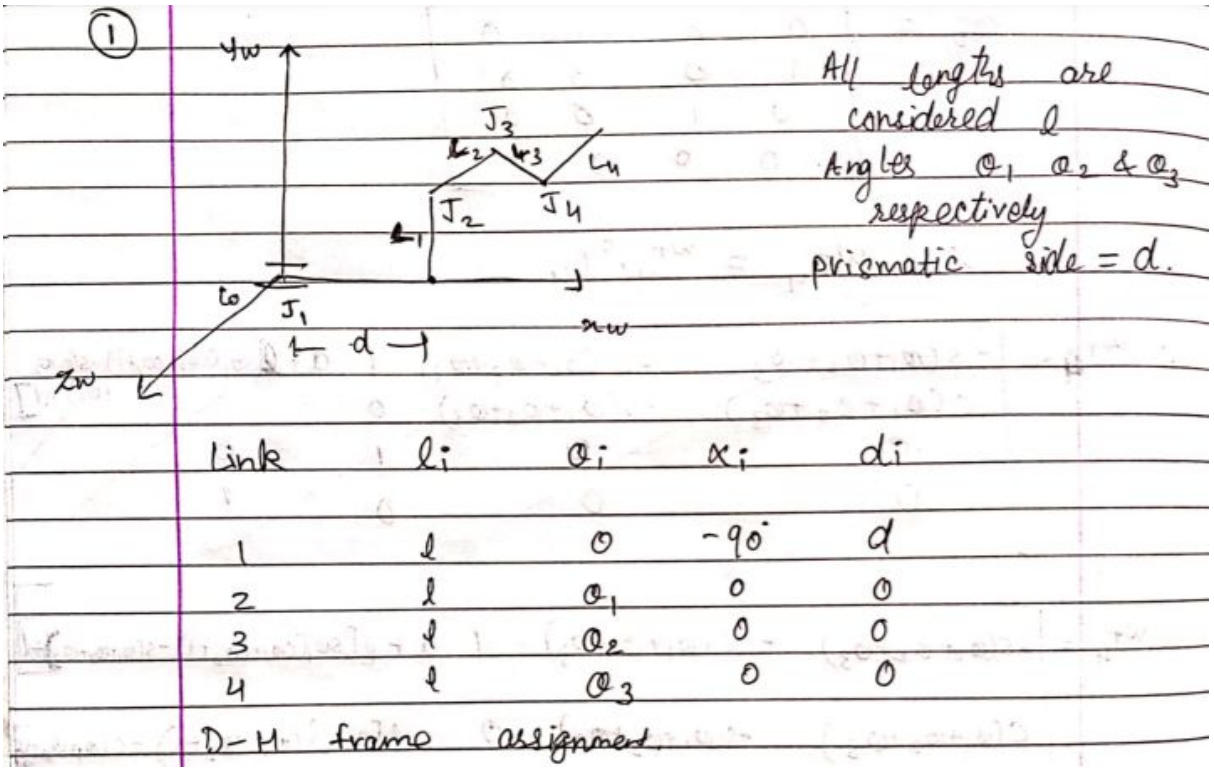
Robotics - BITS F441
Assignment – 1

Date of Submission - 20/2/21

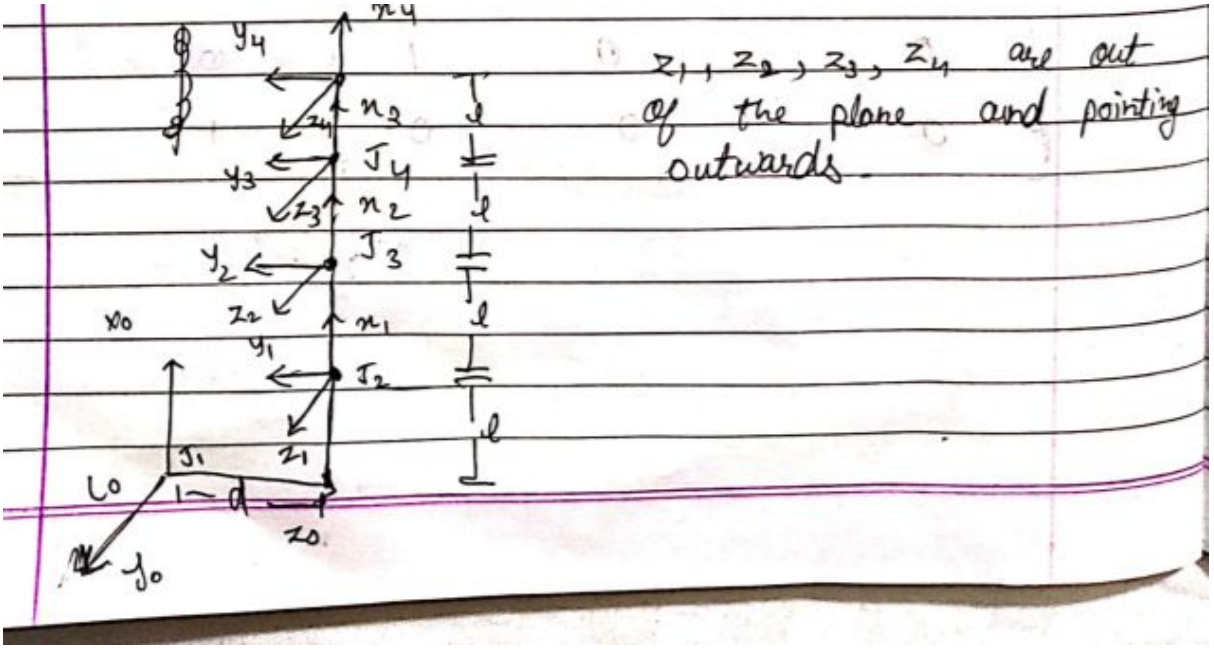
Question 1



All lengths are considered l
Angles θ_1 θ_2 & θ_3 respectively
prismatic side = d.

Link	l_i	θ_i	α_i	d_i
1	l	0	-90°	d
2	l	θ_1	0	0
3	l	θ_2	0	0
4	l	θ_3	0	0

D-H frame assigned.



z_1, z_2, z_3, z_4 are out of the plane and pointing outwards.

$$^{0}T_4 = {}^{0}T_1 \, {}^{1}T_2 \, {}^{2}T_3 \, {}^{3}T_4$$

$$\therefore {}^{0}T_4 = \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & 0 & l[c\theta_1(c\theta_1+c\theta_2+1)+c(\theta_1+\theta_2+\theta_3)] \\ 0 & 0 & 1 & 0 \\ -s(\theta_1+\theta_2+\theta_3) & -c(\theta_1+\theta_2+\theta_3) & 0 & l[s\theta_1(c\theta_1+c\theta_2+1)-s(\theta_1+\theta_2+\theta_3)+d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For world frame

