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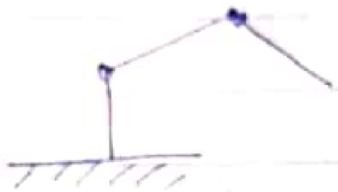
JASH SHAH
2018A8PS0507P

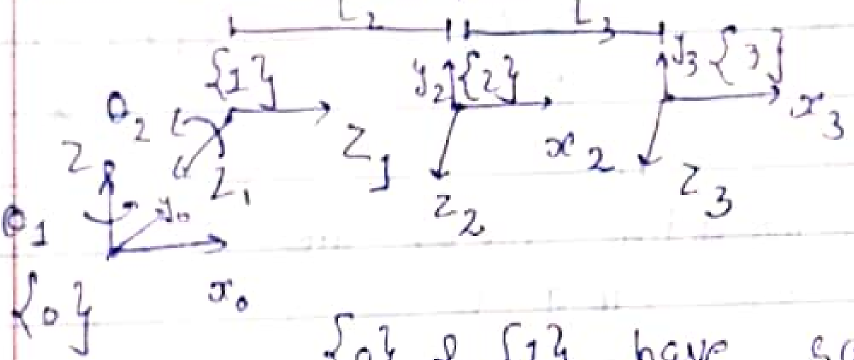
Robotics - Assignment 2

A1



Each link has its CoM at centre.

DH Frame Assignment



{0} & {1} have same origin

Link i	a_i	α_i	d_i	θ_i	q_i	$\cos\alpha_i$	$\sin\alpha_i$
1	0	90°	0	θ_1	θ_1	0	1
2	L_2	0	0	θ_2	θ_2	1	0
3	L_3	0	0	θ_3	θ_3	1	0

$$\rightarrow\ {}^{0}T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{2}T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & L_3c_3 \\ s_3 & c_3 & 0 & L_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^{0}T_2 = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 & L_2c_1c_2 \\ s_1c_2 & -s_1s_2 & -c_1 & L_2s_1c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Finally, $^{0}T_3 = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3$

$$\Rightarrow \begin{bmatrix} c_1 C(\theta_2+\theta_3) & -c_1 S(\theta_2+\theta_3) & S_1 & c_1[L_3 C_{23} + L_2 C_2] \\ s_1 C(\theta_2+\theta_3) & -S_1 S(\theta_2+\theta_3) & -C_1 & S_1[L_3 C_{23} + L_2 C_2] \\ S(\theta_2+\theta_3) & C(\theta_2+\theta_3) & 0 & L_3 S_{23} + L_2 S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ Now, for interactive forces, we have :-

$$F_{0,1} - F_{1,2} + m_1 g = 0$$
$$F_{1,2} - F_{2,3} + m_2 g = 0$$
$$F_{2,3} - F_{3,4} + m_3 g = 0$$

⊛ ⇒ Now, for simplification, we assume $L_2 = L_3 = a$ and $m_1 = m_2 = m$.

So, we have $\eta_{i-1,i} = \eta_{i,i+1} - {}^{0}R_i(-{}^{i}r_i)$

$$* (-F_{i,i+1}) - {}^{0}R_{i-1}({}^{i-1}r_i + {}^{i}r_i) * F_{i-1,i}$$

→ Now finding all "r" vectors,

$${}^{3}r_3 = {}^{2}r_2 = \begin{bmatrix} -a/2 \\ 0 \\ 0 \end{bmatrix} ; \; {}^{1}r_1 = \begin{bmatrix} 0 \\ -a/2 \\ 0 \end{bmatrix}$$

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$${}^{2}r_3 = \begin{bmatrix} aC_3 \\ aS_3 \\ 0 \end{bmatrix} \qquad {}^{0}r_1 = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}$$

$${}^{1}r_2 = \begin{bmatrix} aC_2 \\ aS_2 \\ 0 \end{bmatrix}$$

$\Rightarrow F_{3,P} = \begin{bmatrix} +1 \\ 0 \\ 0 \end{bmatrix}$ kgF $\quad l_{3,P} = \begin{bmatrix} +1 \\ 0 \\ 0 \end{bmatrix}$

All wrt $\{0\}$.

