-ISAC system. In the simulations, we set $M = 8$, $N = 16$, $\sigma_b^2 = \sigma_g^2 = \sigma_w^2 = \sigma_r^2 = -90$ dBm, $R_g^{\min} = 1$ bps/Hz, $\eta_n^2 = \eta^2$, $\forall n$, $c_1 = c_2 = \xi_1 = 10^{-2}$, $\xi_2 = \xi_2 = 10^{-4}$, $\Delta\theta = 10^\circ$, and $Q = 3$ with the targets located in $\hat{\theta}_1 = -35^\circ$, $\hat{\theta}_2 = 0^\circ$, and $\hat{\theta}_3 = 35^\circ$, respectively. Alice, Bob, Grace, Willie, and active RIS are located at (0, 0) m, (80, 10) m, (90, 0) m, (100, 5) m, and (80, 30) m, respectively, in a two-dimensional coordinate space. The path-loss is modeled by $\mathcal{L} = \mathcal{L}_0 d^{-\chi}$ with $\mathcal{L}_0 = -30$ dB and d denoting the distance between the two terminals. We set $\chi = 3.5$ for the direct links and $\chi = 2.2$ for the channels associated with the active RIS [12].

In Fig. 2, we plot an example of the optimal transmit beampattern $P(\theta)$ achieved by the proposed algorithm versus the azimuth angle over $[-90^\circ, 90^\circ]$ for a single Monte Carlo realization. The passive RIS scheme is also presented as

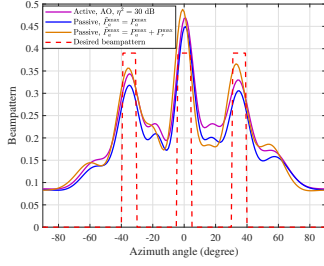


Fig. 2. Transmit beampattern ($P_a^{\max} = 30$ dBm, $P_r^{\max} = 30$ dBm, $\varepsilon = 0.1$)

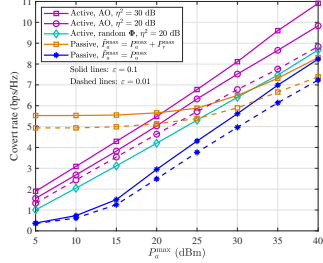


Fig. 3. Covert rate versus transmit power ($P_r^{\max} = 30$ dBm)

the benchmark. We consider two scenarios for the passive RIS scheme where Alice has a power budget of either $\tilde{P}_a^{\max} = P_a^{\max}$ or $\tilde{P}_a^{\max} = P_a^{\max} + P_r^{\max}$ for comparison. The curves of Fig. 2 show that the passive RIS scheme with $\tilde{P}_a^{\max} = P_a^{\max} + P_r^{\max}$ achieves a higher beampattern compared with the active RIS scheme, which is quite predicted owing to the availability of the total transmit power budget at Alice. However, the active RIS scheme has a more preferred shape than the passive RIS scheme when the transmit power budget is the same. It is verified that deploying the active RIS to assist communications in the NOMA-ISAC system can save more transmit power of Alice for target sensing, thereby obtaining better sensing performance than the passive RIS scheme.

In Fig. 3, we investigate the impact of the maximum transmit power budget on the covert rate in different schemes when $P_r^{\max} = 30$ dBm. In addition to the passive RIS scheme, we also compare the covert rate achieved by the proposed AO algorithm to the random Φ scheme of the active RIS. The curves in Fig. 3 reveal that the passive RIS scheme with $\tilde{P}_a^{\max} = P_a^{\max} + P_r^{\max}$ achieves a higher covert rate than the active RIS scheme in the low transmit power region, while the active RIS scheme performs better with the increasing of P_a^{\max} . The active RIS scheme always outperforms the passive RIS scheme with $\tilde{P}_a^{\max} = P_a^{\max}$ even through applying a random reflection beamforming. Additionally, the curves in Fig. 3 also confirm that a higher η^2 leads to a higher covert rate when P_r^{\max} remains constant. The proposed active RIS-assisted NOMA-ISAC system can achieve a considerable covert rate in the high transmit power region mainly thanks to the additional covert rate gain of the active RIS and the shield of the NOMA public user's signal and sensing signal.

V. CONCLUSIONS

In this paper, we have designed the AO-optimized covert beamforming to maximize the covert rate for the active RIS-

assisted NOMA-ISAC system. The penalized Dinkelbach approach has been proposed to achieve the high-quality rank-one solutions for the decoupled sub-problems of the transmission and reflection beamforming with non-convex fractional characteristics. The numerical results have verified the superiority of the proposed AO algorithm. The significant advantages of deploying active RIS in the NOMA-ISAC system have also been demonstrated in terms of the covert rate and sensing beampattern similarity.

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