$$^{w}T_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore {}^{w}T_4 = {}^{w}T_0 \cdot {}^{0}T_4$$

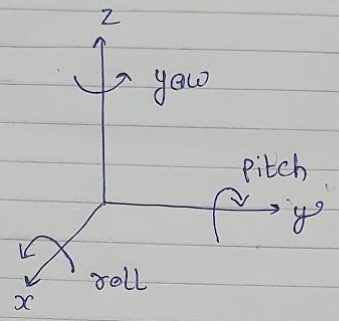
$$^{w}T_4 = \begin{bmatrix} -s(\theta_1+\theta_2+\theta_3) & -c(\theta_1+\theta_2+\theta_3) & 1 & d+l[s\theta_1(c\theta_1+c\theta_2+1)-s(\theta_1+\theta_2+\theta_3)+d] \\ c(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & 0 & \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{w}T_4 = \begin{bmatrix} -s(\theta_1+\theta_2+\theta_3) & -c(\theta_1+\theta_2+\theta_3) & 1 & d+l[s\theta_1(c\theta_1+c\theta_2+1)-s(\theta_1+\theta_2+\theta_3)+d] \\ c(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & 0 & l[c\theta_1(c\theta_1+c\theta_2+1)-c(\theta_1+\theta_2+\theta_3)+d] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2