$F_{23} = F_{3P} - m_3 g \Rightarrow g = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$

So, $F_{23} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ m \end{bmatrix}$ kg force

Similarly, $F_{1,2} = F_{23} - mg = \begin{bmatrix} 1 \\ 0 \\ 2m \end{bmatrix}$ kg

& $F_{091} = F_{12} - mg = \begin{bmatrix} 1 \\ 0 \\ 3m \end{bmatrix}$ kg.

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(*) Now, taking home position simplification,

$\theta_i = 0 \quad \forall \ i$

So, η_{34} becomes $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Also, $\eta_{23} = \eta_{34} - {}^{0}R_3 \left[-{}^{3}r_3 * -f_{34} \right] - {}^{0}R_2 \left({}^{2}r_3 + {}^{3}r_3 \right) f_{23}$

$$\Rightarrow \eta_{23} = \begin{bmatrix} 1 \\ 0 \\ am/2 \end{bmatrix} \underline{\text{kg-cm}} \quad \text{Ans}$$

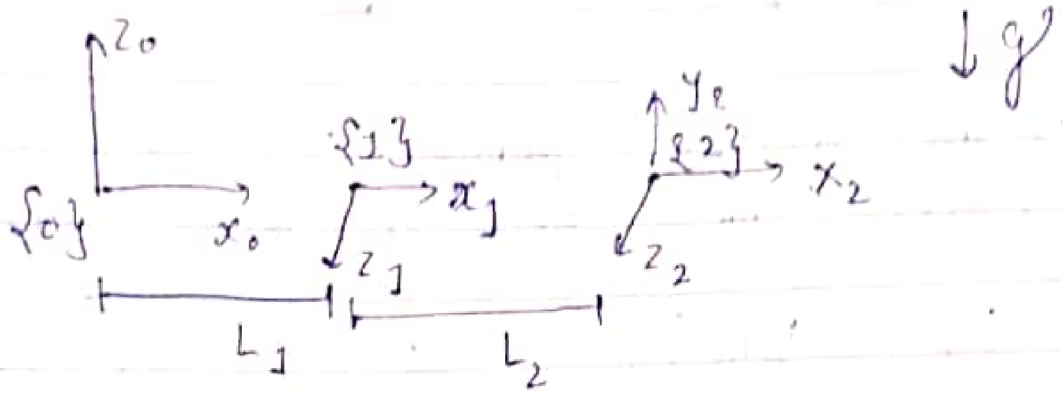
Similarly $\eta_{12} = \begin{bmatrix} 1 \\ 0 \\ am \end{bmatrix}$ kg-cm. Ans

& $\eta_{01} = \begin{bmatrix} \frac{5am}{2} + 1 \\ -a/2 \\ am - a/2 \end{bmatrix}$ kg-cm Ans

* ¶ The general form of $\underline{\eta_{01}}$ comes as :-

$$\begin{bmatrix} 1 + L_2 m_3 C_2 S_1/2 \ - L_3 m_3 S_1 (2C_{23} - C_2) + \frac{L_1 m_{23}}{2} - \frac{L_2 S_1 \sigma_2 m_2}{2} \\ L_1/2 - \frac{L_3 S_{23}}{2} - \frac{L_2 S_2}{2} + \frac{L_3 S_2}{2} + \frac{L_2 C_1 \sigma_2 m_{23}}{2} + \frac{L_3 S_2 C_1 \sigma 1}{2} \\ -\frac{L_1}{2} - \frac{S_1}{2}\left[L_2 - L_3 C_{23}\right] - \frac{S_1 L f_2}{2} + \frac{L_3 C_2 S_1}{2} \end{bmatrix}$$

A2. i) Lagrange Euler Approach



D-H params

i	a_i	α_i	d_i	θ_i	q_i	$C\theta_i$	$S\theta_i$
1	L_1	90°	0	θ_1	θ_1	C_1	S_1
2	L_2	0	0	θ_2	θ_2	C_2	S_2

$$\Rightarrow {}^0T_1 = \begin{bmatrix} C_1 & 0 & S_1 & C_1 \\ S_1 & 0 & -C_1 & S_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} C_1S_2 & -C_1S_2 & S_1 & L_2C_1C_2 + L_1C_1 \\ S_1C_2 & -S_1S_2 & -C_1 & L_2S_1C_2 + L_1S_1 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_1 = Q_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ Since, $\{1\}$ & $\{2\}$ are R.

→ The Inertial frames I_1 & I_2 for 2 slender links with COM at centre are:-

$$I_1 = \begin{bmatrix} \frac{1}{3} m_1 L_1^2 & 0 & 0 & -\frac{1}{2} m_1 L_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} m_1 L_1 & 0 & 0 & m_1 \end{bmatrix}$$

$I_{xx} = 0$
$I_{xy} = I_{xz} = I_{yz} = 0$
$I_{yy} = m_1 L_1^2/3$
$I_{zz} = m_1 L_1^2/3$

$$I_2 = \begin{bmatrix} \frac{1}{3} m_2 L_2^2 & 0 & 0 & -\frac{1}{2} m_2 L_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} m_2 L_2 & 0 & 0 & m_2 \end{bmatrix}$$

$I_{xx} = 0$
$I_{xy} = I_{yz} = I_{xz} = 0$
$I_{yy} = m_2 L_2^2/3$
$I_{zz} = m_2 L_2^2/3$

→ $$d_{11} = \partial\, {}^0T_1 / \partial\theta_1 = \begin{bmatrix} -S_1 & 0 & C_1 & -L_1 S_1 \\ C_1 & 0 & S_1 & L_1 C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d_{12} = \frac{\partial\, {}^0T_1}{\partial \theta_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$d_{21} = \frac{\partial\, {}^0T_2}{\partial \theta_1} = \begin{bmatrix} -S_1 C_2 & -S_1 S_2 & C_1 & -L_2 S_1 C_2 - L_1 S_1 \\ C_1 C_2 & -S_1 S_2 & S_1 & L_2 C_1 C_2 + L_1 C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$d_{22} = \frac{\partial^0 T_2}{d\theta_2} = \begin{bmatrix} -C_1S_2 & -C_1C_2 & 0 & -L_2C_1S_2 \\ -S_1S_2 & -S_1C_2 & 0 & -L_2S_1S_2 \\ C_2 & -S_2 & 0 & L_2C_2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow M_{ij} = \sum_{P=\max(i,j)}^{n} Tr\left[d_{Pj}\, I_P\, d_{Pi}'\right]$$

$$\Rightarrow M_{11} = Tr(d_{11} I_1 d_{11}') + Tr(d_{21} I_2 d_{21}')$$
$$= m_1L_1^2/3 + m_2L_2^2 + m_2L_1L_2C_2 + 2m_2L_2^2C_2/3$$

$$\Rightarrow M_{22} = Tr(d_{22} I_2 d_{22}')$$
$$= m_2L_2^2/3$$

$$\Rightarrow M_{21} = M_{12} = Tr(d_{22} I_2 d_{21}')$$
$$= 0$$

$\Rightarrow$ The coriolis & centrifugal force coefficient can be calculated as

$$h_{ijk} = \sum_{P\max(i,j,k)}^{n} \left[\frac{\partial(d_{pk})}{\partial q_k} I_p\, d_{pi}'\right]$$

$\{$Here $n = 2\}$

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$\Rightarrow h_{111} = h_{222} = 0$

$h_{122} = 0 \quad \{ Tr \, [\partial d_{22}/\partial\theta_2 \, I_2 \, d_{21}'] \}$

$h_{211} = Tr\left[\frac{\partial d_{21}}{\partial\theta_1} I_2 d_{22}'\right] = \frac{m_2 L_2 S_2}{6}(3L_1 + 2L_2 C_2)$

$h_{221} = h_{212} = Tr\left(\frac{\partial d_{21}}{\partial\theta_i} I_2 d_{22}'\right)$

$= \frac{m_2 L_2 S_2}{6}[3L_1 + 3L_2 C_2]$

$h_{121} = h_{112} = 0$

$\Rightarrow H_i = \sum_{j=1}^{n} \sum_{k=1}^{n} h_{ijk} \dot{\theta}_j \dot{\theta}_k$

$H_1 = 0$

$H_2 = h_{211}\dot{\theta}_1^2 + h_{212}\dot{\theta}_1\dot{\theta}_2 + h_{221}\dot{\theta}_1\dot{\theta}_2 + h_{222}\dot{\theta}_2^2$