Hand Calculation

A2) Any body can be rotated about 3 axis : Roll, Pitch and yaw as shown below :-



{ Frame of reference }

Roll angle → Φ
pitch angle → θ
yaw angle → ψ

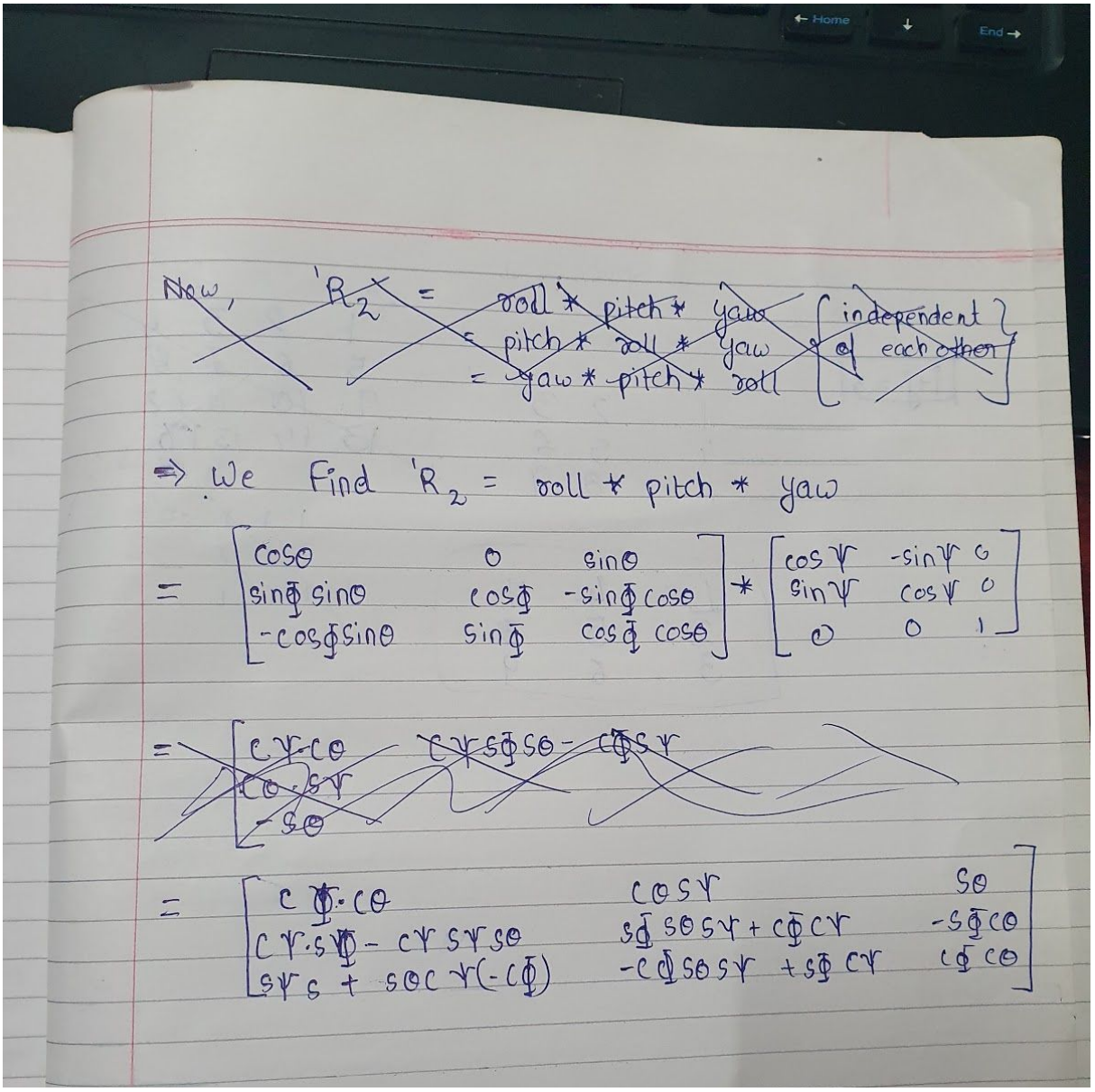
Viewing perspective

As the 3-axis are free, this is a 3-DOF system and hence, we can define 3 independent rotation matrices :-

1) $\text{Roll}\{R_x(\Phi)\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi & \cos\Phi \end{bmatrix}$

2) $\text{Pitch}\{R_y(\theta)\} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

3) $\text{Yaw}\{R_z(\psi)\} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Matlab Code

```
clear all
clc

syms phi;
syms theta;
syms si;

Rz = [cos(phi) -sin(phi) 0;sin(phi) cos(phi) 0;0 0 1];
Ry = [cos(theta) 0 -sin(theta);0 1 0;sin(theta) 0 cos(theta)];
Rx = [1 0 0;0 cos(si) -sin(si);0 sin(si) cos(si)];

R12 = Rx*Ry*Rz;
```

Matlab Result

```
>> R12

R12 =

[                          cos(phi)*cos(theta),                          -cos(theta)*sin(phi),         -sin(theta)]
[cos(si)*sin(phi) - cos(phi)*sin(si)*sin(theta), cos(phi)*cos(si) + sin(phi)*sin(si)*sin(theta), -cos(theta)*sin(si)]
[sin(phi)*sin(si) + cos(phi)*cos(si)*sin(theta), cos(phi)*sin(si) - cos(si)*sin(phi)*sin(theta),  cos(si)*cos(theta)]

>>
```

We get the same result !!!!