$\Rightarrow {}^1\bar{r}_1 = {}^2\bar{r}_2 = \left[\frac{-L_1}{2} \ 0 \ 0 \ 1\right]' = \left[\frac{-L_2}{2} \ 0 \ 0 \ 1\right]'$

If Gravity is in $-z_0$ direction

$g = [0 \quad 0 \quad -g \quad 0]$

$G_1 = [m_1 g d_{11} \, {}^1\bar{r}_1 + m_2 g \, d_{21} \, {}^2\bar{r}_2]$

$= 0$

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Now, Jacobian using principle of virtual work to find joint torques of payload.

$$J = \begin{bmatrix} -L_2S_1C_2 - L_1S_1 & -L_2C_1S_2 \\ L_2C_1C_2 + L_1C_1 & -L_2S_1S_2 \\ 0 & L_2C_2 \\ 0 & S_1 \\ 0 & -C_1 \\ 1 & 0 \end{bmatrix}$$

$$\& \quad F_{ext} = [0 \;\; 0 \;\; -W \;\; 0 \;\; 0 \;\; 0]'$$

$$T_{ext} = J(q)' \cdot F_{ext} = [0 \quad -L_2C_2W]'$$

$$\Rightarrow \boxed{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \frac{m_1L_1^2}{3} + m_2l_2^2 + m_2L_1L_2C_2 + \frac{2}{3}m_2L_2^2C_2 & 0 \\ 0 & m_2L_2^2/3 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{m_2gL_2C_2}{2} \end{bmatrix} + \begin{bmatrix} 0 \\ -WL_2C_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{m_2L_2S_2}{6}(3L_1 + 2L_2C_2)(\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}_2) \end{bmatrix}}$$

Ans

{ Using EOM }

ii) Newton - Euler Method.

Assuming initial conditions.

$$ {}^0v_0 = {}^0\omega_0 = {}^0\dot{\omega}_0 = 0 = [0 \quad 0 \quad -g]' $$

The transformation & inertial matrix remain same as in previous part.

- Forward iteration $i = 1$

$$ {}^1\omega_1 = {}^1R_0 \cdot [{}^0\omega_0 + \hat{z}_0\dot{\theta}_1] = [0 \quad 1 \quad 0]\,\dot{\theta}_1' $$

$$ {}^1v_1 = {}^1R_0\,{}^0v_0 + {}^1\omega_1 \times {}^1R_0\,{}^0D_1 + {}^1\omega_1 \times [{}^1\omega_1 \cdot ({}^1R_0\,{}^0D_1)] $$

$$ = [-L_1\dot{\theta}_1^2 \quad g \quad -L_1\ddot{\theta}_1]' $$

$$ {}^1\dot{v}_1 = [-L_1\dot{\theta}_1^2/2 \quad g \quad -L_1/_2\,\ddot{\theta}_1]' $$

- Iteration $i = 2$:

$$ {}^2\omega_2 = {}^2R_1 [{}^1\omega_1 + \hat{z}_0\dot{\theta}_2] = [S_2\dot{\theta}_1 \quad C_2\dot{\theta}_1 \quad \dot{\theta}_2]' $$

$$ {}^2\dot{\omega}_2 = {}^2R_1 [{}^1\dot{\omega}_1 + \hat{z}_1\ddot{\theta}_2 + {}^2\omega_1\,\hat{z}_0\dot{\theta}_2] $$

$$ = \begin{bmatrix} C_2\dot{\theta}_1\dot{\theta}_2 + S_2\ddot{\theta}_1 \\ -S_2\dot{\theta}_1\dot{\theta}_2 + C_2\ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} $$

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$$^2V_2 = {}^2R_1\,{}^1\dot{v}_1 + {}^2\omega_2\left({}^2R_0\,{}^1P_2\right) + {}^2\omega_2\left({}^2\omega_2\,{}^2R_0\,P_2\right)\Big]$$

$$= \begin{bmatrix} -L_1\dot{\theta}_1^2 C_2 - C_2^2 L_1\dot{\theta}_1^2 - L_2\dot{\theta}_2^2 + gS_2 \\ L_2\ddot{\theta}_2 + L_1\dot{\theta}_1^2 S_2 + S_2C_2L_2\dot{\theta}_1^2 + gC_2 \\ -L_1\ddot{\theta}_1 - L_2C_2\ddot{\theta}_1 + 2S_2L_2\dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$$^2\dot{V}_2 = \left[0 \quad -L_2/_2\dot{\theta}_2 \quad L_2/_2 C_2\dot{\theta}_1\right]'$$

·> <u>Backward iterations i=2</u>

Payload $w \Rightarrow {}^3F_3 = [0 \ -w \ 0]'$

$$^2F_2 = m_2 = {}^2\dot{V}_2 = m_2\begin{bmatrix} -L_1\dot{\theta}_1^2 C_2 - \frac{L_2}{2}L_2\dot{\theta}_1^2 + gS_2 \\ \frac{L_2}{2}\ddot{\theta}_2 + L_1\dot{\theta}_1^2 S_2 + \frac{L_2}{2} + gC_2 \\ -L_1\ddot{\theta}_1 - \frac{L_2C_2\ddot{\theta}_1}{2} + L_2S_2\dot{\theta}_1\dot{\theta}_2 \end{bmatrix}$$

$$^2N_2 = \begin{bmatrix} C_2\dot{\theta}_1\dot{\theta}_2 + S_2\ddot{\theta}_1 \\ S_2\dot{\theta}_1\dot{\theta}_2 \\ -C_2S_2\dot{\theta}_1^2 \end{bmatrix} \left\{ I_2\,{}^2\omega_2 + {}^2\omega_2 I_2\,{}^2\omega_2 \right\}$$

$$^2f_2 = {}^2F_2 + {}^2R_3\,{}^3F_3 \qquad {}^2R_3 = [I]_{3\times3}$$

$$= \begin{bmatrix} m_2\left(-L_1\dot{\theta}_1^2 C_2 - \frac{L_2}{2}C_2^2\dot{\theta}_1^2 - \frac{L_2}{2}\dot{\theta}_2^2 + gS_2\right) \\ -W + m_2\left(L_2/_2\,\ddot{\theta}_2 + L_1\dot{\theta}_1 S_2 + \frac{L_2}{2}C_2S_2\dot{\theta}_1^2 + gC_2\right) \\ m_2\left(-L_1\ddot{\theta}_1 - \frac{L_2C_2\ddot{\theta}_1}{2} + L_2S_2\dot{\theta}_1\dot{\theta}_2\right) \end{bmatrix}$$

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$${}^{2}n_2 = {}^{2}R_3\, {}^{2}m_3 + {}^{2}R_0\, {}^{1}D_2 \times {}^{2}R_3\, {}^{3}f_3 + \left({}^{2}R_0\, {}^{1}D_2 + {}^{2}R_0\, {}^{2}\bar{r}_2\right) + {}^{2}f_2\, {}^{2}N_2$$

$$= \begin{bmatrix} -\omega L_2 S_{12} \\ 0 \\ +\omega L_2 C_1 \end{bmatrix} + \frac{L_2}{2} \begin{bmatrix} 2C_1 - C_{12} \\ 2S + S_{12} \\ -2S_{12} + S_1 \end{bmatrix} \times A$$

o) $${}^{1}f_1 = m_1 \bar{\dot{v}}_1 \quad \{ \text{for } i = 1 \}$$

$$m_1 = \left[-\frac{L_1}{2}\dot{\theta}_1^2 \quad g \quad -\frac{L_1}{2}\dot{\theta}_1\right]'$$

$${}^{1}N_1 = [0\ \ 0]'$$

$${}^{1}F_1 = {}^{i}f_1 + {}^{1}R_2\, {}^{2}f_2$$

$$\tau_1 = {}^{1}n_1'\, {}^{1}R_0\, \hat{z}_0 \quad \text{and}$$

$$\tau_2 = {}^{2}n_2'\, {}^{0}R_1\, \hat{z}_0 \quad \{\text{simplifying}\}$$

Ans

$$\boxed{\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -WL_2C_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{2} m_2 g L_2 C_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{3} m_2 l_2 S_1 (2l_2 C_2 - 3l_1)\dot{\theta}_1\dot{\theta}_2 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} m_1 l_1^2 + \frac{1}{3} m_2 L_2^2 C_2^2 + m_2 L_1 L_2 C_2 + m_2 l_1^2 & 0 \\ 0 & \frac{1}{3} m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}}$$

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